Electric Charge Separation in Subfreezing Cumuli

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(Manuscript received November 22, 1956)

Abstract

It is shown that charges may be separated in large cumuli as a result of negative charging of growing ice particles by collision with supercooled droplets. The magnitude of the effect has been calculated and it has been shown that charges comparable to those involved in thunderstorms can develop, providing the water content and updraught velocity in a cloud are high. The hypotheses lead to conclusions which agree, qualitatively and quantitatively, with the known physical characteristics of thunderclouds.

1. Introduction

The generation of the large electric charges in thunderstorm clouds has been the subject of many theoretical and experimental investigations. Although several theories have been advanced to explain thunderstorm charge-generation processes, most of these theories were expressed only in qualitative form and all seem unsatisfactory when examined quantitatively. In a recent review MASON (1953a) has outlined and discussed the classicial theories of charge generation and has also described the main physical features of a thunderstorm as deduced from the observations of Byers, Braham, Chapman, Gunn, Kuettner, Malan, Reynolds, Schonland, Simpson, Wilson, Workman, and others. The more important of these features are as follows.

(1) The average electrical moment destroyed in a lightning flash is about 110 coulomb kilometres and the electrical charge involved is 20 to 30 coulombs.

(2) Lightning flashes occur at average intervals of about 20 seconds, so that the current dissipated and the charging current are of the order of 1 amp.

(3) A thundercloud has a bipolar electrical structure, the upper positive charge being centred in the vicinity of the $-30°$ C isotherm, and the lower negative charge centred in the vicinity of the $-8°$ C isotherm. A subsidiary and smaller charge (about 4 coulombs) may also be found below the 0° C isotherm.

(4) The first lightning flash appears within 12 to 20 minutes of the appearance of precipitation particles large enough to be detected by radar.

(5) It is probably necessary for the cloud base to be warmer than 0° C and for the top of the radar echo from the cloud to approach the $-30°$ C level.

(6) An incipient thundercloud is characterized throughout its volume by updraughts averaging 7 m sec^{-1} but values of 15 m sec^{-1} have been frequently found.

(7) The average duration of precipitation and electrical activity is about 30 minutes.

In a subsequent paper MASON (1953b) outlined a theory for charge generation associated with graupel formation, and showed that charges of the order of 500 coulombs could be separated during the later stages of growth of large ice particles, when the particle radii approached 1 cm. Although the mechanism described may undoubtedly give rise to large charges it does not seem sufficient for the following reasons to account for most occurrences of thunderstorms.

(1) As Mason himself points out, hailstones of radius 1 cm or more are probably not a normal feature of thunderstorms.

(2) The volume distribution of particles computed by Mason, and their associated charges, must give rise to very high values for precipitation intensity and precipitation current density – of order 2 mm $\sec^{-1}$ and 2×10^{-10} amp cm^{-2}, respectively. Both values appear excessive: a precipitation rate of 100 mm per hour is unusual, while CHALMERS and LITTLE (1939) have reported one occurrence of a precipitation current of 7×10^{-12} amp cm^{-2} during a hailstorm as a very unusual event.

In the present paper a different approach has been adopted, based on actual observations of natural cloud element electrification. The charge-separation process postulated is similar to that assumed by Mason, but it will be shown that large charges can be separated without the formation of giant hailstones.

2. The mechanism of electrification

Experiments by FINDEISEN (1940), WORKMAN and REYNOLDS (1948) and several other workers show that the freezing of supercooled water is accompanied by electric charge separation. It now seems certain that the impingement and freezing of supercooled droplets on an ice surface results in a negative charge being acquired by the collecting surface. The balancing positive charge is presumably carried away, either by the surrounding air or by minute droplets. From experiments in natural supercooled cloud, LUEDER (1951) concluded that the charge received by the collecting ice surface increased in direct proportion to the volume of supercooled water collected until it reached a limiting value determined by the surface potential gradient. The charge was given by Lueder as -0.2 coulombs (-6×10^8 e.s.u.) per cm^3 of supercooled water collected, and the limiting surface potential gradient was approximately 700 volts cm^{-1}.

Experimental measurements of cloud element electrification carried out on a mountain top (TWOMEY, 1956) showed that cloud droplets were normally positive, the mean charge being approximately 10^{-9} d^2 e.s.u. (where d was the droplet diameter in microns). When ice crystals were present, however, many of the cloud elements carried negative charges, and it was found that the surface potential gradient, calculated on the assumption that the particles were spherical, appeared to have an upper limit close to 700 volts cm^{-1}. This suggested that the negative charges arose as a result of collisions between ice particles and supercooled cloud droplets and that charging ceased when the surface potential gradient attained a value near 700 V cm^{-1}. It should be noted that, on the basis of a charge of 0.2 coulombs (6×10^8 e.s.u.) per cm^3, the collection of a supercooled droplet of radius 1 micron is more than sufficient to charge a 300 micron particle to the limiting charge determined by the restriction of the surface potential gradient to 700 V cm^{-1}.

3. Assumptions

In order to compute the charge separation which may occur in a cumulus cloud by the mechanism described, the following assumptions have been made.

(1) In an ascending parcel of air, the number of ice crystals which have appeared in the parcel when it has been cooled to a temperature $-T°$ C is the same as the number of freezing nuclei active at that temperature.

(2) The freezing nucleus spectrum is such that a tenfold increase occurs in the number of effective nuclei for each temperature increment of $-4°$ C. This assumption gives a spectrum close to experimentally observed values (BIGG, private communication). Hence if the lapse rate in a cloud is $\gamma°$ C cm^{-1}, the number of nuclei active at or below a height z above the 0° C isotherm is

$$N(z) = A e^{\alpha Z} \qquad (1)$$

where $\alpha = 0.25\ \gamma \ln 10$. The number A is a function of the concentration of freezing nuclei. On the average about 1 nucleus per litre is found at $-20°$ C; under such conditions A has the value 10^{-8}. However, although the shape of the freezing nucleus spectrum does not vary greatly, concentrations often vary enormously from day to day. Hence A may take values ranging from about 10^{-7} to 10^{-9},

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and occasionally may be even as low as 10^{-10} or as high as 10^{-6}.

(3) The vapour pressure in that part of the cloud between the 0° C isotherm and the level at which complete glaciation of the cloud occurs (i.e. above which the liquid phase is absent) is the equilibrium vapour pressure above water. The supersaturation with respect to ice at level z, expressed in degrees C, is then approximately $T/10$ at temperature $-T$° C giving for the supersaturation

$$S=\gamma \frac{Z}{10}\ {}^\circ C \tag{2}$$

(4) A simplified equation for the growth of spherical ice particles by sublimation has been derived from that given by SQUIRES (1952, p. 64) for the growth of cloud droplets by condensation. The equation used is as follows:

$$r\frac{dr}{dt}=4\times 10^{-8}S \tag{3}$$

(r = radius of spherical ice particle in cm, t = time in seconds, S = supersaturation in degrees C with respect to ice).

(5) The particles are assumed to obey Stokes' Law, so that the free fall velocity is

$$u=kr^2 \text{ cm sec}^{-1} \tag{4}$$

where k is approximately 1.2×10^6 cm^{-1} sec^{-1}. Temperature variations of viscosity and density have been neglected.

(6) A uniform updraught of V cm sec^{-1} and a uniform lapse rate $\gamma=5\times 10^{-5}$ ° C cm^{-1} is taken to exist throughout the cloud.

(7) The collision of a supercooled droplet of volume v cm^3 with an ice particle is capable of imparting a charge of $6\ v\times 10^8$ e.s.u. to the particle, but the charge which can be imparted to a particle of radius r has a maximum value of $7/3\ r^2$ e.s.u., this being the charge which produces a surface potential gradient of $7/3$ e.s.u. (700 V cm^{-1}).

(8) Charging ceases when no supercooled droplets are available or when the ice particle melts or acquires a covering of unfrozen water.

Although the above assumptions represent a much simpler model than is found in nature, they do not seem likely to introduce any gross errors in the final deductions.

4. The growth of ice particles

Ice particles grow in cloud by two distinct mechanisms: firstly by sublimation (i.e. diffusion of water vapour from the supersaturated environment) and secondly, by collection of water droplets and ice particles. From equation (2) the rate of increase of volume by sublimation is seen to be given by

$$\left(\frac{dv}{dt}\right)_s=16\pi\times 10^{-8}Sr.$$

Growth by collection of smaller ice particles and water droplets is given approximately by

$$\left(\frac{dv}{dt}\right)=\pi kEw_c r^4.$$

(w_c = water content of cloud, including liquid and solid phases, and
E = collection efficiency)

Hence

$$\frac{\left(\frac{dv}{dt}\right)_c}{\left(\frac{dv}{dt}\right)_s}=\frac{kEw_c r^3}{16\times 10^{-8}S}.$$

Substituting values for typical cloud conditions with w_c = 1 g m^{-3} and S = 2° C this ratio is found to become unity when the radius approaches 100 μ. Smaller particles grow mainly by sublimation, while larger particles grow mainly by accretion.

If accretion is neglected, the equations of motion for an ice particle above the 0° C isotherm can readily be solved. By combining equations (2), (3) and (4), one obtains

$$\frac{d^2z}{dt^2}=-\omega^2 z$$

where $\omega=(8k\times 10^{-9}\gamma)^{1/2}\approx 7\times 10^{-4}$ sec^{-1}. The solution of this equation is

$$\left.\begin{aligned} &z=Z\sin(\omega t+\varphi)\\ &\text{from which}\\ &r=k^{-1/2}\{V-\omega Z\cos(\omega t+\varphi)\}^{1/2}\end{aligned}\right\}. \tag{5}$$

If the ice particle came into existence at a height ζ at time $t = 0$, the constants become

$$\varphi = \tan^{-1}\frac{\omega\zeta}{V}$$

$$Z = \zeta \operatorname{cosec} \varphi.$$

(Here, and everywhere in the subsequent discussion, height is measured from the 0° C isotherm.) Equation (5) describes the trajectory and time dependence of radius for an ice particle growing by diffusion alone; it is not, of course, applicable for negative values of z or ζ.

The probability of a particle undergoing a collision with a cloud droplet between t and $t + \Delta t$ is readily found to be

$$p(t)\,\Delta t = \pi k E N r^4\,\Delta t$$

where N is the number of cloud droplets per unit volume and E the corresponding collection efficiency. Hence the probability of a particle originating at time $t = 0$ undergoing at least one collision within a time t is given by

$$P(t) = 1 - e^{-\pi k \int_0^t E N r^4 dt} \tag{6}$$

The quantity $P(t)$ represents that fraction of the total number of particles originating at time $t = 0$ which have undergone one or more collisions at time t. The exponent in the expression for P(t) is a function of the height at which the particle originated and also of the supersaturation and other cloud parameters. However when $P(t)$ was evaluated for several sets of values of the various parameters it was found that the time τ which gave $P(t) = 0.5$ did not vary greatly, ranging from about 50 to 70 seconds; in this time the particle grew to about 30 microns radius. The calculations were made assuming growth from zero radius at $t = 0$.

5. The collection of charge by growing ice particles

As indicated earlier, the assumptions made concerning the charging mechanism imply that the collection of a single cloud droplet is capable of imparting the maximum charge to all but the largest ice particles. Hence when a growing particle experiences the first collision with a cloud droplet (which occurs in the average some 60 seconds after its formation) it will acquire the maximum charge, which is a function of particle radius. Thereafter frequent collisions will occur, which are potentially capable of separating relatively large charges; the actual charge imparted, however, is limmited by the restriction that the charge carried by a particle of radius r shall not exceed $7/3\ r^2$ e.s.u. Consequently once a particle has undergone its first collision the charge carried will be determined by the particle radius. Hence for $r > 30\ \mu$, the charge is taken as

$$q \approx -2.33\, r^2 \text{ e.s.u.} \tag{7a}$$

and

$$\frac{dq}{dt} = -4.66 \times r\frac{dr}{dt}. \tag{7b}$$

If a rising parcel of cloud air of unit volume at level z is considered, $A\,e^{\alpha z}$ ice crystals have formed during the ascent from the 0° C isotherm; the particles originating at lower levels have grown and fallen from the parcel, but have been replaced by an approximately equal number falling into the parcel from above. Thus there are approximately $A\,e^{\alpha z}$ ice particles per unit volume at level z and those particles which have been in existence for a time τ or more have undergone several collisions with supercooled droplets and therefore carry charges the magnitude of which are given by equation (7a). The number of such particles is clearly the total number of particles formed when the rising air parcel was at a height $z - \tau V$. The charging rate for such a particle is given by equation (7b). Combining these results with equations (2) and (3), one finds that the total charging rate per unit volume of ascending cloud air at level z is given by

$$\frac{dQ}{dt} = A e^{\alpha(z-\tau V)} \times 4.66\,(4 \times 10^{-9}\gamma)\, z$$
$$\approx 2 \times 10^{-8} \gamma A z\, e^{\alpha(z-\tau V)}.$$

But the parcel is ascending at velocity V, hence

$$Q = 2 \times 10^{-8}\frac{\gamma A e^{-\alpha\tau V}}{V}\int_{\tau V}^{z} z e^{\alpha z} dz$$

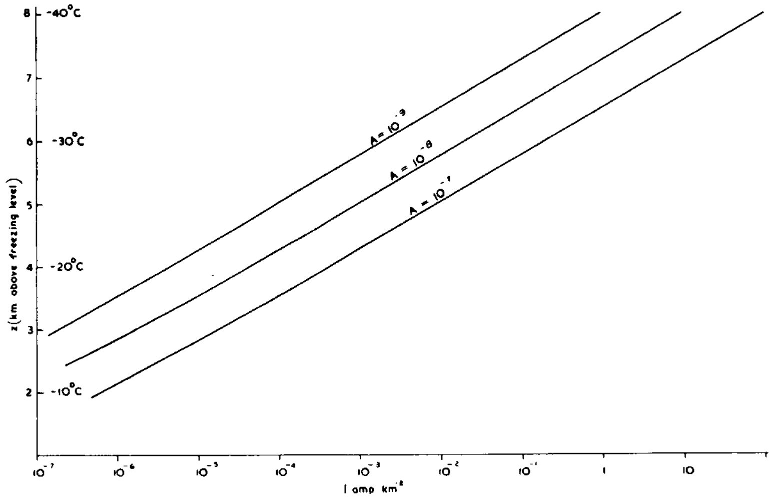

Fig. 1. Graph showing the variation of the function I (z) with height (z). I(z) represents the current flowing at or below the glaciation level z_M; for $z < z_M$ the current is constant and equal to $I(z_M)$.

and

$$Q(z) \approx 2 \times 10^{-8} \frac{\gamma A}{V\alpha^2} (\alpha z e^{\alpha z} - e^{\alpha z})$$

providing $z \gg \tau V$.

Therefore the current per unit area being carried by the cloud air at level z is found to be

$$i(z) = 2 \times 10^{-8} \gamma A \alpha^{-2} (\alpha z e^{\alpha z} - e^{\alpha z}) \text{ e.s.u. cm}^{-2}. \quad (8)$$

This function converted to amps km^{-2} has been plotted in fig. 1. If at some level z_M charging of the ice particles ceases, the charged ice particles rapidly fall out and the ascending parcel of air is left with a positive charge which is carried aloft and is equivalent to a current $i(z_M)$. It is clear from equation (8) that this current depends critically on z_M, and accordingly it is necessary to determine the value of z_M from the initial assumptions.

6. Evulation of the glaciation level

On the basis of the charging mechanism which is being considered, charge separation in a rising parcel of cloud air will cease when liquid droplets are no longer present, i.e. when complete glaciation of the cloud occurs. Supercooled water droplets have been observed in cumulus clouds at temperatures as low as $-40°$ C, while complete glaciation has occasionally been reported at temperatures as high as $-15°$ C. It may be said that the temperature of complete glaciation ranges from $-15°$ C to $-40°$ C, and is most frequently around $-30°$ C. However, it does not seem desirable to use these experimental data to determine what value of z is to be substituted in equation (8) to obtain the maximum current density; it is preferable to calculate this level on the basis of the assumptions which led to equation (8). This approach has a twofold advantage in that it permits the evaluation to be completed in a self-contained form and also in that the results of a mathematical evaluation of the glaciation level can be compared with the results of observations, thus providing an independent test of the validity of the assumptions made.

In order to determine the glaciation level, it is necessary to compute the total mass of ice per unit volume in a rising parcel of cloud air; complete glaciation is taken to occur at the level at which the mass of ice per unit volume is equal to the mass of available water per unit volume.

Substitution of $dm = 4\pi r^2 dr$

and $dz = V dt$

in equations (2) and (3) gives, for an ice

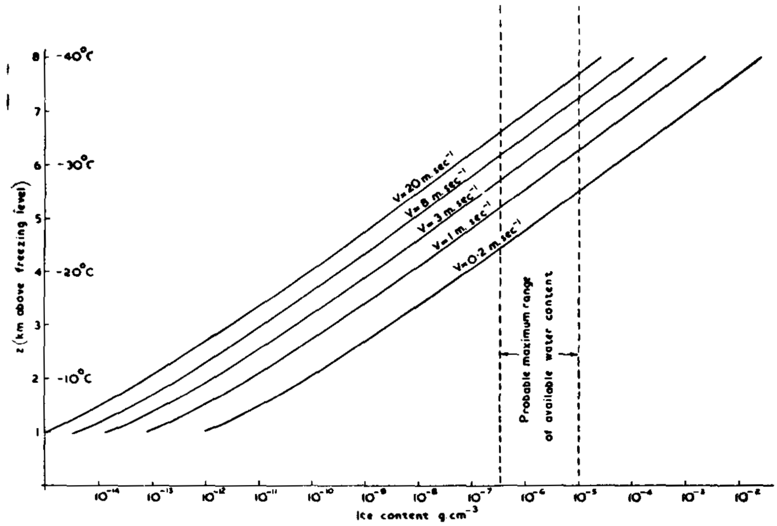

Fig. 2. Graph showing variation of computed ice content with height (z) for values of updraught velocity V from 0.2 m sec^{-1} to 20 m sec^{-1}, and $A = 10^{-8}$ (i.e. 1 freezing nucleus per litre active at — 20° C).

particle originating at a height ζ and growing from zero radius,

$$dm = (16\pi\, 10^{-9}\gamma\, V^{-1})\, rz\, dz$$

Hence

$$m = (16\pi \times 10^{-9}\gamma V^{-1}) \int_{\zeta}^{z} rz\, dz.$$

Also

$$r dr = 4 \times 10^{-9}\gamma\, V^{-1} z\, dz,$$

whence

$$r^2 = 4 \times 10^{-9}\gamma\, V^{-1}(z^2 - \zeta^2)$$

and

$$\int_{\zeta}^{z} rzdz = (4 \times 10^{-9}\gamma V^{-1})^{1/2} \int_{\zeta}^{z} z(z^2 - \zeta^2)^{1/2} dz$$
$$= \frac{1}{3}(4 \times 10^{-9}\gamma V^{-1})^{1/2}(z^2 - \zeta^2)^{3/2}.$$

Thus the mass at a height z of a particle which originated with zero radius at level ζ is given by

$$m = \frac{32\pi}{3}\left(\frac{10^{-9}\gamma}{V}\right)^{3/2}(z^2 - \zeta^2)^{3/2}.$$

But the number of particles originating per unit volume between ζ and $\zeta + d\zeta$ is $A\alpha e^{\alpha\zeta} d\zeta$, from which one finds the total mass per unit volume at level z to be

$$\mu(z) = \frac{32\pi}{3}\left(\frac{10^{-9}\gamma}{V}\right)^{3/2} A\alpha \int_{0}^{z} e^{\alpha\zeta}(z^2 - \zeta^2)^{3/2} d\zeta. \quad (9)$$

The integral here is to be evaluated with z held constant and ζ as the variable of integration. If this integral is denoted by $I(z)$,

$$I(z) = \int_0^z e^{\alpha\zeta}(z^2 - \zeta^2)^{3/2} d\zeta$$
$$= \frac{z^4}{2} \int_0^1 e^{\alpha z x^{1/2}} x^{-1/2}(1 - x)^{3/2} dx$$
$$= \frac{z^4}{2} \sum_{r=0}^{r=\infty} \left\{ \frac{(\alpha z)^r}{r!} \int_0^1 x^{1/2(r-1)}(1 - x)^{3/2} dx \right\}$$
$$= \frac{z^4}{2} \sum_{r=0}^{r=\infty} \frac{(\alpha z)^r}{r!} B\left(\frac{r+1}{2}, \frac{5}{2}\right),$$

where $x = \frac{\zeta^2}{z^2}$ and $B(p, q)$ is the complete Beta-function $\int_0^1 y^{p-1}(1 - y)^{q-1} dy$. When this result is substituted in equation (9), one obtains

$$\mu(z) = \frac{16\pi}{3}\left(\frac{10^{-9}\gamma}{V}\right)^{3/2} A\alpha z^4 \sum_{r=0}^{r=\infty} \frac{(\alpha z)^r}{r!} B\left(\frac{r+1}{2}, \frac{5}{2}\right). \quad (10)$$

Using the last result, the mass of ice per unit volume as a function of height has been calculated; the variation of computed mass with height is shown in fig. 2. According to equation (10), the mass per unit volume of ice is in direct proportion to the concentration of freezing nuclei. Hence if the freezing nucleus spectrum is such that there are 10 nuclei per litre active at $-20°$ C, the masses given in fig. 2 should be multiplied by 10, while for 10^{-1} nuclei per litre active at $-20°$ C the masses should be divided by 10.

Under adiabatic conditions most of the water vapour in an ascending parcel of air condenses to liquid water in the first few km above the condensation level. Hence at higher levels in a cloud the amount of available water is determined mainly by the water vapour content (mixing ratio) of the air at cloud base. This in turn is determined primarily by the cloud base temperature; in warm moist air up to 12 g m^{-3} may be available and in cold dry air only 2 g m^{-3}. It has however been found that the measured liquid water content in cumulus clouds is only a fraction of the adiabatic value. The observations of WARNER (1955) showed that at the higher levels in cumuli about 3 km deep the liquid water content was about one-fifth of the adiabatic value. If this holds in the case of large cumuli, the available water in the upper parts of such clouds might better be taken as ranging from 0.4 g m^{-3} to 2.5 g m^{-3}.

The glaciation level has been computed for several values of V, A and available water content w_c, and the results are set out in table 1. A lapse rate of 5×10^{-5}° C cm^{-1} has been assumed here and in all other computations involving lapse rate.

From table 1 it is seen that the computed glaciation levels agree well with observational results; the table shows that the computed glaciation level may range from 3.75 km ($-18.75°$ C), for small updraught, low water content and high freezing nucleus concentration, to 8.4 km ($-42°$ C) for large updraught and water content and low freezing nucleus concentrations.

From equations (8) and (10) it follows that both current and ice content at any height are directly proportional to A (which determines freezing nucleus concentration), which implies that an increase in A results in an increase in current at any given height, but also results in a lowering of the glaciation level. The latter gives the value z_M which must be used to determine the maximum current i_M from equation (8), so that the maximum current does not vary appreciably with A, but the level at which it is attained is lowered significantly as A increases. For example, if the updraught and water content are taken as 3 m sec^{-1} and 2 g m^{-3} respectively, the glaciation level increases from 5.55 km for $A = 10^{-7}$ to 7.0 km for $A = 10^{-9}$, but the resulting change in i_M is insignificant – from 4.5×10^{-2} amp km^{-2} to 5×10^{-2} amp km^{-2}. The variation of liquid water content with height is very slow several km above cloud base and can be ignored. The above considerations imply that maximum charge separation can occur in smaller clouds if the freezing nucleus concentration is high.

Table 1. Computed values for glaciation level (km above 0° C isotherm

(a) $A = 10^{-8}$ cm^{-3} (1 freezing nucleus per litre effective at —20° C)

V (m sec^{-1})	w_c (g m^{-3})				
	0.4	1.0	2.0	4.0	10.0
0.2	4.45	4.75	4.95	5.2	5.45
1.0	5.2	5.5	5.75	5.95	6.25
3.0	5.75	6.05	6.25	6.5	6.75
8.0	6.2	6.5	6.7	6.95	7.25
20.0	6.6	6.95	7.15	6.4	7.7

(b) $A = 10^{-7}$ cm^{-3} (10 freezing nuclei per litre effective at —20° C)

V (m sec^{-1})	w (g m^{-3})				
	0.4	1.0	2.0	4.0	10.0
0.2	3.75	4.05	4.25	4.45	4.75
1.0	4.45	4.8	5.0	5.2	5.5
3.0	5.0	5.3	5.55	5.75	6.05
8.0	5.45	5.8	6.0	6.2	6.5
20.0	5.9	6.2	6.4	6.6	6.95

(c) $A = 10^{-9}$ cm^{-3} (0.1 freezing nucleus per litre effective at —20° C)

V (m sec^{-1})	w (g m^{-3})				
	0.4	1.0	2.0	4.0	10.0
5.2	5.2	5.45	5.7	5.9	6.2
1.0	5.95	6.25	6.45	6.7	7.0
3.0	6.5	6.75	7.0	7.2	7.5
8.0	6.95	7.25	7.45	7.65	7.95
20.0	6.4	7.7	7.9	8.1	8.4

Table 2. Current at glaciation level (amp km^{-2}) for several values of V and w_c

V (m sec^{-1})	w_c (g m^{-3})				
	0.4	1.0	2.0	4.0	10.0
0.2	0.00018	0.00045	0.00085	0.0018	0.004
1.0	0.0018	0.005	0.01	0.018	0.05
3.9	0.01	0.025	0.05	0.10	0.20
8.0	0.04	0.10	0.18	0.40	1.0
20.0	0.13	0.40	0.70	1.7	4.0

In table 2, the maximum current density has been tabulated for several values of updraught velocity V and available water content w_c. It will be noticed that current densities sufficient to provide the charging rate of about 1 amp required in thunderclouds are given by the higher values of updraught velocity and available water content. For example if the available water content is 4.0 g m^{-3}, a current of 1 amp is provided by an updraught of 3 m sec^{-1} extending over 10 km^2, or an updraught of about 15 m sec^{-1} over an area of 1 km^2.

7. Electrical moment separation

On the basis of the theory described, charge separation commences as soon as the ice crystals begin to form in a growing cumulus, but does not become significant until the top of the cloud is 5—6 km above the 0° C isotherm (i.e. until the cloud top temperature is −25° C to −30° C). Thereafter rapid charge separation occurs and within 10—20 minutes several thousands of coulombs may have been separated. Much of this charge, however, may be located in that part of the cloud where the positive charge on the air is largely balanced by the presence of negatively charged ice particles. It is readily seen that settling of the growing ice particles leads to a bipolar charge distribution of the correct sign, and the resulting electrical moment separation can be estimated by considering the falling velocity of the ice particles. In 10 minutes ice particles can grow by sublimation alone to a radius of order 100 μ, so that the falling velocity of the particles as given by Stokes' Law is of order 1.3 metres per second. If for example a total charge of 2,500 coulombs has been generated, one obtains for the moment separation a value of about 3.25 coulomb kilometres per second. Thus the 110 coulomb km destroyed in an average lightning flash is separated in about one half-minute.

8. The charge on precipitation and the distribution of negative charge

If the liquid droplets in the warmer parts of the cloud carry positive charges of the order of $4 \times 10^{-1}\, a^2$ e.s.u. (a = cloud droplet radius in cm), as was found in experimental observations, the negative charge acquired by a growing ice particle in the upper part of the cloud will tend to be neutralized when it settles into the lower parts of the cloud. Once an ice particle has melted or acquired a liquid covering (LUDLAM, 1950), the negative charging process will cease and the falling particle will commence to acquire a positive charge by collection of positive cloud droplets. Of for example an ice particle melts at radius r_0 while carrying the postulated maximum charge of $-2.33\, r_0^2$ e.s.u., and falls through a cloud containing 200 droplets per cm^3 of radius 10 μ, which carry positive charges of order 4×10^{-7} e.s.u., it can readily be shown that the charge originally carried by the ice particle will be neutralized in a fall of about 0.1 km.

It follows from the above considerations that the final charges on precipitation elements depend on the electrification of the cloud droplets near cloud base and bear little relation to the main charge-separation process. In fact either positive or negative charges may be found on precipitation. A more important consequence is that the many thousands of coulombs involved in thunderstorm processes need not be removed from the cloud itself by precipitation: if the negative particles become neutralized by collection of positively charged cloud droplets, the negative charge on the cloud air, which originally was balanced by positive charges carried by the cloud droplets, becomes "exposed" so that a negative charge is acquired by the cloud air near the 0° C isotherm. This negative charge will be carried up with the rising air, hence the negative charge centre should extend up from the 0° C isotherm. MALAN and SCHONLAND (1951 a, 1951 b) reported that the negative charge involved in a lightning flash to earth was contained in a vertical column up to 6 km deep and that successive strokes tapped higher and higher regions of the negative column. These observations are consistent with the negative charge distribution suggested.

9. Discussion

Before discussing the conclusions to which the development of the initial assumptions have led, it is desirable to refer to the effect of varying the parameters which have been treated as constants. It is found that the most critical assumptions are those involving the growth equations of ice particles and the freezing nucleus spectrum.

The numerical factor 4×10^{-8} in equation (3) is not in fact a true constant. It decreases with decreasing temperature (SQUIRES, loc. cit.), so that the growth rate at higher levels may have been overestimated. However, the departure of the particles from sphericity and the contribution of accretion to the growth rate would tend to have the opposite effect, and these factors also have not been taken into account. It is probable that the best test of the assumptions made concerning growth rate is obtained by comparing the computed glaciation levels with the results of observations. The close agreement found suggests that the growth equation adopted was a fair approximation.

The shape of the natural freezing nucleus spectrum is not constant, so that the quantity α is also likely to vary in nature. The assumed value of α was 2.88×10^{-5}, which corresponded to a logarithmic freezing nucleus-temperature spectrum such that the total number of effective nuclei increased by a factor of ten for a temperature decrement of 4° C. Experimental observations suggest that the temperature interval so designated may on occasions range from 2° C to 6° C, i.e. α may range from 5.75×10^{-5} to 1.92×10^{-5}. When the effect of varying the value of α is calculated, it is found that the maximum current $i(z_M)$ is almost independent of α, whereas the glaciation level (z_M) varies markedly with α. For example, table 1 shows that for w_c in the range $0.4 - 10.0$ g m^{-3} and V in the range $0.2 - 20$ m sec^{-1}, $\alpha = 2.88 \times 10^{-5}$ and $A = 10^{-8}$, the computed glaciation level ranged from 4.45 to 7.7 km. If the above extreme values of α are employed, one finds the range of z_M to be $2.6 - 4.2$ km for $\alpha = 5.75 \times 10^{-5}$ and $6.2 - 10.8$ km for $\alpha = 1.92 \times 10^{-5}$. It is seen that the computed values for the glaciation level are still feasible, while the maximum currents are not appreciably altered. It follows that the conclusions which have been reached are not drastically modified by altering the value of α, within the bounds of its observed variations.

On the basis of the initial assumptions, which were largely based on experimental evidence, a model has been set up for a thunderstorm cell. This has been shown to lead to certain conclusions, of which the more important are repeated below.

(1) In a large cumulus extending well above freezing level, large electric charges can be separated as a result of the collection of supercooled droplets by growing ice particles.

(2) The resulting charge distribution is such that a positive charge is accumulated in the upper part of the cloud, with a lower negative charge extending from just below the 0° C isotherm to several km above.

(3) The upper charge in average conditions will mainly be centred 6 km to 7 km above freezing level (i.e. between the −30° C and the −35° C levels).

(4) The rate of charge separation increases rapidly with increasing height above freezing level but ceases when the glaciation level is reached. The current therefore attains a maximum just below glaciation level.

(5) The maximum current density increases with increasing updraught velocity and water content, the computed values ranging from 0.0018 amp km^{-2} for an updraught of 0.2 metres per second and a water content of 0.4 g m^{-3} to 4 amp km^{-2} for an updraught of 20 metres per second and a water content of 10 g m^{-3}. This dependence on updraught and water content implies that vigorous cumuli, the bases of which extend well below the 0° C isotherm, are the most likely to become thunderclouds.

(6) The level at which current reaches its maximum value depends on the freezing nucleus concentration (assuming a one-to-one correspondence between freezing nuclei and ice crystals). The cloud must penetrate to this level to achieve maximum electrical activity; this critical level becomes lower as freezing

nucleus concentration increases, being lower by about 1.5 km for a hundredfold increase in nucleus concentration. This suggests that in otherwise similar conditions thunderstorms are more likely to occur when the freezing nucleus concentration is high.

(7) The level to which a cloud must penetrate in order to realize its maximum electrical activity is under average conditions, 6—7 km ($-30°$ C to $-35°$ C).

Despite the simplifications made in developing the theory, it is seen that the above conclusions agree well with the main known physical features of a thunderstorm as given by MASON (1953 a) and summarized in the introductory paragraph. The close agreement between the computed glaciation level and the results of observations suggest that the assumptions made were not grossly in error. The suggested dependence of minimum cloud top height for maximum electrical activity on freezing nucleus concentration, and the consequent implication that thunderstorms are more likely when freezing nucleus concentrations are high, may provide a further test of the validity of the hypotheses which have been made.

10. Conclusions

It seems likely that charge separation during the collection of supercooled water droplets by growing ice particles plays an important role in thunderstorm charge generation. It has been shown that current densities of order 1 amp km^{-2} can be produced as a result of the charging of ice particles in this fashion and that the conditions necessary for the generation of large charges are similar to those which have been deduced from the known properties of thunderstorms.

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