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LETTER TO THE EDITOR

Comments on "A note on time averages in turbulence with reference to geophysical applications" by P. C. Chatwin and C. M. Allen

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It appears that the authors have fully ignored the increasing amount of understanding in geophysical time series collected since the times of say, Bravais 1849, Kämtz 1860, Kelvin 1877 (see Hann, 1915) and which has been documented by Blackman and Tukey (1959) and later on by several good textbooks. The topic of ergodicity has been treated in the context of turbulence at least by Obukhov (1954), if not earlier.

To start with a minor item. The statement on line 12 of the paper by Chatwin and Allen, that each realisation has to be taken at the same position and time is debatable. If the statement would be taken at face value, we would not have an ensemble of realisations, but only one unique function. It probably would be more reasonable to say that for a given set of governing equations, we take different experiments as realisations of the same process, if initial conditions (if any) and boundary conditions are the same. Here it is understood that initial conditions and/or boundary conditions will contain a certain degree of randomness.

The present attitude towards ergodicity seems to be that everybody, who has some experience in time series analysis, thinks of, but does not care too much for ergodicity

The reason for this attitude can best be seen if time series analysis is discussed in terms of spectral analysis. Spectral estimates (within the limits of filter width) are uncorrelated. This probably was already known to Fourier when he invented Fourier series, since it is a consequence of orthogonality of sines and cosines. Hence, non-stationary data (non-stationary in the first moment) appear as low frequency spectral components or "trends". In fact, for good reasons it is common practice to eliminate trends before doing Fourier transforms. Yet, the substitution of the "true" mean by the short time mean is of little relevance. This is seen if we consider that for a short-term average of time T_n, the corresponding width of the spectral window centered at zero frequency is proportional to $1/T_n$. However, since we usually substitute time averaging for ensemble averaging, we use spectral windows of width proportional to $1/T_m$, where $K = T_n/T_m = N/M$ are the equivalent degrees of freedom, usually $K \sim 60$, but at least $K > 20$. That means, the spectral window for zero frequency is always much broader than that related to short-term mean substituted for long-term mean. Hence, trend elimination is tailored towards dealing with lower frequencies which otherwise would leak through the somewhat broad window with width proportional to $1/T_m$. (As an aside, let us note that one could make $K = 1$ or 2, do the Fourier transforms and afterwards average over spectral estimates to obtain statistical stability — this method is computer time consuming but provides for a sharp window.)

To put it another way. The problem of true mean is not a mathematical but a geophysical problem. With geophysical periods ranging from fractions of seconds to millenium, we could always find a process with periods longer than the ones considered in a given problem. If we investigate turbulence, we do not mind that mean wind speed in central Europe probably was different during the ice age from todays mean (though that may have implications for pollen analysis). What we need to consider is that different realisations must be taken under the same set of conditions. Consequently, in geophysical time series analysis, we have to take into account that the set of parameters which we have selected to characterize initial and boundary conditions is sufficiently complete and relevant.

It appears, that the central statement below eq. (6) is a misunderstanding. The error probably stems from the definition (5). In most books (e.g., Blackman and Tukey (1959) for a famous reference), time averaging is defined for a time interval $-T/2$ to $+T/2$. In eq. (5), the time interval is chosen as $t - T/2$ to $t + T/2$, where t apparently is a variable. In practice, since we always have limited record length, the definition (5) is not practical and instead time averaging over $t_0 - T/2 \leqslant t \leqslant t_0 + T/2$ is used, keeping t_0 fixed. In that case, *the time average of the deviation from the mean is* exactly zero. In that sense, there exists no "basic mathematical error in many authoritative accounts of the fundamental equations of turbulence". There is no physical reason to believe that running time averaging is preferable to fixed span time averaging. The difference, again, is at the low frequencies, and it is a matter of geophysical judgement whether the spectral energy at low frequencies should be included or not.

Non-stationarity of data is a problem, not because of the first moment, but because of the second moments. If there is suspicion that the data are non-stationary, it is advisable to calculate spectral and cross-spectral estimates as a function of time and check for systematic influences. If one is not interested in the full spectrum, but rather in one or a few frequencies, the methods by Bartels (1935, 1940) and numerous other papers, are certainly useful and highly recommended. If one is interested in variations of the spectrum, bispectral analysis is available. So far, bispectral analysis has rarely been used. The reason for this is probably that bispectra are the spectral decomposition of third-order moments, and the sampling conditions in geophysical applications are usually not sufficient to obtain stable estimates. The obvious way out is to describe spectra by a few parameters (total energy) and deal with the time series of these parameters.

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