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The average vertical mixing coefficient for the oceanic thermocline

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ABSTRACT

The new estimate of the average vertical mixing coefficient for the oceanic thermocline, based on all GEOSECS tritium data, is 1.7 ± 0.2 cm^2 s^{-1}.

1. Introduction

Until a reliable general circulation model of the ocean has been developed, those interested in modeling the uptake of fossil fuel CO_2 by the sea and heat buffering exerted by the sea will have to depend on simple transfer models calibrated using radioisotope data. To date, we have only three isotope data sets suitable for global modeling, the distributions of natural radiocarbon and of bomb produced radiocarbon and tritium (at the time of the GEOSECS surveys; Atlantic 1972–73; Pacific 1973–74; and Indian 1978). However, the natural pre-bomb radiocarbon data are limited and the GEOSECS radiocarbon data need to be corrected for the pre-bomb radiocarbon contribution. Therefore, the tritium distribution holds the most valuable information. Also the time scale for the penetration of tritium is nearly matched to the time scales of the CO_2 and heat transients. Thus it is important that the results of the GEOSECS surveys of tritium be summarized in a way that make them useful to those conducting such modeling efforts.

Our approach was to obtain area-averaged depth distributions for tritium in various regions of the ocean. We then sought that mathematical function which would best fit these distributions. It turned out that a Fickian diffusion law is appropriate for this purpose. Because of this, the most convenient means of expressing the tritium distribution is in terms of vertical mixing coefficients with the same unit of normal diffusion coefficient (cm^2 s^{-1}). While these vertical mixing coefficients have no direct physical meaning in that tritium is thought to move into the body of the sea along isopycnal horizons, they do permit the mean distribution of tritium in the sea as a whole or in a given region of the sea to be reproduced. We also realize that these mixing coefficients are probably applicable only to penetration times close to that for the mean age of ocean tritium as of the GEOSECS surveys. Further, we realize that because of differing surface boundary conditions, these mixing coefficients may not apply equally well to CO_2 and heat even for the same penetration time scales. However until more transient tracer data is in hand allowing us to extend the penetration time scale and to carefully intercompare the distributions of various tracers (i.e. tritium, ^{85}Kr and freons), this is the best approximation we can make.

In the one-dimensional box-diffusion model of Oeschger et al. (1975), the well-mixed surface ocean layer is coupled to a deep ocean layer through which material (e.g. fossil CO_2, bomb ^{14}C, ^{3}H, etc.) is transported by finite vertical mixing processes. By fitting the average vertical profile of pre-bomb ^{14}C in the ocean to the model, Oeschger et al. (1975) estimated that the average vertical mixing coefficient ($\bar{K}$) in the deep layer is most

likely about 1.26 $cm^2 s^{-1}$ with an acceptable range of from 0.95 to 1.78 $cm^2 s^{-1}$. As mentioned earlier and cautioned by Broecker et al. (1980), the vertical mixing coefficient K is a purely empirical parameter which takes into account all the processes that transfer tracers across density horizons in the oceanic thermocline, e.g. vertical eddy mixing, and advection along isopycnal-horizons which outcrop at surface ocean.

Using the box-diffusion model of Oeschger et al. (1975) and the tritium concentrations of rain waters at 50° N as the time-dependent tritium input function (Weiss and Roether, 1980), Broecker et al. (1980) first established the relationship between the mean penetration depth of tritium ($= \int_0^\infty C\,dz/C_0$, where C_0 and C are the concentration of tritium at surface and depth z respectively) and K. Then, from the estimation of the global mean penetration depth of tritium from GEOSECS Atlantic and Pacific data, they obtained a $\bar{K}$ of 1.7 $cm^2 s^{-1}$ which is within the range of Oeschger et al. (1975). Broecker et al. (1980) also obtained a $\bar{K}$ in the range of from 1.9 to 3.3 $cm^2 s^{-1}$ using the GEOSECS Atlantic and Pacific ^{14}C data. However, considering the large uncertainty involved in subtracting the pre-bomb ^{14}C from the observed ^{14}C data, the K's estimated from bomb ^{14}C have a large uncertainty.

Based on the GEOSECS tritium data from the norther hemisphere, Viecelli et al. (1981) proposed a $\bar{K}$ value of 5.1 $cm^2 s^{-1}$, which is four times greater than that obtained by Oeschger et al. (1975). Viecelli et al. (1981) treated the bottom tritium maxima in the North Atlantic GEOSECS stations as cross isopycnal vertical mixing inputs from the warm surface in their model fit (instead of large-scale lateral advective inputs from the source areas of the North Atlantic Deep Water). This may explain, in part, their large $\bar{K}$ value.

Considering the importance of the $\bar{K}$ value in predicting the future atmospheric CO_2 concentration (Oeschger et al. 1975) and the global sea level changes (Gornitz et al., 1982), we present here a new estimate of $\bar{K}$, based on all GEOSECS tritium data (Östlund et al., 1976, 1979, 1980) and various approaches. For even geographical coverage in the Atlantic Ocean, the tritium data from stations 3B, 8, 28C and 31B of German cruise GS7309 and 30, 45, 51 of German cruise GS7205 (W. Roether, personal communication) were also included.

2. Results and discussion

The ocean is divided into nine zones as shown in Fig. 1. The average concentration profiles of tritium in each zone (Fig. 1) were obtained by the following procedures: the tritium profiles in each 10° or 20° latitudinal strip were averaged first, then the averages of latitudinal strips within each zone were averaged again but using the area of each latitudinal strip as a weighting factor. In averaging the tritium profiles from the north Atlantic zone (20° N to 70° N), the advective inputs of tritium at the bottom of GEOSECS stations 3, 5, 11 and 23 were subtracted by assuming an exponental decrease of tritium concentrations at depths below 1500 m.

The solid line corresponding to a given surface mixed layer thickness (h) and vertical mixing coefficient (K) in Fig. 1 represents a good model fit to the averaged tritium profile in each zone, using the box-diffusion model of Oeschger et al. (1975). If the mixed layer thickness h is increased (or decreased) by 25 m, one still can obtain a reasonably good fit in Fig. 1 by decreasing (or increasing) the K value by about 0.1 to 0.2 $cm^2 s^{-1}$, except the profile for the North Atlantic. The tritium concentrations of rain water at 50° N and 50° S were used as the tritium input function for northern zones (>20° N) and southern zones (>20° S), respectively (Weiss and Roether 1980). Both time-dependent tritium input functions were expressed as fractions of the total rain inputs between 1952 and 1977 for each hemisphere. For the equatorial zones (20° N to 20° S), we used the average of 50° N and 50° S. The model ran from 1952 to 1973 for the Atlantic, to 1974 for the Pacific and to 1977 for the Indian Ocean. The surface tritium concentration of the model-produced tritium profiles was always normalized to the "observed" tritium concentration of the surface mixed layer. Therefore, we needed to know only the relative magnitude of the time-dependent tritium input function.

The good fit of the model curves to the "observed" data points (Fig. 1) indicates that the tritium transport in the surface 2000 m of the ocean can be represented by a Fickian-type diffusion process on a regional scale.

The global average of the vertical mixing coefficient of the ocean ($\bar{K}$) can be estimated from the relationship:

$$\bar{K} = (\sum \sqrt{K_i} f_i)^2, \qquad (1)$$

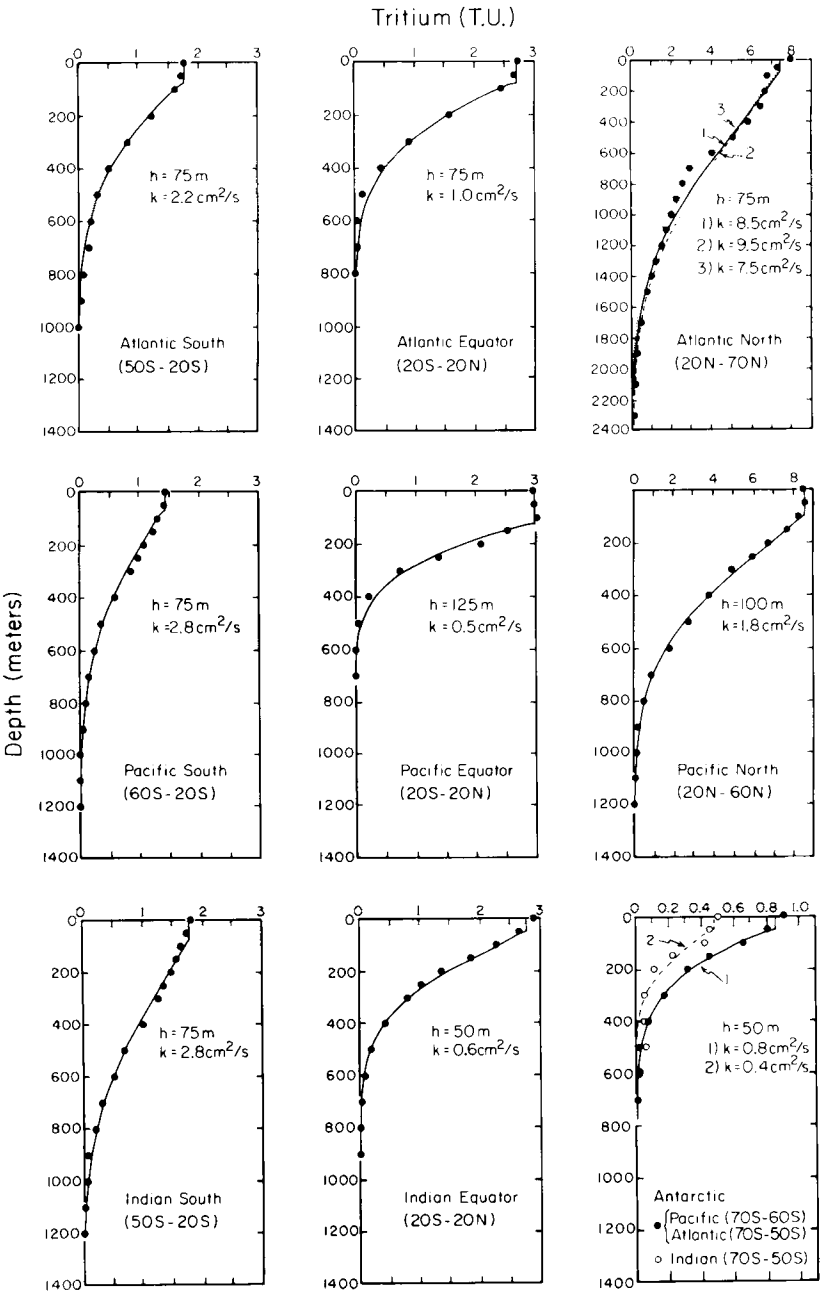

Fig. 1. The box-diffusion model fits of the average tritium concentration profiles in various regions.

where K_1 and f_i are the vertical mixing coefficient and the fraction of the total ocean area in zone i, respectively (Viecelli et al., 1981). The area-weighted average of the square root of the regional vertical mixing coefficients as the global average is justified, since the integrated uptake of tritium for any region is proportional to the square root of the vertical mixing coefficient (Viecelli et al., 1981). Using the K_i values in Fig. 1, we obtain the $\bar{K}$ value of 1.7 $\pm$ 0.2 cm^2 s^{-1} (with the average surface

mixed-layer thickness of about 75 m) which is consistent with the estimates by Oeschger et al. (1975) and Broecker et al. (1980). The uncertainty of the $\bar{K}$ value is mainly contributed by the uncertainty in the surface mixed-layer thickness of ± 25 m. The area-weighted average of the surface mixed-layer thickness over the whole ocean is estimated independently to be about 70 m using monthly temperature and salinity profile data of several hundred stations (Inez Fung, personal communication).

Another way to estimate $\bar{K}$ is to normalize the tritium profiles to the surface concentration in each zone, average the normalized tritium profiles using the area of each zone as a weighing factor, and fit the global average to the box-diffusion model using the average of the tritium input functions of rain water at 50° N and 50° S as in the cases of the equatorial regions (Fig. 2). The best fit $\bar{K}$ is again 1.7 ± 0.2 cm^2 s^{-1} with $h = 75$ m, if the model is run from 1952 to 1974. If the model is run from 1952 to 1973 or to 1975, $\bar{K}$ is 1.8 ± 0.2 or 1.6 ± 0.2 cm^2 s^{-1}.

As of 1973, the total tritium advected into the bottom water of the north Atlantic (20° N to 70° N) was estimated to be about 13% of the surface tritium (0–2000 m) in the same zone. If we assume that an equal amount of tritium was also advected into the Arctic bottom water, then it is equivalent to an increase in the effective surface area of the north Atlantic zone of 26%. Therefore, by eq. (1) the effective $\bar{K}$ can be about 1.8 ± 0.2 cm^2 s^{-1}.

We have also estimated $\bar{K}$ by the average mean penetration depths of tritium from three oceans. We first average the mean penetration depths of tritium for GEOSECS stations within a 10° or 20° latitudinal strip in each ocean. Then the averages of latitudinal strips in each oceanic hemisphere (north and south) were averaged again using the area of each latitudinal strip as a weighting factor. The results are summarized in Table 1. The tritium concentrations usually decrease rapidly with depth and oscillate around zero with the standard error of about ± 0.06 to ± 0.09 T.U. below a certain depth when a 0.02 T.U. blank correction (Östlund et al., 1980) is applied to all GEOSECS tritium data. Therefore, the depth interval(s) where the tritium concentrations oscillate around the standard error (indicating near-zero concentration) are not included in the mean penetration depth calculation for each GEOSECS station. The relationships between K and the mean penetration depth for each oceanic hemisphere (Fig. 3) were obtained by using the box-diffusion model with the tritium concentration of rain water at 50° N and 50° S as

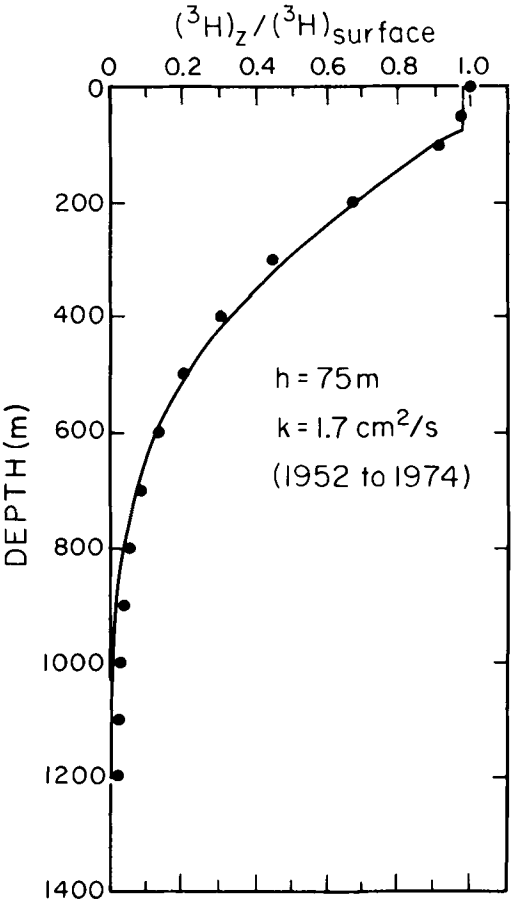

Fig. 2. The box-diffusion model fit of the average tritium concentration of the whole ocean.

Table 1. *The mean penetration depth of tritium (Z) and the corresponding vertical mixing coefficient (K) in the oceanic hemispheres*

	Z (m)	K (cm^2 s^{-1})
Northern hemisphere		
Atlantic (0°–70° N)	700	6.8
Pacific (0°–60° N)	350	1.3
Indian (0°–20° N)	210	0.3
Southern hemisphere (0°–70° S)		
Atlantic	280	1.5
Pacific	310	1.6
Indian	320	1.2

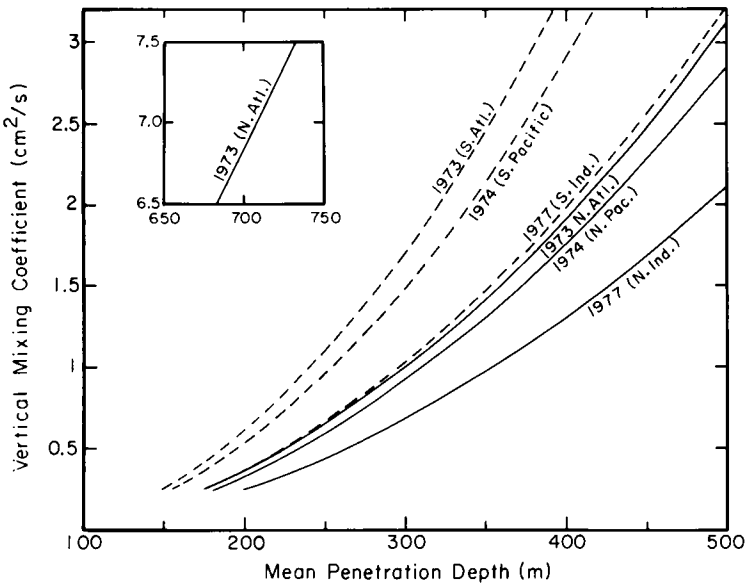

Fig. 3. The relationship between the mean penetration depth of tritium and the vertical mixing coefficient obtained by use of the box-diffusion model.

the tritium input functions for northern and southern hemispheres, respectively. The average surface mixed-layer depth was taken to be 75 m (Fig. 2). As before, the model ran from 1952 to 1973 for the Atlantic, to 1974 for the Pacific and to 1977 for the Indian Oceans. The K's for corresponding mean penetration depth in each oceanic hemisphere, as obtained from Fig. 3, are given in Table 1. The $\bar{K}$ is 1.8 ± 0.2 cm^2 s^{-1} if K's from Table 1 are applied to eq. (1). If the bottom tritium inputs in the North Atlantic GEOSECS stations are subtracted from the mean penetration depth calculation, $\bar{K}$ is reduced to 1.7 ± 0.2 cm^2 s^{-1}.

In short, our refined estimate of $\bar{K}$ is within the range of previous estimates by Oeschger et al. (1975) and Broecker et al. (1980). Therefore, their calculations with regard to the fossil CO_2 uptake by the oceanic thermocline are still appropriate.

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