A Consideration on Earth Tidal Change of Gravity

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Abstract

Gravimetric observations with the portable Worden gravimeter were made at three stations in our country to obtain accurate values for the earth tidal factor of gravity (G) and to detect, if possible, the local characters of these values. The observation period was respectively one month for each station, and the three stations were specially selected from the stand-point of diversity in conditions with regard to the supposed effect by ocean tides and subterranean structure. Judging from the observation results the *G*-values observed at three stations are generally well in accord with each other, but, if the numerals in the third figure of the *G*-value obtained can be assumed to have any significance, it should be said that the problem of the local character of the *G*-value remained unsolved and awaits the future investigation. The effect by ocean tides upon the *G*-value is discussed somewhat in detail, and the highly sensitive gravity variometer of double bifilar suspension type is also introduced.

1. Introduction

Since the first theoretical treatment by LORD KELVIN (1863), the phenomena of earth tides have ever been an attractive and important subject of research in the field of physical geodesy. E. v. REBEUR-PASCHWITZ (1892) was the first to succeed in detecting and instrumentally observing earth tides by the tiltmeter of the horizontal pendulum type, and after him many observations on earth tides have been made, especially with tiltmeters, by many researchers at various places in the world. Thus the study on earth tides has a comparatively long history extending about a century and becomes one of the old research items, but the recent development of techniques in measurement and observation has rejuvenised it and again made it an important research item for investigating the nature of the earth and its interior. In reality, to tap the earth with great earthquakes and to twist the earth with earth tides are certainly two powerful methods for diagnosing the earth's interior, and in this sense the study on earth tides will everlastingly afford us new clues for exploring the earth and advancing our knowledge on the nature of the earth's interior.

The observation of earth tides with tiltmeters have been made by many researchers since the first observation by E. v. REBEUR-PASCHWITZ in 1892, and the problem on the value of the so-called "diminishing factor", $D\ (\equiv 1 + k - h)$, and the secondary effect by ocean tides were fully studied. The dimensionless numbers, k and h, above mentioned are those introduced by A. E. H. LOVE (1909) in his theoretical treatment on earth tides, and they play an important role in the investigation of the earth and the interior by the aid of observational results. On the contrary, there are few examples of the observations of earth tides with gravimeters since the first gravimetric observation by W. SCHWEYDAR (1914), owing mainly to its instrumental difficulty. Consequently the value of $G\left(\equiv 1 - \frac{3}{2}k + h\right)$—which is the ratio of gravimetrically observed tidal change of gravity on the real earth to that observed on the hypothetically perfect rigid earth, and is called "earth tidal factor of gravity" or, in short, "gravimetric factor"—has not yet been accurately determined in comparison with the value of D. Really, the values of G, obtained by some recent provisional observations of short period of from several days to half a month with the portable gravimeter for prospection use, show a considerably large diversity among them, varying from 0.9 to 1.5. The causes of this diversity are considered to be attributed mainly to the following three effects: first, the observational errors and computational errors in analysing the data of short period observation; secondly, the secondary effects caused by ocean tides; and thirdly, the effects introduced by the difference of subterranean structure. Concerning these three points a consideration will be presented in the following.

2. Observational results

(1) Observation stations

In order to examine the above described effects, observation stations were effectively selected to fit the purpose of detecting the maximum effect caused by ocean tides and subterranean structure. Namely, the three stations of Kyoto, Chikubushima, and Shionomisaki were selected whose position, sea shore distance, Bouguer anomaly, and observational period are as listed in the following table and shown in Figure 1.

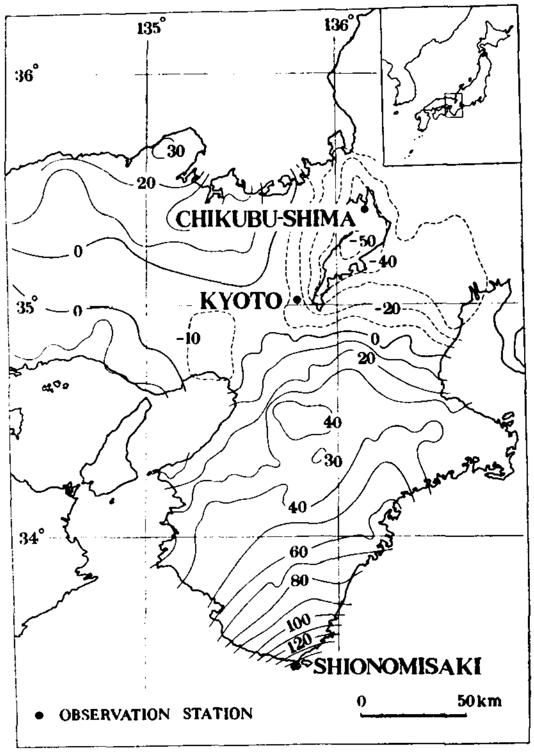

Fig. 1. The position of observation stations and the distribution of Bouguer anomaly in mgal-unit.

(2) Instrument

The instrument used in the present series of observations is the Worden gravimeter, No. 127. Its sensitivity given by the factory is 0.18473 mgal/div, and for practical purposes it can be read to one tenth of the scale division by the vernier. According to the experiments on the reliability of the scale constant, made by the comparison with the North American gravimeter, *AG*-1,108, the accuracies of the scale constants of both gravimeters accord well with each other within the limits of experimental error. Although the problem concerning the absolute value of the gravimeter scale constant is generally very troublesome owing to the difficulties on the method of checking, it may safely be said from various considerations that the scale constant given by the factory is trustworthy at least to the third significant figure. Consequently, when discussing the earth tidal factor of gravity to its third significant figure, the value of the scale constant given by the factory will be regarded as correct in the present observations. Moreover, judging from our own experiments as well as from those made independently by the members of the Geological Institute of Kyoto University, the above described scale constant has remained constant within the limits of experimental error since the time of its importation (1953) to the present; in other words, there is no noticeable time variation in the scale constant of that gravimeter. The plastic flow of the main spring, or simply expressed "drift", of the gravimeter has been farily large, and moreover, the drift speed was never uniform. The average values of drift speed were estimated, in the present case, to be 0.60 mgal/day, 0.94 mgal/day, and 0.86 mgal/day for the

Table

Station	Latitude (N)	Longitude (E)	Height (m)	Bouguer anomaly (mgal)	Distance from the nearest sea shore of Pacific Ocean (km)	Observation period
Kyoto..................	35° 02′	135° 47′	58	— 12	110	June 11—July 13, 1954
Chikubushima	35° 25′	136° 09′	98	— 45	140	Sept. 28—Oct. 30, 1955
Shionomisaki............	33° 27′	135° 46′	74	+ 139	0.4	Nov. 16—Dec. 17, 1955

$H \cos (nt - \zeta)$, observed

Station	M_2		S_2		O_1		K_1	
	H_{obs}	ζ_{obs}	H_{obs}	ζ_{obs}	H_{obs}	ζ_{obs}	H_{obs}	ζ_{obs}
Kyoto.........	59.4 μgal	181°.96	27.2 μ gal	176°.67	33.7 μ gal	178°.07	53.3 μ gal	202°.73
Chikubushima ..	60.2	181 .20	28.5	182 .22	32.8	174 .30	54.1	180 .95
Shionomisaki ...	64.0	182 .42	30.0	185 .37	28.2	179 .61	54.1	199 .62

$H \cos (nt - \zeta)$, theoretical

Station	M_2		S_2		O_1		K_1	
	H_{theor}	ζ_{theor}	H_{theor}	ζ_{theor}	H_{theor}	ζ_{theor}	H_{theor}	ζ_{theor}
Kyoto.........	50.3 μgal	180°.00	23.4 μ gal	180°.00	29.3 μ gal	180°.00	41.2 μ gal	180°.00
Chikubushima ..	49.8	180 .00	23.2	180 .00	29.4	180 .00	41.4	180 .00
Shionomisaki ...	52.2	180 .00	24.3	180 .00	28.6	180 .00	40.3	180 .00

Station	M_2		S_2		O_1		K_1	
	G	$\varkappa$	G	$\varkappa$	G	$\varkappa$	G	$\varkappa$
Kyoto.........	1.18±0.02	+ 1°.96	1.16±0.03	— 3°.33	1.15±0.04	— 1°.93	1.29±0.07	+ 22°.73
Chikubushima...	1.21±0.02	+ 1 .20	1.23±0.02	+ 2 .22	1.12±0.05	— 5 .70	1.31±0.13	+ 0 .95
Shionomisaki ...	1.23±0.02	+ 2 .42	1.23±0.02	+ 5 .37	0.99±0.05	— 0 .39	1.34±0.10	+ 19 .62

periods of the observations at Kyoto, Chikubushima, and Shionomisaki respectively.

(3) Results of the observations

The gravity changes at three stations were read at intervals of half an hour during the respective one month, and the obtained values were corrected by applying drift reduction. After the reduction on drift, the hourly values during one month were harmonically analysed by Darwin's method, giving the following component tides. The values shown in the following table are corrected for the augmenting factor in analysis and reduced to the mean inclination of the moon's orbit to the equator, the phase being referred to the respective meridian of the observation stations.

Corresponding to the above values obtained from the observational data, the theoretical values calculated for a perfectly rigid earth are as follows:

Taking the ratios between the observed values and the theoretical, the following values of G and $\varkappa(\zeta_{obs.} - \zeta_{theor.})$ for the respective component tide at each station are obtained.

As shown in the above table, the values of G and $\varkappa$ at the three stations and for each component tide, especially the M_2-component, are quite consistent with each other within the limits of observational and instrumental error. From these results it may safely be said that the earth tidal factor of gravity and phase difference in a certain small area can correctly be determined within the above mentioned accuracy by one month's observation with a gravimeter of ordinary sensitivity. But whether the same value for the tidal factor ofgravity can be expected or not at places of greatly different latitudes from those of the present three stations, or at the middle of the continent, or at an isolated island in the middle of the ocean is quite another problem to consider, and this sort of observation is highly recommended to be put into effect through the cooperation of many countries at the opportunity of the coming Geophysical Year (1957—1958).

3. Effects of ocean tides

As above stated, the values of the earth tidal factor of gravity obtained at the three stations during the present observations are well in accord with each other, but if, in this case, it is provisionally assumed that the third figure of the numerical G-value in the above listed table is trustworthy and the differences between the three stations have any significance, they should be attributed to the differences in effects of ocean tides and of the subterranean structure upon the G-values obtained at each station. In this section a brief discussion will be made on these secondary effects upon the G-values.

Concerning the effect caused by the difference of subterranean structure which is represented by the distribution of Bouguer anomaly in the present treatment, the theoretical calculation is considerably difficult. But as H. Takeuchi has recently taken up the problem and has been working out the computation, the amount of this effect upon the G-value will be estimated before long. In the present case only the effect caused by ocean tides will be discussed in some detail. Before entering into the treatment on ocean tidal effect, the effect caused by the height differences of observation stations should briefly be mentioned. In the present case, the maximum height difference between the three stations is only 40 meters, and the difference of vertical component of gravity due to this height difference calculated from the tide-generating potential is estimated to be 1/160,000 of the original force. With regard to Love numbers, they are also influenced by height, but the effect of height difference upon them is negligibly small in the similar order of the above described vertical component. Hence the effect of the height difference between the three stations upon the G-value which is the algebraic summation of the vertical component of tidal force and forces relating to Love numbers may safely be disregarded in the present case.

The supposed effects upon the G-value caused by ocean tides are generally divided into three principal parts. One is the effect of the vertical component of attraction by the tidal height change of sea water, the second, the effect of the vertical displacement of the ground at the observation station caused by the tidal load of sea water, and the third, the effect caused by the distortion of gravitational potential field as a result of deformation of the ground and sea bottom. Beside these, the effect originating from the horizontal displacement of the ground at the observation station caused by the tidal load of sea water may also to be taken into account, but this effect was ascertained to be negligibly small.

On the effect of attraction by tidal change of sea water height, the station to be discussed is certainly considered to be only Shionomisaki whose observation point is 74 meters high above the sea level and 400 meters distant from the sea shore of Pacific Ocean. Concerning the other two stations, Kyoto and Chikubushima, the effects concerned with the ocean tides were ascertained to be negligibly small, because the distances from the nearest sea shore of the Pacific Ocean are 110 kilometers and 140 kilometers respectively. The vertical component of attraction of the M_2-period caused by the neighbouring sea water within 1,000 kilometers distance from the observation station for Shionomisaki is calculated as nearly

$$0.9\ \mu\text{gal} \cos (2t - 165°).$$

Secondly, the effect originating from the vertical displacement of ground at the observation point caused by the tidal load of sea water will be calculated. The quantity of vertical displacement is calculated from the formula derived by NAGAOKA (1906) in his study on the elastic deformation of the ground by surface load. Assuming tentatively the values of Lamé's elastic constant as $\lambda = \mu = 5 \times 10^{11}$ c.g.s. units, the vertical ground displacement of M_2-period at the station of Shionomisaki caused by the tidal load of the neighbouring sea water within 1,000 kilometers distance from the station is estimated as nearly

$$7.3 \text{ cm} \cos(2t - 165°).$$

Converting the above value of vertical ground displacement into change of gravity concerned with the M_2-period is

$$22.5\ \mu\ \text{gal} \cos(2t - 165°).$$

The calculated effect concerning the vertical ground displacement is by itself of an exceedingly large amount, and it contains many problems to examine.

And thirdly, the effect originating from the distortion of gravitational potential field as the result of deformation of the ground and sea bottom should be discussed. But it is a hard task to calculate accurately the amount of this effect under practical circumstances, and its amount is supposed to be so considerably large as to nearly cancel the effect caused by the vertical ground displacement by the tidal load of sea water. Because, if only the correction of effect caused by the vertical displacement is applied to the observed result in the present case of Shionomisaki, the earth tidal change of gravity and the G-value derived therefrom show the following absurdly small values: namely the change of gravity of M_2-period is nearly 42.2 μgal cos $(2t - 192°.02)$ and the G-value is reduced to nearly 0.81. From this fact and many other considerations on the normal value of G derived from many other gravimetric observations, it is safely concluded that the large effect upon the earth tidal change of gravity caused by the vertical displacement of the ground at the station near the sea coast should generally be cancelled in balance by the effect originated from the distortion of gravitational potential field caused by the tidal deformation of the surrounding ground of the station. The calculation of the latter effect under each set of practical circumstances is certainly desirable for a complete discussion of secondary effects by ocean tides upon the tidal change of gravity.

In the present case, the theoretical treatment by H. TAKEUCHI (1951) on the deformation of the earth by surface loads will tentatively be applied for correcting the observed value at Shionomisaki. After his theory, the total effect upon the tidal change of gravity by ocean tides which is the algebraic summation of the above described three effects caused by the attraction of sea water, vertical displacement of the station ground, and distortion of gravitational potential field due to deformation of the ground is calculated as nearly 1.1 times the effect caused by the attraction of sea water, when the tidal deformation of the earth by ocean tides is simply expressed by a spherical harmonic of 20th degree, in case of the earth of Bullen's model. Under these assumptions the secondary effect of ocean tides is roughly estimated for the station of Shionomisaki as 1.0 μgal cos $(2t - 165°)$, and consequently the G-value corrected for the ocean tidal effects at Shionomisaki becomes $G = 1.21$. Actually, the total effect of ocean tides at Shionomisaki is reasonably supposed to be a little larger than 1 μgal. Thus the secondary effects of ocean tides upon the earth tidal change of gravity, is in any case, supposed to be small in quantity compared with the original primary value of the tidal change of gravity, and really it plays any role in discussing the significance of numerals in the third figure of the G-value.

When the observed value of G at Kyoto—which is supposed to be little influenced by ocean tidal effects and to have a normal crustal structure of 30 kilometers, as estimated from seismic observations—is representatively adopted as the standard in our region, the numerical values of the Love numbers, h and k, are simply calculated by combining the G-value with the D-value. When adopting the numeral obtained by E. NISHIMURA (1950) as the D-value, the values of the Love numbers become as follows:

$$G \equiv 1 - \frac{3}{2} k + h = 1.18$$

$$D \equiv 1 + k - h = 0.66$$

$$h = 0.66, \qquad k = 0.32.$$

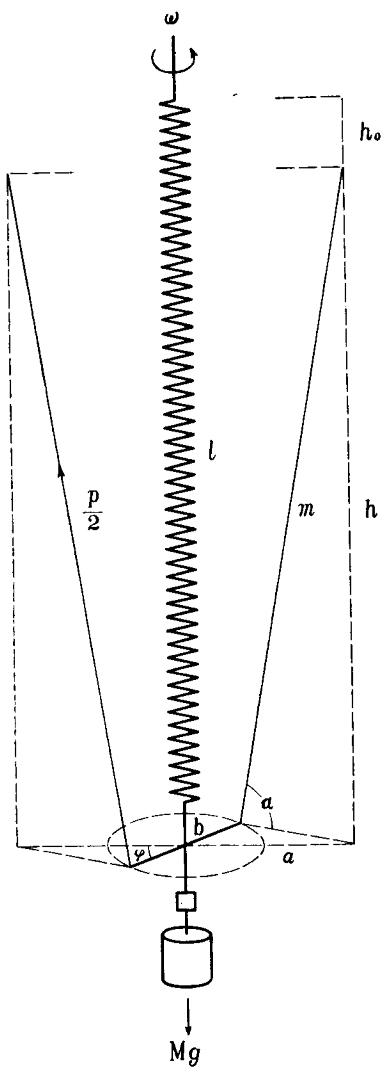

Fig. 2 a. Schema on construction of an ordinary bifilar gravity variometer.

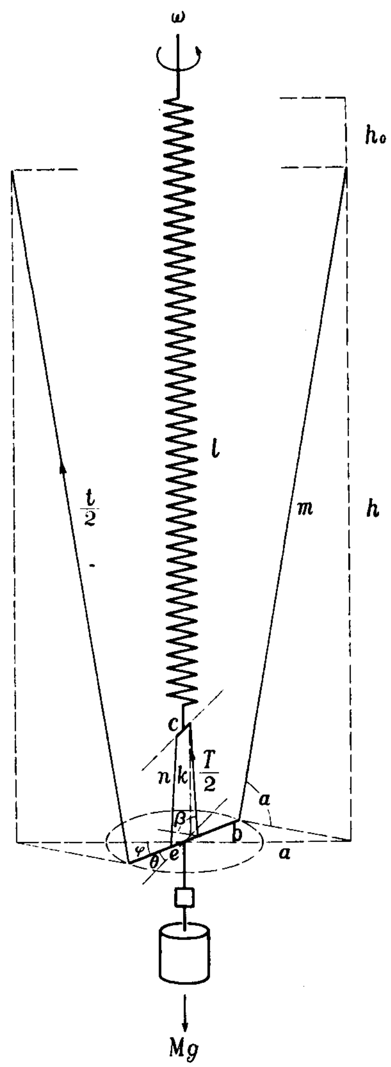

Fig. 2 b. Schema on construction of a double bifilar gravity variometer.

Applying the value of h calculated above to the expression of the radial displacement, $U = h\ W_2/g$ (W_2: tide-generating potential), at Kyoto, the tidal upheaval of the ground is estimated as follows:

Tidal upheaval of the ground at Kyôto for M_2-component: 21.3 cm.

Total tidal upheaval of the ground at Kyoto: 50.5 cm. And the greatest tidal upheaval of the ground that could occur on the earth is calculated as 51.5 cm.

4. Gravity variometer of double bifilar suspension type

In the preceding section, it is repeatedly mentioned that the precise observation of earth tidal change of gravity during a period of more than one year with a more sensitive gravity variometer than 1×10^{-9} g/mm of automatic recording is highly recommended for the development of the study of the earth tidal phenomena and the advancement of our knowledge on the nature of the earth's interior. For this purpose the double bifilar gravity variometer, for example, devised by ICHINOHE (1951, 1955) will certainly be serviceable, and its construction and function are briefly described in this section.

The double bifilar gravity variometer is the same in principle as the ordinary bifilar gravity variometer, but its sensitivity is largely and easily raised, and moreover stabilised compared with that of the ordinary bifilar gravity variometer, by applying the second inner bifilar suspension to the part between the main helical spring and the hanging bob as shown in Figure 2b. These two thin strings composing the second inner bifilar suspension play an important role in greatly raising the sensitivity of the instrument, and in addition, are effectively serviceable for minimizing the undesirable deflection caused by temperature changes and the drift, or plastic flow, of the instrument. This depends on the fact that the torsional force, which has a share in the causes of undesirable deflection and instrumental drift, is the couple produced by the horizontal components of tensional forces of strings in the case of the double bifilar gravity variometer in contrast to the elastic force produced by the torsion of a helical spring in the case of the ordinary bifilar gravity variometer.

The sensitivities for the two types of gravity variometer are approximately expressed as follows:

$$\frac{d\varphi}{dg} \cong -\frac{abM\sin\varphi}{mf+abp\cos\varphi}$$

for the ordinary bifilar suspension type,

$$\left.\begin{aligned} \frac{d\varphi}{dg} &\cong -\frac{\left(abM \pm \dfrac{mf}{gF}\right)\sin\varphi}{mf\left(1 \pm \dfrac{q\cos\varphi}{F}\right)+abqMg\cos\varphi} \\ \text{with } F &\equiv \sqrt{\frac{(cem)^2}{(abn)^2}-(q\sin\varphi)^2} \end{aligned}\right\}$$

for the double bifilar suspension type,

where φ denotes the turning angle of the bob; g the acceleration of gravity; M the mass of the bob; m and n the length of the outer and inner bifilar strings; f the torsional elastic constant of the helical spring; p the tension of the outer strings in the ordinary bifilar suspension type; q the ratio of the tension of outer bifilar strings to that of inner bifilar strings in the double bifilar suspension type; and a, b, c, e are the constructional constants having

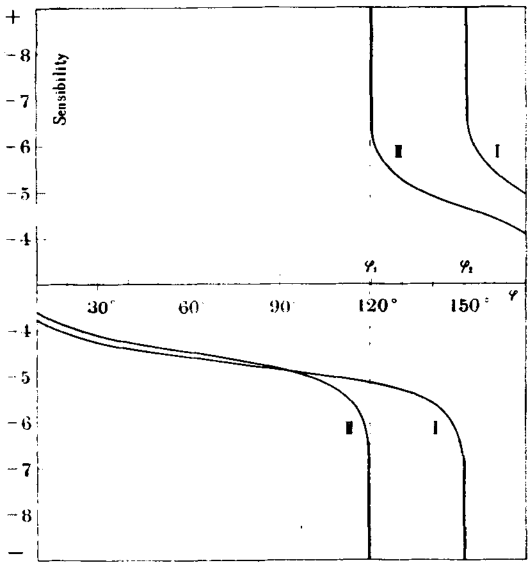

Fig. 3 a. Sensitivity curve of an ordinary bifilar gravity variometer.

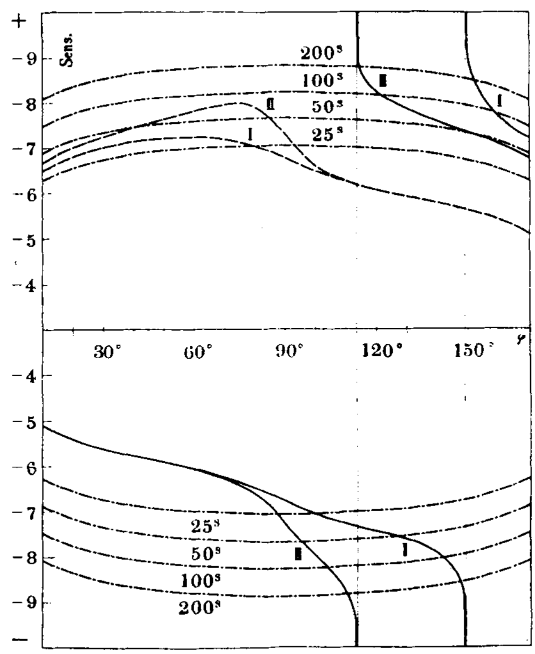

Fig. 3 b. Sensitivity curve of a double bifilar gravity variometer.

the dimension of length as shown in Figure 2b. And the relation between q, φ, and $\frac{d\varphi}{dg}$ in the double bifilar suspension type is represented in Figure 3b, with the sensitivity curve of ordinary bifilar type for comparison in Figure 3a, the numerical values of constructional constants adopted in the calculation being as follows: $g = 980$ cm/sec^2; $M = 50$ gr; $m = 130$ cm, $n = 10$ cm; $f = 1{,}000$ dyne cm; $a = 6$ cm, $b = 3$ cm, $c = e = 0.1$ cm; and $q = 7.0/1{,}000$ for curve I, and $= 7.2/1{,}000$ for curve II. In the Figure, the full-line curves correspond to the plus sign of the above sensitivity formula and the broken-line curves to the minus sign. The numerals in the ordinate indicate dg/g in the common logarithms, corresponding to 1 mm displacement of the optical image on the recording paper with 5 meters optical distance. The chain-line curves show the relation between the free oscillation period of the bob and $\frac{d\varphi}{dg}$ in case of $r^2 = 2$ cm^2, r being the radius of gyration of the bob. From the comparison of Figure 3b with Figure 3a, it is clearly understood that the sensitivity of the double bifilar gravity variometer is easily raised up with a great stability compared with that of ordinary bifilar type. A double bifilar gravity variometer was early in 1949 constructed for testing at the ordinary underground room of the building of the Abuyama Seismological Observatory attached to Kyoto University; its constructional constants were nearly the same as mentioned above in Figure 3b, and the sensitivity could be raised up to 3×10^{-9} g/mm/5 m with the free oscillation period of the bob of 200 seconds, but in practical observation the instrument was inevitably used in considerably reduced sensitivity by reason of the great disturbances due to microseisms, local ground tremors, and minute but restless oscillations and pressure changes of the surrounding air. And, moreover, the drift of the instrument caused by the plastic flow of the main spring was also greatly enlarged according to the raise of instrumental sensitivity, but the problem of drift was temporisingly solved by applying the artificial control at the part of outer bifilar suspension for recovering the permanent flow of the main spring. An example of record thus obtained is inserted in

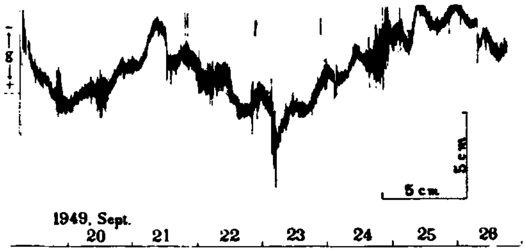

Fig. 4. An example of records obtained with a highly sensitive gravity variometer of double bifilar suspension type (sensitivity: 1×10^{-8} g/mm/2 m).

Figure 4, in which the effect of microbarometric change upon the instrument is largely observed.

Under such circumstances as above stated, observations with highly sensitive gravity variometer has again recently been planned, and, in the present schedule, the instrument should be evacuated at least to 0.1 mm Hg for avoiding buoyancy changes and convection of the surrounding air, and should be set up in a deep seated thermostatic room of more than 500 meters depth for avoiding the microseisms and ground tremors. The drift of the instrument is to be controlled by any suitable method of artificial regulation. In this way the sensitivity of the double bifilar gravity variometer will be raised to the degree of 3×10^{-10} g/mm/5 m by the multiple reflection method of recording, and the gravimetric observation with it is expected to be greatly useful for studying the earth tidal change and, if existing, non-periodic micro-variation of gravity. Gravimetric observations with super high sensitive gravity variometers at two stations are planned for the international cooperative observation period during the coming Geophysical Year (1957—1958), and they are also scheduled to be operated in conjunction with concurrent gravimetric observations with the station gravity variometers of the double bifilar suspension type having an ordinary sensitivity of 1×10^{-8} g/mm/2 m at ten suitable stations in our country, and with two portable pen recording gravimeters of the Askania Gs-11 type at more than thirty points in our country for detecting the local character of the G-values. On the results of these observations, a detailed report will be made in due time.

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