Hydrodynamical Methods of the Short- and Long-Range Weather Forecasting in the USSR

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I. Short-range forecasting (by I. A. Kibel)

1. The development of the hydrodynamical methods for short-range weather forecasting was initiated more than 15 years ago, when the possibility was demonstrated of resolving the problem by means of the expansion in powers of the small parameter that multiplies time derivatives in the equation of motion. This parameter, let us denote it by ε, is defined as $\varepsilon = \frac{1}{lt_0}$, where l is the Coriolis parameter and t_0 is the characteristic time of the problem (of the order 10^5 sec.).

By performing the expansion in powers of ε and neglecting the terms of the second and higher order, we obtain from the horizontal equation of motion the horizontal components u, v of the velocity, and from the continuity equation — the vertical velocity w, in functions of the time- and space derivatives of pressure p and temperature T.

Substituting these expressions of u, v, w, in the equation of energy (above the boundary layer of the earth in the troposphere, this equation coincides with the adiabatic condition) we receive a single equation for the determination of two functions: p and T; using the hydrostatic approximation we can represent p by means of p_0 (sea level pressure) and T. The equations may be put into form

$$
\begin{aligned}
T\frac{\partial}{\partial z}\left(T\frac{\partial}{\partial t}\int_0^z \frac{dz}{T}\right) &+ \frac{RT}{gp_0}\left[\gamma_a \frac{p_0}{p} - \gamma\left(\frac{p_0}{p} - 1\right)\right]\frac{\partial p_0}{\partial t} = \\
&= u_g \frac{\partial T}{\partial x} + v_g \frac{\partial T}{\partial y} + (\gamma_a - \gamma)\, w' + \\
&+ u'\left(\frac{\partial T}{\partial x} - \frac{\varkappa - 1}{\varkappa}\,\frac{T}{p}\,\frac{\partial p}{\partial x}\right) + \\
&+ v'\left(\frac{\partial T}{\partial y} - \frac{\varkappa - 1}{\varkappa}\,\frac{T}{p}\,\frac{\partial p}{\partial y}\right)
\end{aligned}
\tag{1}
$$

where

$$u_g = -\frac{RT}{l}\left(\frac{1}{p_0}\frac{\partial p_0}{\partial y} - \frac{g}{R}\frac{\partial}{\partial y}\int_0^z \frac{dz}{T}\right),$$

$$v_g = \frac{RT}{l}\left(\frac{1}{p_0}\frac{\partial p_0}{\partial x} - \frac{g}{R}\frac{\partial}{\partial x}\int_0^z \frac{dz}{T}\right)$$

$$w' = -\frac{1}{\varrho}\int_0^z \varrho\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) dz$$

$$u' = -\frac{1}{l}\left(\frac{\partial v_g}{\partial t} + u_g\frac{\partial v_g}{\partial x} + v_g\frac{\partial v_g}{\partial y}\right)$$

$$v' = \frac{1}{l}\left(\frac{\partial u_g}{\partial t} + u_g\frac{\partial u_g}{\partial x} + v_g\frac{\partial u_g}{\partial y}\right)$$

Here x, y are the horizontal coordinates, z is the vertical coordinate, R the gas constant, γ_a the adiabatic lapse rate, γ the temperature lapse rate, $\varkappa$ the specific heats ratio, g the acceleration of gravity.

Later we shall call the equation (1) "the prognostic relation". We have to add to this relation one boundary condition; in the first work, the condition of non-penetrability of the air masses through the tropopause was adopted.

The first work-formula has been obtained from the prognostic relation by making two essential simplifications.

Firstly, the expansion in powers of ε was done also in the prognostic relation, in order to derive the value $\frac{\partial p_0}{\partial t}$ as functions of the space derivatives of p_0 and T. (Taking into consideration terms that do not contain ε was called "the first approximation"; the terms with ε in the first power gave "the second approximation".) On the other hand, the prognostic relation was not satisfied for all levels, but only at sea level.

Various versions of this method have been investigated in a number of papers. Thus it was proposed to replace the condition at the tropopause by the empirical condition relating the time derivatives of the pressure and the temperature at some special level (N. MALKIN, 1945, B. USPENSKI, 1948, G. MORSKOI, 1955, and others); the prognostic relation has been applied at the level of the tropopause instead of at the sea level (N. BULEJEV, 1948); the prognostic relation was averaged over the whole troposphere (KIBEL, 1948) ond so on.

In all these versions the work-formulae were constructed for a polytropic model.

In the first approximation the result was that $\frac{\partial p_0}{\partial t}$ was proportional to the Jacobian $J(T_0, p_0)$, where T_0 is the temperature at the top of the earth's boundary layer. These expressions looked like the well known EXNER's relations (1906), but the coefficients were essentially different. In the second approximation the linear combinations of the Jacobians $J(p_0, \Delta p_0)$, $J(T_0, \Delta T_0)$, $J(T_0, \Delta p_0)$, $J(p_0, \Delta T_0)$ and of the expression

$$\frac{\partial T_0}{\partial x} J\left(T_0, \frac{\partial K}{\partial x}\right) + \frac{\partial T_0}{\partial y} J\left(T_0, \frac{\partial K}{\partial y}\right)$$

have to be added (K denotes the pressure at the mid-level).

2. In the first studies time derivatives of p_0 at each point of space have been expressed in terms of the above mentioned Jacobians, calculated for the same point of space. The next step was to introduce "the dispersion". This removed the chief inaccuracy in the derivation of work-formulae from the prognostic relation, i.e. the expansion of $\frac{\partial p_0}{\partial t}$ in power series of the parameter ε. At the same time the prognostic relation was simplified in the sense that the ageostrophic components of the wind were neglected in the terms giving the advection of temperature. Finally, the boundary condition at the tropopause was replaced by the condition that the vertical momentum of mass vanishes at the top of the atmosphere.

With this new treatment of the prognostic relation the time derivatives of pressure (geopotential) have to be determined from a second order differential equation with partial space derivatives. Taking as independent variables x, y, p and t the prognostic relation leads to the now well known equation:

$$\left(\Delta + l^2 \frac{\partial}{\partial \zeta} \frac{\zeta^2}{c^2} \frac{\partial}{\partial \zeta}\right) \frac{\partial \varphi}{\partial t} = R l^2 \frac{\partial}{\partial \zeta} \left(\frac{\zeta A_T}{c^2}\right) - l A_\Omega \quad (2)$$

where φ is the geopotentional, $\zeta = \frac{p}{P}$ (P is the standard sea level pressure),

$$A_T = \frac{\zeta}{Rl} J\left(\frac{\partial \varphi}{\partial \zeta}, \varphi\right), \quad A_\Omega = \frac{1}{l} J\left(\varphi, \frac{\Delta \varphi}{l} + l\right),$$

$$c^2 = \frac{\gamma_a - \gamma}{g} R^2 T_1$$

T_1 is the mean temperature.

Equating to zero the vertical velocity at the earth surface (we investigate here the case of a plane surface) leads to the condition

$$\frac{\partial^2 \varphi}{\partial \zeta \partial t} + \frac{c^2}{R T_1} \frac{\partial \varphi}{\partial t} = R A_T \quad \text{at} \quad \zeta = 1 \quad (3)$$

At infinity ($z = \infty$, $\zeta = 0$) we assume the value of

$$\zeta \frac{\partial^2 \varphi}{\partial \zeta \partial t} \quad (4)$$

to be finite

The equation (2) with the boundary condition (3) permits one to determine the values of $\frac{\partial \varphi}{\partial t}$ in terms of linear combinations of

Jacobians $J\left(\frac{\partial \varphi}{\partial \zeta}, \varphi\right)$, $J\left(\varphi, \frac{\Delta \varphi}{l} + l\right)$ integrated throughout the space; in this manner the dispersion is taken into account.

In the first studies only the dispersion along the vertical has been taken into account—the integration through horizontal in (2) being replaced by some approximate operations. So in A. CHISTIAKOV's work (1951) the Laplacian $\Delta \frac{\partial \varphi}{\partial t}$ was replaced by the approximative expression $-\frac{4}{L^2}\left(\frac{\partial \varphi}{\partial t}\right)_M$ where L is a characteristic length; in A. OBUKHOV's work (1952) only one wave term was investigated by computing $\Delta \frac{\partial \varphi}{\partial t}$ and so on. A general solution of the equation (2) was obtained by N. BULEJEV and G. MARCHUK (1951). Adopting a constant (height average) value for c, Bulejev and Marchuk obtained $\frac{\partial \varphi}{\partial t}$ at the point (x, y, ζ) at a moment t, in the form

$$\frac{\partial \varphi}{\partial t} = \frac{l}{2\pi} \int_0^1 \iint_{-\infty}^{\infty} g_\Omega^{(\varphi_t)} A_\Omega \, dx' dy' d\zeta' - \frac{Rl^2}{2\pi c^2} \int_0^1 \iint_{-\infty}^{\infty} g_T^{(\varphi_t)} A_T \, dx' dy' d\zeta' \tag{5}$$

where

$$g_\Omega^{(\varphi_t)}(\tau_1, \zeta, \zeta') = \frac{1}{\sqrt{\zeta\zeta'}} \left[\frac{1}{2} \{\sigma(a, \tau_1)\}_{a=\ln\frac{1}{\zeta\zeta'}}^{a=\left|\ln\frac{\zeta}{\zeta'}\right|} + X_\alpha\left(\ln\frac{1}{\zeta\zeta'}, \tau_1\right) \right]$$

$$g_T^{(\varphi_t)} = -\zeta' \frac{\partial g_\Omega^{(\varphi_t)}}{\partial \zeta'}; \; \sigma(a, \tau_1) = \frac{1}{\sqrt{a^2 + \tau_1^2}} e^{-\frac{1}{2}\sqrt{a^2 + \tau_1^2}}$$

$$X_\alpha(a, \tau_1) = \sigma(a, \tau_1) + \left(\frac{1}{2} - \frac{c^2}{RT_1}\right) e^{-\left(\frac{1}{2} - \frac{c^2}{RT_1}\right)a} \int_a^\infty e^{\left(\frac{1}{2} - \frac{c^2}{RT_1}\right)a} \sigma(a, \tau_1) \, da$$

$$\tau_1^2 = \frac{l^2}{c^2}\left[(x - x')^2 + (y - y')^2\right]$$

The Green's functions $g_\Omega^{(\varphi_t)}$ and $g_T^{(\varphi_t)}$ are obtained here in the explicit form. This permits one to construct also explicitly the Green's functions for many other quantities (KIBEL, 1954, LEONOV, 1955, and others). The vertical velocity w, for instance, is expressed as follows

$$w = \frac{RT_1 l}{2\pi c^2 g} \int_0^1 \iint_{-\infty}^{\infty} g_\Omega^{(w)} A_\Omega \, dx' dy' d\zeta' + \frac{R^2 T_1^2 l^2}{2\pi g c^2} \int_0^1 \iint_{-\infty}^{\infty} g_T^{(w)} A_T \, dx' dy' d\zeta' \tag{6}$$

where

$$g_\Omega^{(w)} = \frac{1}{2\sqrt{\zeta\zeta'}} \left\{ \left(\frac{c^2}{RT_1} - \frac{1}{2}\right) \sigma(a, \tau_1) + \frac{\partial \sigma}{\partial a} \right\}_{a=\ln\frac{1}{\zeta\zeta'}}^{a=\ln\frac{\zeta}{\zeta'}}$$

$$g_T^{(w)} = \frac{1}{2\sqrt{\zeta\zeta'}} \left\{ \frac{\partial^2 \sigma}{\partial a^2} - \frac{1}{2}\left(\frac{c^2}{RT_1} + \frac{1}{2}\right)\sigma + \frac{c^2}{RT_1}\frac{\partial \sigma}{\partial a} \right\}_{a=\ln\frac{1}{\zeta\zeta'}}^{a=\ln\frac{\zeta}{\zeta'}}$$

On the base of the relation (2) the various rules of forecasting "by hand", or by graphical methods, have been constructed. One of the first graphical methods was proposed by BULEJEV (1951); it deals with the prognosis at the mid-level (here $g_T^{(\varphi_t)} \approx 0$) and consists in propagating values of φ along the lines of constant B, where

$$B = a(\bar{\varphi} + by)$$

Here a and b are constants, and $\bar{\varphi}$ is an arithmetic mean of the φ-values taken in four points around the point at which B is calculated. Bulejev's method is similar to a known set derived by Fjörtoft.

Bulejev takes the values of B as constant with time; A. OBUKHOV and A. CHAPLYGINA (1953) proposed to extrapolate in time the B-lines and to transfer φ along the extrapolated lines of constant B. M. YUDIN (1955) proposed to use for the prognosis in the mid-level the transfer along the lines of the type B, constructed not only with the φ values at mid-level, but also with φ taken at other levels.

The methods, also have to be mentioned, based upon the *linearization* of the prognostic relation. These methods were elaborated for the short-range forecasting use at the mid-level

(KIBEL, 1950) and in the three-dimensional case (MARCHUK, 1954, S. NEMCHINOV, 1955); they are analogous in technique to some methods for the long-range forecasting (see the second part of this paper).

Many authors have dealt with the synoptic interpretations of the prognostic relation, or investigated the possibility of deriving some practical rules with one-time-step formula (B. IZVEKOV, M. SHWETZ, A. DUBUQUE, N. TABOROVSKI, V. BUGAEV, B. USPENSKI, S. GUNIA and others).

3. The complete solution of the nonlinear problem of the short-range forecasting with the aid of (5) was first realized using soviet electronic computing machines. The formulae (5) permits easy construction of a number of approximate schemes for the prognosis of φ at various arbitrary chosen levels.

As some examples let us demonstrate the one-level, two-level and four-level models.

In the one-level model (it stays close by the so called "Barotropic model") the relation (5) is written for the level $\zeta = 0.5$ (or nearly that), the pressure averaged values of A_Ω and A_T are assumed; these values are attributed to the level $\zeta' = 0.5$; then the integration with respect to ζ' is carried out in the right side of (5).

Now, the terms that contains A_T proves to be negligible and the equation (5) becomes

$$\left(\frac{\partial \varphi}{\partial t}\right)_{\zeta=0.5} \approx \frac{l}{2\pi} \iint\limits_{-\infty}^{\infty} \bar{g}_\Omega^{(\varphi_1)} (A_\Omega)_{\zeta'=0.5} dx' dy' \quad (7)$$

where

$$\bar{g}_\Omega^{(\varphi_1)} = \int_0^1 (g_\Omega^{(\varphi_1)})_{\zeta=0.5} d\zeta'$$

The function $\bar{g}_\Omega^{(\varphi_1)}$ has a logarithmic singularity at the point $r_1 = 0$ and becomes very small as r_1 is increased. The integral (7) may be extended over a sufficiently large but finite area and the integration may be approximately replaced by a summation.

The one-level model (the "mid-level" one) was realized by S. BELOUSOV (1954) using the soviet high-speed electronic computer "BESM" in the Academy of Sciences of the USSR, and later on the soviet electronic computor "Arrow". An example is shown in Fig. 1.

Fig. 1 a. 700 mb contours 06h 5 July, 1956 (in decameters).

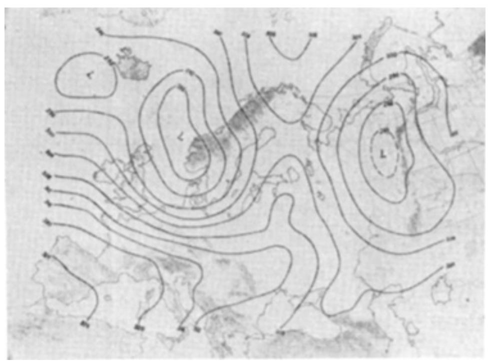

Fig. 1 b. Observed 700 mb contours 06h 6 July, 1956.

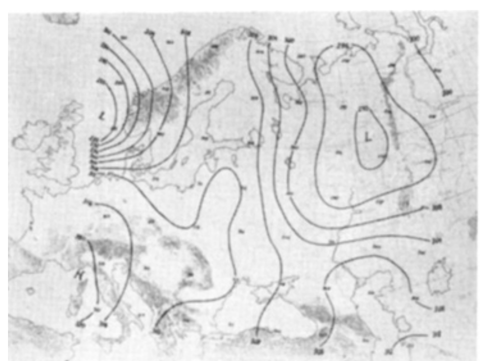

Fig. 1 c. 24-hour barotropic forecast 06h 5—6 July, 1956.

In the two-level scheme the right side of (5) have been written for the polytropic atmosphere: in this case the geopotential of any isobaric surface will be expressed by the heights of two arbitrary chosen surfaces; the solution (5) will be rewritten for these two surfaces. The realization of the problem using the "BESM"

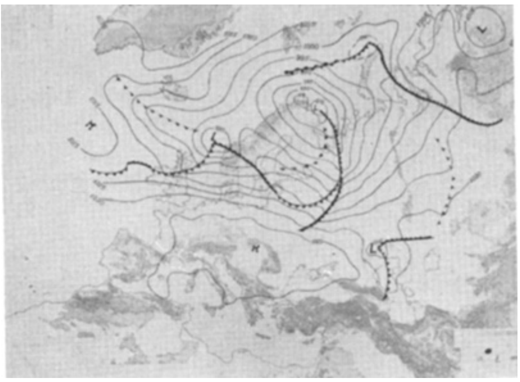

Fig. 2 a. Sea-level pressure 09h 13 Oct., 1954, (observed).

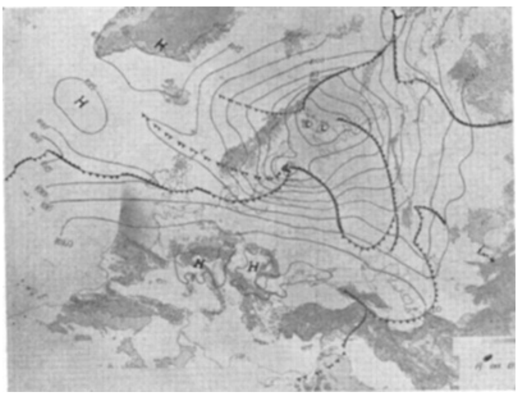

Fig. 2 b. Sea-level pressure 09h 14 Oct., 1954.

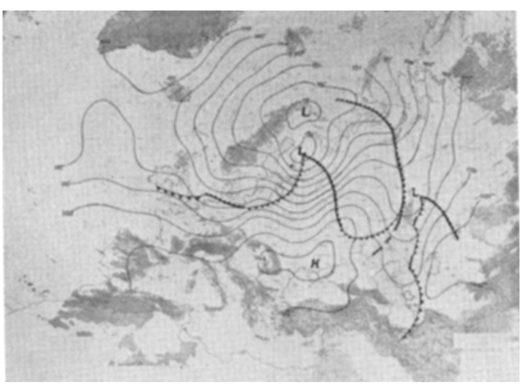

Fig. 2 c. 24-hour 2-level model forecast of the sea-level pressure 09h 13—14 Oct., 1954.

computor has been conducted by S. MASHKOVITCH (1955). An example is shown in Fig. 2.

In the case of the four-level model the formula (5) itself is written down for four levels; the integration with respect to ζ' in the right side of (5) is divided in four intervals with the approximate quadrature in each inerval. The computation may be carried outt with "BESM" (BELOUSOV, 1956). An example is shown in Fig. 3.

4. In all the above considerations the earth surface was supposed to be horizontal. Taking into account the orographical features makes in some cases the prognosis more precise. This is true as well for the sea-level prognosis as for the mid-level one. For the mid-level solution it is possible to replace the geostrophic relation by the relation of the type:

$$u = -\frac{1}{\eta}\frac{\partial \psi}{\partial y}, \quad v = \frac{1}{\eta}\frac{\partial \psi}{\partial x}, \tag{8}$$

where $\eta = \frac{p_h}{P}$, p_h is the pressure at the top of the mountain (p_h may be considered as a standard value for the given point on the surface). The values (8) can be inserted, further, into the equation of vorticity advection at the mid-level. Then an equation is obtained in which the derivative $\frac{\partial \varphi}{\partial t}$ enters not only under the Laplacian operator (as it is in a common case) but also under the first space derivatives.

The analysis of the solution was given by V. BYKOV (1954). BYKOV and BELOUSOV (1956) have realized the solution using the "BESM" computor.

Another way of solution was proposed by V. SADOKOV and SH. MUSAELIAN (1954) and consists of the following. The wind is assumed to be quasi-geostrophic in all the space, but the equation (2) is resolved with the boundary condition

$$\frac{\partial^2 \varphi}{\partial \zeta \partial t} + \frac{c^2}{RT_1}\frac{\partial \varphi}{\partial t} = RA_T - \frac{c^2}{l} J(\varphi, \eta) \quad \text{at } \zeta = 1 \tag{9}$$

where η is as previously denoted divided by P, standard pressure at the top of the mountain. Owing to the linearity of the equation (2) and of the boundary condition it will be sufficient now to add to the solution (5) the term:

$$\frac{1}{2\pi\sqrt{\zeta}} \iint\limits_{-\infty}^{+\infty} X_\alpha\left(\ln\frac{1}{\zeta}, \tau_1\right) J(\varphi, \eta)_{\zeta=1} dx'dy'$$

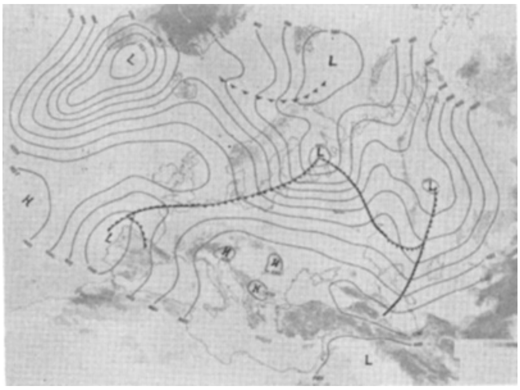

Fig. 3 a. Sea-level pressure 09h 5 Dec., 1953.

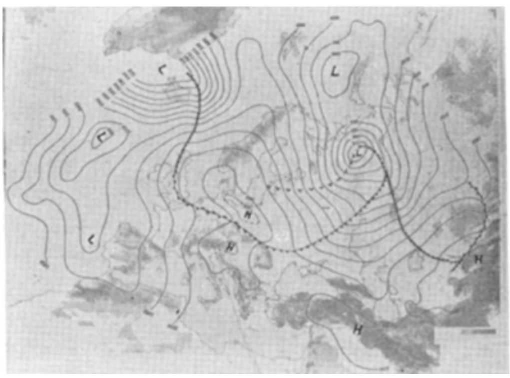

Fig. 3 b. Observed sea-level pressure 09h 6 Dec., 1953.

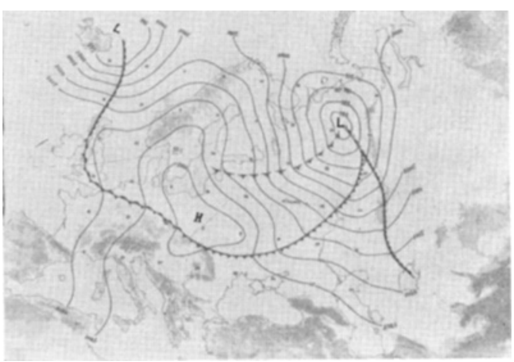

Fig. 3 c. Exploratory 24-hour 4-level model forecast 09h 5—6 Dec., 1953.

The effect of the viscosity, as well as of the thermal transformation, can be taken into account by an analogous way (MARCHUK, 1953, KIBEL, 1956). Thus, taking into account the viscosity (the earth surface is assumed, as before, to be horizontal) the atmospheric planetary boundary layer may be gathered into the pellicle, the boundary condition (3) for resolving (2) being simultaneously replaced for the free atmosphere by the condition

$$\frac{\partial^2\varphi}{\partial\zeta\partial t}+\frac{c^2}{RT_1}\frac{\partial\varphi}{\partial t}=RA_T+\frac{c^2gk}{RT_1 l}\Delta\varphi \text{ at } \zeta=1$$

Here k is the dimension-of-length parameter, expressed by means of the quantities that characterize the turbulent viscosity.

The viscosity can introduce some corrections in the values of $\frac{\partial\varphi}{\partial t}$ and w. Thus, for instance, owing to the viscosity we have to add to the expression (6) the term

$$\frac{kl}{2\pi c^2\sqrt{\zeta}}\iint\limits_{-\infty}^{+\infty}\left(\frac{\partial\sigma}{\partial a}\right)_{a=\ln\frac{1}{\zeta}}(\Delta\varphi)_{\zeta'=1}\,dx'dy'$$

where σ is given above.

5. The nongeostrophic influences have to be carefully estimated at the frontal zones. The shear of the horizontal velocities makes it necessary to take into account the term with the vertical velocity in the equations of motion (I. VETLOV, 1954); the great absolute values of the horizontal temperature lapse rate oblige us to consider the nongeostrophic components of the horizontal velocities in the energy equation. Instead of the equation (2) we shall now have a system of equations (KIBEL, 1955):

$$\Delta\frac{\partial\varphi}{\partial t}-\frac{l}{p}\left(l\frac{\partial\overline{\omega}}{\partial\zeta}-\frac{\partial v_g}{\partial\zeta}\frac{\partial\overline{\omega}}{\partial x}+\frac{\partial u_g}{\partial\zeta}\frac{\partial\overline{\omega}}{\partial y}\right)-$$

$$-\frac{R}{g\zeta}\Delta T\overline{\omega}=-lA_\Omega$$

$$\frac{\zeta^2}{l}\left(l\frac{\partial^2\varphi}{\partial\zeta\partial t}-\frac{\partial v_g}{\partial\zeta}\frac{\partial^2\varphi}{\partial x\partial t}+\frac{\partial u_g}{\partial\zeta}\frac{\partial^2\varphi}{\partial y\partial t}\right)+$$

$$+\frac{\tilde{c}^2}{p}\overline{\omega}=-R\zeta\tilde{A}_T$$

where $\tilde{c}$, $\tilde{A}_T$ are the expressions similar to c and A_T respectively, and

$$\overline{\omega}=\frac{dp}{dt}$$

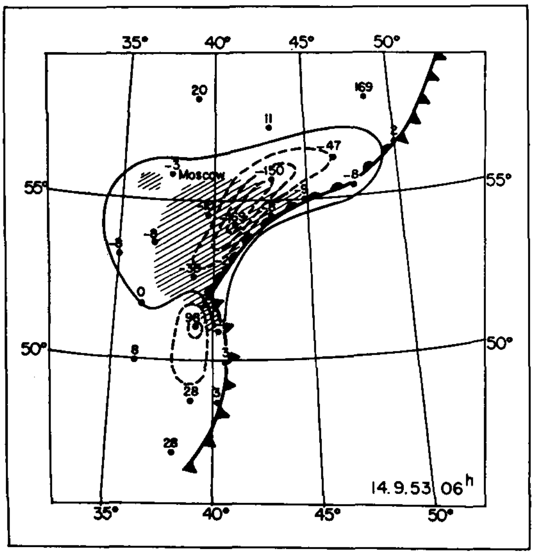

Fig. 4. Computed vertical velocity $\bar{\omega} = \frac{dp}{dt}$ due to the presence of a frontal surface (in mb/12 hours) at the 800 mb level 06h 14 Sept., 1953, (computations made with the use of sea-level and upper level charts for 06h 14 Sept., 1953). Isoplets are drawn at 50 mb/12 hours intervals (dashed lines); the zero-lines (full lines). Observed precipitation areas are shaded.

Introducing the new variables $\xi = x + \frac{1}{l} v_g$, $\eta = y - \frac{1}{l} u_g$ (Kibel, Yudin) it is possible, by a simple transformation, to reduce the problem to the solution of the equation of the type (2) for $\frac{\partial \varphi}{\partial t}$ with corresponding Green's functions. These last functions are now not symmetrical as regards to vertical axis through the point for which $\frac{\partial \varphi}{\partial t}$ is computed.

Much more tedious is the determination of the solution when frontal surfaces are present (Sadokov, 1955). The equations of the type (2) have to be written for the cold and warm masses separately; the variation of the position of the frontal surface is predicted in the course of the solution. The refinement of the solutions for vertical velocities near frontal surfaces is especially essential. In Fig. 4 examples are shown of the vertical velocity computations after Sadokov. The problem of the frontogenesis (V. Gubin, 1956) may be put and its practical resolution may be obtained by means of electronic computors.

Nowadays a new approach is incepted of taking into account nongeostrophic influences, namely, on the base of some theoretical considerations about the geostrophic adoptation of motion (Obukhov, 1949, Kibel, 1955). The scheme may be constructed for prediction of the geopotential, in which the quasi-geostrophicity is not postulated in advance, but is obtained in the course of the solution. Here the formula of type (5) may also be obtained with the essential difference that the horizontal integration will be extended to the finite area, the radius of which essentially depends on time; the corresponding Green's functions are, certainly, also dependent on time.

II. Long-range forecasting (by E. N. Blinova)

6. As early as the thirties of this century Lev Vasilievitch Keller suggested that extended-range weather forecasts may be constructed as a result of the analysis of the solution of a non-stationary problem of non-zonal disturbances superimposed on the zonal atmospheric circulation. However, at that time no electronic computors were available. Therefore, efforts have been made to look for the solutions of the linearized hydrodynamical equations. After many tests a model was selected in which small disturbances were superimposed on the W—E zonal current. In this model the velocity components V_Θ and V_λ along the axis (Θ) and (λ) of the spherical coordinate system (Θ colatitude, λ longitude) and the pressure p are in the form:

$$V_\Theta = v'_\Theta, V_\lambda = \bar{v}_\lambda + v'_\lambda, \quad p = \bar{p} + p'$$

where v'_Θ, v'_λ, p' (functions of Θ, λ, the time t and the height z above the sea level) are small quantities of which the second order terms can be neglected; $\bar{v}_\lambda$ and $\bar{p}$ charactirize the basic pure zonal motion.

Let us write down $\bar{v}_\lambda$ in the form

$$\bar{v}_\lambda = \alpha a_0 \sin \Theta \tag{1}$$

where a_0 is the earth's radius, α the angular velocity of the air motion relative to the earth "circulation index" (here we use the terminology introduced by C.-G. Rossby and

B. Haurwitz). The circulation index depends, generally speaking, on z, Θ and t. In the first scheme of solution the hydrodynamical equations have been written for the mid-level of the atmosphere; α has been taken (for this level) as a function of time only.

For the mid-level a stream function ψ can be introduced, such as

$$v'_\Theta = -\frac{\partial \psi}{a_0 \sin \Theta \partial \lambda}, \qquad v'_\lambda = \frac{\partial \psi}{a_0 \partial \Theta} \tag{2}$$

The movement is a solenoidal, but not a geostrophic one. The non-stationary non-zonal part p' of the pressure p is related with ψ by the expression:

$$\frac{1}{\tilde{\varrho}} \frac{\partial p'}{\sin \Theta \partial \lambda} =$$

$$= -\left(\frac{\partial}{\partial t} + \alpha \frac{\partial}{\partial \lambda}\right) \frac{\partial \psi}{\partial \Theta} + 2(\alpha + \omega) \operatorname{ctg} \Theta \frac{\partial \psi}{\partial \lambda} \tag{3}$$

(the simplified Euler's equation along the λ axis) in which $\tilde{\varrho}$ is the standard density for the mid-level.

As for the stream function itself, it satisfies the equation:

$$\left(\frac{\partial}{\partial t} + \alpha \frac{\partial}{\partial \lambda}\right) \Delta \psi + 2(\alpha + \omega) \frac{\partial \psi}{\partial \lambda} = 0 \tag{4}$$

(vertically projected simplified Friedman's equation, determining the vorticity change for a particle), where

$$\Delta \psi = \frac{1}{\sin \Theta} \frac{\partial}{\partial \Theta} \left(\sin \Theta \frac{\partial \psi}{\partial \Theta}\right) + \frac{1}{\sin^2 \Theta} \frac{\partial^2 \psi}{\partial \lambda^2}$$

The sinusoidal solution of (4) was obtained first, as is known, for the case $\frac{\partial \alpha}{\partial t} = 0$ by HAURWITZ (1940).

Let us give the general solution of the prognostic problem.

Suppose that for an initial moment ($t = 0$) the pressure p can be expanded in series of spherical harmonics:

$$(p')_{t=0} =$$

$$= \sum_{n=1}^{\infty} \sum_{m=1}^{n} (A_n^m \cos m\lambda + A'^m_n \sin m\lambda) p_n^m (\cos \Theta) \tag{5}$$

Then, resolving the system (3) and (4) we shall obtain, for an arbitrary moment of time:

$$p' = 2(\alpha + \omega)\tilde{\varrho}$$

$$\sum_{n=1}^{\infty} \sum_{m=1}^{n} [D_n^m \cos (m\lambda_1 + \sigma_{1n}^m t_1) +$$

$$+ D'^m_n \sin (m\lambda_1 + \sigma_{1n}^m t_1)] H_n^m (\cos \Theta) \tag{6}$$

with

$$\sigma_{1n}^m = \frac{2\omega m}{n(n+1)}, \qquad \lambda_1 = \lambda - \int_0^t \alpha(t)\, dt,$$

$$t_1 = \int_0^t \left(1 + \frac{\alpha}{\omega}\right) dt \tag{7}$$

H_n^m is the linear combination of the associated Legendre's polynomials:

$$H_n^m (\cos \Theta) = \frac{n(n+1-m)}{(2n+1)(n+1)} p_{n+1}^m (\cos \Theta) +$$

$$+ \frac{(n+1)(n+m)}{(2n+1)n} p_{n-1}^m (\cos \Theta) \tag{8}$$

The coefficients D are related to the coefficients A from (5) by the expressions:

$$2(\alpha_0 + \omega)\bar{\varrho}\left[D_{n-1}^m \frac{(n-1)(n-m)}{(2n-1)n} + \right.$$

$$\left. + D_{n+1}^m \frac{(n+2)(n+m+1)}{(2n+3)(2n+1)}\right] = A_n^m \tag{9}$$

(an analogous relation exists between D' and A'). Having obtained D and D', the prediction of ψ may be done using the formula:

$$\psi = \sum_{m=1}^{\infty} \sum_{n=1}^{m} [D_n^m \cos (m\lambda_1 + \sigma_{1n}^m t_1) +$$

$$+ D'^m_n \sin (m\lambda_1 + \sigma_{1n}^m t_1)] p_n^m (\cos \Theta) \tag{10}$$

For the prediction in the northern hemisphere it is permissible to put $v_\Theta = 0$ at the equator. In this case the differences $n - m$ in the equations (6) and (10) must be odd; but in (5) they must be even. The coefficients D and D' are to be determined one after another from (9) (beginning with the small numbers of n).

For a detailed representation of ψ by means

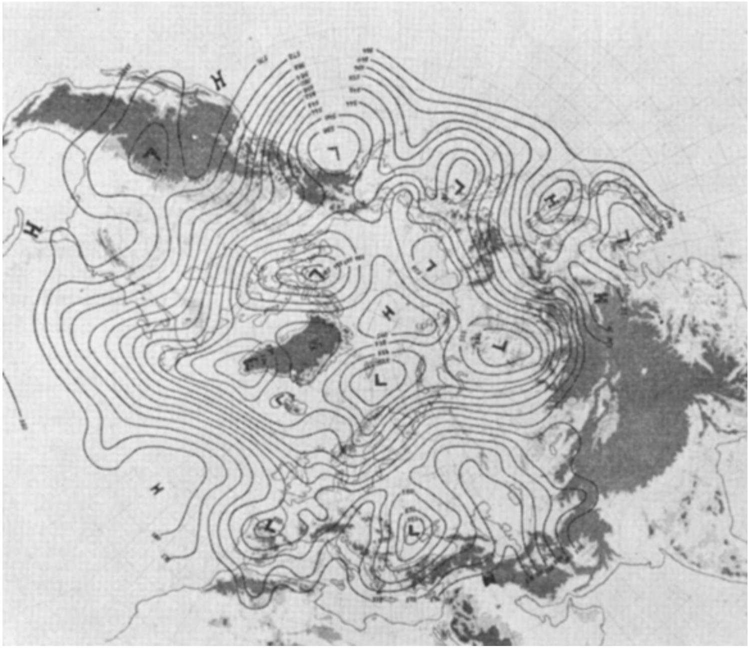

Fig. 5 a. 500 mb contours 06h 10 June, 1949, (in decameters).

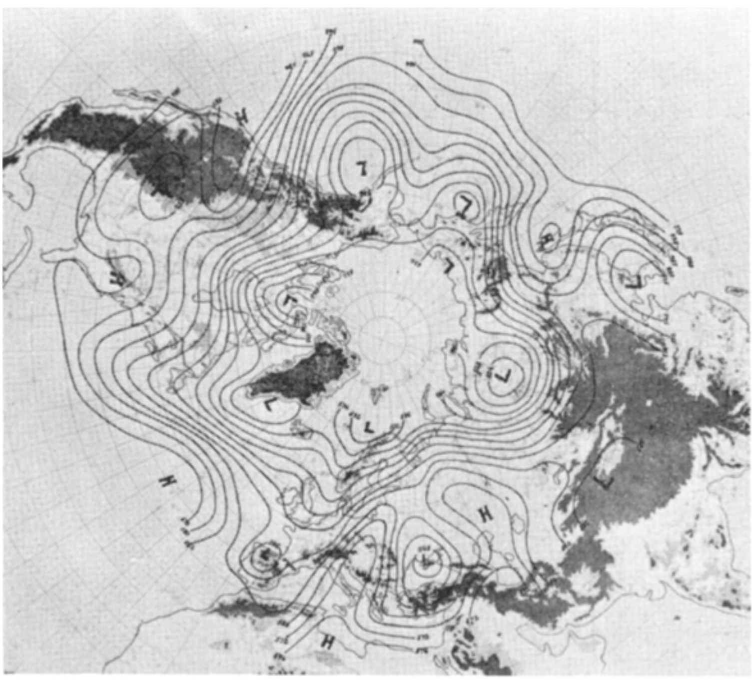

Fig. 5 b. 500 mb contours 06h 10 June, 1949, represented by means of spherical functions with $m \leq 12$, $n \leq 20$. (The initial field for 8-day forecast.)

of (10) we shall need a great number of the terms. It is therefore interesting to get the solution of the equation (3) for ψ in another form—by means of the influence function. This solution may be written in the form:

$$\psi(\Theta, \lambda_1, t_1) = \psi_0(\Theta, \lambda_1) + \\ + \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} g_0(\Theta, \Theta', \lambda_1, \lambda_1', t_1) \\ \psi_0(\Theta', \lambda_1') \sin\Theta' d\Theta' d\lambda'_1 \qquad (11)$$

where

$$\psi_0(\Theta, \lambda_1) = [\psi(\Theta, \lambda_1, t_1)]_{t_1=0}$$

$$g_0 = \sum_{n=1}^{\infty}\left(n+\frac{1}{2}\right)$$

$$[p_n(\cos\gamma_n) - p_n(\cos\bar{\gamma}_n) - p_n(\cos\gamma) + p_n(\cos\bar{\gamma})]\cdot$$

with

$$\cos\gamma_n = \cos\Theta\cos\Theta' + \\ + \sin\Theta\sin\Theta'\cos\left(\lambda_1' - \lambda_1 - \frac{2\omega t_1}{n(n+1)}\right),$$

$$\cos\bar{\gamma}_n = -\cos\Theta\cos\Theta' + \\ + \sin\Theta\sin\Theta'\cos\left(\lambda_1' - \lambda_1 - \frac{2\omega t_1}{n(n+1)}\right),$$

$$\cos\gamma = (\cos\gamma_n)_{t_1=0}, \quad \cos\bar{\gamma} = (\cos\bar{\gamma}_n)_{t_1=0}$$

p_n are the ordinary Legendre polynomials.

7. By means of the formulae (6) and (10) many forecasts of the 500- and 700-m height patterns of the northern hemisphere have been issued for the time periods from several days to several tens of days. The accuracy of the forecasts depends essentially on the quantity of meteorological data throughout the northern hemisphere. In particular, the accuracy of the forecast is considerably reduced by the lack of initial data from the vast areas of the Atlantic and Pacific oceans. The accuracy of the forecasts, naturally, depends upon the quality of the initial field representation by means of the series of the type (5). For those forecasts the tables of the associated Legendre polynomials for $n \leq 20$, $m \leq 12$ were computed (J. HEIFETZ, 1950); these tables were later extended so as to cover values for $n \leq 56$, $m \leq 36$ (S. BELOUSOV, 1956). Examples of the prognosis are shown in Fig. 5.

The techniques for obtaining the 500- and 700-m height patterns were worked out in 1947—1949 by J. Heifetz, S. Mashkovitch, N. Bagrov, A. Monin and others; special tables were computed for the expansion in series of functions H_n^m, for the influence functions and so on.

In the Central Forecasting Institute (Moscow) the values of α are computed for the northern hemisphere daily for several levels; to determine the values of α the data from latitudes 65°, 60°, 55°, 50°, 45°, 40° are used. The values of α are computed from a standard formula based upon the least-squares rule.

After prognosis of p' values at the mid-level was done, we can obtain the prognosis of the non-stationary part of sea level pressure p_0', "descending down" with the barometric formula of the polytropic atmosphere:

$$p_0'(\Theta, \lambda, t) = \\ = \frac{\tilde{p}(0)}{\tilde{p}(h)} p'(\Theta, \lambda, t) - \frac{gh}{T_1}\tilde{\varrho}(0)\, T'(\Theta, \lambda, t) \quad (12)$$

where $\tilde{p}(z)$ and $\tilde{\varrho}(z)$ are standard pressure and density at the level z respectively, h the height for which the values of p' are computed, T_1 the mean temperature at the height h, $T'(\Theta, \lambda, t)$ the non-stationary non-zonal part of the temperature. In the first approximation, for the medium-range forecasting it is permissible to assume that T' satisfies the linearized advection equation, the terms with the vertical velocity not being taking into account:

$$\frac{\partial T'}{\partial t} + \alpha\frac{\partial T'}{\partial\lambda} - \frac{2M\cos\Theta}{a_0^2}\frac{\partial\psi}{\partial\lambda} = \frac{K}{a_0^2}\Delta T' \quad (13)$$

Here K is the coefficient of turbulent thermometric conductivity describing the lateral turbulent diffusion, M the parameter characterizing the zonal temperature distribution between the equator and the pole; the zonal temperature distribution T is taken as follows:

$$\overline{T} = \text{const} + M\sin^2\Theta$$

The equation (13), in which ψ is determined by (10), may be solved without difficulty, the solution having the form of series in spherical

Fig. 5 c. Observed 500 mb 06h 12 June, 1949.

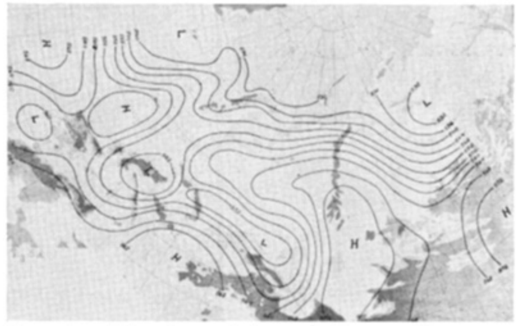

Fig. 5 d. 2nd day of 500 mb forecast 06h 10—12 June, 1949.

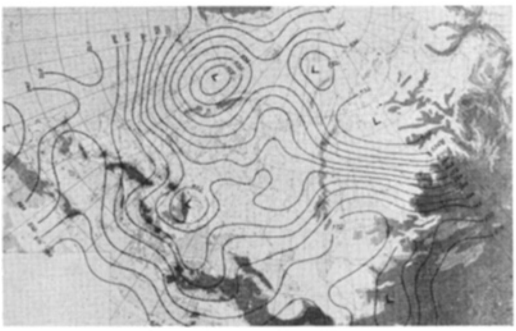

Fig. 5 e. Observed 500 mb 06h 14 June, 1949.

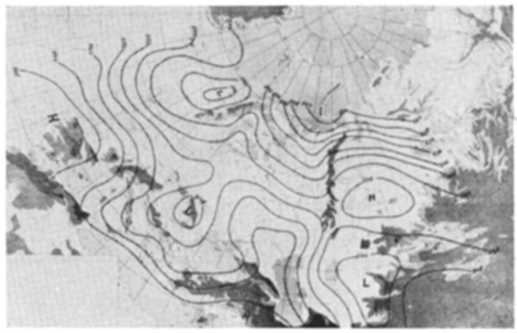

Fig. 5 f. 4th day of 500 mb forecast 06h 10—14 June, 1949.

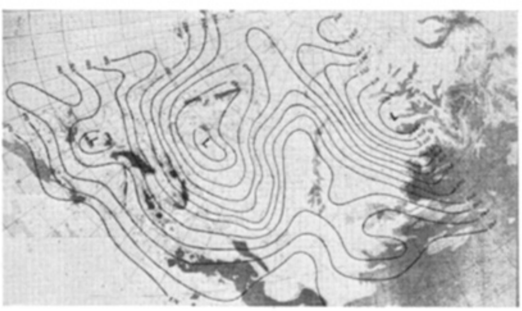

Fig. 5 g. Observed 500 mb 06h 16 June, 1949.

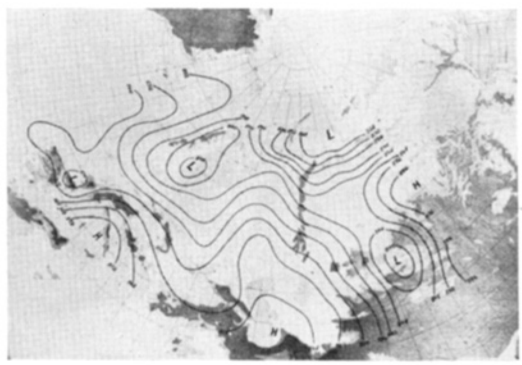

Fig. 5 h. 6th day of 500 mb forecast 06h 10—16 June, 1949.

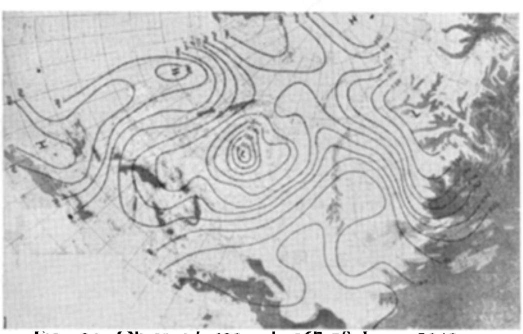

Fig. 5 i. Observed 500 mb 06h 18 June, 1949.

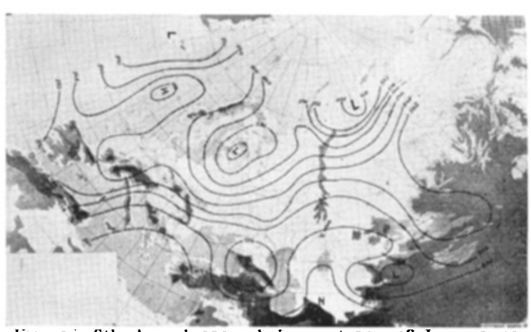

Fig. 5 j. 8th day of 500 mb forecast 10—18 June, 1949.

harmonics. The coefficients of this series are represented by the coefficients A and A' of the series (5) and by the coefficients B and B' of the following series

$$(T')_{t=0} = \sum_{n=1}^{\infty} \sum_{m=1}^{n} (B_n^m \cos m\lambda + B_n'^m \sin m\lambda)\, p_n^m$$

An example of the forecast using the above technique (from the paper by Blinova and Mashkovitch, 1952) is shown in Fig. 6.

The full three-dimensional model taking into account vertical velocities and assuming the atmosphere not to be polytropic was investigated later (Blinova, 1953, Heifetz, 1953). Here the linearization was performed with α depending only on the altitude. The motion was assumed to be approximatively solenoidal at all levels (not only at the mid-level); this means that v_Θ' and v_λ' are expressed after (2) with the non-stationary part of ψ depending now on z also. The problem is reduced to the solution of the equation:

$$\left(\frac{\partial}{\partial t} + \alpha \frac{\partial}{\partial \lambda}\right) \frac{\partial \psi}{\partial \xi} - \frac{d\alpha}{d\xi} \frac{\partial \psi}{\partial \lambda} =$$

$$= -\frac{\Gamma}{\xi^2} \int_1^{\xi} \left[\left(\frac{\partial}{\partial t} + \alpha \frac{\partial}{\partial \lambda}\right) \Delta \psi + 2(\alpha + \omega) \frac{\partial \psi}{\partial \lambda}\right] d\xi \tag{14}$$

(the turbulent diffusion is not taken into account), where

$$\xi = \frac{\tilde{p}(z)}{\tilde{p}(0)}, \qquad \Gamma = \frac{R^2 T_1}{4 a_0^2 \omega^2 g} \left(\frac{\gamma_a - \gamma}{\cos^2 \Theta}\right)_{\text{average}} \tag{15}$$

γ_a the adiabatic lapse rate, γ the mean temperature lapse rate, R the gas constant, T_1 the mean temperature of the atmosphere, ω the angular velocity of the rotation of the earth.

As a boundary condition the equality

$$\int_0^1 \left[\left(\frac{\partial}{\partial t} + \alpha \frac{\partial}{\partial \lambda}\right) \Delta \psi + 2(\alpha + \omega) \frac{\partial \psi}{\partial \lambda}\right] d\xi = 0 \tag{16}$$

has been assumed.

Various types of approximate solutions of the equation (14) have been tested.

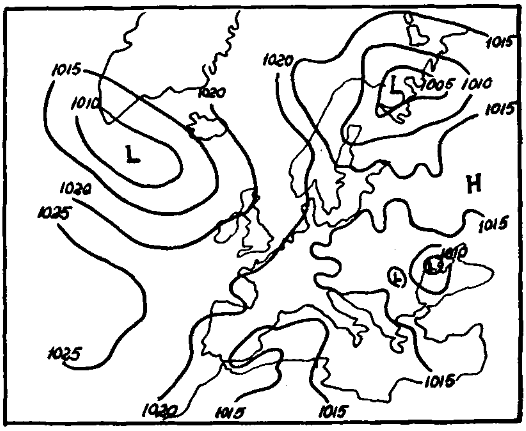

Fig. 6 a. Sea-level pressure 06h 10 June, 1949.

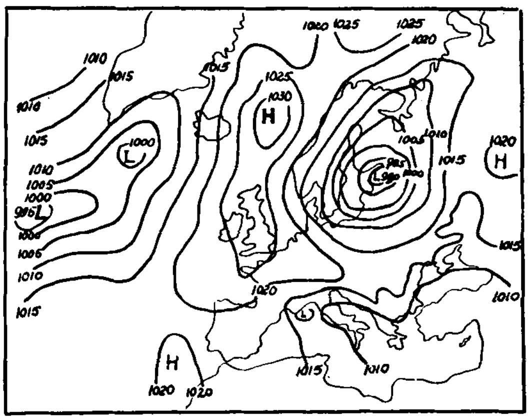

Fig. 6 b. Observed sea-level pressure 06h 18 June, 1949.

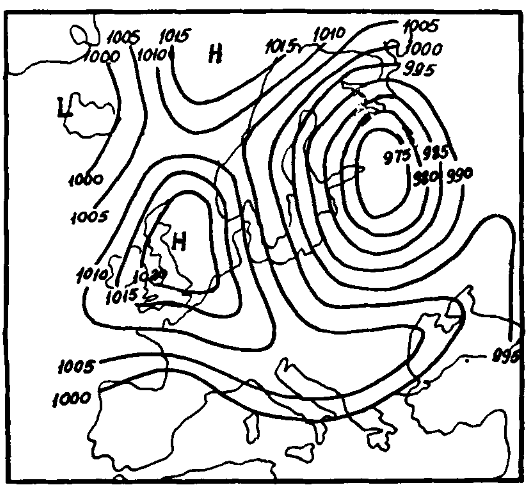

Fig. 6 c. Forecasted sea-level pressure 06h 10—18 June, 1949, computed by "descending down" with (12).

8. The long-range forecast of the sea level temperature anomalies may be given by means of the solution of the linear problem in the planetary layer of the earth. In the case of the sea level pressure prognosis, accomplished by "descending down" in the free atmosphere, it was sufficient to consider only the horizontal diffusion of the temperature; but in forecasting the temperature anomalies in the planetary boundary layer, it becomes necessary to introduce also the turbulent diffusion in the vertical direction. In the simpliest scheme we find the temperature anomalies from the equation:

$$\frac{\partial T'}{\partial t} + \alpha \frac{\partial T'}{\partial \lambda} - \frac{2M \cos \Theta}{a_0^2} \frac{\partial \psi}{\partial \lambda} = = \frac{K}{a_0^2} \Delta T' + \frac{\partial}{\partial z}\left(K' \frac{\partial T'}{\partial z}\right) \quad (17)$$

introducing the mid-level values of the parameter α and of the ψ function. Here K' is the coefficient of vertical turbulent thermometric conductivity.

The scheme describes, in general, the intrusion of heat and cold along meridians, the advection by the basic W—E flow, the smoothing due to the horizontal mixing and, finally, the transforming influence of the underlying surface.

To take into account influence of the underlying surface by resolving equation (17) it is necessary to fulfill the boundary condition of the heat balance at the earth surface.

We assume:

$$-\Lambda' \frac{\partial T'}{\partial z} + \lambda^{\star} \frac{\partial T'^{\star}}{\partial z} = S - \mu T'^{\star} \quad \text{at} \quad z = 0 \quad (18)$$

where $T'^{\star}$ and $\lambda^{\star}$ are temperature anomaly and coefficient of thermal conductivity of the ground respectively, Λ' is the coefficient of turbulent thermal conductivity of the air; the therm $\mu T'^{\star}$ characterizes the loss of heat due to the long-wave radiation of the earth surface; S describes, principally, the influence of the anomalies of cloudiness.

The unknown temperature $T'^{\star}$ is determined in parallel with T' from the equation

$$\frac{\partial T'^{\star}}{\partial t} = \frac{\partial}{\partial z}\left(k^{\star} \frac{\partial T'^{\star}}{\partial z}\right) \quad (19)$$

($K^{\star}$ is coefficient of the thermometric conductivity of the ground) with additional boundary conditions:

$$T' = T'^{\star} \quad \text{at} \quad z = 0$$

$$T'^{\star} \to 0 \quad \text{as} \quad z \to -\infty$$

Having determined ψ with the aid of (10), the system (17) and (19) may be resolved in a general way, with the arbitrary initial values of T' and $T'^{\star}$ (K. DOBRYSHMAN, 1951). But practically interesting for the extended-range prognosis is the forced "stationary regime" established under the influence of varying intrusions of heat and cold and the continuous transformation by the underlying surface (BLINOVA, 1950).

Similarly, by introducing the boundary layer for the velocities it is possible to take into account the viscosity influence upon wind variations in the boundary layer; one can also compute vertical velocities in the friction layer. These computations were conducted by DOBRYSHMAN (1956) in his work upon the long-range forecasting of the humidity.

After testing several variants of the solution of systems (17) and (19) one of them was adopted to prove under operational conditions. The computational schemes and the auxiliary tables have been prepared. Since 1952 at the Central Forecasting Institute (Moscow) the experimental operational prognosis of the mean monthly temperature anomalies (based on the simpliest linear hydrodynamical model) are issued covering the whole territory of the USSR for the periods from 40th to 70th day in advance. An example of the prognosis is shown in the Fig. 7. The mean goodness of these forecasts is about 76 %. (This means the share of forecast area where the difference between observed and computed mean monthly temperature anomalies do not exceed 20 % of the mean monthly climatological temperature amplitude.)

The computations for the hydrodynamical linear long-range forecasting problem are now performed on the specialized electronic computor "Weather" in the Central Forecasting Institute.

9. The chief difficulty to be overcome in any linear problem of the long-range forecasting lays in the determination (for the basic

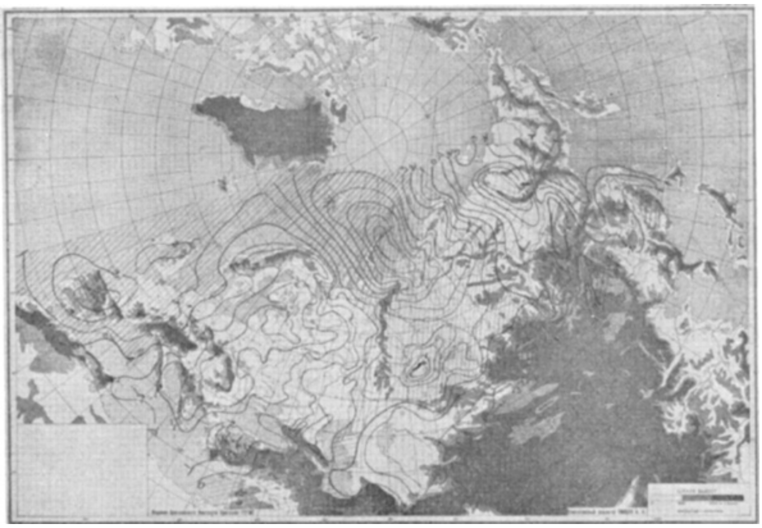

Fig. 7 a. Observed mean monthly sea-level temperature anomalies in April, 1955. Isoplets are drawn at 1° C intervals. Dashed areas—positive anomalies.

flow) of variations in time of the circulation index. A quantitative theory of the annual variation of α (in its climatic aspect) has been investigated in the works of BLINOVA and G. MARCHUK (1951) and S. MASHKOVITCH (1956). Besides the annual variation the circulation index has also non-regular, non-periodical variations, too.

For the prediction of the variation of the circulation index various ways have been proposed. One of them consists in the application of the "periodogram analysis"; by this way one came, in particular, to the 11-day period of α variation (MONIN 1955). The other way is based upon the hydrodynamical technique (BLINOVA, 1949). Namely, it is easily proved that for a mid-level we have the relation:

$$\frac{\partial\alpha}{\partial t} = -\frac{1}{2\pi a_0^2 \sin^3\Theta}\frac{\partial}{\partial\Theta}\left(\sin^2\Theta\int_0^{2\pi} v'_\Theta v'_\lambda \, d\lambda\right) \qquad (20)$$

Substituting in the right side of (20) v'_Θ and v'_λ taken from the linear solution of the problem one can receive the possibility of the prognosis of α by performing the time quadratures.

This formula was tested previously on the

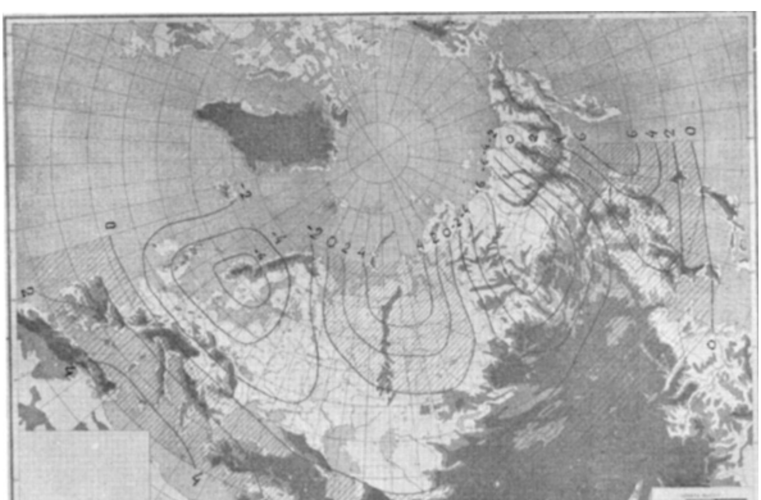

Fig. 7 b. Computed mean monthly sea-level temperature anomalies, April, 1955. Computations based on initial data of 18—20 Febr., 1955. Isoplets are drawn at 2° C intervals.

prognosis for several days in advance (Fig. 8); then it was tested in the long-range forecasting of the mean monthly values of α (S. MASHKOVITCH, N. PETUNINA, I. SMIRNOV, 1954). This simple method permits evaluation of the nonperiodic corrections of the annual variation of α.

The circulation index at the mid-level possibly depends also upon Θ. This case was investigated by MASHKOVITCH (1952), who used the expansion upon a small parameter, and, after an example by N. Krylov and N. Bogoljubov, constructed simultaneously the expansions both of the amplitude and such of the periods of the solution.

For the three-dimensional linear case it is particularly essential to specify the dependence

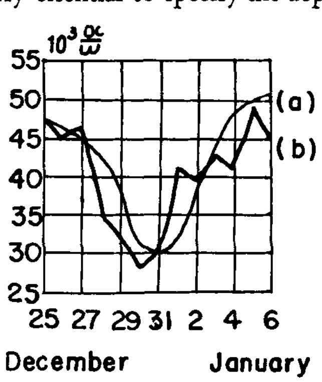

Fig. 8. The circulation index forecast. Observed (a) 1,000 $\frac{\alpha}{\omega}$ values at 500 mb level during theperiod 25 Dec., 1949—6 Jan., 1950, and computed (b) ones.

of α upon the height z (BLINOVA, 1953, HEIFETZ, 1953, MONIN, 1953). Conforming to the results of Kuo, Fleagle and many other authors the linear problem may have the non-stable solutions for some special kind of the dependence of α upon height. The physical meaning of this instability may be clarified only by the analysis of the solutions of the corresponding non-linear problem.

Before proceeding to the non-linear problem as the last part of our communication we shall mention the works of MASHKOVITCH (1953, 1955) in which the non-linearity is introduced only as some corrections, as the second approximation to the solution (the first approximation being a linear solution). Examples from the works of Mashkovitch are represented in Fig. 9.

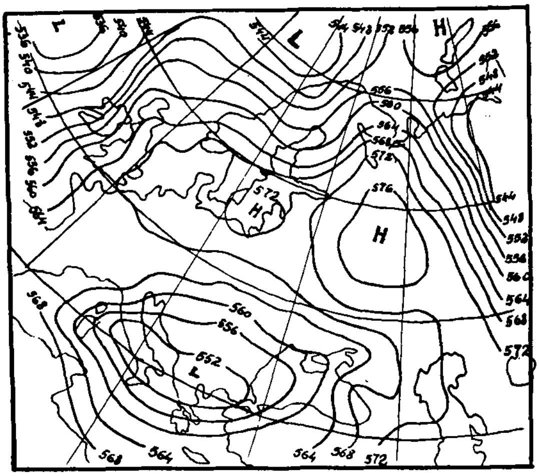

Fig. 9 a. 500 mb chart 06h 9 May, 1954, (in decameters).

10. The study of the full non-linear problem of the long-range forecasting has become possible only with the appearance of the high-speed electronic computing machines. In distinction from the short-range problems, we require now initial data from the whole northern hemisphere. It is particularly essential to take into account the thermal transformation of the air from the ground. This can always be done for the boundary layer of the earth, after the forecast for the free atmosphere is already computed.

The investigation of the long-range forecasting problem in the free atmosphere began with a consideration of the mid-level process

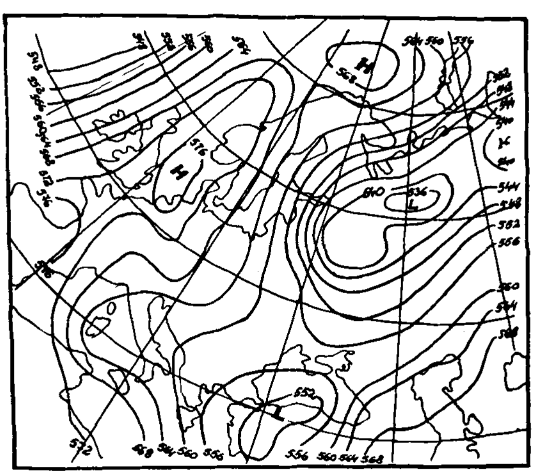

Fig. 9 b. Observed 500 mb contours 06h 12 May, 1954.

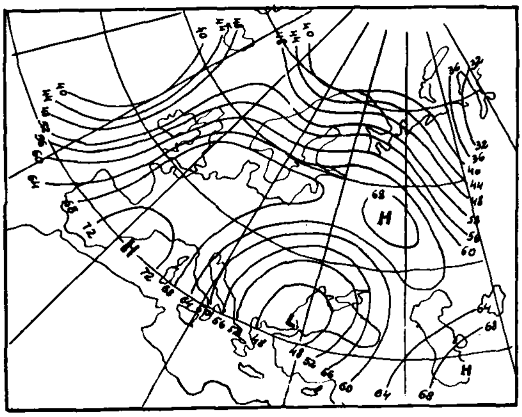

Fig. 9 c. 500 mb contours forecast from 06h 9 May, 1954, to 06h 12 May, 1954, (in decameters), 500 dkm extracted.—Linear theory.

(some different variants of solution have been proposed by BLINOVA (1954), A. CHEKIRDA (1956) and others). In the general three-dimensional case the problem may be approximately reduced to the solution of the equation for the stream function ψ (BLINOVA, 1956):

$$\frac{\partial L}{\partial t} = 2\omega\Gamma\frac{\partial\psi}{\partial\lambda} - \frac{1}{a_0^2\sin\Theta}J(\psi, L) \qquad (21)$$

where

$$L = -\frac{\partial}{\partial\xi}\left(\xi^2\frac{\partial\psi}{\partial\xi}\right) - \Gamma\Delta\psi,$$

$$J(\psi, L) = \frac{\partial\psi}{\partial\Theta}\frac{\partial L}{\partial\lambda} - \frac{\partial\psi}{\partial\lambda}\frac{\partial L}{\partial\Theta}$$

ξ and Γ being determined by (15).

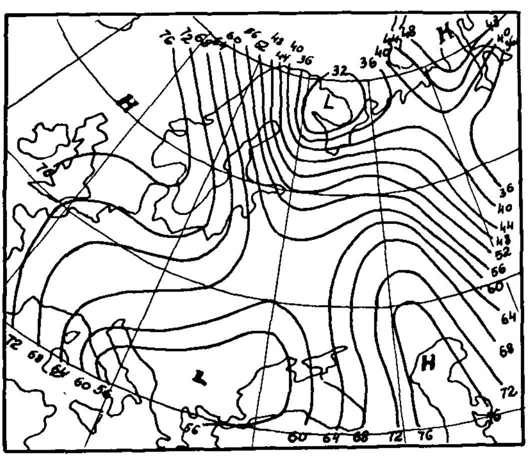

Fig. 9 d. The same as 9 c, but with nonlinear corrections.

Equation (21) have to be resolved with the boundary conditions:

$$\frac{\partial^2\psi}{\partial\xi\partial t} = -\frac{1}{a_0^2\sin\Theta}J\left(\psi, \frac{\partial\psi}{\partial\xi}\right) \quad \text{at} \quad \xi = 1 \qquad (22)$$

$$\xi\frac{\partial^2\psi}{\partial\xi\partial t} \quad \text{is limited at} \quad \xi = 0 \qquad (23)$$

With respect to $\frac{\partial\psi}{\partial t}$ equation (21) is a linear nonhomogeneous equation. The solution of it with the boundary conditions (22) and (23), supposing the non-penetrating of air masses through the equator, is for the northern hemisphere found to be

$$\frac{\partial\psi}{\partial t} = \frac{1}{2\pi a_0^2}\int_0^1\int_0^{2\pi}\int_0^{\frac{\pi}{2}} g_1(\Theta, \lambda, \xi; \Theta', \lambda', \xi')$$

$$\left[J(\psi, \Delta\psi) + 2\omega a_0^2\sin\Theta'\frac{\partial\psi}{\partial\lambda'}\right] d\Theta' d\lambda' d\xi' +$$

$$+\frac{1}{2\pi a_0^2}\int_0^1\int_0^{2\pi}\int_0^{\frac{\pi}{2}} g_2(\Theta, \lambda, \xi; \Theta', \lambda', \xi')$$

$$J\left(\psi, \frac{\partial\psi}{\partial\xi'}\right) d\Theta' d\lambda' d\xi' \qquad (24)$$

$$g_1 = \frac{\Gamma}{4\sqrt{\xi\xi'}}$$

$$\sum_{n=1}^{\infty}\frac{2n+1}{m}\left[\frac{2m+1}{2m-1}e^{-m\ln\frac{1}{\xi\xi'}} + e^{-m\left|\ln\frac{\xi}{\xi'}\right|}\right]$$

$$[p_n(\cos\gamma) - p_n(\cos\bar{\gamma})]$$

$$\Gamma g_2 = -\xi'^2\frac{\partial g_1}{\partial\xi'}$$

$$2m = \sqrt{4\Gamma n(n+1) + 1},$$

$$\cos\gamma = \sin\Theta\sin\Theta'\cos(\lambda' - \lambda) + \cos\Theta\cos\Theta'$$

$$\cos\bar{\gamma} = \sin\Theta\sin\Theta'\cos(\lambda' - \lambda) - \cos\Theta\cos\Theta'$$

The functions g_1 and g_2 have been tabulated.

The calculation of the thermal transformation in the boundary layer of the earth may be carried out in the manner analogous to that in the solution of the linear problem of the long-range prognosis. But now we should have dealt with the approximate solutions of corresponding equations of thermal conductivity (that is by means of the time-steps).

It is necessary to emphasize that with the extension of the forecasting range the role of the skill will be increased by taking into account not only of the turbulent thermal conductivity but also of other kinds of the afflux of heat—the long- and short-wave radiation, the heat of condensation and evaporation and so on. In this connection the works of Soviet scientists (N. Kochin, L. Keller, A. Dorodnitzyn, V. Shulejkin, M. Shwetz, S. Mashkovitch, S. Musaeljan, R. Rakipova, M. Malkevitch, A. Mkhitarjan, G. Kurbatkin and others) play an important role on the construction of the theory of climate; these works appeared to form some touchstone in approval of the hydrodynamic's possibilities before a more difficult problem, that of the long-range weather forecasting, could be solved.

A complete bibliography concerning research in this field in USSR will be given in a later issue of this journal.