An Experimental Study of the Field Variation of the Eddy Conductivity

By ALF NYBERG and LARS RAAB
Swedish Meteorological and Hydrological Institute, Stockholm

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Abstract

By means of thermistors the temperature field is measured in cold air from the sea entering a strongly heated arid surface. The temperature at 1 m level at first increases and then decreases when the air moves further inland. From these measurements the field of the coefficients of eddy conductiv ty is calculated. The result shows that the eddy conductivity not only increases upwards but also in the horizontal direction being 3—4 times larger half a km from the shoreline than at the sea. In the Appendix a formula is deduced for determining the coefficient of eddy conductivity. In doing this the definition of A_z is somewhat modified considering the fact that the scales of temperature and potential temperature may differ, the difference depending upon the reference pressure of the potential temperature. The error of neglecting the effect of the reference pressure of the potential temperature in the definition of A_z is discussed.

1. Introduction

In local meteorological phenomena the eddy conductivity plays an important rôle. It depends on the distribution of local obstacles, on the existing temperature field, which may cause convective phenomena, and finally on the wind field. On the other hand the eddy conductivity influences the temperature and wind fields.

In our investigation the interaction between the fields of eddy conductivity and temperature resulted in a very peculiar effect not earlier observed as far as the authors know. Studying a rather strong and cold see breeze entering a shore line with a hot surface several kilometers inside, we have found that due to the heating from below the temperature at the level 1 m above the ground at first increases as the air moves inland. But several hundred meters inland the temperature again drops at this level. This fact may be due to the increase of convection as the air moves further inland.

If the local meteorological phenomenon to be investigated is small enough (of the dimension of centimeters and decimeters or some few meters) simple measurements can give information as to the temperature, wind and eddy conductivity. In such cases the phenomenon of transition of energy from the surface to the layers of air a few meters above can be studied.

When arriving at the problem of determining energy and eddy conductivity conditions in spaces of a dimension of several hundred meters or some few kilometers in the horizontal and some ten and hundred meters in the vertical, the problems of measuring all the quantities necessary for a correct determination of the eddy conductivity become overwhelming. The only possible way of facilitating the measurements in a field study of eddy conductivity is to find out ideal places of measurements where characteristic phenomena occur, and where the influence of local obstacles is a minimum. If the study of eddy conductivity is restricted to local strongly marked phenomena along some border line the mathematical problem may be simplified to a two-dimensional one.

2. Location of the measurements

In our investigation we have tried to find a place along the sea shore where the problem of determining the eddy conductivity is to a great extent reduced to a two-dimensional problem. Fig. 1 shows the localities in the neighbourhood of Gudesjö at the westcoast of the island of Öland in the Baltic. The coast-line is mainly north-southerly but at our observational site there is a wide bay with the shore line running northeast-southwest. Just at the shore there is a 20 m wide band of flat stones rizing only a few decimeters above the water, then there is a steep stony slope rising 5 m above the water. Then follows a wide sparsely vegetated arid plain, where only some juniper bushes grow. Four masts were placed in a straight line along a small road, not quite perpendicular to the shore line. The effective distance of the masts from the shore line is then reduced but this is partly compensated for by the fact that the wind was mainly southwesterly during the time of observation. The path of the wind overland from mast to mast has its maximum value when the wind direction is 250°.

The purpose was to measure the temperature lapse rate and the variation of it as the air moves inland and from these values and from approximate wind measurements at the third mast in the 10 m level, some conclusions will be drawn as to the variation of the eddy conductivity with the distance from the shore.

3. Apparatus of measurements

The four masts were of a type, very suitable for meteorological observations. They were made by thin aluminium tubes and of a very small weight, thus easy to transport. They were constructed by the Royal Swedish Board of Civil Aviation for air port illumination purposes and were manufactured in sections of 5 meters, easy to put together at the place of observation.

Fig. 2 shows a vertical plane through the four masts. In this figure the thermistors are marked with small dots. In all there were 16 thermistors of type Western Electric 17 A, which were coupled to a Honeywell-Brown 6-point recorder. By means of auxiliary synchronous relays it was possible to record up to 20 points in one cycle on the recorder. On a special recorder also wind speed (10-minutes-mean) and wind direction were obtained.

From the records, the vertical mean-temperature field has been evaluated for periods of one hour. In order to compute the values of eddy conductivity in different heights, some

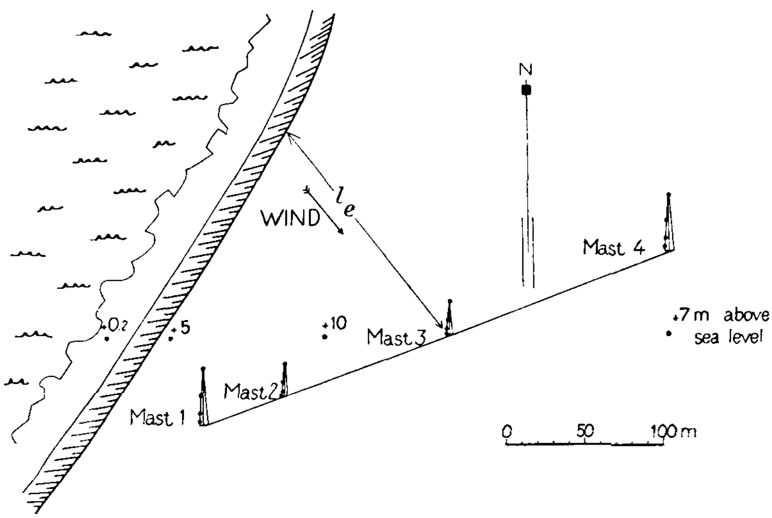

Fig. 1. The Öland-expedition 1954. Observational site. l_e = effective length.

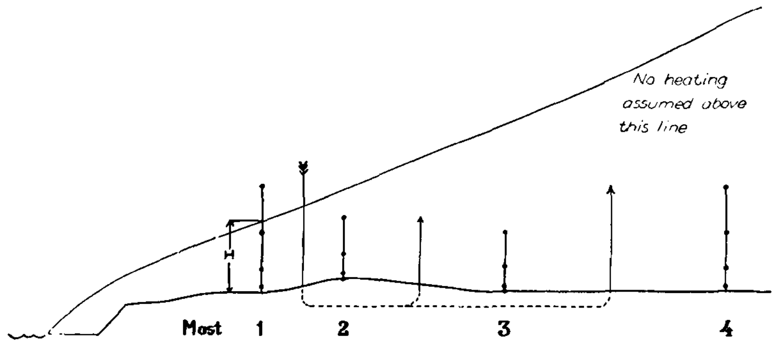

Fig. 2. Vertical section with path of integration. (See the text).

mathematical deductions have to be made. These may be found in the appendix where the case of mean motion along the vertical section is treated. Some other simplifications are also introduced.

In the decuction a somewhat unusual definition of A_z is introduced, this because the scale of potential temperature depends on the standard pressure in question. As c_p refers to absolute temperature the following definition of A_z has to be used

$$\overline{U}_t = -c_p A_z \cdot \frac{\partial \overline{\Theta}}{\partial z} \cdot \frac{\overline{T}}{\overline{\Theta}}$$

where $\overline{T}/\overline{\Theta}$ is some sort of a scale factor. $\overline{U}_t$ is the vertical mean transport of energy. $\overline{T}$ is the absolute mean-temperature $\overline{\Theta}$ is the potential mean-temperature referring to *any* standard pressure.

4. The field of eddy conductivity

By means of the equation (2) in the appendix

$$-A'_z \cdot \frac{\partial \overline{\Theta}}{\partial z} = \int_z^H \varrho u \frac{\partial \overline{\Theta}}{\partial z} dz$$

it is possible to deduce the mean value A'_z at the heights of measurements in the space between the different masts. At first mean values of Θ have to be determined from the series of records obtained. In general 10 measurements per point were obtained in an hour of measurement. From the Θ-values, values of $\frac{\partial \overline{\Theta}}{\partial x}$ and $\frac{\partial \overline{\Theta}}{\partial z}$ were obtained in the regions between the masts. From the wind records u was obtained and so the integral could be evaluated. The integral corresponds to the upward energy flux divided by c_p. Now some of the energy gained from below is transported above the top of the masts, especially for the masts at greater distance from the shore. This vertical transport is smaller near the shore-line and vanishes above a certain level (cf. fig. 2). As we do not exactly know the integral limit $z = H$ we put the energy flux equal to zero at the top of mast nr 1 and integrate down to surface level. We may then introduce an error and the value of A may be too small. If we assume the energy transport at the top of the first mast not to be equal to zero, a rough computation shows that the computed A_z-values will be modified in such a way that if the energy transport at the top is assumed to be half of the energy transport at ground surface the computed values of A_z become twice as large. However, no significant difference could be found between the water temperature and the temperature at the first mast at the level 18 m. The value at the surface level of the integral is then assumed to be the same also between mast nr 2 and 3 and 4 as radiational and ground conditions were equal everywhere. From this integral value we determine the other integral values at the heights of measurement and obtain finally the A'_z-values. The paths of integration are shown in fig. 2.

The 1st and 2nd of July 1954 were days with great insolation. Fig. 3 shows an exampel of a vertical cross-section and also the effective length l_e (Fig. 1) from the shore line of the different masts. The effective length is taken along the direction of air flow and replaces

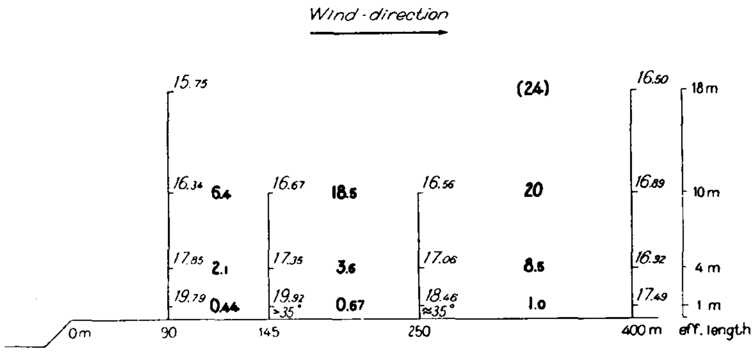

Fig. 3. Mean temperatures at the 4 masts at various heights and mean values of vertical eddy conductivity at different levels between the masts. The measurements are made in the period 11.30—12.30 GCT, July 1954.

x in the equation of A'_z. The figures in the space between the masts are the evaluated values of A'_z. Five vertical cross-sections based upon five hours of measurement were obtained in this way and Fig. 4 shows the mean-values of A'_z obtained in these five cases. The increase of A'_z with distance from the shore is striking, the values being $2-4$ times 500 m from the shore in comparision with the value at a distance of 100 m from the shore. The value of $A'_z = 0{,}2$ cgs-units at the surface level is very uncertain because no accurate measurements of vertical temperature lapse rate were made there.

Sources of errors. There are many sources of errors in the determination of the values of A'_z. The most important ones are listed below.

1) *Time-mean-values.* The temperatures have to be determined with an accuracy of at least 0,05° C. If the time-mean-value of temperature does not include a sufficiently number of measurements one single measured extreme value could damage the whole time-mean-value. In our case we had to use a time interval of more than one hour to get the effect of extremes eliminated. In such a long time the mean-conditions, especially if the wind changes, may cause errors in determining all the quantities in the equation. To overcome this difficulty in all cases a faster multirecorder has to be used. However, in our measurements wind conditions presumably were sufficiently stable.

2) *No measurements of vertical transport at the top of masts.*

The transport of energy above the top level of the first mast causes an error in the determination of A'_z. This error has already been discussed and may be overcome by using some kind of instrument measuring the vertical eddy exchange at the top of the mast. Measurements above the top heights of the masts will also give information as to the correction of the integral value. Such measurements must be made by means of captive balloons or other air-borne means, e.g. radiosondes. However such measurements are very expensive and do not give an accurate value of the temperature field without numerous and systematic measurements.

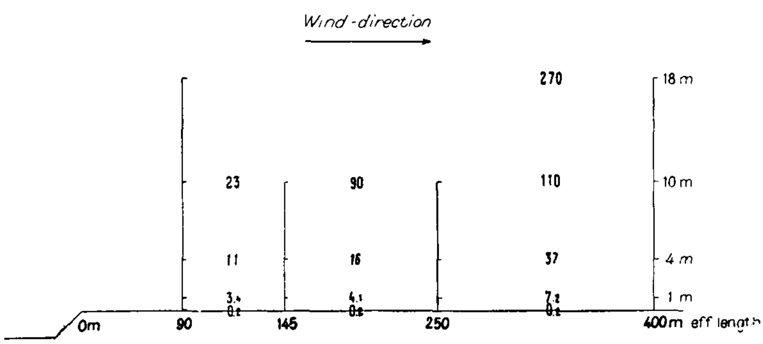

Fig. 4. Mean values of vertical eddy conductivity between masts at different levels.

3) *The measurement of* $\frac{\partial \overline{\Theta}}{\partial z}$. The measurement of the vertical temperature lapse rates in the lowest layers are made from graphs drawn by means of the measured absolute values at the different heights. Instead of measuring the temperature values separately, some suitable temperature-difference instrument would increase the accuracy of measuring the gradient.

4) *The three-dimensional problem.* When the wind is entering in an oblique angle to the shore line the problem is a three-dimensional one. We have reduced it to a two-dimensional one by using the concept of effective length and assuring that surface conditions are uniform over a large area. In order to obtain accurate values of A'_z this procedure is not sufficient, we have to measure also the effect of transport of energy perpendicular to the line of the masts. For this purpose it would be necessary to use a system of double rows of masts with an intermediate distance of about 100—200 m.

Appendix

By LARS RAAB

The equations of energy flux. Regard a non-movable volume in a streaming media with the volume V and a surrounding surface S. An infinitesimal volume element is denoted with dV and the corresponding surface element with dS.

The energy gained through radiation in the total frequency range is per unit mass

$$\int_0^\infty \alpha_\nu E_\nu d\nu$$

where α_ν is the coefficient of absorption and E_ν the black-body radiation in the frequency interval ν to $\nu + d\nu$. For the volume element dV we obtain

$$\varrho dV \int_0^\infty \alpha_\nu E_\nu d\nu$$

We denote the absorbed radiant energy per unit mass and unit time with U_r and we get instead of the last expression:

$$\varrho U_r dV$$

The energy gained through molecular conduction of heat is the surface integral of the heat current U_s (positive outwards) into the volume V

$$-\iint_S U_s dS$$

which can be written (Green's formula):

$$-\iiint_V \left(\frac{\partial Usx}{\partial x} + \frac{\partial Usy}{\partial y} + \frac{\partial Usz}{\partial z}\right) dV =$$

$$= \iiint \frac{\partial U_{si}}{\partial x_i} dV$$

(Einstein's summation symbol is used here.)

The energy gained in the volume element dV is:

$$-\frac{\partial U_{si}}{\partial x_i} dV$$

U_{si} is the heat flow in the direction of the i-axis. The energy gained per unit time in the volume element dV through radiation and conduction is then

$$\left(\varrho U_r - \frac{\partial U_{si}}{\partial x_i}\right) dV = \varrho \frac{\partial Q}{\partial t} dV$$

The internal energy will then increase, a process which is influenced by the fact that the air expands,

$$\frac{\partial Q}{\partial t} \varrho dV = \left(c_v \frac{dT}{dt} + p \frac{d\alpha}{dt}\right) \varrho dV =$$

$$= \left(c_p \frac{dT}{dt} - \alpha \frac{dp}{dt}\right) \varrho dV$$

We introduce the potential temperature and obtain

$$c_p \frac{T}{\Theta} \frac{d\Theta}{dt} = c_p \frac{dT}{dt} = \alpha \frac{dp}{dt}$$

$$\frac{\partial Q}{\partial t} \varrho dV = c_p \frac{T}{\Theta} \frac{d\Theta}{dt} \varrho dV$$

The equation of energy flux is then

$$\varrho U_r dV - \frac{\partial U_{si}}{\partial x_i} dV = c_p \frac{T}{\Theta} \frac{d\Theta}{dt} \cdot \varrho dV$$

Divide with $\varrho\ dV$ and we get the energy gain of unit mass

$$U_r - \frac{1}{\varrho}\frac{\partial U_{si}}{\partial x_i} = c_p \frac{T}{\Theta}\frac{d\Theta}{dt}$$

In case of turbulent motion the gain of energy is divided into a mean-gain of a certain time and space dimension and a gain resulting of the eddy motion. All quantities have to be divided into a time-mean-value and a deviation from it, e.g.:

$$\varrho = \varrho + \varrho'$$
$$\Theta = \overline{\Theta} + \Theta'$$

and so on

We get

$$\overline{U}_r + U_r' - \frac{1}{\overline{\varrho} + \varrho'}\frac{\partial(\overline{U}_{si} + U'_{si})}{\partial x_i} =$$
$$= c_p \frac{\overline{T} + T'}{\overline{\Theta} + \Theta'}\left[\frac{\partial\overline{\Theta}}{\partial t} + \frac{\partial\Theta'}{\partial t} + \overline{v}_i'\frac{\partial\overline{\Theta}}{\partial x_i} + \overline{v}_i\frac{\partial\Theta'}{\partial x_i} + \right.$$
$$\left. + v_i'\frac{\partial\overline{\Theta}}{\partial x_i} + v_i'\frac{\partial\Theta'}{\partial x_i}\right]$$

The fluctuations in U_r and U_{si} are neglected

$$U_r' = U_{si}' = 0$$

The pressure fluctuations are also neglected. Then we have

$$\frac{T'}{T} = \frac{\Theta'}{\Theta} \text{ and } \frac{T + T'}{\Theta + \Theta'} = \frac{\overline{T}}{\overline{\Theta}}$$

Furthermore we assume homogenous conditions in the direction of the y-(2)-axis. In the practical case described in this paper the shoreline is assumed to be parallel with the y-axis. The eddy exchange in this direction is then neglected and we may thus simplify the equation to

$$\overline{U}_r - \frac{1}{\varrho}\left(1 - \frac{\varrho'}{\overline{\varrho}}\right)\frac{\partial\overline{U}_{si}}{\partial x_i} = c_p\frac{\overline{T}}{\overline{\Theta}}\left(\frac{\partial\overline{\Theta}}{\partial t} + \frac{\partial\Theta'}{\partial t} + \right.$$
$$+ \overline{u}\frac{\partial\overline{\Theta}}{\partial x} + \overline{u}\frac{\partial\Theta'}{\partial x} + u'\frac{\partial\overline{\Theta}}{\partial x} + u'\frac{\partial\Theta'}{\partial x} + \overline{w}\frac{\partial\overline{\Theta}}{\partial z} +$$
$$\left. + \overline{w}\frac{\partial\Theta'}{\partial z} + w'\frac{\partial\overline{\Theta}}{\partial z} + w'\frac{\partial\Theta'}{\partial z}\right);$$

Repeating the mean-value-process we get:

$$\overline{U}_r - \frac{1}{\varrho}\frac{\partial\overline{U}_{si}}{\partial x_i} = c_p\frac{T}{\overline{\Theta}}\left(\frac{\partial\overline{\Theta}}{\partial t} + \overline{u}\frac{\partial\overline{\Theta}}{\partial x} + \overline{w}\frac{\partial\overline{\Theta}}{\partial z} + \right.$$
$$\left. + \overline{u'\frac{\partial\Theta'}{\partial x}} + \overline{w'\frac{\partial\Theta'}{\partial z}}\right)$$

or

$$\overline{U}_r - \frac{1}{\varrho}\frac{\partial\overline{U}_{si}}{\partial x_i} - c_p\frac{\overline{T}}{\overline{\Theta}}\left(\overline{u'\frac{\partial\Theta'}{\partial x}} + \overline{w'\frac{\partial\Theta'}{\partial z}}\right) =$$
$$= c_p\frac{\overline{T}}{\overline{\Theta}}\left(\frac{\partial\overline{\Theta}}{\partial t} + \overline{u}\frac{\partial\overline{\Theta}}{\partial x} + \overline{w}\frac{\partial\overline{\Theta}}{\partial z}\right)$$

The fourth term on the left hand side corresponds to the eddy vertical transport of energy. Quite formally it can be looked upon as a mechanism of the same kind as heat conduction. The eddy transport in the direction of the x-axis per unit time and unit surface is formally written:

$$U_t = -c_p A_x \frac{\partial\overline{\Theta}}{\partial x}\cdot\frac{\overline{T}}{\overline{\Theta}} \text{ (no summation)} \quad (1)$$

where A_x is the coefficient of horizontal eddy conductivity. In the definition of A_x the right side of the formula is multiplied with the factor $\frac{\overline{T}}{\overline{\Theta}}$. Often this factor is neglected which of course if permissible if $\overline{T} = \overline{\Theta}$, that is studying eddy conditions at surface pressures. If the formula should be correct at *any* pressure we should remember that c_p refers to conventional temperature in computing energy gain in an air parcel at constant pressure. The scale of $\overline{\Theta}$ may not be the same as the scale of conventional temperature and hence $\frac{\partial\overline{\Theta}}{\partial x}$ has to be multiplied with $\frac{\overline{T}}{\Theta}$ to give a correct value of the eddy energy transport. Compare the equation expressing the energy gain in the volume element dV. Here the factor $\frac{\overline{T}}{\Theta}$ is obtained through deduction of the formula.

The fact that $\frac{\overline{T}}{\overline{\Theta}}$ is equal to a scale factor can be seen from the formula of potential temperature

$$\frac{T^{\varkappa}}{p^{\varkappa-1}} = \frac{\Theta^{\varkappa}}{p_0^{\varkappa-1}}$$

were p_0 is the standard pressure.

If p and p_0 are constant we get

$$\frac{dT}{T} = \frac{d\Theta}{\Theta} \quad (p = \text{constant})$$

$$\text{and } \frac{\Theta}{T} = \frac{d\Theta}{dT}$$

which is the ratio between the two scales of temperature.

Now we use the new definition of A_z to compute the energy gained in the volume element dV:

$$c_p \frac{\overline{T}}{\overline{\Theta}} \left[\frac{\partial}{\partial x}\left(A_x \frac{\partial \overline{\Theta}}{\partial y}\right) + \frac{\partial}{\partial y}\left(A_y \frac{\partial \overline{\Theta}}{\partial y}\right) + \right.$$

$$\left. + \frac{\partial}{\partial z}\left(A_z \frac{\partial \overline{\Theta}}{\partial z}\right) dV \right.$$

In cour case is $\dfrac{\partial \overline{\Theta}}{\partial y} = 0.$

If we want to have the energy gain per unit mass, we have to divide with ϱdV and we get

$$\frac{c_p}{\overline{\varrho}} \left[\frac{\partial}{\partial x}\left(A_x \frac{\partial \overline{\Theta}}{\partial x}\right) + \frac{\partial}{\partial z}\left(A_z \frac{\partial \overline{\Theta}}{\partial z}\right) \right] =$$

$$= -c_p \left(\overline{u' \frac{\partial \Theta'}{\partial x}} + \overline{w' \frac{\partial \Theta'}{\partial z}} \right);$$

The equation of energy flux will then be:

$$\overline{U}_r - \frac{1}{\overline{\varrho}} \frac{\partial \overline{U}_{si}}{\partial x_i} + \frac{c_p}{\overline{\varrho}} \frac{\overline{T}}{\overline{\Theta}} \times$$

$$\left[\frac{\partial}{\partial x}\left(A_x \frac{\partial \overline{\Theta}}{\partial x}\right) + \frac{\partial}{\partial z}\left(A_z \frac{\partial \overline{\Theta}}{\partial z}\right) \right] =$$

$$= c_p \frac{\overline{T}}{\overline{\Theta}} \left(\frac{\partial \overline{\Theta}}{\partial t} + \overline{u} \frac{\partial \overline{\Theta}}{\partial x} + \overline{w} \frac{\partial \overline{\Theta}}{\partial z} \right)$$

We now make the following assumptions:

$$\overline{U}_r = 0.$$

We count quite formally the molecular transport of heat into the eddy transport of energy and denote:

$$A'_z = A_z + \frac{\lambda}{c_p}$$

$$A'_x = A_x + \frac{\lambda}{c_p}$$

where λ is the coefficient of molecular heat conduction. Furthermore we assume stationary conditions:

$$\frac{\partial \overline{\Theta}}{\partial t} = 0 \text{ and } \overline{w} = 0$$

We then get the following equation

$$\frac{1}{\overline{\varrho}} \left[\frac{\partial}{\partial x}\left(A'_x \frac{\partial \overline{\Theta}}{\partial x}\right) + \frac{\partial}{\partial z}\left(A'_z \frac{\partial \overline{\Theta}}{\partial z}\right) \right] = \overline{u} \frac{\partial \overline{\Theta}}{\partial x}$$

Some estimates can be made of the order of magnitude of the two terms on the left hand side in this equations assuming A'_x being equal to A_z. The second derivatives of $\overline{\Theta}$ with respect to x and z are of the magnitudes of 10^{-4} and 10^{-1} degrees C per m². The derivatives of A_z with respect to x and z are of the magnitudes of 10^{-1} and 10 cgs-units, obtained by neglecting the term with A_x. Hence the second term is 100 times larger than the first one justifying the neglect of it. We thus may use the simplified formula:

$$\frac{1}{\overline{\varrho}} \frac{\partial}{\partial z}\left(A'_z \frac{\partial \overline{\Theta}}{\partial z}\right) = \overline{u} \cdot \frac{\partial \overline{\Theta}}{\partial x}$$

Integrating from the height H downward to the height z we obtain

$$-A'_z \cdot \frac{\partial \overline{\Theta}}{\partial z} = \int_z^H \overline{\varrho} \overline{u} \frac{\partial \overline{\Theta}}{\partial x} dz \qquad (2)$$

if $\dfrac{\partial \overline{\Theta}}{\partial z} = 0$ for a sufficiently high value $z = H$. The formula (2) is then used to treat the problem of variation of eddy conductivity normal to a coastline with different heating of the surface air-layers.

The measurements described are made at normal surface pressure. In this case $\overline{T} = \overline{\Theta}$ and the use of the more complicated formula (1) is from a practical point unnecessary. However, as soon as Θ and T are different there will be an error in A_x if old formula of the type

$$U_t = -c_p A_x \frac{\partial \Theta}{\partial x}$$

are used. This error (denoted by e) will be equal to $e = (\overline{\Theta} - \overline{T})/\overline{\Theta}$. This means that when studying eddy conductivity at a pressure of 900 mb instead of 1000 mb $\left(\frac{d_p}{p_0} = -10\ \%\right)$ we have an error in $A_z = 3\ \%$ when using the old formula and referring Θ to 1000 mb. At $p = 500$ mb the corresponding error will be $= 15\ \%$. Hence the necessity of using formula (1) in studying upper air eddy conductivity becomes quite clear.

Studying only surface phenomena at normal pressures near the sea level (p between 900 and 1100 mb) the error in $A_z = 3\ \%$ may be neglected and the simple formula may be used without introducing any serious error.

REFERENCES

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