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Effects of an optically thin Rayleigh atmosphere on the absorption of solar radiation by a Lambert surface

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ABSTRACT

Absorption of the solar radiation by a Lambert surface under an optically thin plane-parallel scattering layer is analyzed. Point-symmetry of the scattering phase function is assumed and therefore the study applies directly to a Rayleigh atmosphere. The scatterers produce two opposite effects: a decrease in the absorption results from backscattering to space out of the solar beam, but an increase results from scattering back to the surface of the fluxes reflected from it. The hemispheric reflectivity R_f (spectral albedo) of the surface atmosphere system is formulated as:

$$R_f(a_0, \mu_0, \tau^s) = a_0 + (1 - a_0)\,[(1/\mu_0) - 2a_0]\,(\tau^s/2)$$

where a_0 is the Lambert Law spectral albedo of the surface, μ_0 is the cosine of the solar zenith angle, τ^s is the vertical optical thickness, and where only the first power terms are retained in a series expansion in the optical thickness. Applying this equation, we analyze how the Rayleigh atmosphere mitigates the change in the absorption at the surface which follows from a change in the surface spectral albedo. For a small change in the surface albedo, the change in the daylong absorption at equinox, at latitude z, is smaller by a fraction

$$\tau^s[(\pi/4 \cos z) + 1 - 2a_0]$$

than the comparable change in the absence of an atmosphere. This fraction can be termed the mitigating factor, and is discussed as such.

Analysis of the mitigating effects when a large change in the albedo occurs, that from snow to vegetation, indicates that an adequate specification of a Lambert surface for climate studies requires at least two parameters: the visible and the infrared albedo.

1. Introduction

An unusually extensive and prolonged snow cover and an attendant high surface albedo over a vast region have been suggested as the cause of an unusually cold season (Kukla and Kukla, 1974). A pronounced surface albedo change over an entirely different time scale, that of the entire span of our history, has been hypothesized: the anthropogenic pressures, especially overgrazing, might have increased the albedo of the steppe in the desert-fringe areas to that of the bare sand (Otterman, 1977). Because changes in the surface albedo entail possible climatic consequences, these changes are of strong interest to us. The driving force for a possible climatic consequence is a change in the absorption of the solar radiation that results from an albedo change. In this study, we analyze the changes in the absorption at the surface considering that the surface is permanently veiled by the Rayleigh atmosphere.

In our simplified analysis, the aim is to consider the realities of the surface characteristics. We stress

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here that for some types of terrain the surface reflectivity can differ by an order of magnitude from one part of the solar spectrum to another. For a thick canopy of vegetation for instance, the hemispheric reflectivity (spectral albedo) can be in the 0.4 to 0.6 range at the infrared wavelengths, 0.7–1.3 μm, as compared to the 0.05 to 0.10 range in the 0.6–0.7 μm band.

Even though our treatment of the radiative transfer through the atmosphere is greatly simplified, we treat the transmission (or the backscatter) of the solar beam from above the atmosphere, as different from the transmission from below of the fluxes reflected from the surface. In fact, our results stem from the difference in the transmission that follows from the different effective path length for the backscattering of the solar beam and and for the backscattering of the fluxes reflected from the surface. Thus, our results could not be obtained by applying the approximation used by Rasool and Schneider (1971) in their study of effects of aerosols on global climate. In deriving their eq. (2) for the total albedo of the combined surface–aerosol system, they assumed that the transmission through the aerosol layer either from above or from below can be represented by the same expression $1 - r - \alpha$, where r is the backscattered fraction and α is the fraction absorbed in the layer.

The explicit expressions linear in the optical thickness τ^s that are presented in the subsections 2.1 to 2.5 are a reasonable approximation only for an optically thin scattering layer, $\tau^s \ll 1$. The fractional inaccuracies are of the order of τ^s in each of the two main terms in the solution, one associated with the backscattering from the direct beam and the other associated with backscattering of the fluxes reflected from the surface. These two inaccuracies tend to compensate for each other, because the two main terms carry opposite signs. Yet, the fractional inaccuracy in the net effect can be large when the two main terms are of about the same magnitude, that is, when the net effect is close to zero. Thus, these simplified equations should not be used for actual radiative transfer calculation except on a very limited basis, as an approximate assessment. Their main value lies in the insight they provide, offering guidelines to the parameterization of the atmosphere and of the surface. In the subsection 2.6 the limitations of the analysis based on expressions linear in τ^s are further discussed.

2. Analysis

2.1. *Hemispheric reflectivity of the surface–atmosphere system linear in τ^s*

Our surface–atmosphere system consists of a planar surface with a Lambert Law spectral albedo a_0 and a plane-parallel scattering layer with a vertical optical thickness τ^s, $\tau^s \ll 1$. Point-symmetry is assumed for the phase function of the scatterers, that is, scattering into any two opposite directions is of an equal probability. Isotropic scattering and Rayleigh scattering are well known examples of such a phase function. The solar irradiance on a horizontal surface at the top of the atmosphere is represented as $\mu_0 = \cos\theta_0$, where θ_0 is the solar zenith angle. The fraction backscattered to space from the direct beam is denoted as β_d and the fraction backscattered to the surface of the reflected flux is denoted as β_r. The surface irradiance g can now be stated as:

$$g(\mu_0, \tau^s, a_0) = \mu_0[1 - \beta_d(\mu_0, \tau^s)]\{1 + a_0\beta_r(\tau^s) + [a_0\beta_r(\tau^s)]^2 \ldots\} = \mu_0 \frac{1 - \beta_d(\mu_0, \tau^s)}{1 - a_0\beta_r(\tau^s)}$$

$$= \mu_0 - \mu_0 \frac{\beta_d(\mu_0, \tau^s) - a_0\beta_r(\tau^s)}{1 - a_0\beta_r(\tau^s)}. \qquad (1)$$

We express β_d and β_r as a function of τ^s, using the approach of simplified single scattering (SSS). In this approach, all the photons scattered by a purely scattering atmosphere are accounted for and included in the solution, but are assigned the direction of the first scattering. Thus, photons scattered twice or more are included in the scattered fluxes as if the second and higher order scattering events did not occur (Otterman, 1978). For a point-symmetrical phase function, the fractional inaccuracy of the SSS approach in calculating fluxes scattered from the direct beam is of the order of $(\tau^s)^2/\mu_0$ only, whereas the single scattering approximation leaves a fraction $\tau^s/2\mu_0$ out of the solution.

Under the SSS approach for a point-symmetrical phase function, the flux scattered backward equals the flux scattered forward, that is, each flux equals half of the total scattered fluxes. (The backscattered flux is thus underestimated and the forward scattered flux is overestimated.) The backscattered fractions calculated on this basis are denoted β_{de} and β_{re} and can be stated as

$$\beta_{de}(\tau^s/\mu_0) = \frac{1 - \exp(-\tau^s/\mu_0)}{2}. \quad (2)$$

and

$$\beta_{re}(\tau^s) = \int_0^{\pi/2} \cos\phi \sin\phi\,[1 - \exp(-\tau^s/\cos\phi)]\mathrm{d}\phi$$
$$= \int_0^1 \mu[1 - \exp(-\tau^s/\mu)]\mathrm{d}\mu = C_1(\tau^s), \quad (3)$$

where ϕ denotes the zenith angle of a pencil of radiation $\cos\phi/\pi$ reflected from the surface, $2\pi \sin\phi\, \mathrm{d}\phi$ is a solid angle for the pencils at the same zenith angle $\phi, \mu = \cos\phi$, and $[1 - \exp(-\tau^s/\cos\phi)]/2$ is the backscattered fraction for these pencils. $C_1(\tau^s)$ denotes the value of the integral which is related to the exponential integral. The integral C_1 has been tabulated for several values of τ^s by Otterman (1978) and is discussed in a later subsection here. In this subsection, we approximate the backscattered fractions by β_{df} and β_{rf}, where only the first term in the expansion of β_{de} and β_{re} as power series in τ^s is retained. Introducing these approximate fractions $\beta_{df} = \tau^s/2\mu_0$ and $\beta_{rf} = \tau^s$ into eq. (1), and approximating $[1/(1 - a_0\beta_{fr})] \approx 1 + a_0\beta_{rf}$, we obtain for the surface irradiance:

$$g_f(\mu_0, \tau^s, a_0) = \mu_0\left[1 - \left(\frac{1}{\mu_0} - 2a_0\right)\frac{\tau^s}{2}\right]. \quad (4)$$

The term in the inner parenthesis is the difference between the airmass (or the path length) for scattering from the direct beam $(1/\mu_0)$, and the product of the effective airmass (2) for scattering from the reflected fluxes and of the surface spectral albedo (a_0). The effect of the scatterers on the surface irradiance is the product of this difference and $\tau^s/2$. We attempt to offer a physical meaning for each expression, and $\tau^s/2$ can be termed the backscattering optical thickness.

We note from eq. (4) that the effect of the scatterers in decreasing the surface irradiance is largest for large zenith angles, that is, a large airmass for scattering out of the direct solar beam, and for a dark surface, $a_0 \ll 1$. For a very bright surface and sun close to the zenith, $a_0 > (1/2\mu_0)$, the scatterers induce an *increase* in the surface irradiance. This occurs when the numbers of photons that strike the surface for the second time (reflected and then backscattered to the surface) is larger than the number backscattered to space from the direct beam (Otterman, 1978).

We compare now our approximate calculations of the global surface irradiance, eq. (4), with the calculations that account for all orders of scattering in the Rayleigh atmosphere. The global irradiance data from Deirmendjian and Sekera (1954) for values of albedo of 0.0 and 0.8, for cosine of the solar zenith angle of 1.0 and 0.6, and for vertical optical thickness ranging from 8.5×10^{-3} to 0.15, are presented in Table 1. Below these data, denoted DS, the corresponding data calculated by our eq. (4) are presented. In assessing the accuracy of our equation for computing the atmospheric effects, we should ask what is the difference between our data and the DS data as a fraction of the difference

Table 1. *Surface irradiance tabulated by Deirmendjian and Sekera (1954) and as calculated by applying eq. (4)*

μ_0	τ^s	0.15	0.06	0.02	8.5×10^{-3}
			Albedo = 0.00		
1.00	DS	0.9301	0.9709	0.9901	0.9958
	eq. (4)	0.9250	0.9700	0.9900	0.99575
0.60	DS	0.5332	0.5714	0.5902	0.5958
	eq. (4)	0.5250	0.5700	0.5900	0.59575
			Albedo = 0.80		
1.00	DS	1.0282	1.0142	1.0054	1.0025
	eq. (4)	1.0450	1.0180	1.0060	1.00255
0.60	DS	0.5894	0.5969	0.5993	0.5999
	eq. (4)	0.5970	0.5988	0.5996	0.5998

between the DS data and the value of μ_0 (μ_0 is the global irradiance in the absence of the atmosphere). When examined along these lines for $a_0 = 0.0$, eq. (4) gives a reasonably accurate answer up to and including $\tau^s = 0{\cdot}06$ for both values of μ_0. For $a_0 = 0.8$, eq. (4) is reasonably accurate only up to $\tau^s = 0.02$, and only if $\mu_0 = 1.0$. For $a_0 = 0.8$ and $\mu_0 = 0.6$, our assessment is not very meaningful, because both our and DS irradiance data are very close to the value of μ_0. The net effect of the scatterers is then essentially zero for all the four values of τ^s. This occurs because $(1/\mu_0) = 1.67 \approx 2a_0$.

The absorption at the surface is a product of the surface absorptivity $1 - a_0$ and the surface irradiance g_f. Inasmuch as there is no absorption in the atmosphere, the hemispheric reflectivity[1] R_f is the solar irradiation above the atmosphere μ_0 less the absorption at the surface, divided by μ_0:

$$R_f(\mu_0, \tau^s, a_0) = \frac{\mu_0 - (1 - a_0) g_f(\mu_0, \tau^s, a_0)}{\mu_0}$$
$$= a_0 + (1 - a_0)\left(\frac{1}{\mu_0} - 2a_0\right)\frac{\tau^s}{2}, \qquad (5)$$

where g_f from eq. (4) was introduced as the surface irradiance.

A simple physical significance can be offered for the two terms of the $R_f - a_0$ difference. The term $(1 - a_0)(\tau^s/2\mu_0)$ describes the increase in the reflectivity by backscattering from the direct beam. The backscattered photons, a fraction $\tau^s/2\mu_0$ of the solar beam, by escaping to space avoid a possible absorption at the surface, which for the photons that impinge on it has a probability of $1 - a_0$. On the other hand, the once-reflected and then backscattered fraction $2a_0(\tau^s/2)$ of the solar beam is subject to a *second* encounter with the surface, that is, subject to an *additional* probability $1 - a_0$ of absorption. The second term, which carries a minus sign, describes this effect. For $a_0 > (1/2\mu_0)$ the second term is larger than the first, and $R - a_0$ is negative. For a very bright surface, with the sun close to the zenith, the scattering layer *reduces* the hemispheric reflectivity of the surface–atmosphere system.

This curious effect might actually take place over the snows of Kilimanjaro, when the sun is at the zenith there. Such a very small reduction in the reflectivity under a limited range of zenith angle, optical thickness and surface albedo conditions cannot be regarded as important. However, associated with this effect is the finding that under certain conditions the surface irradiance and therefore the absorption at the surface are essentially independent of the presence or the optical thickness of the scatterers. These conditions are that the airmass for the direct solar beam be approximately equal to the product of the surface albedo and the effective airmass for scattering the reflected fluxes (see DS data for $\mu_0 = 0.6$ and for the surface albedo of 0.8 in Table 1). The weak effect of the scatterers under these conditions is further discussed later.

2.2. *Averages of the surface–atmosphere system reflectivity linear in τ^s*

Turning our attention to a daylong absorption/reflection of the solar radiation, we designate $\bar{R}_f(z, \tau^s, a_0)$ as the fraction of the solar radiation reflected by the surface–atmosphere system at equinox and at a latitude z. At equinox, the solar irradiance at the top of the atmosphere is $\mu_0 = \cos z \cos t$, where the time t is measured from the local noon in units $\pi/12$ radians per hour. Thus,

$$\bar{R}_f(z, \tau^s, a_0) = \frac{\int_{-\pi/2}^{\pi/2} \mu_0 R_f(\mu_0, \tau^s, a_0)\, dt}{\int_{-\pi/2}^{\pi/2} \mu_0 dt}$$
$$= a_0 + (1 - a_0)\left(\frac{\pi}{2\cos z} - 2a_0\right)\frac{\tau^s}{2}, \qquad (6)$$

where we introduced R_f from eq. (5). At low latitudes the scatterers can produce a decrease of the effective, daylong reflectivity, but only for a very high albedo, $a_0 > (\pi/4 \cos z)$. Even at the equator, $z = 0$, the required surface albedo of 0.8 is so high that a decrease can naturally occur only over snow. At a latitude $z \geqslant \cos^{-1}(\pi/4) = 38°$, a decrease becomes impossible for any surface albedo.

The reflectivity of a planet $\bar{R}_{pf}$ for monodirectional radiation can be simply derived by applying the circular symmetry to integrate the flux

[1] Spectral albedo was defined in the Introduction as synonymous with hemispheric reflectivity. From here on, we use reflectivity for the hemispheric reflectivity of the surface–atmosphere system and albedo for the spectral albedo of the surface.

reflected from the planetary hemisphere with the sub-solar point at its center. Dividing this computed reflected flux by the irradiance, the reflectivity $\bar{R}_{pf}$ is obtained:

$$\bar{R}_{pf}(\tau^s, a_0) = a_0 + (1 - a_0)^2 \tau^s. \tag{7}$$

The coefficient of τ^s is the square of the surface absorptivity, obviously non-negative for any a_0.

Integrating both the reflected flux and the irradiance not to the terminator, but only to a specific $\theta_c < (\pi/2)$, we obtain the system reflectivity R_{cf} of a sub-solar cap:

$$\bar{R}_{cf}(\tau^s, a_0, \theta_c) = a_0 + (1 - a_0)\left(\frac{1}{1 + \cos\theta_c} - a_0\right)\tau^s. \tag{8}$$

2.3. *Mitigating influence of scatterers on consequences of a small surface albedo change, for latitude z at equinox*

An increase of da_0 in the surface albedo reduces the intake of the solar radiation by the surface, which is proportional to $1 - a_0$ for a specified insolation. In the absence of an atmosphere, the product $-da_0$ times the solar irradiation μ_0 describes the change in the absorption by the surface.[1] For a surface veiled by a scattering atmosphere, the product $-dR\mu_0$ describes this change, where R is the atmosphere–surface reflectivity and where $dR = (\partial R/\partial a_0)\, da_0$. A change in the intake of the solar radiation over a small area modifies locally the temperatures and the climate near the ground. A change over a large enough area can modify the regional and the global climate. The scatterers reduce "at the source" the sensitivity of climate to a possible modification, inasmuch as an increase da_0 in a_0 does not cause an equal increase $dR = da_0$ in the atmosphere–surface reflectivity R. The increase dR (or $d\bar{R}$ for the daylong average) corresponding to an increase da_0, is almost always smaller than da_0. An insignificant exception is discussed below.

In studies of mesoscale phenomena, we are concerned with the surface heating at a specific time of day. To assess the effects of scatterers on the surface absorption at a specific solar zenith angle, we differentiate the system reflectivity as given by eq. (5) with respect to the surface albedo:

$$\frac{\partial R_f(\mu_0, \tau^s, a_0)}{\partial a_0} = 1 - \left(1 + \frac{1}{2\mu_0} - 2a_0\right)\tau^s. \tag{9}$$

We note that $(\partial R_f/\partial a_0) < 1$, except for very unusual conditions of sun near the zenith and $a_0 > 0.75$. Thus, the scatterers almost always have a mitigating influence on consequences of a surface change.

For climatic analysis, the daylong average $\bar{R}$ is more relevant than R. To study small changes in a_0, we differentiate $\bar{R}$ as given by eq. (6) with respect to a_0:

$$\frac{\partial \bar{R}_f(z, \tau^s, a_0)}{\partial s_0} = 1 - \left(\frac{\pi}{4\cos z} + 1 - 2a_0\right)\tau^s. \tag{10}$$

The factor MF $= 1 - (\partial\bar{R}_f/\partial a_0) = \tau^s[(\pi/4\cos z) + 1 - 2a_0]$ which describes the mitigating influence of the scatterers on change in the daylong absorption by the surface (at equinox) for a given da_0, can vary from an insignificant fraction (for a low τ^s, high a_0 and a low z), to almost one. MF approaching one, describes a near cancellation of the effects of a surface change. This occurs at a high $\tau^s/\cos z$ ratio. Eq. (10) in such a case overestimates the mitigating influence. Eq. (10) breaks down and the above expression for MF becomes meaningless when its value is larger than one.

The following numerical examples indicate the large range in the value of MF depending on the values of a_0, z and τ^s. Over a sandy desert at the equator ($z = 0$), assuming an infrared surface albedo of 0.55 and a Rayleigh optical thickness of 0.02, MF $= 0.014$. At a latitude of 60° and over a dark soil with an albedo of 0.1 at visible wavelengths, with the Rayleigh optical thickness $\tau^s = 0.10$, MF $= 0.16$. At the same latitude and the same wavelengths but over snow, with an albedo in the visible of 0.8, MF is only 0.0185. The mitigating influence increases with latitude, but eq. (10) becomes, as indicated, inaccurate for a large $\tau^s/\cos z$. The influence depends on the average airmass, and thus is obviously larger in the winter and smaller in the summer than at equinox (for latitudes larger than 23.5°). When the noon sun is at the zenith, MF can become negative, that is, with an increase da_0 in the surface albedo, the reduction in the daylong absorption of the solar radiation is larger than in the absence of an atmosphere. This

[1] Without an atmosphere, the fractional change in the surface absorption is $-da_0/(1 - a_0)$. Thus, the *fractional* effect of a given change da_0 in the albedo is larger for a bright surface.

happens only at an extremely high surface albedo of 0.9. A negative MF, an enhancement of the consequences of a change in the surface albedo, is a physically possible phenomenon, but it appears to be insignificant from a practical viewpoint.

This analysis indicates that the surface albedo changes are a more powerful climate affecting factor at equatorial and near equatorial regions as compared to higher latitudes, not only because of a higher average solar irradiance but also because of a lower mitigating influence of the atmosphere. The increase in the surface albedo that possibly occurred since the prehistoric times in the near equatorial arid regions (Otterman, 1977) could have altered significantly the regional and even the global radiation balance. The mitigating influence is lower for a high reflectivity, and an albedo change in the infrared part of the solar spectrum, in which sandy deserts have a reflectivity exceeding 0.5 (Otterman and Fraser, 1976) is translatable proportionately into a change in the absorption, inasmuch as the MF in the case is only of the order of 1 %.

2.4. *Mitigating influence of scatterers on consequences of a small change in the surface albedo, sub-solar cap*

We analyze here the mitigating effect of the scatterers for a small change da_0 in the surface albedo over a sub-solar cap. Differentiating the system reflectivity R_{cf} given by eq. (8), we obtain

$$\frac{\partial R_{cf}(\tau^s, a_0, \theta_c)}{\partial a_0} = 1 - \left(\frac{1}{1 + \cos\theta_c} + 1 - 2a_0\right)\tau^s. \tag{11}$$

The mitigating factor MF_c for the absorption by the surface of the sub-solar cap increases from $(1.5 - 2a_0)\tau^s$ when $\theta_c \to 0$ to $(2 - 2a_0)\tau^s$ when the entire hemisphere is considered. The value for a zero θ_c is obviously identical to MF given by eq. (9) for $\mu_0 = 1$.

2.5. *Mitigating influence of scatterers on consequences of a large albedo change: snow versus vegetation*

Consider now the change in the surface absorption when a large change in the surface albedo occurs, that from a snow covered terrain (visible albedo 0.8, infrared albedo 0.5) to a thick plant cover (visible albedo 0.1, infrared albedo 0.5). We assume that the solar energy in the visible and the infrared each constitutes half of the total solar irradiance outside the atmosphere. The no-atmosphere average albedo is then 0.65 for the snow and 0.30 for the vegetation. The surface absorptivity by the snow is $1 - 0.65 = 0.35$ and by the vegetation $1 - 0.30 = 0.70$, for a difference of 0.35.

Under an atmosphere, the effective absorptivity of the surface for the daylong irradiation at equinox is $1 - \bar{R}_f$. Introducing $\bar{R}_f$ from eq. (6), the absorptivity is

$$(1 - a_0)\left[1 - \frac{\tau^s}{2}\left(\frac{\pi}{2\cos z} - 2a_0\right)\right].$$

Assuming $\tau^s = 0.1$ in the visible and $\tau^s = 0.02$ in the infrared, at a latitude of 60° the daylong average of the absorptivity by the snow is,

VIS	$0.5 \cdot 0.2[1 - 0.05(\pi - 1.6)]$	$= 0.092$
IR	$0.5 \cdot 0.5[1 - 0.01(\pi - 1.0)]$	$= 0.245$
for a total of		0.337

and the absorptivity by the vegetation is

VIS	$0.5 \cdot 0.9[1 - 0.05(\pi - 0.2)]$	$= 0.384$
IR	$0.5 \cdot 0.5[1 - 0.01(\pi - 1.0)]$	$= 0.245$
for a total of		0.629

The atmospheric effect of reducing the surface absorption is thus $0.350 - 0.337 = 0.013$ over the snow and $0.700 - 0.629 = 0.071$ over the vegetation. The much larger reduction over the vegetation stems from the fact that vegetation is dark ($a_0 \ll 1$) in the visible, where τ^s of Rayleigh scattering is relatively high. The fractional difference in the absorption by the two surfaces stands now as $0.629 - 0.337 = 0.292$. This represents a considerable mitigating influence, of $0.350 - 0.292 = 0.058$, from the difference 0.350 that would have occurred in the absence of the atmosphere.

We analyze now the same change in the surface, applying the same expression for the absorptivity, but averaging separately the surface and the atmosphere over the solar spectrum. The average optical thickness is $0.5(0.1 + 0.02) = 0.06$. The absorptivity by the snow (average albedo 0.65) is

$$0.35[1 - 0.03(\pi - 1.3)] = 0.331,$$

and by the vegetation (average albedo 0.3) is

$$0.70[1 - 0.03(\pi - 0.6)] = 0.647,$$

for a difference $0.647 - 0.331 = 0.316$. The atmospheric mitigating influence on the difference stands now as $0.350 - 0.316 = 0.034$. It thus is underestimated by $0.058 - 0.034 = 0.024$, or by over 40% of its correctly computed value.

The above example is equivalent to computing the effective daylong hemispheric reflectivities of the Rayleigh atmosphere–Lambert surface system. This example is meant to point out that the combined surface–atmosphere system has to be analyzed as such. In averaging over the solar spectrum, spectral intervals in which both the atmosphere and the surface have approximately constant parameters should be separately analyzed.

2.6. *Hemispheric reflectivity of the surface–atmosphere system nonlinear in τ^s*

In this subsection the effects of the scatterers are re-examined in a more quantitative analysis. The reflectivity of the surface–atmosphere system is based here on the backscattering terms that result from the SSS formulation. We also point out where the linear expression for the system reflectivity can be used with a reasonable accuracy, and where the linear analysis becomes untenable.

Introducing the terms $\beta_{de}(\tau^s/\mu_0) = [1 - \exp(-\tau^s/\mu_0]/2$ and $\beta_{re}(\tau^s) = C_1(\tau^s)$ that were obtained as eqs. (2) and (3) respectively into the eq. (4) for the surface irradiance, we express now the surface irradiance as

$$g_e(\mu_0, \tau^s, a_0) = \mu_0 \frac{1 - \beta_{de}(\tau^s/\mu_0)}{1 - a_0\beta_{re}(\tau^s)} = \mu_0 \frac{1 + \exp(-\tau^s/\mu_0)}{2[1 - a_0 C_1(\tau^s)]} \tag{12}$$

The hemispheric reflectivity R_e is obtained, just as in eq. (5), as the difference between the solar irradiation above the atmosphere μ_0 and the absorption at the surface $g_e(1 - a_0)$, divided by μ_0:

$$R_e(\mu_0, \tau^s, a_0) = a_0 + \frac{\beta_{de}(\tau^s/\mu_0) - a_0\beta_{re}(\tau^s)}{1 - a_0\beta_{re}(\tau^s)} = a_0 + \frac{\dfrac{1 - \exp(-\tau^s/\mu_0)}{2} - a_0 C_1(\tau^s)}{1 - a_0 C_1(\tau^s)}. \tag{13}$$

Similarly to eq. (6) for $\bar{R}_f$, we integrate R_e to form its daylong average $\bar{R}_e$ at latitude z, at equinox:

$$\bar{R}_e(z, \tau^s, a_0) = \int_{-\pi/2}^{\pi/2} \cos t\, R_e(\cos z \cos t, \tau^s, a_0)\, dt/2 \tag{14}$$

We form the difference $\bar{R}_e - a_0$, which expresses the effect of the scatterers for a given τ^s, and divide this difference by τ^s, to obtain the proportionality factor for the effect. This proportionality factor $(\bar{R}_e - a_0)/\tau^s$ is plotted versus a_0 for the equator and for the latitude of 60° as Figs. 1 and 2, respectively. All the graphs indicate a pronounced reduction in the enhancement of the system reflectivity by the scatterers for a higher surface albedo. The effect of the scatterers at latitude 60° is reduced by a factor of 4 when the surface albedo increases from $a_0 = 0$ to $a_0 = 0.55$. The corresponding reduction at the equator is by a factor of 7. For $a_0 > 0.55$, at the equator the values of $(\bar{R}_e - a_0)/\tau^s$ are very small; the scatterers have virtually no effect on the absorption. Analyzing the dependence of the effect of the scatterers on the latitude, we note for $a_0 = 0$ an increase by a factor of about 2 from $z = 0$ to $z = 60°$; the corresponding factor is 3.5 for $a_0 = 0.55$.

The presentation of the effect of the scatterers in Figs. 1 and 2 as a proportionality factor to τ^s makes it possible to assess the accuracy of $\bar{R}_f$, the linear

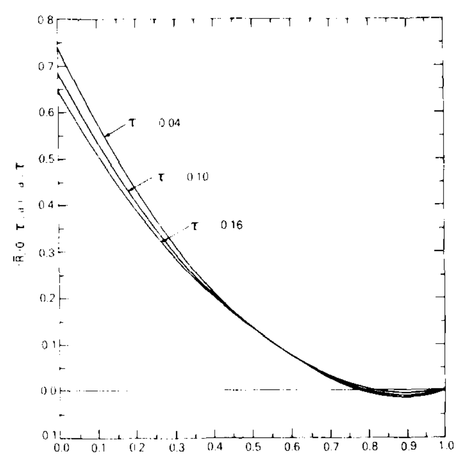

Fig. 1. The effects of a scattering layer on the effective reflectivity of the surface–atmosphere system at the equator, $[\bar{R}_e(0°, \tau^s, a_0) - a_0]/\tau^s$, plotted versus a_0 for three values of the vertical optical thickness τ^s.

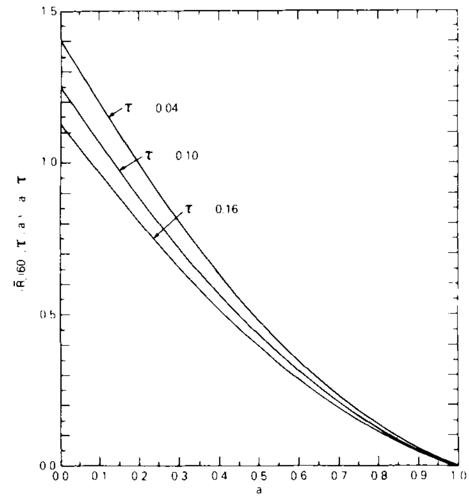

Fig. 2. The effects of a scattering layer on the effective reflectivity of the surface–atmosphere system at latitude of 60°, $[\bar{R}_e(60°,\tau^s,a_0) - a_0]/\tau^s$, plotted versus a_0 for three values of the vertical optical thickness τ^s.

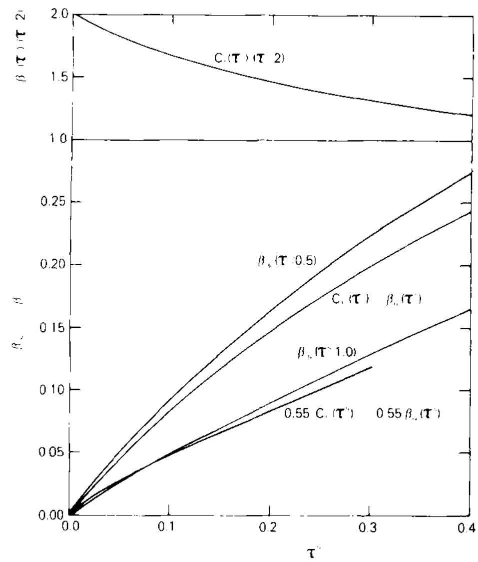

Fig. 3. The effective path length of backscattering to the surface, $C_1(\tau^s)/(\tau^s/2)$, versus τ^s; the fraction backscattered to space out of the solar beam $\beta_{de}(\tau^s/\mu_0)$ for $\mu_0 = 0.5$ and $\mu_0 = 1.0$, versus τ^s; and the fraction backscattered to the surface $\beta_{re}(\tau^s) = C_1(\tau^s)$ and the product $0.55\ C_1(\tau^s)$, versus τ^s.

expression for the effective daylong reflectivity, eq. (6). By inspecting Fig. 2, we note that when using the linear expression we overestimate the effect of the scatterers by about 11% for $\tau^s = 0.04$, by 20% for $\tau^s = 0.10$ and by 28% for $\tau^s = 0.16$. These fractional inaccuracies are calculated for $a_0 = 0$, but hold approximately true for the entire range of plausible a_0. The comparable inaccuracies in Fig. 1, $z = 0$, assessed at $a_0 = 0$, are 6% for $\tau^s = 0.04$, 12% for $\tau^s = 0.10$ and 17% for $\tau^s = 0.16$. However, this assessmant holds true for only a very limited range of a_0. For a_0 in the range 0.5 to 0.6, the linear expression is highly accurate both on a fractional basis and an absolute basis. For still higher a_0, the linear expression remains accurate on an absolute basis, but becomes quite inaccurate on a fractional basis: the effect of the scatterers is very small, and can be either in the direction of enhancing or reducing the surface–atmosphere system reflectivity. The linear treatment can give an erroneous answer as to the direction of this small effect.

The linearization in τ^s simplifies the analysis, enabling us to formulate readily an assessment of the effects of an optically thin scattering layer. The analysis is based on a comparison of the path length for backscattering from the direct solar beam, and the path length for backscattering to the surface of the fluxes reflected from the surface. With the help of Fig. 3 we examine now how the effective path length or airmass changes as a function of the vertical optical thickness τ^s. At the top of Fig. 3, the effective path length for backscattering to the surface is plotted, expressed as $C_1(\tau^s)/(\tau^s/2)$, where $(\tau^s/2)$ should be regarded as the vertical optical thickness for backscattering. We note that the path length decreases from 2 for $\tau^s \to 0$ to only 1.2 for $\tau^s = 0.4$. In the lower part of Fig. 3, the backscattered fractions $\beta_{de}(\tau^s/\mu_0) = [1 - \exp(-\tau^s/\mu_0)]/2$ and $\beta_{re}(\tau^s) = C_1(\tau^s)$ are plotted versus τ^s. The net effect of the scatterers depends on the difference $\beta_{de} - a_0\beta_{re}$. Examining the plots of β_{de} for $\mu_0 = 1$ and of the product $a_0\beta_{re}$ for $a_0 = 0.55$, we note that the difference $\beta_{de} - 0.55\beta_{re}$ is negative only between $\tau^s = 0.0$ and $\tau^s = 0.08$. The anti-intuitive effect of the *increase* in the surface absorption by the scattering layer is limited to this small range of τ^s. The limitation stems from the fact that $C_1(\tau^s)$ increases more slowly with τ^s than the fraction $[1 - \exp(-\tau^s/1.0)]/2$ backscattered from the direct beam.

3. Discussion and conclusions

Our aim in this study was to gain an insight into the meteorological/climatological effects of an optically thin scattering atmosphere. Toward this aim, we derived hemispheric reflectivities of the surface–atmosphere system as explicit expressions. In these simple solutions each term corresponds to an identifiable physical quantity.

The scattering atmosphere modifies the amount of solar radiation absorbed at the surface by producing two opposite effects: scattering to space out of the solar beam reduces the absorption, while scattering back to the surface of the flux reflected from the surface increases the absorption. The magnitude of each of these two effects depends on the effective path length through the atmosphere. The first effect, reduction of the surface absorption, is thus smallest with the sun at the zenith and becomes larger with increasing zenith angle. The net effect depends on one hand on the solar zenith angle and on the other on the magnitude of the reflected flux, that is, on the surface albedo. Analyzing our explicit expressions, we conclude that at small and moderate solar zenith angles the surface absorption is approximately independent of the presence of the scatterers for a high-albedo surface. However, for a low-albedo surface at any solar zenith angle or for any surface at a large solar zenith angle, the scatterers reduce the absorption and mitigate significantly the meteorological/climatological consequences of changes in the surface albedo. The mitigating factor varies from an insignificant fraction to a large one.

The explicit solutions that we present and on which we base this discussion are accurate only under conditions of low optical thickness. Thus, these solutions apply more accurately to the Rayleigh atmosphere at the longer solar wavelength. With increasing optical thickness, the effective airmass for backscattering for the reflected flux decreases more sharply than the effective airmass for backscattering from the direct beam. The scattering to space of the solar beam thus becomes the dominant influence of the scatterers as compared to the scattering back to the surface of the reflected flux.

Our solutions are not meant to obtain numerically accurate answers, except in some special situations. Accurate computations of radiative transfer through a Rayleigh atmosphere were successfully accomplished about 30 years ago (Deirmendjian and Sekera, 1954). These computations represented a significant challenge at that time. Since then, new approximation techniques that facilitate the computational treatment also for strongly anisotropic phase function have been developed (Coakley and Chýlek, 1975), that are quite accurate for low optical thickness conditions. The speed of digital computation has progressed by several orders of magnitude, so that what constituted a challenge 30 years ago, is an almost facile task today. It is on the basis of these developments as well as on the basis of our analysis that we stress the need for a spectral partitioning in an appropriate treatment of the surface–atmosphere system.

The effective optical thickness of Rayleigh atmosphere in the solar infrared wavelengths is by an order of magnitude smaller than that in the visible. The reflectivity of vegetation can be higher by a factor of 5 or 10 in the infrared as compared to the visible. The reflectivity of snow declines pronouncedly in the infrared. The dangers of averaging the atmosphere and the surface separately were pointed out above by a numerical example. The possible inaccuracies stem from the fact that the net effect of scatterers, when linearized in τ^s, should be regarded as a difference between a term in τ^s and a term in $a_0 \cdot \tau^s$. Obviously when averaging over the solar wavelengths, the second term should be the average of the product, $\overline{a_0 \cdot \tau^s}$, and not the product of the averages, $\overline{a_0} \cdot \overline{\tau^s}$. The relative inaccuracy in the computed net effect can be especially serious if the two terms are of a comparable magnitude. Parameterization of the atmosphere (such as presented by Lacis and Hansen, 1974) based on the integration over all wavelengths of the radiative transfer equations and a subsequent application of an a_0 also averaged over the entire solar spectrum, can therefore be inappropriate for some terrain types.

Treating the hemispheric reflectivities of the surface–atmosphere system separately in the visible and in the infrared entails parameterization of the atmosphere in these *two* spectral intervals as well as specification of the surface by *two* parameters: the visible albedo and the infrared albedo. Inasmuch as recent compilations of albedo rely in an increasing measure on the satellite data (Kukla and Robinson, 1980), and most satellite-borne radiometers contain separate visible and infrared bands (for the Landsat data, see Otterman and Fraser, 1976; Otterman, 1981), such a two-parameter specification of the surface is quite practicable.

In this study of the earth–atmosphere system we treated the atmosphere in a primitive fashion, while pointing to the realities of the surface, namely, that the spectral albedo can vary sharply from one spectral interval to another. It is appropriate to conclude this study by suggesting that for many types of terrain, plants protruding from the soil invalidate the Lambert plane characterization. For these complex surfaces, the absorption depends pronouncedly on the direction of the irradiation (Otterman, 1983). Such surfaces deserve a separate study.

4. Acknowledgement

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