Upper-Air Analysis over Ocean Areas[1]

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Abstract

From the results of a test of numerical analyses of 500-mb charts over an ocean area it is concluded that the number of ocean station vessels is a minimum and that supplementary upper air data is required to ensure continuously reliable upper air analyses. A method of determining the mean lapse rate by interpolation is presented by means of which heights of upper pressure surfaces can be extrapolated from observed values of surface pressure and temperature. An attempt is made to determine statistically the true mean error in the numerical analysis of upper-air charts.

Introduction

Internal consistency in analyses of the many upper-level charts has always been the aim of the analyst but the recent development in multi-level models for numerical forecasting has made this a definite requirement. In the past decade the density of radiosonde observations has been greatly increased but the density of upper-air stations over the ocean areas is still insufficient to ensure a continuously reliable analysis of the upper atmosphere which is internally consistent. The importance of the weather ships in the North Atlantic has been repeatedly shown by the differences in conventional analyses when ship reports were missing. The numerical analyses, as developed by BERGTHORSSON and DÖÖS 1955, seemed to be another good basis for measuring the adequacy of the upper-air data over the ocean area and at the same time furnish a check on the reliability of such analyses. Figure 1 shows a series of numerical analyses of the 500-mb level for 1500 GMT, 16 April 1956. The solid contours denote the analysis computed with all available radiosonde data including the eight weather ship reports. The analyses resulting from the removal of one weather ship are shown by the dashed contours. Figure 1a was made without weather ship C (52.7 N-35.5 W), figure 1b without weather ship D (44.0 N-41.0 W), figure 1c without weather ship E (35,0 N-48.0 W) and figure 1d without weather ship K (45.0 N-16.0 W). The removal of the remaining weather ships influenced the analysis to a much less degree and therefore are not shown here. Whereas the conventional analyses might not show such radical differences by the removal of weather ship reports, the importance of these ocean station vessels is clearly emphasized in the above numerical analyses.

Since the autumn of 1955 the Royal Swedish Air Force has used numerical analyses in conjunction with a program in routine computations of barotropic forecasts. As a measure of the reliability of these analyses over an area of the North Atlantic Ocean a comparison was made between the numerical analyses and the conventional analyses of the Swedish Weather Bureau covering a period of five months. The average of the root mean square of the differences (RMS) between the heights of the 500-mb surface of the two analyses at corresponding gridpoints (one grid unit =

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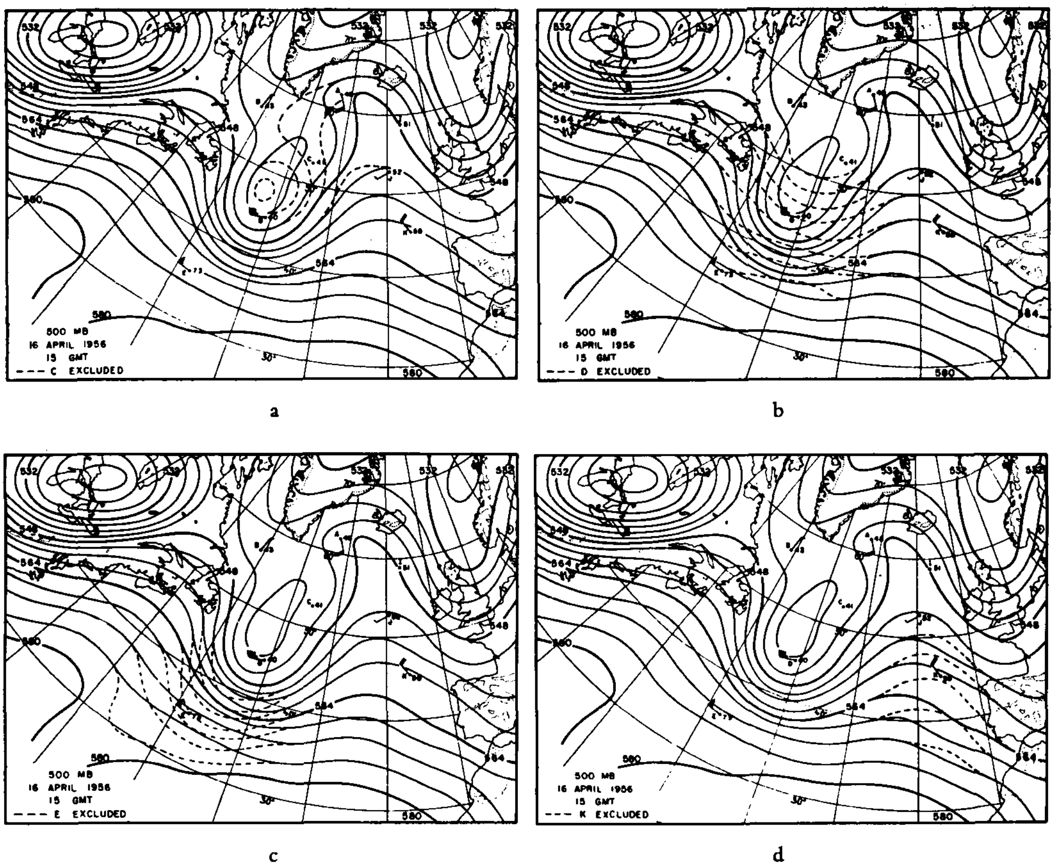

Fig. 1. Numerical analyses of the 500-mb chart, 16 April 1956, 1500 GMT. Solid contours represent the analysis with all radiosonde data. Dashed contours represent analyses without a weather ship observation.

300 km) in the area shown in fig. 2 are given in table 1.

It was found that the root mean square differences were fairly consistent except for occasional fluctuations. Further investigation revealed that the poorest analyses were usually due to missing data from the regularly reporting weather ships but that in some instances there was simply insufficient coverage of upper-air data to provide a reliable analysis.

Table 1. The average of the root mean square of the differences between the numerical analyses (*N*) and the conventional analyses (*C*) from the Swedish Weather Bureau

	OCT	NOV	JAN	MAR	APR
RMS (N—C) (meters)	35	34	37	38	28

The upper-air data regularly available over the North Atlantic consist of radiosonde and reconnaissance flight observations. Since the commercial aircraft flight reports are not regularly available nor is their exactness sufficient for direct use, the only remaining means of supplementing upper-air data is to use the heights of upper-level pressure surfaces extrapolated from surface ship reports. Because of the emphasis on the barotropic model at the International Meteorological Institute, it is natural that the 500-mb chart should be of special interest, and investigations reported here were limited to this pressure level. The results and conclusions are generally applicable to pressure surfaces below the 500-mb level.

Heights of upper pressure surfaces extrapolated from known lower surfaces have been used to supplement observational data since the very beginning of upper-air analysis. Extrapolation techniques are fundamentally the same in that they are all based on the assumption of hydrostatic equilibrium. Any variations in these techniques stem from the differences in the method of selecting the proper mean lapse rate or mean temperature of the atmospheric layer in question. In the literature it is generally suggested that the mean lapse rate can be estimated from local indications in the surface observations of cloud and weather types and from the history of the air mass. It is apparent that an approximation of the mean lapse rate from such considerations is very subjective and the accuracy will depend chiefly on the skill and experience of the analyst.

A preliminary study of the cloud and precipitation types and the corresponding lapse rates showed such a random relationship that further investigations along this line were not continued. In an investigation of the relationship between the mean lapse rate and the dynamic features of the 500-mb chart it was observed that the conservatism of the lapse rate pattern associated with features of the 500-mb chart was insufficient to be used as a criterion for determining the mean lapse rate at specific points. It was concluded that some method of interpolation between known lapse rates would be the most practicable and that an investigation of this method should be made. Results of the investigation are described in the following section.

Method

The height z_1 of the pressure surface p_1 is obtained by integrating the hydrostatic equation

$$z_1 = -\frac{R^*}{mg}\int_{p_0}^{p_1} T\frac{dp}{p} \qquad (1)$$

where T is the temperature, R^* the universal gas constant, m the apparent molecular weight of atmospheric air, g the acceleration of gravity and p_0 the sea level pressure. Using the mean temperature in terms of the mean lapse rate γ between the pressure surface p_1 and the sea level pressure p_0 and the sea level temperature T_0, we obtain after integration

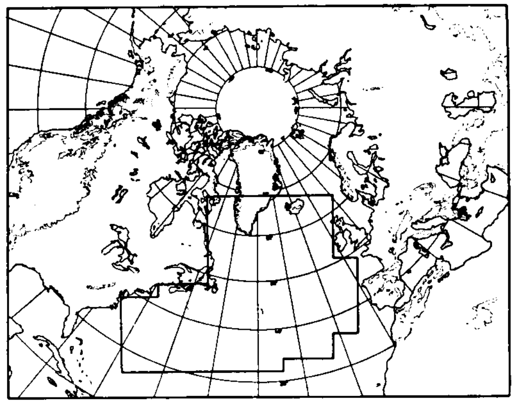

Fig. 2. Location of the grid used in numerical analyses. The Atlantic Ocean area is defined by the inner solid lines.

$$z_1 = \frac{R^*}{mg}\left[T_0 - \frac{\gamma}{2}(p_0 - p_1)\right]\log\frac{p_0}{p_1} \qquad (2)$$

Thus knowing p_0, T_0 and γ it is possible to calculate the height of the pressure surface p_1 = 500 mb. Both p_0 and T_0 are observed quantities while a proper value for γ may be interpolated between computed lapse rates at nearby reference stations (soundings). At the reference stations z_1, p_0 and T_0 are observed quantities. The lapse rate can therefore be obtained from (2). It is obvious that for the present purpose, a lapse rate calculated in this way is more accurate and representative than a lapse rate calculated from the temperatures T_0 and T_1 and the surface pressure p_0.

The magnitude of the errors in the extrapolated heights of the 500-mb surface obtained with the aid of (2) corresponding to errors in the surface pressure and temperature and mean lapse rate are shown in table 2. Another source of error in extrapolating from ship reports is the time difference between surface and upper-air observations. Without a frontal passage it is reasonable to assume no temperature change

Table 2

Error in	Error in extrapolated 500-mb height
Surface Pressure 1 mb......	± 5 meters
Surface Temperature 1°C....	± 20 »
Mean lapse rate 0.2°C/100 mb	± 10 »

during this three-hour interval but the change in pressures with some synoptic situations may introduce large errors. These may be reduced by modification of the surface pressure by an amount based on pressure tendency and continuity between synoptic charts. Fortunately this difficulty will be removed when the times of surface and upper-air observations become synchronous.

An interpolated value for the lapse rate γ_i at a point can be determined by a weighted mean of the computed lapse rates $\gamma_o\ (r_k)$ at the surrounding reference stations. This can be expressed as

$$\gamma_i = \frac{\sum_{k=1}^{N} w_k \gamma_o(r_k)}{\sum_{k=1}^{N} w_k} \tag{3}$$

where the subscript o denotes observed values, N the number of reference stations, the subscript i denotes interpolated values, the argument r denotes the distance of the reference station from the point, and $w_1, w_2 \ldots w_N$ are the weights. As can be found in books of statistics (e.g. WHITTAKER and ROBINSON 1954) the weights which give the most probable value of γ_i are

$$w_1 = ch_1^2, \quad w_2 = ch_2^2, \ldots \ldots w_N = ch_N^2$$

where c is an arbitrary constant and $h_1, h_2, \ldots h_N$ are the corresponding moduli of precision of the observed lapse rates $\gamma_o\ (r_1), \gamma_o\ (r_2) \ldots \ldots \gamma_o\ (r_N)$. Considering the variation of γ in space and the variation of γ due to observational errors the weights have been determined as a function of r in the following way. Denoting the true value of the lapse rate at the reference stations by $\gamma(r)$ and the true value of the lapse rate at other points by γ, the deviation is

$$\xi = \gamma - \gamma(r) \tag{4}$$

From the Central Limit Theorem one can assume that the deviations should have a gaussian distribution. An inspection of the deviations showed that this is a reasonable assumption, therefore

$$\gamma = \gamma(r) + n(0, \sigma_r) \tag{5}$$

or in words, the true value of the lapse rate at any point is equal to the true value at the reference station at a distance r plus a difference which has a normal frequency distribution. Due to observational errors the true value of the lapse rate at the reference station is not known but the observed value can be written

$$\gamma_o(r) = \gamma(r) + n_1(0, \sigma_o) \tag{6}$$

where $\gamma_o(r)$ is the lapse rate at a reference station calculated from the observed values according to (2) and $n_1(o, \sigma_o)$ is a variable with normal frequency distribution with a mean value zero and a standard deviation σ_o. Combining (5) and (6) the deviation will be

$$\xi = \gamma - \gamma_o(r) = n(0, \sigma_r) - n_1(0, \sigma_o) \tag{7}$$

with the mean value m and the standard deviation σ expressed by

$$m = E\{\xi\} = 0 \tag{8}$$

$$\sigma^2 = E\{(\xi - m)^2\} = E\{\xi^2\} = \sigma_r^2 + \sigma_o^2 \tag{9}$$

or

$$\sigma = \sqrt{\sigma_r^2 + \sigma_o^2} \tag{10}$$

The relation between the standard deviation and the modulus of precision is

$$\sigma = \frac{1}{\sqrt{2}\,h} \tag{11}$$

or

$$h^2 = \frac{1}{2\sigma^2} = \frac{1}{2(\sigma_r^2 + \sigma_o^2)} \tag{12}$$

Here σ_r^2 is a function of r and σ_o^2 is constant. Since $w_1 = ch_1^2$, $w_2 = ch_2^2, \ldots \ldots w_N = ch_N^2$, from (12) the weights as a function of r are

$$w(r) = \frac{c_1}{\sigma_r^2(r) + \sigma_o^2} \tag{13}$$

The function $w(r)$ was obtained in the following way. From upper-air observations over the Atlantic Ocean and the British Isles, lapse rates were determined for every fifth day during one year and the differences between values at pairs of stations were computed. The difference between two observed lapse rates may be written

$$\xi_1 = \gamma_o - \gamma_o(r) \tag{14}$$

According to (6) the observed value at $r = 0$ can be expressed as

$$\gamma_o = \gamma + n_2(0, \sigma_o)$$

so that (14) can be written

$$\xi_1 = \gamma + n_2(0, \sigma_o) - \gamma_o(r) \qquad (15)$$

Together with (7) the difference between the lapse rates at stations is then

$$\xi_1 = n(0, \sigma_r) - n_1(0, \sigma_o) + n_2(0, \sigma_o) \qquad (16)$$

As with (9) the standard deviation is

$$\sigma_0^2(r) = E\{\xi_1^2\} = \sigma_r^2(r) + 2\sigma_o^2 \qquad (17)$$

Combining (13) and (17) the weighting function $w(r)$ for different values of r will be

$$w(r) = \frac{c_1}{\sigma_o^2(r) - \sigma_o^2} \qquad (19)$$

With σ_o estimated to be 0.6° C/100 mb and with the standard deviation $\sigma_o(r)$ obtained from observed values at different values of r, then the function $w(r)$ was approximated to be

$$w(r) = \frac{4}{r^8 + 10} + \frac{10}{r^4 + 20} + 0.1 \qquad (20)$$

A graphical representation of this weighting function is shown in figure 3. To test the sensitivity of extrapolated heights to the shape of this weighting function, another weighting function was tested where w was twice as large for $r = 0$. In recomputing 25 extrapolated heights for one day the difference between the corresponding heights were, at the most, 10 meters.

The relative topography 1000—500 mb at a surface station can also be determined by interpolation between the surrounding upper air stations using the same weighting function. The height of the 500-mb surface can then be obtained by adding the height of the 1000-mb surface, determined from the reported surface pressure, to the relative topography 1000—500 mb.

Figure 4 illustrates the actual use of the interpolation technique to determine an approximation of the lapse rate over a ship, S, located between the four weather station ships, A, B, C and D. The observed lapse rates are plotted at each reference station and the interpolated lapse rate at S is shown in parentheses.

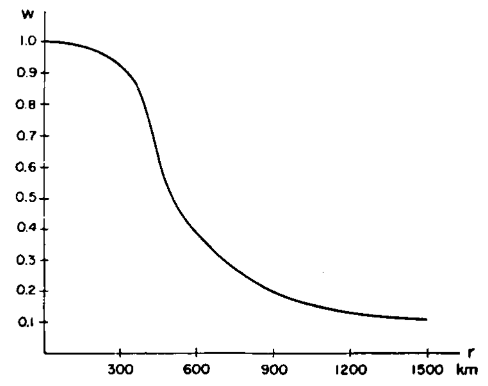

Fig. 3. A graphical representation of the weighting function.

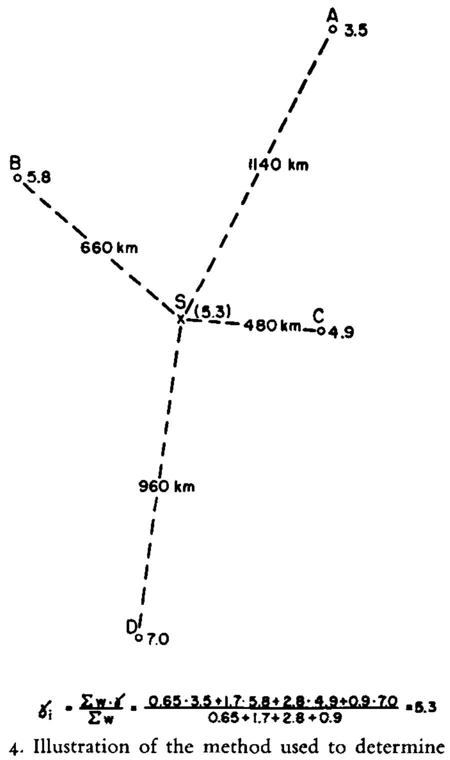

Fig. 4. Illustration of the method used to determine the lapse rate over a ship by interpolation between the ocean station vessels A, B, C, and D.

Results

Because the upper-air analysis over the Atlantic Ocean is of special importance for forecasts over the European continent, tests

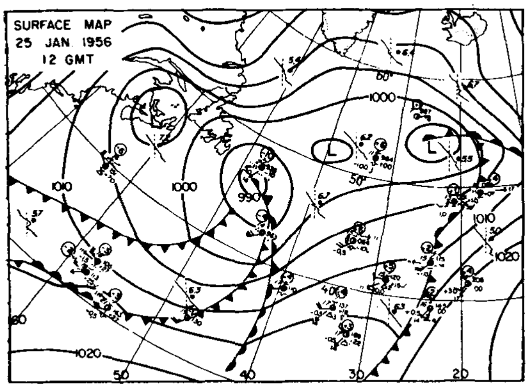

Figure 5. Surface synoptic chart, 25 January 1956, 1200 GMT.

of the extrapolation technique described above were limited to the Atlantic Ocean area. The mean lapse rates over ships reporting surface observations were determined by interpolation between the computed mean lapse rates at the eight weather ship stations Lagens, Azores (08509); Funchal, Madeira (08521); Bermuda (78016); Sable Island (72600); and at dropsonde positions. Because the reduction of temperatures and pressures to sea level can result in nonrepresentative values, mean lapse rates computed from coastal radiosonde stations cannot be considered reliable. The elevations and exposures to the sea at the four island stations above are such that sea level temperatures and pressures were considered to be representative. The steep lapse rate gradients generally existing along the eastern coast of continents make it impracticable to determine by interpolation the mean lapse rates within approximately 10° of longitude from the coast of North America. Although Sable Island is in the coastal region it was found that interpolated lapse rate values at points between Bermuda and Sable Island were more accurate with the inclusion of the latter as a reference station.

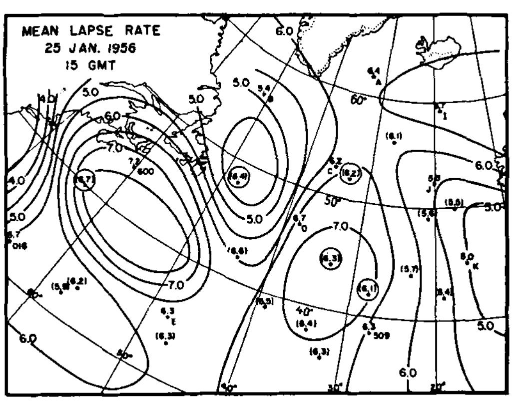

Figure 6. Lapse rate chart, 25 January 1956, 1500 GMT.

The surface synoptic chart for 1200 GMT, 25 January 1956, is shown in figure 5. The temperature-pressure curve at each of the eleven reference stations has been determined from the 1500 GMT radiosonde observations. The mean lapse rates in degrees/100 mb have been computed hydrostatically from the known values of the sea level temperature and pressure and the height of the 500-mb surface, and are plotted at each of the reference stations. The encircled number above each ship report is the difference in decameters between the extrapolated heights of the 500-mb surface and the values at the corresponding points on a conventional analysis prepared by the Swedish Weather Bureau. The difference is positive if the extrapolated height is greater than that of the conventional analysis and negative if the extrapolated height is less. The average root mean square of these differences is 43 meters. Inspection of figure 5 shows that the largest differences between extrapolated heights and conventional analysis values are not necessarily near frontal zones but tend to be within the same air masses. These differences can be due to errors in the surface temperatures and pressures, the time difference and/or to the interpolated values of the mean lapse rates.

The mean lapse rate map in figure 6 is constructed from the values of the 500-mb heights and corresponding surface temperatures and pressures taken from the Swedish Weather Bureau analyses. Note that the mean lapse rates are in units of degrees centigrade per 100 mb. Mean lapse corresponding to the moist adiabatic lapse rate have the approximate values 5.7, 7.0, 8.0° C/100 mb for surface temperatures of 20, 10 and 0° C respectively. The mean lapse rates, sea level to 500 mb, computed from radiosonde data are plotted at the reference stations. The interpolated mean lapse rates are shown in parentheses. The lapse rates are encircled at the five ships where the difference between the extrapolated height and the height from the conventional analysis was greater than 40 meters. From

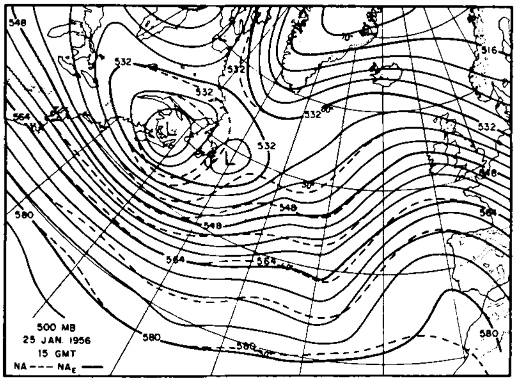

Fig. 7 a. Numerical analyses of the 500-mb chart, 25 January 1956, 1500 GMT. Solid contours represent the numerical analysis with the addition of extrapolated heights of the 500-mb surface. Dashed contours (every 80 meters) represent the numerical analysis without extrapolated heights.

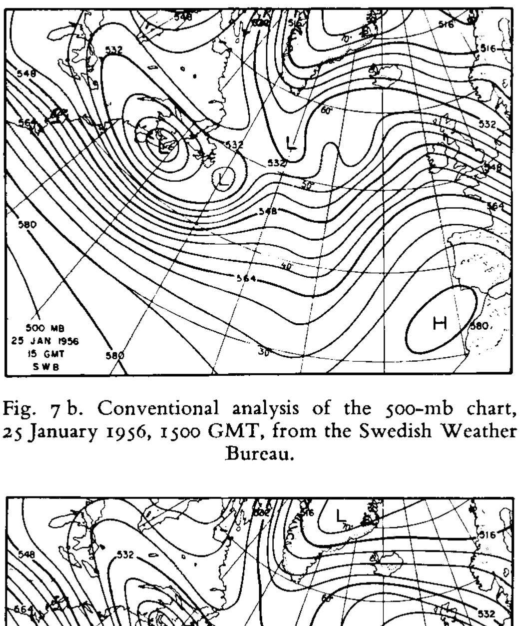

Fig. 7 b. Conventional analysis of the 500-mb chart, 25 January 1956, 1500 GMT, from the Swedish Weather Bureau.

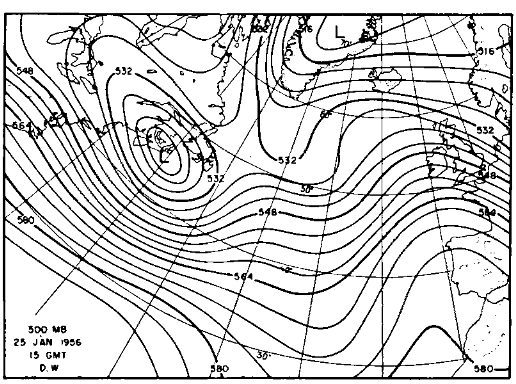

Fig. 7 c. Conventional analysis of the 500-mb chart, 25 January 1956, 1500 GMT, from the Deutsches Wetterdienst.

the lapse rate analysis it can be seen that, although part of these differences may be due to temperature errors and/or time difference in observations, errors in the lapse rates are largely responsible for the differences. The three ships located in the area between 40 and 25 degrees west longitude are nearly centered in an unstable air mass. Since the lapse rates have been obtained by interpolation between reference stations situated on the outer boundaries of this air mass, it is readily seen why the interpolated values are in error. Similarly, the ships at 48° N—48° W and 40° N—63°W are so situated with respect to the lapse rate gradients that the interpolated values show relatively large errors. Attention is called to the fact that these two ships are within 10° longitude from the coast and therefore should be expected to have inaccurate interpolated lapse rates but are included here to illustrate the situation under which any method of interpolation is limited.

To measure the value of the extrapolated 500-mb heights two numerical analyses (BERGTHORSSON and DÖÖS 1955) were computed for the 500-mb chart of 1500 GMT, 25 January 1956. The first analysis utilized only the radiosonde and reconnaissance flight data while the second was based on the same data plus the extrapolated values of the 500-mb heights. Figure 7a has the numerical analysis with extrapolated data in solid contours and the numerical analysis with only the radiosonde and aircraft data in dashed contours. The conventional analyses of the Swedish and German Weather Bureaus are shown in Figs. 7b and 7c for comparison. Addition of the extrapolated values for the numerical analysis has produced the double low structure in the region of Newfoundland similar to the Swedish Weather Bureau analysis. Over the remaining areas the analysis remains essentially unchanged.

The numerical analysis of the 500-mb chart, 1500 GMT, 1 November 1956, shown in figure 8 a again illustrates the influence which extrapolated values of the 500-mb heights have on a numerical analysis. As in figure 7 a, the numerical analysis with extrapolated data is shown in solid contours and the dashed contours at 80 meter intervals show the numerical analysis without the extrapolated data. Observed heights of the 500-mb are

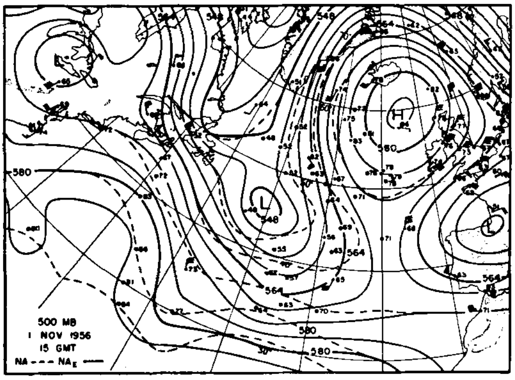

Fig. 8 a. Numerical analyses of the 500-mb chart, 1 November 1956, 1500 GMT. Solid and dashed contours represent analyses as denoted in figure 7 a.

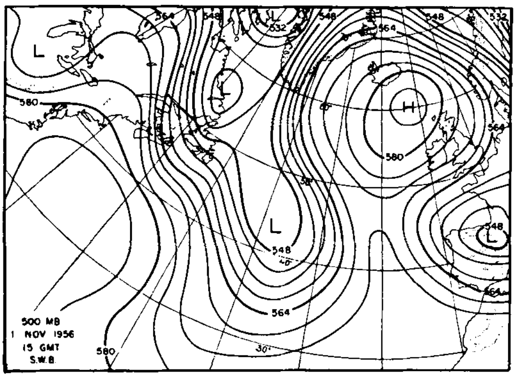

Fig. 8 b. Conventinal analysis of the 500-mb chart, 1 November 1956, 1500 GMT, from the Swedish Weather Bureau.

plotted in decameters at the radiosonde stations and reconnaissance flight positions indicated by the solid points. The extrapolated heights are shown at the encircled points. Wind reports are plotted in the conventional manner. The Swedish Weather Bureau conventional analysis is shown in figure 8 b for comparison. Addition of the extrapolated data has shifted the trough west of the blocking ridge to a more north-south orientation similar to the conventional analysis. Note also that the numerical analysis in solid contours conforms more closely with the observed height values than the original analysis shown by dashed contours.

Figures 9 and 10 are additional examples of numerical analyses made with and without the extrapolated heights. The most noticeable effect of the addition of the extrapolated heights is the adjustment in the positions of the troughs and ridges. The smaller scale features on the final analysis is better described as seen in figure 10 where the trough off the east coast of North America has been sharpened and intensified.

As a check of the accuracy of the heights extrapolated from ship reports the values were compared with the heights taken at the corresponding points on analyses from the Swedish Weather Bureau and the Deutsches Wetterdienst. The root mean squares of the differences between the values are tabulated in Table 3 where q denotes the number of extrapolated heights, SWB denotes the values from the Swedish Weather Bureau analyses, and DW denotes the values from the Deutsches

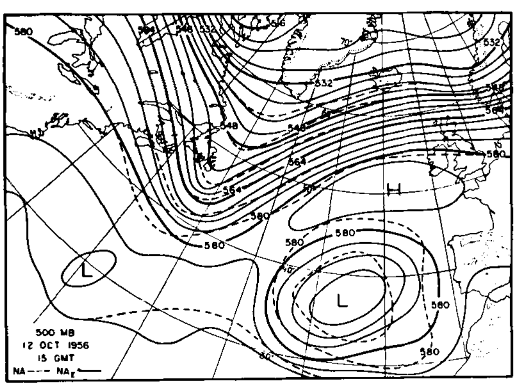

Fig. 9. Numerical analyses of the 500-mb chart, 12 October 1956, 1500 GMT. Solid and dashed contours represent analyses as denoted in figure 7 a.

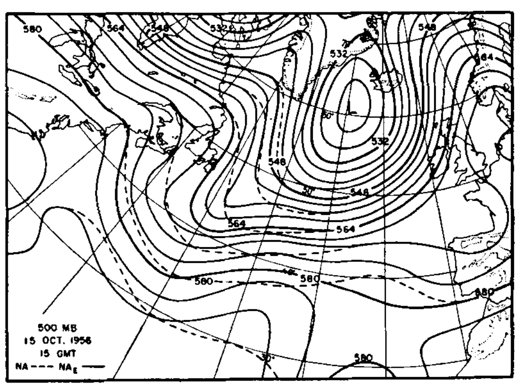

Fig. 10. Numerical analyses of the 500-mb chart, 15 October 1956, 1500 GMT. Solid and dashed contours represent analyses as denoted in fig. 7 a.

Table 3

Date	q	RMS E-SWB (m)	RMS-E-DW (m)	RMS RT-SWB (m)	RMS RT-DW (m)	RMS SWB-DW (m)
24/1-56 15 ...	25	35	38	34	33	18
25/1-56 15 ...	21	43	51	38	35	19
17/4-56 15 ...	26	61	51	58	51	30
18/4-56 15 ...	22	45	49	46	46	22
11/10-56 15 ..	37	42	39	45	44	25
12/10-56 15 ..	39	45	42	45	45	24
15/10-56 15 ..	24	58	55	76	65	37
1/11-56 15 ..	29	41	39	42	36	21
Mean	28	46	46	48	44	25

Wetterdienst analyses. As previously mentioned, 500-mb heights might be determined by interpolating values of the relative topography 1 000—500 mb. A similar check of the 500-mb heights determined by this method was made and the results are also tabulated in Table 3 where RT denotes the height of the 500-mb surface determined from interpolated values of the relative topography. From Table 3 it is seen that where the root mean square of the difference between the extrapolated heights and the heights from analyses are large, there is also a large difference between the conventional analyses. Heights determined by interpolation between either mean lapse rates or relative topographies show no significant differences. Since the surface temperature is not a critical factor in the latter method, it would probably be more adaptable to areas where coastal radiosonde stations are required for reference stations, e.g. Eastern North Pacific Ocean.

The question arises "Is an extrapolated height thus obtained closer to the correct value than a height determined by straight interpolation of the 500-mb heights?" To answer this question the 500-mb heights were calculated by interpolation when actual values of the 500-mb height were substituted for the computed mean lapse rates at the reference stations. Since the root mean square of the differences between these values and the heights taken from conventional analyses was 70 m it was concluded that the proposed method of extrapolation is a definite improvement over simple interpolation of heights.

The reliability of numerical analyses was tested by comparing with conventional analyses and from these comparisons an estimate of the true error was obtained. The root mean square of the differences between the numerical analyses without extrapolated heights (N), numerical analysis with extrapolated heights (N_E), and two independent conventional analyses (C_1 and C_2) from the Swedish Weather Bureau and the Deutsches Wetterdienst were used as a basis for judging the reliability of the numerical analyses over an ocean area. The root mean squares of the differences $(N - N_E)$, $(N - C_1)$, $(N_E - C_1)$, $(N - C_2)$, $(N_E - C_2)$ and $(C_1 - C_2)$ at corresponding gridpoints over the Atlantic Ocean area shown in figure 2 are tabulated in Table 4.

Over continental regions where the network of upper-air observations is dense, the root mean square of the differences between numerical analyses and conventional analyses RMS $(N - C_1)$ varies between 10 and 20 meters. It is not surprising that, over an area where the density of the upper-air network is small, the differences between numerical and conventional analyses is much greater as seen by the values in columns 2 and 4, Table 4. From column 6, Table 4, it is seen that even the difference between conventional analyses is relatively great. The values in columns 3 and 5 show that in general the addition of extrapolated heights has improved the numerical analyses Where the numerical analysis, (N), is already relatively good the addition of extrapolated heights gives only a slight inprovement but where the numerical analysis is relatively poor the addition of

Table 4

Date	1 RMS $N-N_E$ (m)	2 RMS $N-C_1$ (m)	3 RMS N_E-C_1 (m)	4 RMS $N-C_2$ (m)	5 RMS N_E-C_2 (m)	6 RMS C_1-C_2 (m)
25/1-56 15 ...	20	34	34	38	40	31
18/4-56 15 ...	12	32	31	30	30	25
11/10-56 15 ..	14	42	41	52	50	31
12/10-56 15 ..	16	40	38	37	32	36
15/10-56 15 ..	20	53	43	39	31	27
1/11-56 15 ..	28	63	45	61	43	23
Mean	18	44	39	43	38	29

supplementary data makes a marked inprovement of the analysis, (N_E).

Using the mean values of the root mean square differences (Table 4), an estimate of differences between the several analyses and a fictitious true analysis can be obtained. Denoting the true height of the 500-mb surface as Z_T, the heights at a corresponding point on the two conventional analyses and the numerical analysis, Z_{C_1}, Z_{C_2} and Z_N, may be expressed as

$$\left.\begin{aligned} Z_{C_1} &= Z_T + n(0, \sigma_{C_1}) \\ Z_{C_2} &= Z_T + n(0, \sigma_{C_2}) \\ Z_N &= Z_T + n(0, \sigma_N) \end{aligned}\right\} \tag{21}$$

where $n(0, \sigma_{C_1})$, $n(0, \sigma_{C_2})$ and $n(0, \sigma_N)$ denote the analysis errors which are assumed to have a normal distribution with a mean value equal to zero for a finite number of points, and have the standard deviations σ_{C_1}, σ_{C_2} and σ_N. If one can assume that the errors of the different analyses are uncorrelated the mean square of the differences between the several analyses can be expressed as

$$\left.\begin{aligned} E\{(Z_{C_1} - Z_{C_2})^2\} &= \sigma_{C_1}^2 + \sigma_{C_2}^2 \\ E\{(Z_N - Z_{C_1})^2\} &= \sigma_N^2 + \sigma_{C_1}^2 \\ E\{(Z_N - Z_{C_2})^2\} &= \sigma_N^2 + \sigma_{C_2} \end{aligned}\right\} \tag{22}$$

Substituting the mean squares of the differences given in Table 4 in these equations the standard errors, σ_{C_1}, σ_{C_2} and σ_N were determined to be

$$\sigma_{C_1} = 22, \quad \sigma_{C_2} = 19 \quad \sigma_N = 38,$$

There exists a similar set of equations involving numerical analyses with extrapolated heights (N_E). Substituting the corresponding mean

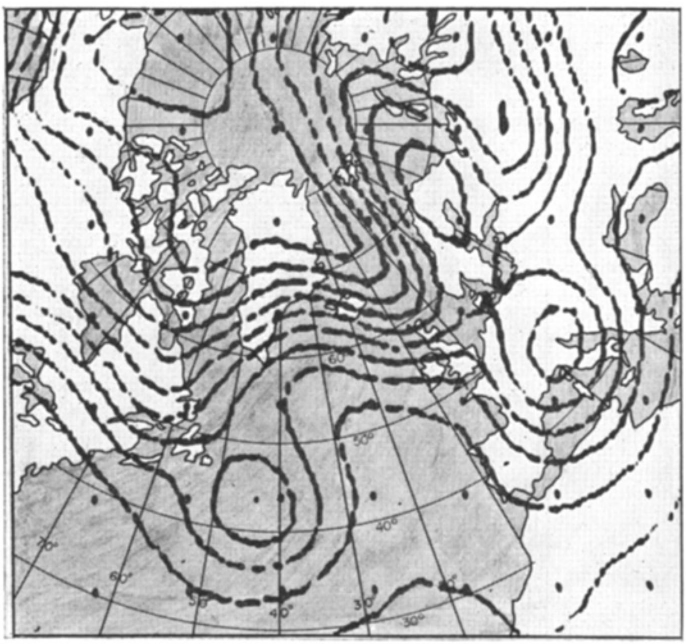

Fig. 11. Numerical analysis of the 500-mb chart, 28 October 1956, 1500 GMT, as displayed on the screen of a cathode ray tube. The cathode ray tube attached to the electronic computer, BESK, has a resolving power of 512 × 512 points. For the construction of the map above, a grid of points 256 × 256 were used. At each of these points a height value was computed by bilinear interpolation between the descrete height values resulting from the numerical analysis. If this interpolated value satisfied one of the inequalities

$$Z_0 + n \cdot \Delta Z - \delta < Z < Z_0 + n \cdot \Delta Z + \delta$$

where

$$n = 0, \pm 1, \pm 2 \ldots .$$

a point was displayed on the tube screen. The range, δ, was made proportional to the gradient of the height field in order to get the contour lines displayed with approximately constant width. In the actual case $Z_0 = 5040$ meters and $\Delta Z = 80$ meters. The time consumed to diplay the analysis over an area shown in fig. 11. is less than one minute.

squares of the differences given in Table 4 results in the following values

$$\sigma_{C_1} = 21, \quad \sigma_{C_2} = 20, \quad \sigma_{NE} = 33$$

These values represent the mean errors of the several analyses. Because the amount of data is insufficient there is a slight difference between the two errors for C_1 and also for C_2 but in spite of this it may be possible to conclude that the numerical analysis has been definitely improved by the addition of heights of the 500-mb surface extrapolated from surface observations.

Since numerical analyses have been used extensively during the above investigation it is considered appropriate here to mention an improvement in the presentation of these analyses recently developed by A. Bring, International Meteorological Institute, Stockholm. One disadvantage common to methods of numerical analysis has been the time gap which exists between the results of the machine computations and the presentation of the analysis on an aerological chart. A numerical analysis is given by a set of values of the height of the pressure surface at each grid point over a prescribed area. Generally a presentation of the analysis involves the plotting of height values at 1,230 grid points and drawing the contours. A graphical representation of analyses and forecasts by typewriter output has been used (Staff Members, Institute of Meteorology, University of Stockholm, 1954), but unless a very high speed typewriter is available this method neither completely solves the time gap problem and nor does it show a detailed representation. Bring has developed a method whereby the values at the grid points are interpolated and displayed on a screen of a cathode ray tube as contours of the pressure surface. An illustration of this analysis display and a brief description of the method is shown in Fig. 11.

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