Spectrum and Cross Spectrum Analysis of Hemispheric Westerly Index[1]

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Abstract

The following tentative conclusions are drawn from the results of cross spectrum analysis applied to the hemispheric 500-mb zonal index at latitudes of 25° N, 40° N, and 60° N.

1. Most of the variations of the indices at 40° N and 60° N are produced by oscillations with periods of the order of 25 days. This result is obtained from two independent samples of data, each at least $2^1/_2$ years in length. At latitude 25° N, longer periods seem to be predominant. High-frequency fluctuations with periods of the order of five days may also be important, but have been filtered out by the analysis.

2. The indices at latitudes 60° N and 40° N are correlated negatively. Most of this correlation is again contributed by oscillations with a period of the order of 25 days. The correlation between latitudes 25° N and 40° N is also negative, but so small as to be hardly significant. Again, both sets of observations agree fairly well in furnishing these results.

3. There is an indication of a slight lag of the long-period variation of index at latitude 40° N behind that at 60° N.

I. Introduction

It has been suspected since the turn of the century that significant correlations exist between the strengths of the westerlies at various latitudes. For example, Sir Gilbert Walker found negative correlations between air flow over the northern and southern portions of the oceans of the Northern Hemisphere. RIEHL, YEH and LASEUR (1950) analyzed the progression of hemispheric indices between various latitudes qualitatively, indicating both northward and southward migrations of high westerly index. Their figures also clearly indicate a good negative correlation between indices near the Arctic Circle and those in middle latitudes. Similar correlations at sea level were found by LORENZ (1951).

The relationship between westerly indices at various latitudes is well suited to a technique of analysis called "cross spectrum analysis". Cross spectrum analysis not only aims to determine the existence of correlations between two variables, but also simultaneously breaks each of two time series into its harmonic components, and shows how much each component contributes to the correlation between the variables. In principle, the total correlation coefficient between two variables could be zero; yet the cross spectrum would indicate that significant correlations between the high frequency components of each series exist which are cancelled by equally good correlations of opposite sign between the low frequency components. In such a case, each time series could be used to give a good estimate of the other, a fact not suspected from ordinary correlation analysis.

Cross spectrum analysis not only gives

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information regarding the coherence between two time series, but also about possible lags of one series behind the other. Further, differences in the lags of the various harmonic components are readily determined. ESTOQUE (1955) has described the theory of this type of analysis in this journal.

This paper deals with cross spectrum analysis applied to the relations between hemispheric 500-mb westerly indices at latitudes 25° N, 40° N, and 60° N. Incidental to cross spectrum analysis, the *spectra* of the indices at the individual latitudes are first found; these show how much oscillations of different frequencies or periods contribute to the variation of individual indices. The computation of the spectra begins with the formation of autocorrelation coefficients of the indices, followed by cosine transform and subsequent smoothing. Correlations were computed with lags from 0 to 30 days.

Cross spectra between two time series are determined by first forming cross correlation coefficients between two sets of data. These in themselves are of some interest, when applied to indices, and some of them will be given in the report. Cross spectra consist of two parts, which will be explained later: cospectra and quadrature spectra. The former are obtained by averaging cross correlation coefficients with lag L and lag $-L$, and subjecting these averages to a cosine transform. Quadrature spectra are determined from a sine transform of half the difference between cross correlation coefficients with lags L and $-L$.

Finally, the coherence between two time series (also explained later) is formed from the formula:

$$CO = (Q^2 + CS^2)/S_1 S_2;$$

where CO is the coherence; Q the quadrature spectrum; CS the cospectrum; and S_1 and S_2 the spectral estimates of the two variables.

II. Basic data

The basic data for this study were furnished by the report "Analyzed Zonal Wind Profiles" made by U.S. Navy Bureau of Aeronautics Project AROWA. Westerly hemispheric indices were given in this report for two periods, January 1949 through February 1952, and October 1945 through March 1948. Groups of data taken from these periods will be referred to as Group I for the later period, and Group II for the earlier period. The information from both groups was analyzed in exactly the same manner, so as to produce a check on the statistics derived from either group.

The tabulated values represent three-day moving average westerly flow. They are given in two forms: one, in meters per second; the other, with the annual variation removed, is coded as MB, B, N, A, MA for "much below normal", "below normal", etc., through "much above normal". Since the annual variation was of little interest in this study, the coded data were analyzed. Quantities were formed proportional to the mean wind in latitude belts of 10 degree widths centered at latitudes 25° N, 40° N, and 60° N. Thus, for example, quantities proportional to the index at 25° N were derived by adding the values given at 22.5° N and 27.5° N, coded in the manner that MB was counted as 0, B as 1, N as 2, A as 3, and MA as 4. Hence, the sums representing the 10° wide belts varied from 0 to 8. These numbers were punched on IBM cards and analyzed by the IAS computer at Princeton, N.J., by a previously developed code which yields autocovariances, cross covariances, spectra and cross spectra for any two time series. The code was developed according to suggestions made by John W. Tukey. The computations were supervised by James Cooley.

III. Results

When plotting spectra, mathematicians make use of frequency (the reciprocal of period) as abscissa. Meteorologists have plotted the period itself as abscissa in similar work (ESTOQUE, 1955). The author prefers a logarithmic scale of period. This seems appropriate, since the spectrum of atmospheric motions covers many orders of magnitude, from fractions of a second to many years. The choice of a particular abscissa, however, has an effect on the ordinate. The graph of a spectrum of any quantity should be constructed in such a way that the area between two periods or two frequencies represents the fraction of the quantity's variation brought about by oscillations with periods in this range. In particular, if the abscissa shows period on a logarithmic scale, the spectrum estimates have to be

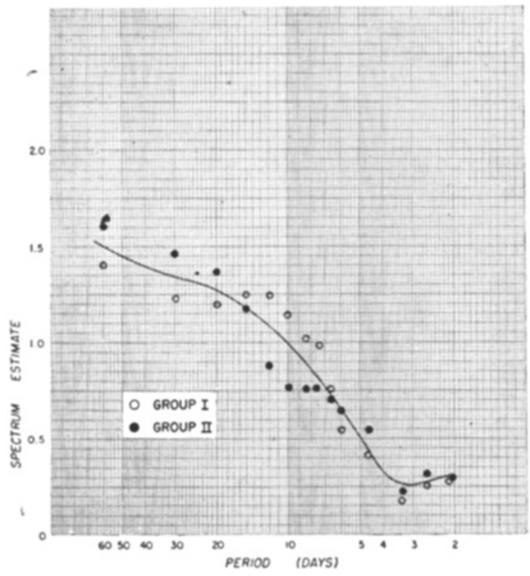

Fig. 1a. Power spectrum estimate of westerly index for latitude 25° N.

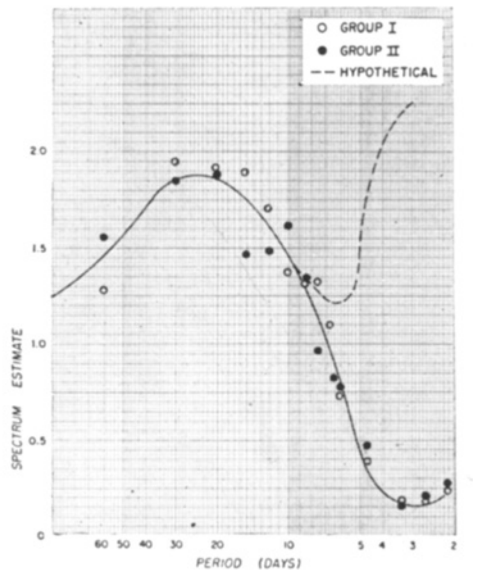

Fig. 1b. Power spectrum estimate of westerly index for latitude 40° N.

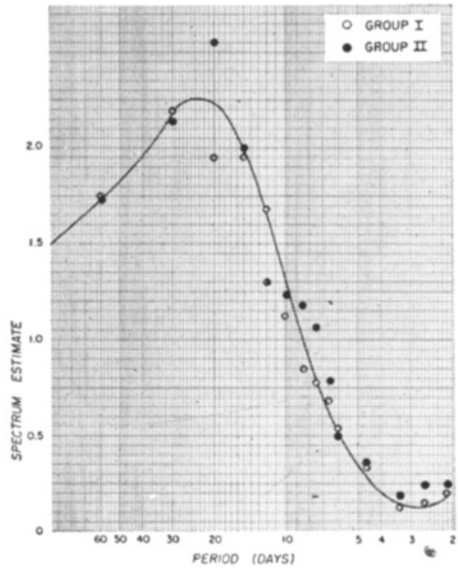

Fig. 1c. Power spectrum estimate of westerly index for latitude 60° N.

multiplied by frequency. Thus, Figures 1a, 1b, and 1c show the spectrum estimates of westerly index (multiplied by frequency), at latitudes 25° N, 40° N, and 60° N, plotted on a logarithmic scale of periods.

Estimates of spectra and cross spectra are available at equal intervals of frequency, that is, at equal intervals of the *reciprocal* of period. Thus, the first estimate was found at 60 days, the second at 30, then at 20, 15, 12 days, etc. Thus, estimates are obtained at extremely small intervals of period when the periods are small. For this reason, spectral and cross spectral estimates were averaged in groups of five when the periods were below 6 days.

Figures 1a, 1b, and 1c show first, that spectra derived from the data of Group I are similar to those obtained from Group II, particularly at latitudes 40° N and 60° N. At these two latitudes most of the variation of the westerlies originates from fluctuation with periods around 24 days. At 25° latitude, on the other hand, even longer periods produce most of the energy.

Of course, since the original data consisted of three-day moving averages, which were further smoothed by averaging over adjacent latitude bands, the spectrum lacks power at high frequencies. The dashed curve represents a hypothesis for the spectrum at 40° N for unsmoothed day-by-day fluctuations of the index. Whether the gap between long-period fluctuations and short-period fluctuations indicated in this hypothetical spectrum is real must be verified by analysis of the unsmoothed daily indices. The reality of the gap would indicate two distinct modes of variation of the general circulation – one as reflected by daily maps, the other with period of about 20 days or more.

It is interesting to note that ESTOQUE (1955) in his analysis of the cross spectrum of horizontal heat flux and momentum flux also finds two main contributions, one near 4 days, the other near 20 days. A maximum near 20 days was also found by Adel and Epstein (unpublished) in the spectra of mean radiation temperatures of atmospheric ozone. Four-day periods, incidentally, were found to be predominant in the spectrum of temperature at University Park, Pa. (GRIFFITH, PANOFSKY and VAN DER HOVEN, 1955).

Table I shows a series of cross correlation coefficients between indices at the various latitudes, with various lags. Such cross correlations are computed incidentally during cross spectrum analysis. Subscripts refer to latitudes, the L in parentheses to the lag between the correlated time series. Positive lag means that the index at a given time for the lower latitude is correlated with the index of the higher latitude at a later time.

The correlation coefficients between the indices at latitude 25° N and 40° N are too small to be very informative, although the slight negative correlation for zero lag is probably real. Of much greater interest are the correlation coefficients between indices at latitudes 40° N and 60° N. Both Group I and Group II show similar behavior: a considerable negative correlation for simultaneously measured indices at both latitudes, falling off in magnitude as the lag increases both positively and negatively. However, the coefficients drop off less rapidly in magnitude with negative lags than with positive lags; in other words, the correlation between the index at 60° N now with the index at 40° N tomorrow is almost as good as the simultaneous correlation. This situation suggests that maxima and minima at 60° N might occur slightly earlier than minima and maxima at 40° N, the lag being of the order of a day. More will be said about this later.

Table I. Cross correlations between indices at various latitudes with various lags

	$r_{25,\,40}(L)$		$r_{40,\,60}(L)$	
Lag davs	Group I	Group II	Group I	Group II
10	+ .06	+ .07	0	0
9	+ .05	+ .06	– .02	– .03
8	+ .05	+ .06	– .04	– .07
7	+ .05	+ .07	– .07	– .10
6	+ .05	+ .06	– .10	– .14
5	+ .03	+ .07	– .16	– .19
4	– .02	+ .06	– .24	– .25
3	– .08	+ .04	– .33	– .32
2	– .14	– .02	– .45	– .42
1	– .19	– .09	– .54	– .50
0	– .20	– .11	– .60	– .54
– 1	– .17	– .13	– .59	– .52
– 2	– .13	– .11	– .53	– .45
– 3	– .09	– .10	– .45	– .38
– 4	– .08	– .10	– .37	– .32
– 5	– .08	– .11	– .31	– .28
– 6	– .10	– .13	– .28	– .25
– 7	– .09	– .15	– .26	– .22
– 8	– .07	– .17	– .23	– .19
– 9	– .04	– .18	– .21	– .16
– 10	– .01	– .17	– .19	– .15

Figures 2a and 2b show the variation of "coherence" between indices in the different latitude belts, as function of period or frequency. Coherence is analogous to the square of a correlation coefficient, excepting that it may take different values at different frequencies or periods. The figures show that the coherence between indices at latitudes 25° N and 40° N is much smaller than between 40° N and 60° N. In fact, for Group II the coherence between the latter two indices reaches 0.44 at low frequencies, corresponding to a correlation coefficient of 0.66. The coherence is slightly higher for Group I. The general behavior of the coherence in Groups I and II is similar, confirming its reality; coherence is greatest at long periods and absent for fluctuations with periods less than 4 days. In other words, long-period fluctuations in the indices at latitudes 40° N and 60° N are strongly coupled, whereas short-period fluctuations are independent.

The coherence between indices at latitudes 25° N and 40° N shows a primary maximum near a period of 8 days. This, however, is of questionable significance.

The coherence consists of two parts, the cospectrum and the quadrature spectrum. The former measures the in-phase relationship between the harmonic components of the two time series, the latter the 90° out-of-phase relationship. Thus, the cospectrum indicates

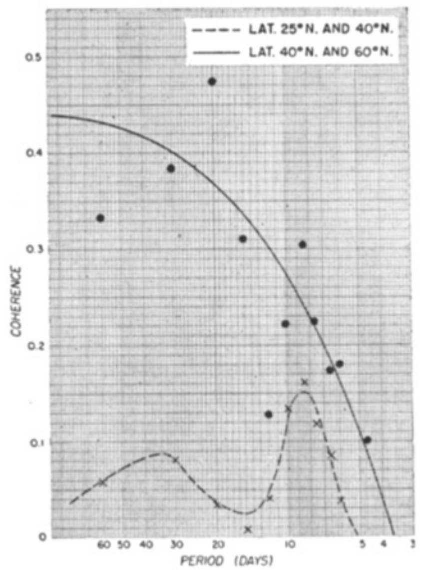

Fig. 2a. Coherence between Indices, Group I.

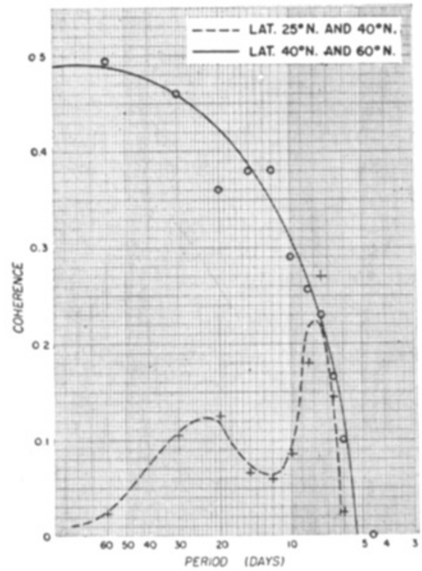

Fig. 2b. Coherence between Indices, Group II.

to what extent fluctuations with a given range of period contribute to the covariance between the two time series, and therefore to the correlation coefficient. The quadrature spectrum shows how much would be contributed to the covariance if each harmonic of one time series was shifted by 90°. For example, if two time series represent a sine and a cosine, respectively, the covariance would vanish, but the quadrature spectrum would be large.

Since the coherence between indices of latitudes 25° N and 40° N was small, no further analysis is given for these two latitudes. The following section refers entirely to the relation between westerly indices at latitudes 40° N and 60° N. Figure 3 shows cospectra and quadrature spectra for the data at these latitudes. The figure shows that the behavior of the cospectrum is quite similar for Groups I and II, indicating that it is not a statistical accident: maximum contribution to the correlation originates from fluctuations with periods about 23 days. Of course, the negative sign signifies a negative correlation between the indices at these two latitudes. Little correlation is contributed at periods less than six days.

The quadrature spectra are generally near zero, indicating that the variation at the two latitudes are generally in phase in the negative sense; in other words, 180° out of phase. There is some indication that the quadrature spectrum is negative at low frequencies, although Groups I and II do not agree well in detail. The meaning of negative quadrature spectra can best be estimated from the fact that the ratio of the quadrature spectrum to

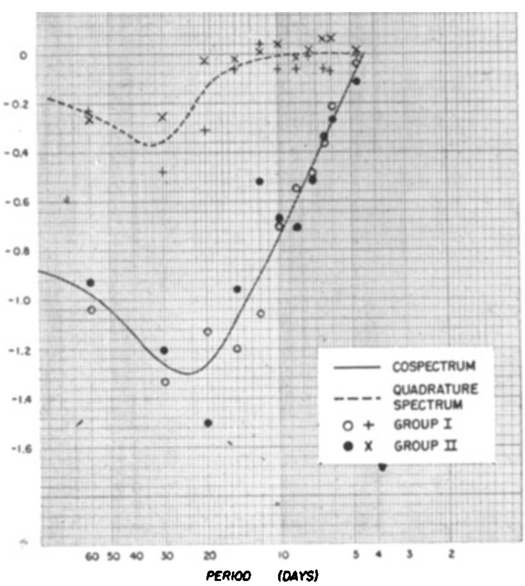

Fig. 3. Cospectrum and Quadrature Spectrum for Latitudes 40° N and 60° N.

the cospectrum is the tangent of the phase angle between the indices at the two latitudes. Here, a positive phase angle indicates a lag of the index at 40° N behind the index at 60° N. Obviously, little is gained in computing this angle at high frequencies where both numerator and denominator are small.

Table II shows the variation of the lag of the oscillations of index at 40° N latitude behind the oscillations at 60° N latitude, as function of period.

Table II. Lag of index at 40° N behind that at 60° N latitude

Period (days)		60	30	20	15
Group I	Phase Angle (Degrees)..	13	20	15	3
	Days..	2.2	1.7	0.8	0.1
Group II	Phase Angle (Degrees)..	16	12	9	2
	Days..	2.7	1.0	0.5	0.1

The two groups agree only in that the lag is positive and decreases with decreasing period. Therefore, this lag is not particularly useful for forecasting. However, it may be useful in explaining the mechanism of latitudinal transport of momentum.

IV. Recommendations for future work

This report summarizes an arbitrarily picked small sample of cross spectrum analysis carried out on index data. The results suggest that similar methods can be used for the investigation of other useful problems:

1. The analysis carried out so far gives little detail in the interesting region of the spectra and cross spectra near 25 days. This information could easily be furnished by repeating the analysis with every third reported three day mean only.

2. The relations are probably different in different seasons. Hence, possibly larger coherences would be found if the indices were treated separately by season.

3. The zonal index is a large-scale meteorological variable which should be compared with the variation of solar data, or the related geomagnetic character figure. Again, cross spectrum analysis should give the most information. The periods important in the variation of index noted here, near 25 days, often appear in analysis of solar data due to the period of rotation of the sun.

4. It would be useful to analyze daily unsmoothed indices in order to determine whether the suspected gap around periods of 10 days is real.

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