On the Accuracy of 500 mb Analysis with Special Reference to Numerical Forecasting

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(Manuscript received April 18, 1957)

Abstract

The result of an analysis experiment is discussed. From 29 different analyses of one and the same 500 mb chart certain parameters were calculated. The standard deviation of the height of the 500 mb surface had an average value of 15 gdm, determined over an area of 4,200 × 8,100 km. The correlation coefficient between the observed changes and the changes forecast with a numerical barotropic forecast varied between 0.53 and 0.77 (24h), 0.77 and 0.84 (48h), respectively. Some other verification parameters are given. Calculations of the geostrophic wind are carried through for all analyses, both at points, over areas and along certain tracks. The deviations due to the differences in analysis are sometimes considerable.

Introduction

During the last decade the problem of numerical methods in making upper air forecasts has received much attention. Quite a few papers have been published giving detailed information about numerical forecasts: CHARNEY, 1949; BOLIN, 1955; CHARNEY-PHILLIPS, 1953; SAWYER-BUSHBY, 1953; Staff Members, Department of Meteorology, University of Stockholm, 1954; BERGTHORSON ET AL., 1955; GATES ET AL., 1956; DÖÖS, 1956. In most of these papers the success or failure of a forecast is demonstrated by certain numerical values such as the correlation coefficient between the observed and forecast changes, the root-mean-square difference between the same quantities, the ratio between this root-mean-square value and the standard deviation of the observed changes and so on. Many authors have expressed their disbelief in these quantities and have asked for other methods of indicating the degree of success of an upper air forecast.

The numerical methods are tested in different ways and in all cases the need for a good measure of the success of the forecast is acute. In some places the methods are used on an operational basis because one wants to have an indication of how the methods work when faced with practical problems such as communication troubles, machine breakdown, personnel requirements and so on. Inherent in these tests are of course tests on the possibility of the numerical forecasts to indicate changes in the upper air pattern, breakdown of blocking highs, occurrence of cut-off lows etc. Sometimes the methods are used on specially selected cases, when one wants to know how an interesting change in the upper air pattern is handled by the atmospheric model used in the method in question. Such an interesting case was observed during the first part of November 1949, when a radical change of the circulation pattern over the North Atlantic and Europe occurred. This situation was first studied by the author for an entirely different reason (BERGGREN, 1952), but was later picked out as a good trial case. The author hopes to be able to give an account of this test in a later paper.

During the work with the test it became evident that some specific questions had to be discussed before the final work could be done. One of the problems can simply be stated as follows: What are the limits within which the so called known (observed) changes of the height of the 500 mb surface can vary? When calculating the correlation coefficient between observed and forecast changes one often treats the values of observed changes as fixed values that are known with all required accuracy and discusses then differences between observed and forecast changes that may be well within the limits of accuracy.

The question of how an upper air numerical forecast is influenced by the analysis used and of how much the forecast would be changed if another base analysis was used, has been discussed by some authors. NEWTON (1954) and BEST (1956) have shown examples of how radical a numerical forecast may be changed, if another base analysis is used.

During the work with the November 1949 case the author got quite convinced that different analyses could give significantly different verification figures (correlation coefficients, errors and so on), i.e. besides the above discussed effect of the base analysis on the forecast the effect of differences in analysis of the verification map has to be taken into account. For this reason an experiment was carried out along the following lines.

Method of analysis experiment

Out of some investigated days in the early part of November 1949 the seventh was chosen. For this day one 700 and one 300 mb forecast were available together with two 500 mb forecasts made according to slightly different schemes. For reasons that may be evident in a forthcoming paper about the result of the tests as mentioned in the introduction this was a great advantage. Furthermore, the situation over the Atlantic was rather clear and the resulting differences ought to represent minimum rather than maximum values. If a situation with a deep Atlantic low had been picked out and resulted in large difference the argument against the result might have been the following: a) the chosen situation is such that maximum differences could be expected and b) if another situation had been chosen, the differences might well have turned out to be insignificant. Finally, it was necessary for the further work to choose one of the days already studied.

The observations from the author's working chart were plotted on tracing paper and each station was checked thoroughly for plotting errors. Then ozalid copies were taken in order to give the analysts identical copies. In fig. 6 the stations used are indicated. The outer boundary was the limit of the analysis and stations were plotted a certain distance outside this boundary, which depended on the

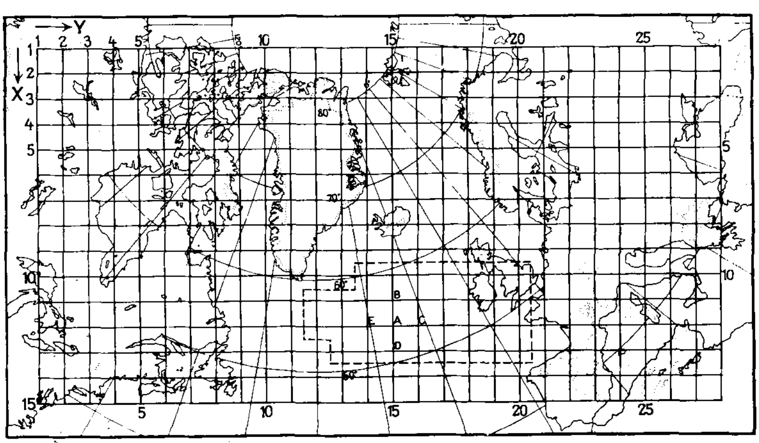

Fig. 1. Location of the grid used for computation of verification figures, geostrophic wind etc. The boundary indicates the region within which the analysis was performed.

amount of data available in the boundary area and varied between 1,000—3,000 km.

The analysts were contacted by the author one at a time and the chart to be analysed, the 500 mb for the 6th (fig. 3a) and the sea-level chart for the 7th (fig. 4) were given to them. The analyst was given instructions, containing among other things a statement that the analysis must not be discussed with other analysts before the trial period was over. In order to get as homogeneous a sample as possible all analysts were told to make "a careful routine analysis", i.e. not to spend too many hours on the analysis and use the same technique as they use in the daily analysis work. On an average the analysts worked with the analysis about two hours. One difficulty was that some of the analysts would have preferred to use the differential analysis technique, but this possibility was deliberately taken away by the author. The reason for this being that in a case like the one used with a rather simple analysis over the ocean the deviations between analyses ought to be smaller if all used a direct analysis method instead of having, say, 50 per cent of the analyses made in accordance with the differential technique. All analysts are accepted as analysts at the Swedish Met. and Hydr. Institute in Stockholm (SMHI).

The material on which the trial analysis was based consisted of reevaluated data. This means that transmission errors were eliminated. This type of error may reach any value and the possibility to trace the error is depending among other things on the density of the station network. The base material was better than the one used in the daily routine work and the expected deviations between the different analyses were thus reduced. In some cases there remained doubt as to whether a certain sounding was correct or not. This question will be discussed somewhat later on in the paper.

It was possible to get 26 different analyses within the time limit set for the trial period. When the analyses were handed over to the author, he personally interpolated height values in each of the 420 grid points indicated in fig. 1. When making such an interpolation it is inevitable that a personal error of interpolation comes in. This error was, however, minimized when all charts were read off by the same person. There was no indication that the interpolator learned some values by heart. On the contrary, it was astonishing how easy it was to avoid such an influence!

The author's working chart for the 7th was read off twice with nearly two years interval. The mean difference was 1.9 gdm[1], and the differences were distributed thus: zero difference in 206 points, ± 5 gdm in 164 points, ± 10 gdm in 49 points, ± 15 gdm in 1 point. The standard deviation was 4.3 gdm. It is very reasonable to assume that the standard deviation of interpolation varies with the isohypse gradient. The material at hand did not, however, permit an investigation of this question.

The first obvious thing to do with the collected analysis material was to calculate, at each grid point, the standard deviation of the height of the 500 mb surface (σ_h). Fig. 5 gives the result of these calculations together with the mean 500 mb chart, calculated from the 26 analyses. One thing is clear from a first sight on fig. 5 viz. the non-relationship between the standard deviation and the upper air pattern. The largest standard deviation, around 44 gdm, is observed over the North West Territories of Canada, near the radiosonde station 72938 Coppermine and at northeasternmost Greenland. Other maximum areas are found east of Newfoundland (38 gdm), at the southwestern shore of Hudson Bay (37 gdm), over the Black Sea (32 gdm), over Italy (29 gdm), over Baffin Island (28 gdm), south and southeast of the southern tip of Greenland (26 gdm), and over the outer Bay of Biscay (25 gdm). Minimum values are observed over northeasternmost USA and adjacent Canadian areas (2—3 gdm), the British Isles, eastern Russia and northeasternmost part of Baffin Island (3—4 gdm).

The overall mean of σ_h for all 420 grid points is 14.9 gdm. The values of σ_h ought to be corrected for the standard deviation due to the interpolation. This correction has not been applied in this case, however, because all charts discussed have been read off in the same way. Thus, all charts are affected in the same manner, if we disregard the variability due to different isohypse gradients. Furthermore, the technique of reading off values at grid points

[1] All through this paper the heights are given in geodynamic meters (gdm).

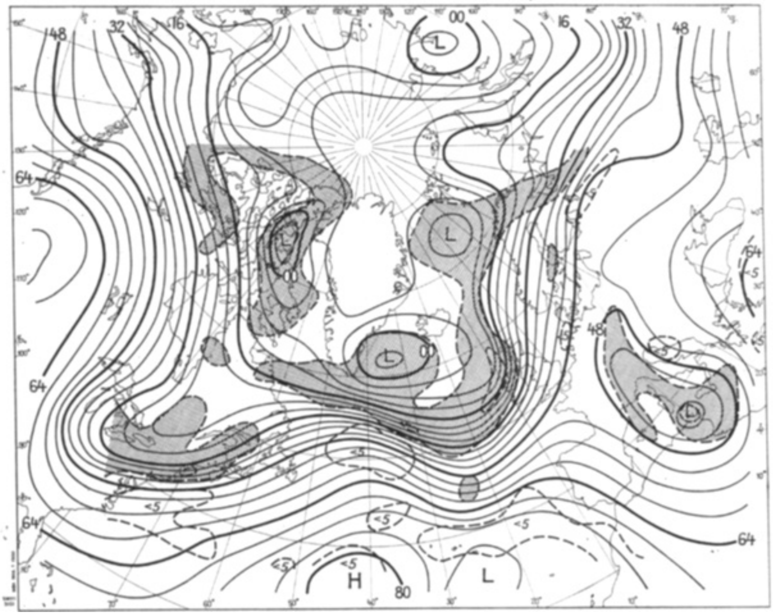

Fig. 2 a. 500 mb, 0300 GMT 5 November 1949, with isohypses for every 4 geodynamic decameters (solid lines) and isolines for the absolute vorticity in 10^{-5} sec^{-1} (dashed lines). Hatched areas indicate vorticity values larger than $15 \cdot 10^{-5}$ sec^{-1}. Isoline interval 10^{-4} sec^{-1}.

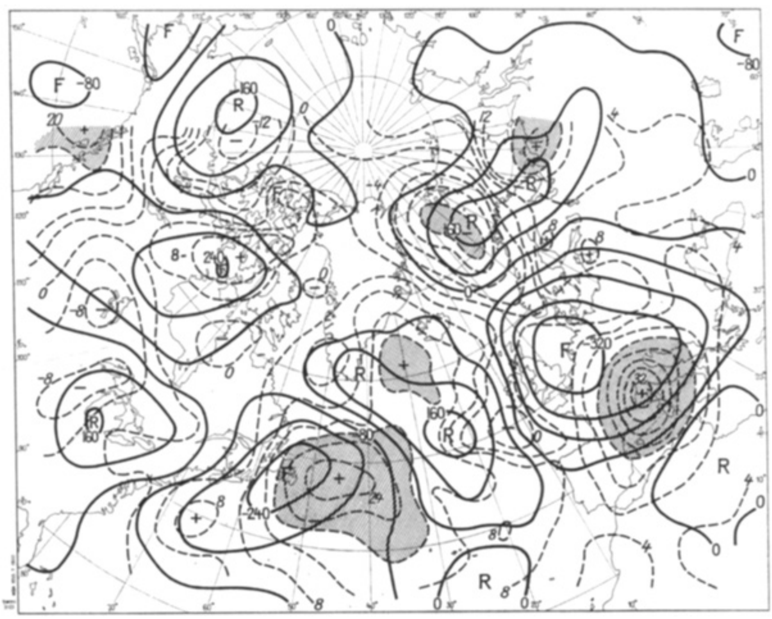

Fig. 2 b. The observed changes (solid lines; isallohypse interval 80 gdm) of the height of the 500 mb surface between 0300 GMT 5—0300 GMT 7 November 1949 and the difference between observed and forecast changes (dashed lines; isoline interval 4 decameters) for the same period. Hatched areas indicate values of the difference *numerically* larger than 16 geodynamic decameters. *F* and *R* indicate extreme values of the observed changes; plus and minus signs indicate extreme values of the difference between observed and forecast changes.

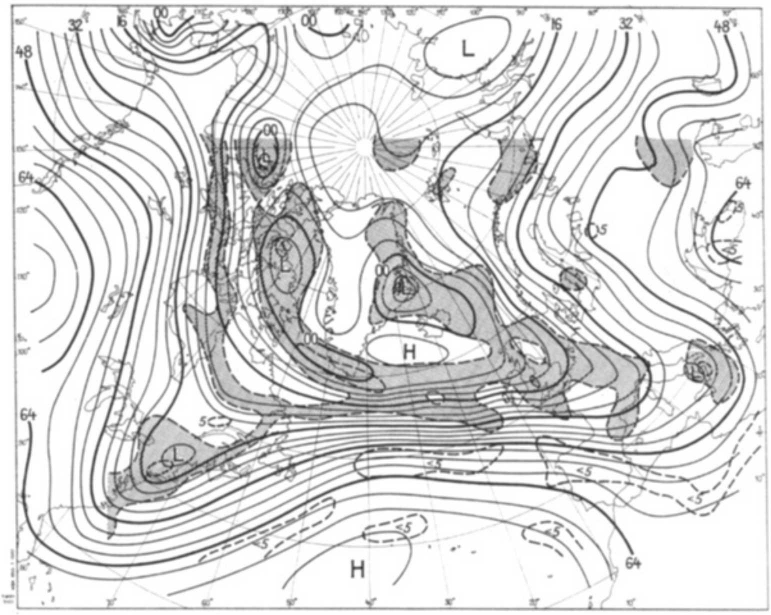

Fig. 3 a. 500 mb, 0300 GMT 6 November 1949, with isohypses for every 4 geodynamic decameters (solid lines) and isolines for the absolute vorticity in 10^{-5} sec^{-1} (dashed lines). Hatched areas indicate vorticity values larger than $15 \cdot 10^{-5}$ sec^{-1}. Isoline interval 10^{-4} sec^{-1}.

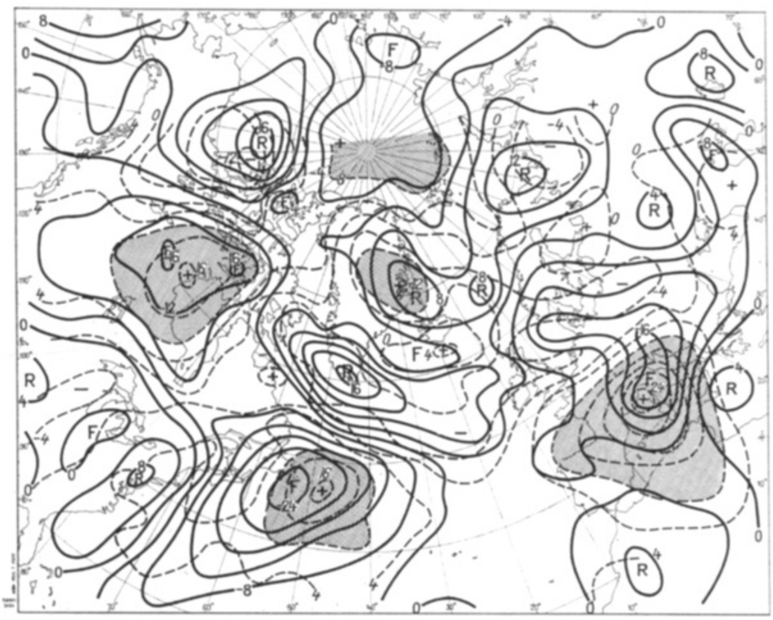

Fig. 3 b. The observed changes (solid lines, isohypse interval 4 decameters) of the height of the 500 mb surface between 0300 GMT 6—0300 GMT 7 November 1949 and the difference between observed and forecast changes (dashed lines; isoline interval 4 decameters) for the same period. Hatched areas indicate values of differences *numerically* larger than 8 geodynamic decameters. *F* and *R* indicate extreme values of the observed changes; plus and minus signs indicate extreme values of the difference between observed and forecast changes.

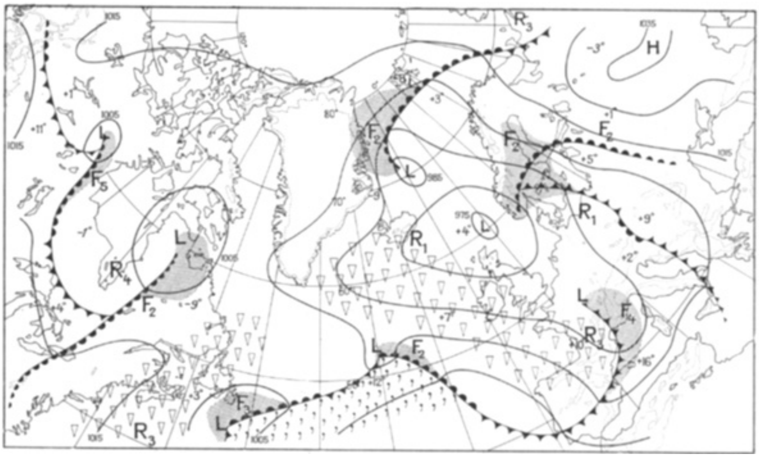

Fig. 4. Sea-level chart for 0000—0300 GMT 7 November 1949.

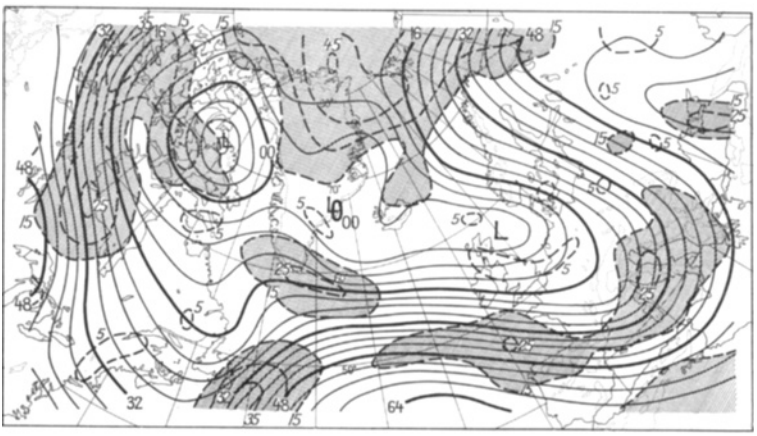

Fig. 5. Standard deviation (dashed lines) of the height of the 500 mb surface calculated from 26 different analyses (σ_h) based on identical material for 0300 GMT 7 November 1949. Hatched areas indicate values larger than 15 gdm. The average 500 mb chart calculated from the 26 analyses is given by the solid lines.

is the most common one so far. Numerical analysis technique eliminates this type of error.

The distribution of the standard deviation looks a little erratic and as stated above no relation with the isohypse pattern can be seen. It ought to be more fruitful to look for a relationship between the position and number of radiosonde stations and the standard deviation. Three different approaches were tried out. According to the first method the number of radiosonde stations in a square of 3×3 grid widths was counted and plotted at the central grid point, the grid width being 1.5 cm or approx. 300 km since the working chart was of the scale $1:2 \cdot 10^7$ (polar stereographic chart with the scale factor equal to one at 50° N). The maximum value of the station density, 14 stations per square, was observed over western Germany, the Netherlands and southeastern North Sea. The area with 6 stations or more per square covered Europe from the latitudes of the Alpes to northern Scandinavia and from the British Isles to eastern Russia. Within the grid area (fig. 1) the value of 6 or larger was also found over northeastern USA and at a few isolated points over the North West Territories of

Canada. A network density equal to zero was observed over northeasternmost part of Greenland, over Spitzbergen and the region between and north of those areas, over the central part of the Atlantic at the boundary of the grid, at one isolated point off the Newfoundland coast, at one point on the southwestern shore of Hudson Bay and finally at some grid points over Yugoslavia, Albania and Greece.

Comparing the distribution of the standard deviation as given in fig. 5 with the statements above about network density one can immediately see that the relationship between the standard deviation and the network density is high in some areas. At a first look one is therefore tempted to calculate a correlation coefficient between these variables. The coefficient turns out, however, to be as low as 0.31.

The explanation of the low coefficient is the fact that some areas have high values of network density together with high values of the standard deviation due to erroneous or questionable observations while for other areas the opposite is true due to the fact that the isohypse gradients are very weak in those areas. These two effects will be discussed later.

When plotting the pair of values at all grid points in a scatter diagram one can see that a straight line or any other curve is not able to take all pair of values reasonably into account.

Next, another network density was calculated taking upper wind stations into consideration. The simplest way possible to do this is to give radiosonde stations and wind stations equal weight. Other more elaborate schemes can be developed, but in this case they would lead nowhere. The isolines for the network density defined in the new manner coincide practically everywhere with the isolines discussed above. No improvement could be obtained this way.

If one tries to analyse how an analyst works when making an upper air analysis, one arrives to the conclusion that only the nearest-by stations are used when a certain area is analysed. Let us take one example: when the area just off the Scottish and Irish coasts is analysed the stations 03026 (Stornoway) on the Hebrides and 03917 (Aldergrove) and 03953 (Valentia) on Ireland are used together with the two nearest weather ships but not at all or to a much lesser degree 03485 (Downham Market) and 03808 (Camborn) in England. The latter stations are mostly used as checks on the more near-by stations. This reasoning leads one to the conclusion that the important factor is not the network density but rather the distance from the analysed area to the nearest station. When applying this reasoning to the problem at hand, one encounters the difficulty that has already been discussed. Which is the best way to take both radiosonde stations and wind stations into account? In order to get a first idea of the important factors the wind stations were disregarded. This is not as bad as it may look at a first glance. Over large areas where the number of radiosonde stations is low as over the Atlantic, a radiosonde station quite often is a wind station at the same time. On the other hand a single wind station in a vast area without radiosonde stations is not as valuable as an additional radiosonde station.

The simple scheme used was to determine the distance from each grid point to the nearest radiosonde station. The distribution of this variable (R) is given in fig. 6.

The overall maximum value (50 mm $\simeq$ 1,000 km) is found at the northeasternmost tip of Greenland and in the lower right corner of the grid. The highest value over the Atlantic (42 mm $\simeq$ 840 km) is obtained in the bottom row and in the column that passes through Spitzbergen and Iceland. Over the Adriatic Sea R reaches the value 35 mm ($\simeq$ 700 km) and just southwest of Hudson Bay 32 mm ($\simeq$ 640 km). The maximum value east of Newfoundland is 37 mm ($\simeq$ 740 km) and southeast of Greenland 30 mm ($\simeq$ 600 km). If one compares the distribution of R with the standard deviation (σ_h) for the 26 different analyses, one can immediately see, that a rather high correlation is at hand. A calculation of the correlation coefficient between these two variables gives the result 0.90. The equation for the regression of the standard deviation (σ_h in gdm) on the distance from a grid point to the nearest radiosonde station (R in mm; 1 mm corresponding to approx. 20 km) reads as follows

$$\sigma_h = 0.84\ R + 3.2$$

The standard error of estimate is 3.1 gdm. It is of interest to investigate where the above

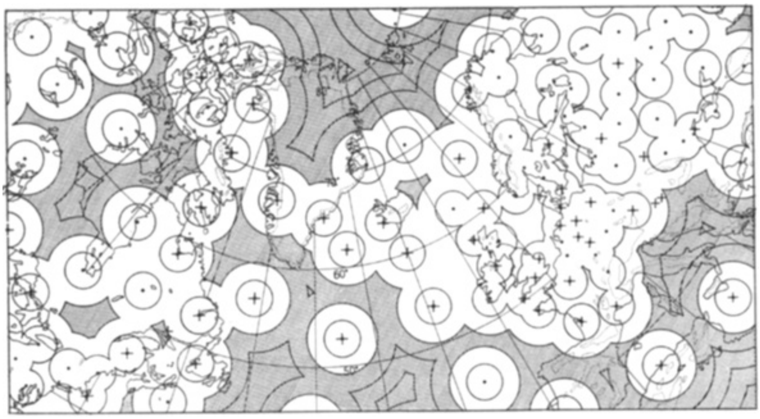

Fig. 6. The distribution of the parameter R, defined as the distance (in mm, 1 mm $\simeq$ 20 km) on the working chart, a polar stereographic chart (1 : 20 mill.), from a certain point to the nearest-by radiosonde station. The circles are drawn for R equal to 10 mm, 20 mm, 30 mm and so on. Hatched areas indicate values of R greater than 20 mm ($\simeq$ 400 km). Dots and crosses give the positions of radiosonde stations and wind stations, respectively.

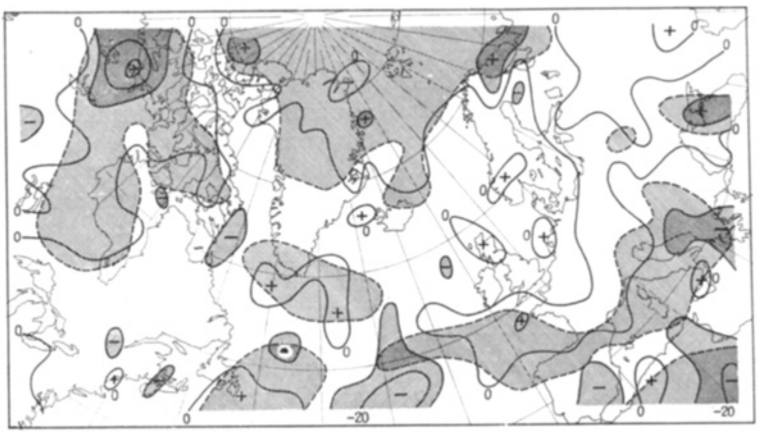

Fig. 7. The distribution of the difference between observed (σ) and estimated values of the standard deviation of the height of the 500 mb surface. Hatched areas with lines sloping from lower left to upper right corner indicates differences numerically larger than 10 gdm. Hatched areas with lines sloping from lower right to upper left corner indicate values of σ_h larger than 15 gdm (cf. fig. 5). Plus and minus signs indicate extreme values of the difference.

regression equation is valid and in which areas it gives large errors. In fig. 7 the difference between the observed and the estimated standard deviation is given. Except for some small areas containing only one grid point an absolute value of this difference greater than 10 gdm is observed over the following areas: the North West Territories of Canada (maximum value +37 gdm), the Canadian archipelago (+15 gdm), southernmost Baffin Island and areas south thereof (−12 gdm), the central Atlantic (−26 gdm), Spain (−15 gdm), Tunisia (−22 gdm), Greece and Yugoslavia (−15 gdm), the Black Sea (−24 gdm) and finally northern Finland and the Kola Peninsula (+12 gdm).

Large positive differences occur where large values of the standard deviation are found near radiosonde stations. One marked area of this type is situated over the North West Territories of Canada (figs. 6 and 7). Here a grid point as near as 4 mm ($\simeq$ 80 km) from a radiosonde station has a standard deviation of about 44 gdm.

The reason for this peculiar discrepancy can be found in the fact that the radiosonde station in question is one of the very few on the actual 500 mb chart, the height value of which may be questioned. About fifty per cent of the analysts considered the value to be too low and made a smoothed analysis in this area. A thorough check of this station has shown that the height value may be taken as correct.

The large difference observed over the Black Sea is due to another type of difficulty. The Crimean station had a 500 mb value that definitely was too high. Some of the analysts disregarded the station altogether while others tried to take it into account as much as possible with a varying degree of smoothing.

The small positive area over the Canadian archipelago may be explained by the fact that this area is situated at the outskirts of vast areas around the north pole completely lacking stations. Even if the grid points in question are situated not too far away from a radiosonde station the lack of stations north and west of the area puts the analyst into difficulties.

The area with positive differences between observed and calculated standard deviations that is found over northern Finland and the Kola Peninsula is due to the fact that the height values given by the stations 02836 (Sodankylä) and 22113 (Murmansk) were incompatible. Some of the analysts tried to take both stations more or less into account while others struck out one of them, the result being an increased value of σ_h.

The four areas with negative differences between the observed and estimated standard deviations in the lower right corner of fig. 7 may all be explained in the same way. The upper air pattern in these regions is rather clear cut and in some cases the gradients are not too strong. It is not too difficult to get a good picture of the isohypse configuration near the radiosonde stations in the lower right corner. It is then easy to extrapolate this pattern into areas with scattered data.

We have now discussed the areas with large differences between observed and calculated values of the standard deviation. It is, however, interesting to note in fig. 7 that practically all areas with a standard deviation equal to or larger than 15 gdm fall outside the areas discussed above. As a whole one can state that the estimated values of the standard deviation coincide rather well with the observed ones, the mean deviation (i.e. the average value of the absolute values) being 5.4 gdm.

A comparison between fig. 6 and 7 shows that the absolute value of the difference between observed and estimated standard deviation generally does not exceed 10 gdm in areas with a station distribution such that no grid point is more than 20 mm ($\simeq$ 400 km) away from a radiosonde station.

The discussion so far has given some figures of standard deviations, that are due to different analyses of one and the same observation material. These values may be compared with the standard error of the radiosoundings. According to NYBERG (1952) the standard deviation of height values (500 mb) between two radiosondes was about 30 gdm, a value found during the international radiosonde comparisons at Payerne in 1950. Before this standard deviation was calculated the differences between the radiosonde values had been corrected for systematic differences. The average standard deviation of height values for 500 mb given by a single sonde was about 18 gdm. It is very difficult to apply these values in one case because in some areas, such as the British Isles and the United States of America, the radiosonde network is rather homogeneous while in others, such as Central Europe, the network is composed of a great variety of radiosonde types. A reasonable overall mean value of the standard deviation of the height of the 500 mb surface originating from instrument errors of different types might be 25 gdm. The same numerical value of the standard deviation for the 26 analyses corresponds to a distance of 26 mm between a grid point and the nearest-by radiosonde station (R) i.e. a distance of about 520 km at the surface of the earth. On the other hand the observed mean value of R corresponds to a standard deviation of 17 gdm. Thus, the deviations due to different analyses attain values that cannot be neglected in comparison with the instrument errors.

Finally it might be of interest to investigate the distribution of the range of the height values (i.e. the difference between the highest

Table I. Frequency distribution of the range of the height of 500 mb surface originating from 26 different analyses of the upper air data for 0300 GMT 7 November 1949. The range is determined at each of the 420 grid points in Fig. 1

range	number of points
0—20 gdm	27
21—40	149
41—60	93
61—80	54
81—100	46
101—120	25
121—140	14
141—160	7
161—180	2
181—200	3
	Total 420

and the lowest value) as interpolated from the 26 analyses at each grid point. This distribution is given in table I. The range varied between 10 and 195 gdm. The maximum value was observed at the north-easternmost tip of Greenland and the minimum value over the Irish Sea and at the grid point over the northernmost part of Baffin Island ($x=8$, $y=4$). The mean value was 60 gdm.

It ought to be emphasized here that the investigated analyses were based on checked data without transmission errors. Furthermore, it has been stated earlier that the upper air pattern was not a difficult one to analyse.

Verification of numerical forecasts

The main object of the analysis experiment discussed in this paper was to see how among other things verification figures for numerical forecasts were changed if another verification analysis was used. In order to arrive at this goal a number of verification figures were computed using a high speed computer (BESK in Stockholm).

The following variables were computed:

$\overline{X}$ the mean value of the 24h and 48h observed changes (x) of the height of the 500 mb surface from the 5th and 6th, respectively, to the 7th of November 1949

$\overline{|X|}$ the mean deviation of x

σ_x the standard deviation of x

ε_0 the root-mean-square of the difference between observed and forecast changes

r_{xy} the correlation coefficient between observed and forecast changes

$\overline{d_0}$ the mean value of the geostrophic wind that corresponds to the observed changes

$\overline{d_e}$ the mean value of the geostrophic wind that corresponds to the error; i.e. the difference between verification analysis and forecast

δ_0 the root-mean-square of d_0

δ_e the root-mean-square of d_e

$(d_0)_{max}$ the maximum value of all the d_0's

$(d_e)_{max}$ the maximum value of all the d_e's

$r_{d_0 d_p}$ the correlation coefficient between geostrophic wind values corresponding to the observed and forecast changes.

All the above values were calculated for the 26 analyses over the grid in fig. 1, which contains 420 points. The maximum and minimum values for each variable were picked out. These extracted values are given in table II. In addition to the 26 analyses the author had access to a routine analysis made at the Swedish meteorological and hydrological institute, his own analysis, and a numerical analysis made according to the scheme described by BERGTHORSON and DÖÖS (1955) and DÖÖS and EATON (1957).

The first two analyses were made with differential analysis technique, more completely carried through in the latter case.

Verification figures were calculated for two different numerical barotropic forecasts, one 24h forecast and one 48h forecast, both forecast periods ending at 0300 GMT on the 7th of November. Thus, the two forecasts were not based on the same analysis. The forecasts were made according to the methods described by Staff Members, Dept of Met., Univ. of Stockholm (1954) and BOLIN (1955).

Figs. 2a and 3a give the 500 mb chart for the 5th and the 6th, respectively. The observed changes of the height of the 500 mb surface are given in fig. 2b and 3b. In the two latter figures the errors, i.e. the difference between observed and forecast changes also are given.

The values of some observed quantities as given in table II have to be compared with the corresponding forecast values. These are given in table III.

a) 24h forecast

Already in the mean value of the observed changes we observe a rather large variability.

Table II. Some verification parameters computed from 29 different verification 500 mb analyses (0300 GMT 7 November 1949), a given 500 mb chart for 0300 GMT 6 November 1949 and a numerical barotropic forecast. The lower part gives the corresponding values for the 48h forecast based on the 500 mb chart for 0300 GMT 5 November 1949. Values of maximum, minimum, mean and range are taken from 26 analyses

24h	$\overline{X}$	$\overline{\|X\|}$	σ_x	ε_0	r_{xy}	δ_0	δ_e	$\overline{d_0}$	$\overline{d_e}$	$(d_0)_{max}$	$(d_e)_{max}$	r_{d_0, d_p}
max......	—26	65	76	68	0.651	12.0	11.0	10.6	9.6	35.2	30.6	0.485
min.......	—38	55	63	57	0.564	10.8	9.4	9.7	8.3	25.3	23.6	0.343
mean.....	—32.1	60.6	70.0	63.7	0.609	11.5	10.2	10.2	9.0	29.1	27.5	0.414
range.....	12	10	13	11	0.087	1.3	1.6	0.9	1.3	9.9	7.0	0.142
routine ...	—27	58	66	60	0.609	11.6	10.5	9.9	8.7	33.6	38.2	0.465
author....	—32	62	71	69	0.534	12.4	11.4	10.6	9.7	36.5	33.6	0.432
NA.......	—27	55	63	45	0.772	10.2	6.9	8.8	6.0	30.7	15.8	0.698
48h												
max......	—59	114	127	98	0.838	18.8	12.4	16.7	10.7	46.8	36.6	0.731
min.......	—72	104	118	88	0.772	17.8	10.9	15.5	9.5	42.8	25.1	0.630
mean.....	—65.5	109.2	124.0	94.1	0.810	18.5	11.6	16.1	10.1	44.5	28.4	0.676
range.....	13	10	9	10	0.066	1.0	1.5	1.2	1.2	4.0	11.5	0.101
routine....	—61	106	121	90	0.812	18.7	12.8	15.7	10.4	55.4	45.5	0.626
author....	—65	109	125	98	0.783	19.0	13.0	16.3	11.0	49.7	34.1	0.622
NA.......	—60	102	118	90	0.804	17.5	10.6	15.2	9.1	44.1	28.1	0.718

The deviation from the overall mean value (-32.1 gdm) is $\pm$ 6 gdm or 19 per cent. The deviation of an individual value from the mean of the standard deviation may be 10 per cent. The root-mean-square of the error may vary within a range that is 17 per cent of the mean value.

Quite often one is not interested in the correlation coefficient, but rather in its squared value. The extreme values for this quantity is 0.318 and 0.424, respectively, a rather large variation.

From table II we can also see that the geostrophic wind field corresponding to the error has a mean value that is nearly as large as the one corresponding to the observed changes. Furthermore, the range is 14 per cent of the mean value. The maximum value at any grid point of these two quantities varies considerably, especially the maximum value of the wind corresponding to the observed changes.

As already mentioned above the author has had access to three additional analyses. The first one is the routine analysis made at SMHI on 7th November 1949. The second analysis was made by the author in 1951. It is interesting to notice that these two analyses that were made with the differential analysis technique differ considerably in some respect from the 26 analyses discussed so far. From table II we find that the author's analysis gave a correlation coefficient of 0.53 which ought to be compared with the maximum value 0.65. Using Fischer's z-transformation we find that the probability to get such a difference out of chance is 6 in 100.

The low correlation coefficient obtained with the author's analysis is explained in the following way. The trough over the British Isles, Central Europe and Italy on the 6th (fig. 3a) moved very little during the next 24 hours (cf. fig. 5), but deepened somewhat. The forecast, however, brought the southern part of the trough to the northeast, thus causing a forecast error in the Mediterranean area. The author's analysis has the lowest value in the bottom of the trough of all analyses, where, by the way, eleven of the twentysix analyses showed a closed center. The difference between the verification chart and forecast chart obtained in this area using the author's analysis is larger than all those obtained with the twentysix analyses. As the changes in this area play an important role when one calculates the correlation coefficient for the whole verification area, this results in a low value of the correlation coefficient when the author's analysis is used.

Table III. Some forecast values for the 24h (48h) forecast based on the 500 mb of the 6th (5th) of November 1949, corresponding to the observed values given in table II

	24h	48h
$\bar{y}$	— 12 gdm	— 10 gdm
$\overline{\lvert y \rvert}$	51 »	97 »
σ_y	64 »	122 »
$\bar{d}_p$	9.4 m/s	15.7 m/s
δ_p	11.0 »	18.1 »

Real large differences were introduced in table II, when the values obtained with the aid of the numerical analysis were entered. This analysis is obtained by using among other approximations the 24h forecast ending at the time of verification (see BERGTHORSON and Döös, 1955). When verifying the forecasts with a numerical analysis constructed in the manner developed by the above-mentioned authors, one runs the risk of being a victim of an "a(a+b)"-effect, i.e. one correlates two variables, the one of which to some extent is contained in the other [mentioned by Döös (1956)]. In the special case discussed in this paper, this effect is obvious. While the correlation coefficient for the 26 analyses varies between 0.56 and 0.65, the numerical analysis gives the value 0.77. The mean value of the geostrophic wind corresponding to the difference between forecast and verification analysis ranges from 8.3 to 9.6 m/s, while the numerical analysis gives the very low value 6.0 m/s. The maximum value of this variable varies between 30.6 and 23.6 m/s for the 26 analyses while it is as low as 15.8 m/s for the numerical analysis.

It is interesting to notice that it was possible to increase the value of the correlation coefficient from 0.53 to 0.65 by choosing another analysis and as high as to 0.77 if the numerical analysis is taken into account. The square of these values 0.285, 0.424 and 0.596, respectively, makes the differences look even more discouraging.

The fact is that discussions on the reliability of the numerical forecasts are sometimes based on correlation coefficients and suchlike calculated from conventional verification analyses and sometimes on the same variables calculated using numerical analysis. One has to be very careful while doing so.

The variability of the correlation coefficient between observed and forecast height changes of the 500 mb surface is to some extent depending on the absolute value of the changes. When the changes are small the correlation coefficient may react sluggishly to the variation of the fields of observed and forecast changes originating from different analyses of the same observation material. Thus, it is of interest to investigate, whether or not the situation studied in this paper represents a case with extreme values of observed changes.

The Staff Members, Inst. of Met., Univ. of Stockholm (1954), BOLIN (1955) and Döös (1956) have published figures showing that the mean value given in this paper is not a markedly low one. The values are in most cases for the same season.

b) 48h forecast

The height changes for the 48h period ending at 0300 GMT on the 7th of November were certainly larger than the 24h ones. In the lower part of table II the verification figures for this period are given and in table III the corresponding forecast values are found.

In table II we notice that the correlation coefficients are markedly higher for the 48h forecast than for the 24h forecast. The mean value of the standard deviation of the observed changes was about the same as the ones given by BOLIN (1955) and Döös (1956). The ranges for X, $\overline{|X|}$ σ_x and ε_0 are of about the same value as for the 24h forecast. As the mean values, however, are higher for the 48h forecast, the percentage errors are lower. The range of the correlation coefficient between observed and forecast changes are somewhat lower in this case.

The probability to get a difference between correlation coefficients as the one observed here (0.77 and 0.84, respectively) is about 5 in 100.

For the different wind fields the absolute values of the ranges are about the same as for the 24h forecast and in some columns they are also the same relatively speaking.

There are remarkable differences in some columns between the extreme values obtained from the 26 analyses and the ones obtained from the routine analysis and the author's analysis. This is especially true for the maximum value of d_0 and d_e. The numerical analysis does not, however, differ as markedly from the others, as was the case with the 24h forecast. In this case there is no "a(a + b)"-effect, since the 48h forecast is not directly contained in the numerical analysis. In many columns the numerical analysis shows values within the range of the 26 analyses and the deviations are mostly small.

The conclusion is that for some verification figures in table II the 48h forecast gave lower relative values than did the 24h forecast. In other cases, however, the relative values are about the same and this is also true for the absolute values of the ranges in many columns. It would have been very interesting to widen this investigation to include many more cases. This is, however, impossible for the author because of the great amount of work involved in each case. It has to be emphasized once again that the results obtained in this investigation are only directly valid for these special cases and cannot be used for other cases without much care and further investigation. The purpose of this paper is only to give examples of what may happen due to variability in the analysis of one and the same observation material, i.e. the differences due to the personal factor in chart analysis.

It also has to be emphasized that the sometimes large differences obtained in this investigation are due only to the variability in the analysis of the chart at one end of the period. In the general case both analyses on which the calculation of the observed changes and so on are based are affected. Take for instance one grid point over the Atlantic somewhere halfway between two weather ships. A reasonable value of the standard deviation of the height of the 500 mb surface due to analysis effect may be 25 gdm. Let us further assume that the standard deviation on the two charts are independent of each other. This assumption is a reasonable one, because the analysis differences at one grid point as obtained by one analyst only to some degree may be considered systematic. Furthermore, in the daily routine work the analyses of two consecutive 500 mb charts, 24h apart, are often analysed by different persons. Under these assumptions the standard deviation of the observed changes is about 35 gdm. A standard deviation of the height of 30 gdm gives the value 42 gdm.

Differences in the total wind component along certain tracks

The discussion so far in this paper has been confined to the absolute height values of the 500 mb surface and to the 24h and 48h changes, observed and forecast. Taking the importance of air navigation into account as well as the necessity in many forecast techniques of knowing the upper winds accurately, it would be of interest to investigate the variability of the geostrophic wind due to different analyses.

For that purpose the geostrophic wind field was calculated for each analysis and for each grid point. The calculation was made at the same time when the above-mentioned verification figures were calculated. At each grid point the x- and y-components (see fig. 1) of the geostrophic wind were obtained. The components were computed over two grid widths, i.e. over a distance of about 600 km. This had the effect that no wind values could be obtained for the points along the boundary of the grid. Furthermore, it was technically an advantage to strike out the bottom row. The result was that the wind field was obtained over a grid consisting of 26×12 points.

In order to get a view of the influence of the differences in the analyses on the geostrophic wind field a number of summations were made. The wind components in the x-direction (v_x) were summed in the x- and y-direction (v_{xx} and v_{xy}), respectively. The y-components were treated accordingly. All these sums for the twentysix analyses were put together in four tables and among these values maximum and minimum values were picked out for each column (x) and row (y). These extreme values were divided by the number of points in the row and column, respectively. The resulting extreme values of the mean geostrophic wind components along rows and columns were entered in table IV and V. The mean values and the ranges for the different rows and columns are also given. In the same tables are also included the corresponding values for the routine analysis, author's analysis and the numerical analysis.

The largest column value of the range of the mean x-component of the geostrophic wind (v_{xx}) is 8.9 m/s[1] and for the y-component

Table IV. Geostrophic wind components along the *x*- and *y*-axes (v_x and v_y, respectively, see fig. range are taken from 26

Column		1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\overline{v_{xx}}$	max	24.8	29.2	23.7	18.1	11.4	9.0	4.0	—4.9	— 5.6	—0.5	3.8	2.1	—1.2	
	min	19.6	20.3	17.4	10.7	7.7	4.3	—0.9	—8.9	—11.0	—6.6	—1.5	—2.6	—5.4	
	mean	22.1	24.7	21.2	14.3	9.4	6.6	1.0	—7.0	— 8.1	—3.3	0.6	—0.2	—3.4	
	range	5.2	8.9	6.3	7.4	3.7	4.7	4.9	4.0	5.4	6.1	5.3	4.7	4.2	
	routine	21.3	22.2	20.9	16.1	10.4	6.1	0.5	—6.0	— 6.9	—3.4	—1.2	—0.2	—2.5	
	author	23.4	22.7	19.1	15.5	11.7	7.5	1.4	—7.6	—10.3	—4.2	0.2	—0.3	—2.5	
	NA	20.8	22.8	21.6	17.1	12.3	6.7	0.2	—5.8	— 6.5	—2.3	—0.8	—1.1	—3.3	
$\overline{v_{yx}}$	max	—0.1	0.5	2.1	3.4	3.5	3.1	3.7	5.2	5.9	6.9	8.9	10.3	10.7	
	min	—0.9	—1.0	—0.5	0.5	1.0	1.9	2.7	3.0	3.6	4.7	6.3	7.2	7.8	
	mean	—0.5	—0.2	1.1	2.4	2.7	2.8	3.3	4.0	4.7	6.1	7.5	8.6	9.0	
	range	0.8	1.5	2.6	2.9	2.5	1.2	1.0	2.2	2.3	2.2	2.6	3.1	2.9	
	routine	—0.4	0.2	1.2	2.4	3.0	3.3	4.1	4.7	5.6	6.7	7.7	8.8	9.4	
	author	—0.3	—0.2	0.3	0.9	1.8	2.5	3.5	3.8	4.3	5.4	6.9	8.2	8.7	
	NA	0	0.3	1.2	2.6	2.9	3.4	3.8	4.8	5.3	6.8	7.5	7.9	7.9	

summed in the same direction (v_{yx}) 3.7 m/s. The largest value of the range of the values of the *y*-component averaged along a row (v_{yy}) is 6.3 m/s and the one for the corresponding crosscomponent (v_{xy}) 1.4 m/s. It ought to be emphasized that the columns represent a distance on the surface of the earth of approximately 3,300 km, while the rows correspond to 7,500 km. The standard deviation of the mean geostrophic wind for a single analysis is 1.0 m/s for the column containing the maximum range of the *x*-component. It may be of interest to translate the above mean values into variations in flight time for an aircraft flying at a certain constant speed relative to the air along the tracks given by the rows and columns in fig. 1.

The concept of equivalent head wind may be used in this connection (SAWYER, 1950). In the formula for this theoretical wind the wind component across the flight path may be neglected for moderate values of that component. With a reasonable accuracy the wind retarding effect of a head wind may be taken in tables IV and V. In table IV, column 3, the maximum difference is 8.9 m/s. With a cruising speed of 100 m/s of the aircraft the delay (or gain of time) is 8.9 %. The flying time in a calm atmosphere is about 9.2 hours for a distance corresponding to a column. Thus, the delay is about 48 minutes, corresponding to a considerable operating cost of a modern large aircraft. For a row the corresponding values are: 6.3 m/s (row 12), 20.8h and 1.3h.

It is very reasonable to assume that the error of upper wind forecast is at least as large as the differences that can be obtained by using different analyses. Taking the case investigated in this paper into account and furthermore recognizing the fact that this case represents a rather simple analysis, it seems unlikely to arrive at an error of the 500 wind forecast that is less than 10 per cent on an average.

In tables IV and V it is interesting to notice that the numerical analysis in most columns and rows gives mean winds within the limits given by the 26 analyses.

The estimation of the geostrophic wind at certain points

Sometimes it is necessary to know the accuracy of the geostrophic wind at a particular point. In order to get an estimate of the magnitude of the standard deviation for the situation discussed in this paper, fourteen points were selected and the standard deviation (σ_g) of the absolute value of the geostrophic wind was calculated at each point. The coordinates of the points are given in table VI. Seven of the points represent grid points

[1] The wind values given in this paper are consistently 2 per cent off compared to correct values, the reason being that geodynamic meters were used instead of geopotential meters.

1) averaged along the x-axis (v_{xx} and v_{yx}, respectively). Values of maximum, minimum, mean and analyses. Wind speed in m/s.

15	16	17	18	19	20	21	22	23	24	25	26	27
—3.2	—5.3	—4.7	—1.9	—1.2	—4.6	—6.2	—6.3	— 6.6	— 8.3	—10.8	—11.3	—12.6
—8.4	—8.5	—7.4	—4.9	—4.1	—6.8	—8.3	—8.8	—10.6	—14.8	—14.7	—15.8	—17.7
—5.5	—6.8	—5.9	—3.3	—2.8	—5.5	—7.4	—7.5	— 8.7	—11.3	—13.0	—13.9	—14.6
5.2	3.2	2.7	3.0	2.9	2.2	2.1	2.5	4.0	6.5	3.9	4.5	5.1
—6.1	—7.4	—4.7	—2.9	—4.3	—7.1	—7.3	—5.4	— 5.9	—10.7	—16.9	—21.3	—23.4
—4.1	—5.8	—5.9	—3.6	—3.3	—6.7	—8.7	—6.3	— 3.3	— 5.9	—14.4	—20.0	—19.5
—5.6	—6.8	—6.0	—3.9	—4.0	—6.1	—8.6	—8.6	— 8.6	—12.1	—15.9	—15.4	—12.6
10.1	8.9	7.3	5.7	4.8	3.2	1.7	1.0	0.4	— 0.1	— 0.2	1.3	2.8
7.4	6.3	5.1	3.5	1.1	—0.1	—0.7	—0.8	— 1.1	— 1.5	— 1.4	— 0.4	— 0.3
8.8	7.8	6.3	4.6	2.9	1.6	0.7	0.3	— 0.3	— 0.8	— 0.6	0.2	0.8
2.7	2.6	2.2	2.2	3.7	3.3	2.4	1.8	1.5	1.4	1.2	1.7	3.1
9.4	8.5	7.3	6.1	5.3	4.0	2.5	0.5	— 1.4	— 2.3	— 1.6	0.1	2.2
8.6	7.8	6.7	5.0	3.6	2.1	0.8	0.1	— 0.6	— 1.2	— 1.4	— 1.0	— 0.7
7.7	6.8	5.2	3.9	2.9	1.9	0.9	0.2	— 0.2	— 0.7	— 0.3	0	0.1

with maximum value of the standard deviation of the height of the 500 mb surface (σ_h) in the corresponding areas (the standard deviations are given in table VI). The remaining seven points correspond to minimum values of the same standard deviation (cf. fig. 5).

The overall mean value of σ_g was 2.3 m/s. The mean value of the geostrophic wind itself was 21.2 m/s. Notice that the points ($x=2$, $y=5$; $x=4$, $y=27$) have large values of σ_h in spite of the fact that R is small. These two points have already been discussed in this paper and the discrepancy explained. The reason was that some of the analysts rejected the near-by radiosonde stations while others did not. This fact must have a repercussion also on σ_g and in table VI we can see that one of the points in question has the highest value among all the points.

From table VI we can see that there is a relationship between σ_g and the distance from the grid point to the nearest radiosonde station (R). The material is much too small to permit a calculation of a partial regression equation or suchlike. Theoretically, it ought to be a positive correlation between σ_g on the one hand and R and the absolute value of the geostrophic wind on the other. The more scarce the station network and the tighter the isohypse gradient, the more variable is the analysis. In table VI these effects may explain why the point ($x=2$, $y=13$), with $R=46$ mm ($\simeq$ 920 km), has the lowest value of σ_g among the points with large values of R. The gradients in this area were weak. Furthermore, the point ($x=13$, $y=13$) has a σ_g-value that is 1 unit larger than the corresponding value for the grid point ($x=13$, $y=5$). Its geostrophic wind speed is twice that of the latter while R is the same.

It is interesting to note that in two cases the range is as large as 16 m/s. The overall mean value is 9 m/s.

Finally an area over the Atlantic was selected as it represented a region with a broad, rather homogeneous air current. The area contained 33 grid points and is indicated in fig. 1. The grid points were divided into three groups according to their value of R (cf. fig. 6): $R \leq 10$ mm ($\simeq$ 200 km) (10 points), $10 < R \leq 20$ mm ($\simeq$ 200—400 km) (13 points), and $R > 20$ mm ($\simeq$ 400 km) (10 points). For each group the mean geostrophic wind speed was calculated from the 26 analyses together with the standard deviation of that mean value. As the selected area showed a current quasi-parallel to the y-direction of the grid (see figs. 1 and 5), the calculation was restricted to the wind component in that direction. The mean value of the cross-component was - 1.0, 0.5 and - 0.1 m/s, respectively, showing that this component could safely be disregarded, since the mean y-component was 28.0, 33.9 and 30.6 m/s, respectively. The standard deviation for the three groups was

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Table V. Geostrophic wind components along the x- and y-axes (v_x and v_y, respectively, see fig. 1) averaged along the y-axis (v_{xy} and v_{yy}, respectively). Values of maximum, minimum, mean and range are taken from 26 analyses. Wind speed in m/s

Row	1	2	3	4	5	6	7	8	9	10	11	12	13
$\overline{v}_{xy}$	max	−0.8	−1.2	−1.1	−1.2	−0.8	−0.5	0.1	0.4	0.2	−0.5	−1.0	−1.3
	min	−1.5	−1.9	−2.5	−1.8	−1.4	−1.3	−1.2	−0.9	−0.7	−0.9	−1.7	−2.6
	mean	−1.1	−1.5	−1.7	−1.4	−1.1	−0.9	−0.7	−0.4	−0.4	−0.8	−1.4	−1.8
	range	0.7	0.7	1.4	0.6	0.6	0.8	1.3	1.3	0.9	0.4	0.7	1.3
	routine	−1.3	−1.6	−2.0	−2.1	−1.9	−1.8	−1.6	−1.3	−1.2	−1.2	−1.9	−2.9
	author	−1.4	−1.6	−1.7	−1.5	−1.3	−1.1	−0.9	−0.6	−0.5	−0.9	−1.4	−1.5
	NA	−1.4	−1.6	−1.6	−1.5	−1.2	−0.9	−0.8	−0.7	−0.8	−1.0	−1.4	−2.0
$\overline{v}_{yy}$	max	−0.7	−3.8	−6.8	−6.8	−6.1	−4.0	0.1	6.0	10.3	18.0	26.5	27.9
	min	−4.4	−8.0	−10.6	−11.4	−9.2	−6.7	−2.9	1.6	6.1	13.8	20.2	23.3
	mean	−2.6	−5.6	−8.8	−11.3	−7.5	−5.8	−1.4	4.7	9.9	16.2	22.8	25.7
	range	3.7	4.2	3.8	4.6	3.1	2.7	3.0	4.4	4.2	4.2	6.3	4.6
	routine	−0.9	−2.7	−6.3	−9.1	−9.3	−7.2	−2.8	0.1	9.4	18.2	25.7	27.1
	author	−1.8	−5.4	−9.1	−9.6	−8.1	−6.5	−2.3	3.8	9.9	17.3	23.1	23.6
	NA	−0.7	−5.1	−9.5	−10.2	−7.9	−5.1	−1.0	4.1	10.1	17.5	22.4	23.7

1.3, 1.1 and 1.1 m/s. The conclusion is that in this case no relationship between the standard deviation of the geostrophic wind on the one hand and R and the geostrophic wind speed on the other could be found. For an area containing about ten grid points selected from the region between Newfoundland and the British Isles and at the lower part of the grid the standard deviation is slightly larger than 1 m/s, originating from differences between the opinion of 26 analysts about the most probable solution of the analysis problem. Notice, however, that the analysis was rather simple in this case.

Final discussion

Part of the discussion in this paper has been devoted to the question of the variability of the height of the 500 mb surface as originating from the personal factor in upper air analysis. Furthermore, the influence of this variability on certain verification parameters has been investigated. It must not be forgotten, however, that the height values as given by the radiosoundings have errors too. Take e.g. a grid point near a weather ship. This point may have a standard deviation (σ_h) of 6 gdm. The standard deviation for the radiosounding itself is about 20 gdm. We notice that the

Table VI. Geostrophic wind speed for certain selected grid points (for coordinates see fig. 1). Values of maximum, minimum, mean and range are given for 26 analyses. Values of σ_g, R, σ_h and range of the height of the 500 mb surface as calculated from the 26 analyses are also given. Wind speed in m/s

Coord														
$x =$	13	12	13	17	10	7	9	2	7	2	11	13	10	4
$y =$	5	10	13	13	19	21	22	5	3	13	13	20	25	27
max	16.8	24.3	39.0	24.0	23.8	26.7	14.8	29.3	33.1	9.0	33.7	48.8	28.7	14.4
min	10.8	13.5	29.1	16.3	17.6	22.8	9.0	13.1	21.9	0.8	26.8	33.3	19.5	4.4
mean	14.0	17.3	32.7	19.7	20.0	24.8	12.1	20.7	28.5	3.7	30.3	41.5	23.6	8.3
range	6.0	10.8	9.9	7.7	6.2	3.9	5.8	16.2	11.2	8.2	6.9	15.5	9.2	10.0
routine	10.3	18.9	35.3	16.9	16.4	23.6	10.2	14.0	22.0	3.3	30.5	37.7	24.7	2.3
author	15.6	18.9	28.7	22.6	20.3	24.5	10.7	21.6	26.2	2.0	29.5	38.1	27.8	5.9
NA	15.1	17.5	31.2	19.5	19.2	24.7	14.8	19.2	26.3	4.0	28.6	35.9	24.0	8.3
σ_g	1.8	2.8	3.0	2.0	1.5	1.2	1.5	4.0	2.8	2.0	2.2	3.2	2.5	2.3
R	5	12	5	5	4	4	2	4	32	46	25	20	30	5
σ_h	2.7	6.8	6.2	5.8	3.4	4.1	4.7	43.8	36.4	43.1	25.9	24.6	28.2	32.0
$(\text{range})_h$	20	35	35	30	10	15	15	135	125	185	100	70	115	100

variability due to the personal effect in the analysis gives a standard deviation that is not negligible compared to the standard deviation of the individual sounding. If there is any doubt as to the reliability of the height value given by a sounding the standard deviation (σ_h) may be as large as 45 gdm. These additional errors in the analysis must at times have a serious repercussion on a numerrical forecast based on that analysis. This effect has been discussed by NEWTON (1954) who gives a drastic example of what may happen if a reanalysis is made. BEST (1956) has discussed for three cases the differences between numerical forecasts made from different analyses. It would have been very interesting to make 29 different forecasts based on the 29 analyses discussed above. Unfortunately, this is not possible because of the great cost involved. The author hopes, however, to be able to make a numerical forecast from a small selection of the analyses and to report the result in a later paper.

The error in the determination of the wind is rather small. A reasonable value for radar wind and radio wind equipments seems to be of the order 2 m/s at 500 mb. It is not likely that this error has any noticeable influence on the analysis of the gradients, and the differences in the 26 analyses discussed in this paper may safely be said to be rather independent of the error in the wind observations. The same goes for the variability of the wind due to large eddies. Furthermore, the winds are plotted rounded off to the nearest five knots.

When using the isohypse field to estimate the geostrophic wind as is done in the forecast technique used by the group at the University at Stockholm, some errors creep into the picture. Firstly, the standard deviation of the radiosounding introduces an error in the geostrophic wind field. This effect has been discussed among others by HOVMÖLLER (1952). The standard deviation arising from the personal factor in chart analysis has a similar effect. Let us assume that we want to estimate the geostrophic wind with the aid of four grid points, e.g. over a distance of about 600 km. When we want to estimate the geostrophic wind at a point A (see fig. 1) the grid points B—D and C—E, respectively, are used simultaneously. The instrument standard deviation (σ_i) is supposed to be 20 gdm. The standard deviation of the geostrophic wind (σ_g) is

$$\sigma_g = \frac{g\sigma}{f \cdot \Delta n}$$

where g is the acceleration of gravity, f the coriolis parameter $\simeq 10^{-4}$ sec^{-1} and Δn the distance between grid points $\simeq$ 300 km. The value of σ is determined by the formula $\sigma^2 = \sigma_i^2 + \sigma_d^2$ provided that the instrument error (σ_i) and the standard deviation (σ_d) of the difference between the height values at the grid points B—D and C—E, respectively, are uncorrelated. This is generally not the case. If a radiosounding shows a value that differs markedly from its neighbouring stations, then different analysts have to make an individual interpretation of the height value, thus creating an increased value of σ_h. The effect ought to be more marked when the error is large. The value used here (20 gdm) is a moderate one and for that reason the simple equation may be used in order to make it possible to arrive at a reasonable estimate of σ_g.

The value of σ_d is a function of σ_h at the grid points and of the correlation coefficient between the height values. Giving σ_d the values 0, 15, and 30 gdm, respectively, we obtain the σ_g-values 6.5, 8, and 12 m/s, respectively. A large value of σ_d has the same effect as the instrument error, thus increasing the uncertainty of the geostrophic wind considerably.

The value of σ_d is as already mentioned a function of the standard deviation of the height values at the grid points (σ_h). On the other hand a definite relationship exists between R and σ_h. With a large material at hand it would be possible to arrive at representative values of R, σ_h and the correlation coefficient between the height values at the grid points used for the calculation of the geostrophic wind. Then, the value of σ_g may be determined as a function of R. This is interesting to note because it gives us a possibility to determine that value of R that corresponds to a predetermined value of σ_g. In other words, by making a series of investigations along the lines indicated here one might be able to determine the radiosonde station network necessary to make a certain predetermined accuracy of the analysis possible.

The orientation of the grid may have an

effect of the calculation, but nothing is known by the author about the magnitude of the variation thus introduced.

Finally the difference method introduces an error. Instead of calculating derivatives, we are calculating ratios of finite differences. The result is a smoothing of the field. In the numerical forecast techniques as used for the time being this effect is mostly a wanted one. As a matter of fact the calculations quite often start with a smoothing of the initial field. It must be emphasized, however, that not enough is known about the error introduced by using finite differences and about the exact repercussion of it on the numerical calculations.

Taking all the above effects into account it is obvious that the geostrophic approximation used for forecasting purposes is affected by a rather large total standard deviation.

Not much is known of the overall effect of this error. One way to solve this question is to investigate a number of well-defined streamline patterns and using both a large number of analysts to get conventional analysis and a high-speed computer to calculate a variety of parameters. In this paper, however, only the error of the estimation of the geostrophic wind field has been discussed. In a later paper the variability of the vorticity and the advection of vorticity will be discussed to some extent. The much more difficult question of the error in the geostrophic approximation itself has still to be solved.

Acknowledgement

Thanks are due to Dr A. Nyberg, Director of the Swedish Met. and Hydr. Institute, Stockholm, and to Dr B. Bolin, University of Stockholm, for the great interest they have shown in this work. The author is also indebted to Mr B. Döös, University of Stockholm, and to Mr T. Duvedal, SMHI, Stockholm, for their very great help with the calculations made on the high-speed computer (BESK).

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