NOTES

Note on the Self-Sustained Oscillations of a Simple Thermal System.

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Some years ago the author carried out some convection experiments in which a U-shaped narrow tube filled with water was heated from below and cooled from above. At one occasion, when the ends of the tube were connected with two open water reservoirs (Fig. 1), it was observed that the free surfaces of the reservoirs did raise and lower in a periodic manner, indicating an oscillatory flow through the tube. Repeating the experiment under varying conditions, it was found that the period of this oscillation did change strongly with the dimensions of the tube and the area of the free surfaces. It was also noted that the oscillation did occur only when the area of the free surface surpassed a certain critical value, which depended on the intensity of the heating and cooling. Only by changing the cross-section area of the tube the period of the oscillation could be made to vary from about 20 second to more than 4 hours. In all cases the period was much larger than the corresponding period for free gravity oscillations. Record of a typical oscillation is seen in Fig. 2.

This phenomenon, which does not seem to have described earlier, was recognized as a self-sustained, non-linear oscillation of the kind described in text-books on non-linear mechanics (MINORSKY, 1947). A qualitative explanation of the oscillations is easily given. Suppose that the water is initially at rest and that then, due to the action of some outer disturbance, a small amount of water is displaced, say, from the left to the right shank of the tube. The left shank then gets a small excess of colder water, the right shank a small excess of warmer water, and a buoyancy force appears which drives the flow from the left

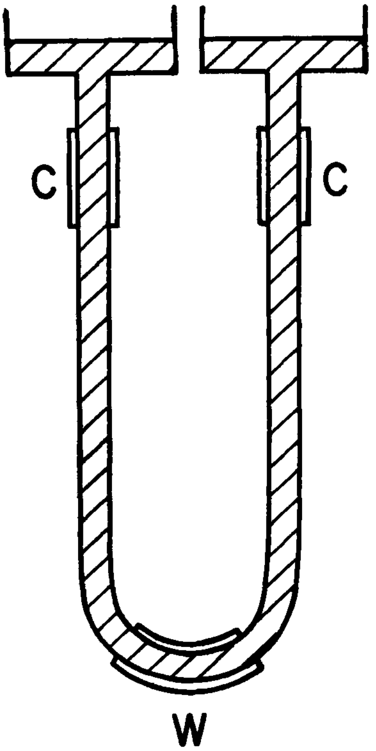

Fig. 1. Scetch of the oscillating system.

to the right shank. As time goes on a difference in level between the reservoirs is built up and slows down the flow. Finally, the buoyancy force and the pressure gradient due to this difference in level balance, and the flow stops. This, however, is not an equilibrium state. When the motion stops, the heating and cooling tend to create a symmetric temperature distribution, thereby reducing the buoyance force, and the flow starts in the opposite direction. On the other hand, when the difference in level is zero the exists still a buoyancy force which sustains the flow, and the motion proceeds periodically.

It can be verified theoretically that no oscillations could occur unless the area of the free surfaces of the reservoirs is sufficiently large. Suppose that due to the initial disturbance a volume dV of water has been displaced from the left to the right shank. If the maximum density difference created by the heating and cooling is $\Delta\varrho$, the difference in pressure between the two shanks due to this displacement cannot exceed $\frac{2g\Delta\varrho dv}{a}$, where a is the cross-section area of the tube and g is the acceleration of gravity. On the other hand, the volume transport creates a difference in level between the reservoirs $\frac{2dv}{A}$, where A is the area of the free surface in one of the reservoirs, and this gives an opposing pressure of magnitude $\frac{2g\varrho dv}{A}$. If oscillations should occur, it is thus necessary that $\frac{a}{A} < \frac{\Delta\varrho}{\varrho}$, or $\frac{a}{A} < 2.\ 10^{-4}\,\Delta\vartheta$, if one introduces the coefficient of thermal expansion for water (at 20°C). If, as example, $\Delta\vartheta = 50°$ one must have $A > 100$ a.

Theoretical expressions may also be derived

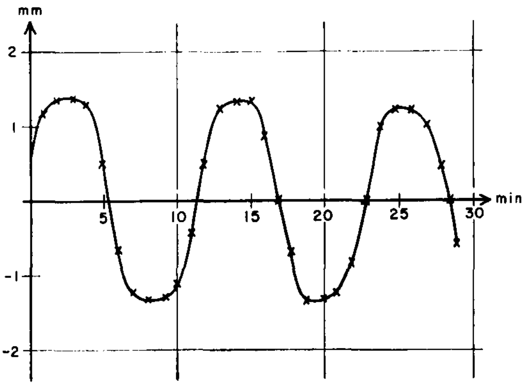

Fig. 2. Typical record of an oscillation. $a = 0.48$ cm² $A = 124$ cm², $\Delta\vartheta \approx 30°$ C.

for the period T of fully developed oscillations. For instance, in the case of large reservoirs and laminar flow through the tube one finds $T = k\,\frac{\nu l A}{g a^4}$ where ν is the kinematic viscosity of water, l is the length of the tube, a and A are the areas defined earlier, and k is a pure number of the order of 1. The observed values of T were found to agree reasonably with the above theoretical result. Some of the theoretical discussions may be presented in a later contribution.

The described experiment may not have any direct application to natural systems since it is an artificially constructed laboratory experiment. Nevertheless, the general information we might collect from this and similar experiments on self-sustained thermal oscillations may provide a base for better understanding of some of the important oscillatory phenomena in the thermal cycles of the earths fluid envelopes, about which we up to now know very little.

REFERENCES

MINORSKY, N., 1947: Non Linear Mechanics, Ann Arbor.