On the Maintenance of the Trade Winds

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Abstract

A relation between the large-scale features of the lower trade stream and the effects of small-scale convective motions is investigated. The small-scale processes impose constraints upon the over-all circulation through non-adiabatic heating and the production of shearing stresses. These constraints may be formulated in terms of a theoretical model describing the mean flow in a section along the northern portion of the Pacific trade, which has previously been studied observationally in some detail.

In a simplified frictionless model with non-adiabatic heating the flow exhibits qualitatively many of the important features of the trades, such as subsidence, and a downstream acceleration, increased vertical wind shear, and pressure drop. The motion and pressure fields become numerically related to the heating as observed, however, only upon the introduction of frictional stresses. It is found possible to describe the frictional stress distribution in the lower trades without resorting to direct measurements, which thus provide a later independent check of the model.

The last sections of the paper consider possible quasi-steady rearrangements between the fields of motion, heating, and stress which might be regarded as steps in the slow fluctuations of the system about its average condition. This is done to inquire whether changes in any one of these properties will produce readjustments in the others acting to restore the mean or to accelerate the departure from it. Although such an analysis is handicapped by lack of knowledge concerning the interaction between the heating and stress distributions, one sample calculation suggests that a reduction of heating acts to relieve the subsidence and might thus be expected to restore the convection and thereby the heating. A stable coupling between small- and large-scale motions in the low-level trades is thus indicated, which may play a role in maintaining the high steadiness of the flow. The fact that the wind steadiness vanishes rapidly above the convective layer is clarified if this model is acceptable.

1. Introduction

As the trade winds flow westward and equatorward they accumulate latent heat in the form of water vapor. This stored energy, when released and exported to distant places plays an important part in creating the temperature gradients which drive the general circulation. In the trades themselves, cumulus convection and convective turbulence below cloud base act as the upward distributors of the moisture, originally supplied to the lowest air by evaporation from the sea. These points were brought out in a budget study of the poleward half of the north-east Pacific trade (RIEHL, YEH, MALKUS, and LA SEUR, 1951), hereafter referred to as I. Energy-wise, and as far as mid-latitudes are concerned, vapor ac-

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cumulation is perhaps the most important function of the trades. Nevertheless, they are also accumulating sensible heat in their low levels, as is evidenced by the increase in potential temperature along the air trajectories. Even in the rather northerly section studied in I, the non-adiabatic warming in the bottom 3 km amounts to 3° C in five days (or 2,500 km horizontal travel) and RIEHL (1956) has recently shown that the consequent sensible heat accumulation amounts to about 25% that of the latent. It will be the purpose of this paper to illustrate the importance of this non-adiabatic heat source in the dynamics of the easterly circulation and to suggest the role played by convection in bringing about and distributing the heating. The concomitant role of convective elements in producing frictional stresses also proves important in constructing a theoretical model of the low-level trades.

In order to set up the primary questions which any theoretical model must attempt to answer, we shall begin by reviewing briefly some of the outstanding features of the low-latitude circulation in the northern hemisphere. Between the equator and about 30° N, the existence of a meridional, Hadley-type circulation cell has been confirmed. The work of PISHAROTY (1955) and others has shown that this cell operates in an energy-producing sense. Their figures show that it not only supplies enough energy to overcome its own losses and drive its own motions, but enough to overcome the net energy consumption in the westerlies. Their calculations provide evidence that large amounts of sensible heat and a significant quantity of kinetic energy are exported northward at high levels across latitude 35°. This "driving" cell differs from the other large branches of the general circulation in exhibiting far greater steadiness and relative freedom from travelling disturbances, particularly in the lower and more northerly portions. Its low-level winds, the northeast trades, are in fact the world's most constant wind system, exhibiting steadiness coefficients of 80—90%. The easterlies generally weaken at upper levels and their steadiness gives way to marked fluctuations aloft. The field of vertical motion is as expected, with subsidence in the poleward portions, giving way to ascent as latitude 15° is passed. North of 15°, interaction between the subsidence and counterworking convection from below produces the striking vertically-layered structure which is one of the outstanding features of tropical meteorology. The lower, convection-dominated layer is moist, only slightly stable statically, and contains disturbances of small horizontal scale (from eddies through trade cumuli to convection cells) possessing marked vertical motions. The upper layer is dry, more stable, and contains disturbances of synoptic scale (such as easterly waves) possessing much smaller vertical motions. The upper limit of the convection constitutes the boundary region; it is normally a few hundred meters in thickness, much more stable than either layer, and is called the "trade inversion".

Only that portion of the poleward branch of the Hadley cell will be treated here which was examined in I. That cross section covered a horizontal distance of 2,500 km along the mean summer trajectory of the Pacific trade from 32° N 136° W to Honolulu, Hawaii (21° N 158° W). Vertically, it extended from the sea surface to 3 km, including thus the bottom convective layer, the transition region, and the lowest part of the upper dry layer. By this restricted choice we hope to have selected a simple enough situation, and one covered by sufficient observational material,

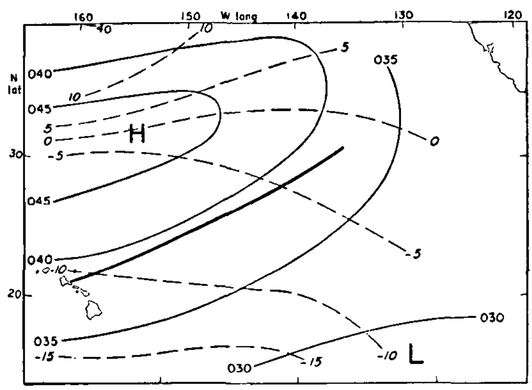

Fig. 1. Map showing orientation of trade-wind section studies. Four sounding stations were spaced out along the heavy solid line, which closely follows the air trajectory for the lowest 3 km. The Hawaiian Islands are shown at the downwind termination, while the U.S. west coast appears in the upper right-hand corner. The lighter solid lines are mean 700 mb contours (10's ft, first digit omitted) while the dashed lines are isopleths of gradient wind speed (m/sec, positive values have component toward east) and thus parallel the sea-level isobars. (After RIEHL et al., 1951.)

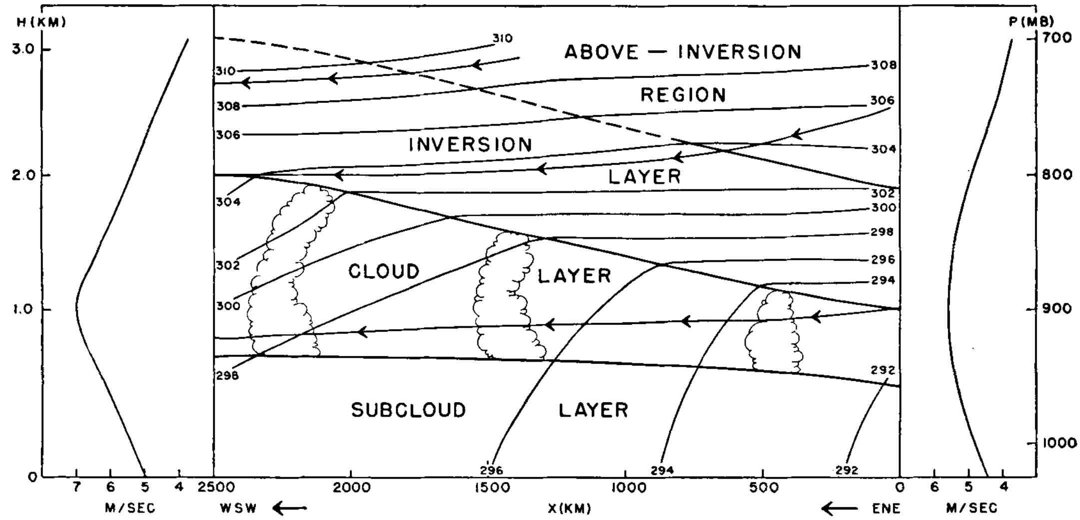

Fig. 2. Vertical structure of the air along the trajectory indicated in Fig. 1. Horizontal distance given in km downstream of entrance end. Vertical coordinate to right in mb, to left in km (approximate). The heavy lines separate the layers described in text. The lines with arrows are trajectories while the lighter solid lines are potential temperature isopleths labelled in degrees absolute. Trade cumulus clouds are entered schematically. To the right is the profile of wind speed (along the section) at the upstream end and to the left is the wind speed profile at the downstream end. Wind speeds are in m/sec.

to frame and test a theoretical model relating several of the predominant processes. To be specific, we shall inquire how the heating, the motions, and the pressure field are related within this section. These relations may in turn provide some clue concerning how this part of the Hadley cell manages to exhibit the structure and perform the transports which the observations tell us that it does. The procedure will be as follows: we shall examine what must happen within such a section to an air-stream which has given properties upon entrance so that it comes out the other end changed in the observed manner. Most importantly, we shall relate the vertical motions to the heating and in so doing demonstrate that the latter is vitally necessary for this portion of the Hadley cell to show subsidence.

In the treatment of a limited section of a larger-scale circulation branch, some features of the over-all flow must be introduced as assumptions. We wish to find out how these over-all features are constrained by the processes going on within this section.

Figures 1—3 summarize the structure of the section, derived from the observations and analyses of I. Those properties which must eventually be related or predicted in a complete theoretical treatment are:

1. The nearly two-dimensional character of the flow. The curvature of the trajectory is not large, and the results of I strongly suggest that the cross-stream divergence is considerably smaller than the downstream divergence.

2. The vertically-layered stability structure. The static stability is close to neutral in the moist layer (nearly dry adiabatic below cloud base and close to moist adiabatic in the cloud layer) with strongest stability in the inversion region, decreasing again above but still positive.

3. The presence of non-adiabatic heating, indicated by the crossing of the parcel trajectories toward higher potential temperature.

4. Downstream increase of east wind speed at all levels below 3 km, with the greatest increase at about 1 km. A vertical profile showing maximum east wind just above cloud base, decreasing upwards and downwards.

5. Subsidence throughout the section, of the order of magnitude of 50 m/day.

6. Extreme wind steadiness in the low levels, decreasing markedly above the cloud tops. Weakness or absence of synoptic disturbances in the summer months. Unlike the westerlies,

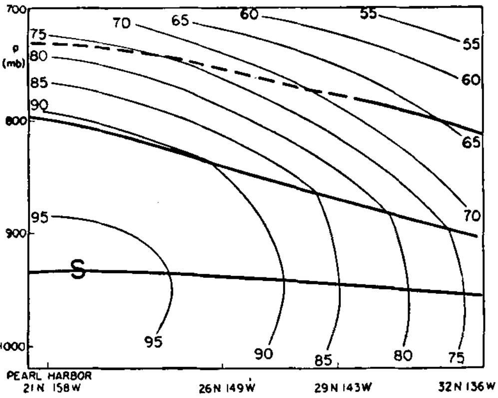

Fig. 3. Vertical cross section of wind steadiness (percent, as defined in I) for the section of Figs. 1 and 2. Heavy lines denote the layers, as in Fig. 2. The vertical coordinate is pressure in mb and the latitude and longitude of the sounding points appear along the bottom. (After RIEHL et al., 1951.)

these can play no role in the maintenance of the flow.

7. A downstream drop in pressure of nearly 5 mb at the ground diminishing upward and disappearing entirely at 3 km. Relative to the trajectory (looking downstream) a counterclockwise rotation of the isobars with height, until they become parallel near the section top. This is another feature of the trades which distinguishes them sharply from the westerlies of middle latitudes.

The problem faced in this paper will be to relate as many as possible of these features theoretically using the hydrodynamic equations. In such a limited treatment as undertaken here, however, it will prove necessary to introduce the first two of the listed properties by assumption and the third, namely the non-adiabatic heating, by "plugging in" observed values. This will result in an overspecified situation, rendering superfluous one equation of motion. The key assumption of the work is probably that of two-dimensional motion, without which the problem would not be soluble. The observations in I, however, provide justification for this limited portion of the atmosphere. The extreme wind steadiness and relatively straight trajectory suggest that the divergence component parallel to the mean motion is very close to the total divergence. This hypothesis was tested in I when the mean vertical motion was computed from this component of the divergence and used satisfactorily in balancing the momentum and water vapor budgets. Estimations of the cross stream divergence (from the slight downstream fanning of trajectories) are in the same sense and about 10 % of the magnitude of the downstream divergence up to the level of the trade inversion, above which it may increase markedly. The reason, if any, for the nearly two-dimensional character of the lower trades must await more fundamental global treatments. Nevertheless, certain important aspects of their structure may be clarified using the restricted approach to be outlined. The types of questions we may hope to shed light upon include the following:

What is the role of the non-adiabatic heating in the dynamics and the thermodynamics of the section? What part do the clouds and smaller-scale turbulent elements play in releasing and distributing the heat? Is the heating vital in maintaining the flow against friction and in accelerating it downstream? Is it purely coincidental that the wind steadiness drops off so rapidly just above the cloud tops and coincidental that the maximum easterly wind is normally observed near cloud base? These questions may be combined in the following one which is tractable to theoretical analysis, namely: Does the heating produced by small-scale convection affect or constrain the larger-scale motions, and if so, is this a stable coupling? For example, if the convection is suppressed will the resulting changes in the large-scale motions be such as to restore it (i.e. perhaps weaken the subsidence rather than strengthen it)?

II. The Premises of the Theory

We are now going to set up a theoretical model for the two-dimensional mean flow in such a section, using the equations of motion, state, continuity, and the first law of thermodynamics containing non-adiabatic heating. Since the velocities solved for occur on a much larger space and time scale than those hypothesized to be responsible for the heating, the latter may, to first order, be introduced as a function of the coordinates only. The fact that the motions in the atmosphere ultimately regulate its heat sources and sinks is of course recognized (MALKUS, 1955 a) and the interaction between motion and sources in maintaining the steadiness of the flow will be considered later. We will first show that the heating is necessary to specify the motion field in the average or steady-state condition. To get the observed motion field and pressure drop downstream related correctly to the observed values of the heating, it will also be necessary to include frictional stresses, as will be seen.

Of course, the necessity for subsidence in a broad, equatorward-flowing stream is obvious from other considerations. If the relative vorticity is small compared to the Coriolis parameter, potential vorticity conservation requires descent. But the rate of descent is not prescribed until the exact trajectory and rate of equatorward flow are known. That is to say, the constraint imposed by the requirement of potential vorticity conservation is not sufficient uniquely to determine the motions: we could, in fact, have an infinite number of equatorward trajectories all obeying it. The law of potential vorticity conservation is derived from combining the first two equations of motion and the equation of continuity. There is another constraint, to be derived here, essentially between the first and third equations of motion and the first law of thermodynamics, which will require a specified distribution of subsidence for a given distribution of heating. In other words, for the air to select between the infinite number of equatorward trajectories, each conserving potential vorticity, it is required to prescribe the relation between heating and vertical motion. *The air must pick an equatorward trajectory such that the heating is adequate to give the subsidence which potential vorticity conservation requires for that particular trajectory.* This conclusion is only quantitatively modified by the presence of friction, which in practice merely intensifies the amount of heating required.

To solve the problem in a truly general and complete manner, all these constraints should be applied to the air at once, without specifying the trajectory; and the trajectory, the steady condition, the two-dimensionality, pressure drop, wind-maximum, and so forth should all be derivable. The writer believes that this is beyond tractable analytical methods, at least at present, and that an experimental or numerical attack like that of PHILLIPS (1956) offers the best hope. The present, simple-minded approach may guide such future endeavors.

Here we shall treat the air flow along the *observed* trajectory of Figure 1, which we know from measurement obeys the vorticity conservation law. We shall relate the heating and motion field therein, to see if by assuming the first from observation we can construct a model giving the second. Thus the latitude does not enter explicitly as it would in a more general formulation, but only implicitly via the choice of coordinate axes.

III. The Frictionless Model

When the coordinates are chosen such that the x-axis points downstream along the surface trajectory (the trajectory rotates negligibly with height up to the top of the layer considered), the y-direction points cross stream, and the z-direction upward to form a right-handed system, the hydrodynamic equations of motion, continuity, and state for steady, two-dimensional flow on a rotating system are

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\varrho}\frac{\partial p}{\partial x} \tag{1}$$

$$u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\varrho}\frac{\partial p}{\partial z} - g \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = -\frac{1}{\varrho}\frac{d\varrho}{dt} \tag{3}$$

$$p = \varrho RT \tag{4}$$

where the symbols all have their conventional meanings. The non-adiabatic heating, V, in cal/gm sec, may be evaluated as follows:

$$\frac{V(x, z)}{c_p} \simeq \frac{d\Theta}{dt} \tag{5}$$

where c_p is the specific heat of air at constant pressure and Θ is the potential temperature. Since the velocities appearing here to be solved for are steady or average values, the mean cross-trajectory velocity v is by definition zero and the Coriolis term in (1) drops out. A Coriolis term would remain in the y-equation of motion, which is, however, not needed because of the number of assumptions and parameters such as V specified from observations. The test of the presently derived results will be their agreement with the observed flow, which patently obeys the second equation of motion and the implied latitudinal relations.

Three final simplifications are made. The first is that of hydrostatic equilibrium, which is surely well approximated for motions of the scale considered. The second is the omission of the $1/\varrho \, d\varrho/dt$ term from the equation of continuity, which has been justified in detail by other writers on the steady-state heating problem (MALKUS and STERN, 1953; SMITH, 1955). The third permits linearization of the equations in this particular situation. This is done by noting that the changes in air properties within the section are small compared with the initial values at the entrance end, and in particular the horizontal windspeed, u, changes by only a fraction of itself (at most about 25 %) in the distance of travel considered. It will be assumed that a uniform wind u of 5 m/sec enters the section at the upstream end and that $u'(x, z)$ is the increment within the section. The other variables are similarly broken down into barred quantities referring to the properties of the current at the entrance end and primed quantities referring to the changes occurring within. The vertical velocity is of the magnitude of a primed quantity, being related to u' through the continuity equation. It can be proved[1] that linearization is not vital to the results presented and that these would be nearly identical were this simplification not made, and qualitatively the same for far longer sections where linearization is clearly not justified. The simplest mathematics is used here which brings out the essentials; more formality is not justified in view of the physical restrictions. With the approximations introduced, equations (1) – (4) become

$$\bar{u}\frac{\partial u'}{\partial x} = -\frac{1}{\bar{\varrho}}\frac{\partial p'}{\partial x} \tag{1 a}$$

$$-\frac{1}{\bar{\varrho}}\frac{\partial p'}{\partial z} = \frac{\varrho'}{\bar{\varrho}}g \tag{2 a}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0 \tag{3 a}$$

$$\frac{\varrho'}{\bar{\varrho}} = -\frac{T'}{\bar{T}} \tag{4 a}$$

An additional small term has been left out of equation (4 a). This omission has also been justified in previous work on non-adiabatic heating (STERN, 1954) and its inclusion here gives results so little different from those actually shown that present-day observations could not test the difference.

The heating function V may now be related to the motion and temperature fields through the first law of thermodynamics, as follows:

$$\frac{V}{c_p} = \bar{u}\frac{\partial T'}{\partial x} + w' \, (\Gamma - \gamma) \tag{5 a}$$

where Γ is the adiabatic lapse rate and γ is the actual (average) lapse rate at the entrance end of the section. Elimination of all other dependent variables is now readily achieved and we obtain a simple differential equation for w', namely

$$\frac{d^2 w'}{dz^2} + \frac{gs}{\bar{u}^2} w' = \frac{gV}{c_p \bar{u}^2 \bar{T}} \tag{6}$$

where s is $(\Gamma - \gamma)/\bar{T}$, the (average) stability at the upstream end. Since the observations show that $d\Theta/dt$ (and hence V) is very nearly independent of x along this section, the height derivative of w' may be written thus as a total derivative. It is readily shown that under conditions where hydrostatic equilibrium is realized, x-variations in w' would follow any x-variations in V exactly as prescribed by the solutions to this equation.

The heating function is evaluated using Figure 4, which was calculated from the observations of I. It shows the height distribution of the downstream rise in potential temperature along the parcel trajectories, which is readily converted into the individual rate of warming. The z-dependence of V is fairly well approximated by an exponential decay, where $V = V_0 e^{-z/h}$ and $h = 1.5$ km. It can be approximated to any higher desired degree of accuracy by a combination of exponential and sinusoidal functions. This was done, with so little difference in result that the added complexity did not justify the labor. If we use the simplest exponential, the differential equation becomes

[1] This has been demonstrated by Melvin Stern, to whom the writer is grateful for many helpful discussions. The formal proof will appear in Stern's Ph. D. dissertation.

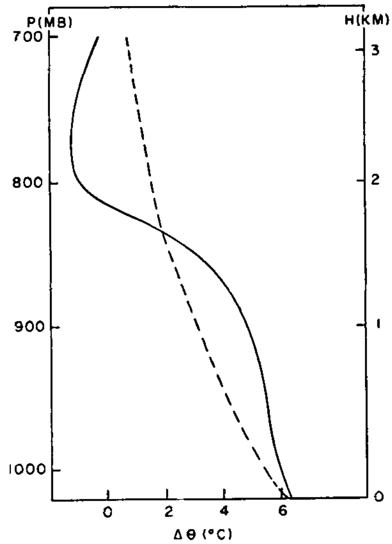

Fig. 4. Graph showing observed net downstream warming, $\Delta\Theta$, of the air parcels, calculated from Fig. 2 as a function of height. Can be converted to the rate of non-adiabatic heating by multiplying by c_p and dividing by the length of time required for the parcels to traverse the section (about 5 days). The potential temperature increment, $\Delta\Theta$, is given in degrees absolute. The dashed curve is the simple exponential function used in the theory to approximate the observed curve.

$$\frac{d^2w'}{dz^2} + B^2w' = Pe^{-z/h} \tag{6 a}$$

where

$$B^2 = \frac{gs}{\bar{u}^2}, \quad P = \frac{gV_0}{c_p\bar{u}^2\bar{T}},$$

and where h is chosen as in Figure 4 to give the correct integrated heating for the layer as a whole and $\bar{T}$ is its mean temperature.

Smith (1955) has shown that the general solution to this equation for a deep air layer bounded below by the ground is given by the real part of the expression

$$\frac{P}{\frac{1}{h^2} + B^2}\left[e^{-z/h} - e^{-iBz}\right]$$

Since h, P, and B are all real, the general solution to (6 a) is

$$w' = \frac{P}{\frac{1}{h^2} + B^2}\left[e^{-z/h} - \cos Bz\right] \tag{7}$$

The divergence field is prescribed as

$$-\frac{\partial w'}{\partial z} = \frac{\partial u'}{\partial x} = \frac{P}{\frac{1}{h^2} + B^2}\left[\frac{e^{-z/h}}{h} - B\sin Bz\right] \tag{8}$$

Since the sine function is zero at the ground, the divergence must always be positive there. This means a downstream acceleration of the surface wind and subsidence in the low levels regardless of the value of B, just so long as the heating is positive and decays with height. Equation (1 a) also shows that the surface pressure must drop downstream. What happens to the motion field at higher levels clearly depends both qualitatively and quantitatively on the stability and wind structure through B. With windspeeds of about 5 m/sec in an atmosphere of very low stability, subsidence occurs through a several km-deep layer. For example, if $B = 0.5 \times 10^{-5}$ cm^{-1}, the vertical distribution of w' in dimensionless units is shown in Table 1.

Since it might appear unrealistic to treat an air layer topped by an inversion as "infinitely" deep, we may alternatively investigate a layer in which the damping effect is simulated by a rigid lid placed somewhat higher than the inversion (this is done because the damping by the inversion is more gradual than that by a rigid surface; the precedents have been discussed by Malkus and Stern, 1953). The solu-

Table 1. Distribution of Vertical Velocity (Dimensionless) with Height for a Deep Layer of Low Stability

Height → (km)	0.2	0.5	1.0	1.5	2.0	2.5	3.0
$\frac{w'(1/h^2+B^2)}{P}$	—.073	—.256	—.365	—.363	—.314	—.125	—.065

tion to (6 a) then contains a sine term with an amplitude about 6 % that of the cosine term and the results shown in Table 1 are imperceptibly altered.

Nevertheless, there are difficulties in applying these results directly to the trade winds. In the first place, the observed magnitude of the heating is much greater than necessary. Evaluating P using Figure 4, we find it to be about 1.7×10^{-10} cm^{-1} sec^{-1}, which would lead to rates of subsidence of 400 m/day! We also find faults in the divergence field, $\partial u'/\partial x$, and wind profile, $\bar{u}+u'$. In this theoretical section, the downstream-end wind profile and the divergence profile must have an exactly similar shape since the wind has a constant profile with elevation at the entrance end. Equation (8) tells us that $\partial u'/\partial x$ must be a maximum at the bottom and that the easterly winds will decrease from there upwards. The observations in I show, to be sure, divergence and wind profiles of similar shape but the maximum in both is well above the surface, at about 900 m. Another problem is that the value of B chosen for Table 1 is probably considerably less than the mean stability of the trade-wind moist layer would permit, if the major portion of the processes in the cloud layer are non-saturated.

The first two of the difficulties suggest that something must be working in opposition to the heat source. Examination of the pressure field provides the clue to what it is and what to do about it. Equation (1 a) tells us that a downstream surface pressure drop of only about 0.05 mb in the 2,500 km is required to produce the observed acceleration of the flow. The observations tell us that a pressure drop of nearly 5 mb in fact occurs. This gradient disappears toward the top of the section at 3 km, along which level the pressure is nearly constant. Using this fact and simple hydrostatic considerations, it is evident that the non-adiabatic heating accounts for 80 % of the observed pressure drop at the bottom surface. Adiabatic compression due to sinking accounts for the remainder of the rise of mean temperature in the layer. It seems that a "frictional drag" has been omitted from equation (1 a) which in practice uses up most of the pressure drop produced by the warming. The fact that the pressure gradient disappears at 3 km, at the top of the heated layer, indicates that the stresses may be provided by the same convective elements responsible for the heating. This suggests a method of introducing frictional stresses into the model, with which the next section will be occupied in detail. Even the present oversimplified analysis, however, illustrates the vital role of non-adiabatic heating in the structure, dynamics, and thermodynamics of the trade system. The refinements of the next section make the predicted numbers agree with measurements and tie some previously unrelated observations to the theory, thus providing independent checks of its plausibility.

IV. Model Including Frictional Stresses

If the frictional forces in the plane of the section are hypothesized to be due to the sea surface drag on the bottom and to vertical exchanges of horizontal momentum by small-scale convective elements within the layer, they may be expressed in terms of a shearing stress τ_{xz}. Equation (1 a) should then be written

$$\bar{u}\frac{\partial u'}{\partial x} = -\frac{1}{\bar{\varrho}}\frac{\partial p'}{\partial x} + \frac{1}{\bar{\varrho}}\frac{\partial \tau_{xz}}{\partial z} \qquad (1\text{ b})$$

Calculations (by SHEPPARD, 1954) from the observations in I and others made in the trades show that the term $1/\bar{\varrho}\; \partial \tau_{xz}/\partial z$ is of the same order of magnitude as $1/\bar{\varrho}\partial p'/\partial x$ and therefore about two orders of magnitude larger than $\bar{u}\partial u'/\partial x$. With the exception of a negligibly small term in the first law of thermodynamics, the remaining equations used here are unchanged. If the shearing stresses within the layer are produced by the same convective and turbulent elements also responsible for the heating, τ_{xz} and its derivatives may be considered functions of the coordinates only, and the elimination carried out as before. Eventually, of course, the shearing stress must be related to the mean wind profile, and be consistent with it.

After elimination, the modified differential equation for w' becomes

$$\frac{d^2 w'}{dz^2} + \frac{gs}{\bar{u}^2} w' = \frac{gV}{c_p \bar{u}^2 \bar{T}} - \frac{1}{\bar{\varrho}\bar{u}}\frac{\partial^2 \tau_{xz}}{\partial z^2} \qquad (9)$$

We could now proceed with this equation in the straightforward manner of attempting to evaluate $\partial^2 \tau_{xz}/\partial z^2$ from the observations and expressing it in some simple functional form in the equation, as was done earlier for the heating function V. The results would be little better than guesswork, however, for the following combination of reasons: the best observational evidence to date indicates that the two inhomogeneous terms in (9) are very nearly the same size, and the small difference between them is the actual forcing function remaining to determine w'. Although the height distribution of τ_{xz} itself is fairly well depicted by recent measurements, its first derivative is not so certain, and its second derivative even less reliably specified. Therefore an indirect procedure will be followed which employs two independent calculations. Although each of these carries considerable uncertainty, consistency in the results of the two, if it occurs, would prove persuasive.

V. Two-Layer Model with Friction

It will be recalled that the trade-wind moist layer is itself subdivided vertically into two layers of quite different static stability. The subcloud layer, reaching from the sea up to about 650 m, has an average stability of zero. The cloud layer, although its lapse rate is steeper than moist adiabatic is almost surely stable on the average, since active clouds occupy only 10—20 % of the space. If we suppose that 78 % of the processes occurring within the layer are unsaturated, $gs/\bar{u}^2 = B^2 \cong 2 \cdot 10^{-10}$ cm^{-2}. Applying equation (9) to this two-layer system will prove extremely illuminating. For the lower neutral region it takes the simple form

$$\frac{d^2w'}{dz^2} = \frac{gV}{c_p\bar{u}^2\bar{T}} - \frac{1}{\bar{\varrho}\bar{u}}\frac{\partial^2\tau_{xz}}{\partial z^2} \qquad (10)$$

The two terms on the right are functions of z only and the left-hand term is related to the divergence through the equation of continuity. The height derivative of the divergence in the subcloud layer may thus be expressed as

$$\frac{d}{dz}\left(\frac{\partial u'}{\partial x}\right) = F_1(z) - F_2(z) = F(z) \qquad (11)$$

where

$$F_1(z) = \frac{1}{\bar{\varrho}\bar{u}}\frac{\partial^2\tau_{xz}}{\partial z^2} \text{ and } F_2(z) = \frac{gV}{c_p\bar{u}^2\bar{T}}.$$

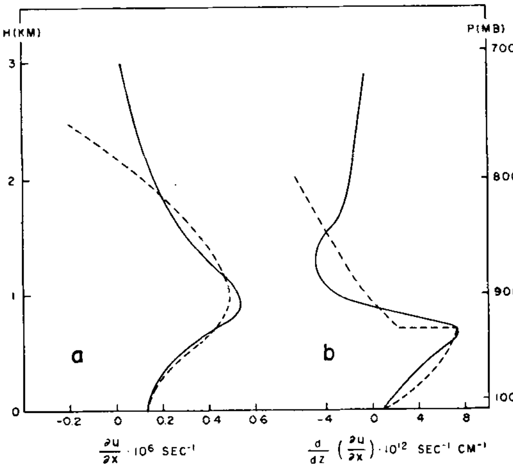

Fig. 5

a. Solid curve is the velocity divergence, $\partial u'/\partial x \cdot 10^6$ sec^{-1}, calculated from the observed wind profiles. It is the component of divergence along the trajectory and averaged over the whole length of the section. The dashed curve is the divergence calculated from the equations of the model.

b. Height derivative of divergence, $d/dz\ (\partial u'/\partial x) \cdot 10^{12}$ sec^{-1} cm^{-1}. The solid curve is calculated from the observed divergence (solid curve in 5a). The dashed curve up to 650 m is constructed (from two exponentials) to fit the observed curve. Its continuation above 650 m is calculated from the results of the model.

Now the observations in I led to a rather good profile of the mean divergence distribution in the section. It was calculated from several wind soundings daily over the four months and was used with consistent results in the previous paper in calculating w' and the transport budgets. This mean divergence profile and its first height derivative are plotted in Figure 5 and will be used here in the following outlined procedures:

(1) Recalling that we know $F_2(z)$ from the heating function, we will establish a functional form of $F(z)$ as a whole from the measurement-calculated profile of $d/dz\ (\partial u'/\partial x)$ in the subcloud layer (that part of the solid curve up to 650 m in Figure 5 b). We will apply the $F(z)$ so obtained to the entire two-layer system via equation (9) written for each layer with appropriate joining boundary conditions. We will solve for w', the divergence field, $d/dz(\partial u'/\partial x)$ above 650 m (not assumed as given), and the pressure field, and compare these with observations.

(2) We will then subtract from the $F(z)$ we took from Figure 5b and used, the $F_2(z)$ evaluated previously from the heating function. This will give a differential equation for τ_{xz}. We may then solve that differential equation and test the results for consistency, both with existing measurements of τ_{xz} in the trades and with the wind profiles.

VI. Solution for the Two-Layer Model

The height derivative of the divergence up to 650 m is well approximated by the dashed curve in Figure 5 b. It is the difference between two exponentials,

$$F(z) = A_1 e^{-z/a_1} - A_2 e^{-z/a_2} \qquad (12)$$

where $A_1 = 1.74 \times 10^{-10}$ cm^{-1} sec^{-1}, $a_1 = 1.715$ km; $A_2 = 1.73 \times 10^{-10}$ cm^{-1} sec^{-1}, $a_2 = 1.5$ km. This expression for $F(z)$ is substituted into equation (9) and the solution is carried out for the two-layered system. The boundary conditions are that w' is zero at the lower surface; that it and its height derivative are continuous at the interface between layers (giving a continuous divergence and wind profile); and that a rigid lid is placed at 4 km. It may be shown that the solution is nearly the same if the upper layer is allowed to be infinitely deep.

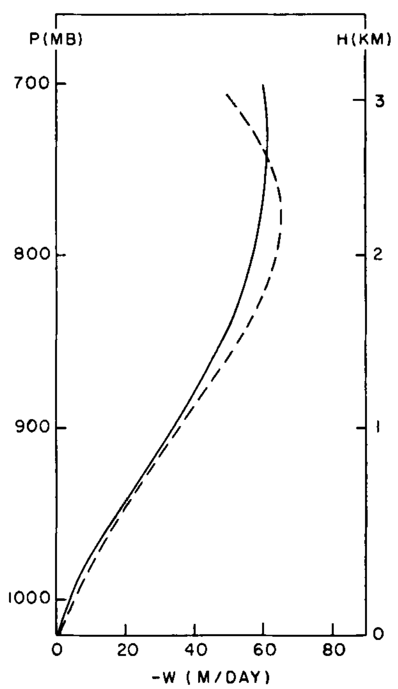

Fig. 6. Solid curve shows mean vertical motions (m/day) as a function of height and pressure calculated from the observed divergence distribution for the section. The dashed curve shows the corresponding vertical motion distribution given by the solution to the differential equation for the two-layer model.

The results for $\partial u'/\partial x$, $d/dz(\partial u'/\partial x)$, and w' are shown and compared with the observations in Figures 5 and 6. On the whole, the agreement is good, although the theoretical divergence shifts over to convergence above 2 km, while the observed curve trails off more slowly. This is believed to be due to the decrease in the two-dimensional character of the flow aloft as the wind steadiness falls off and thus the model begins to break down at higher levels. In practice, an increasing $\partial v'/\partial y$ probably accounts for the difference between the two curves above 2 km. Otherwise, many of the significant aspects of the flow are reproduced and elucidated by the model. Reserving discussion of the pressure field for the section below on shearing stresses, we can now call attention to some important features of the velocity structure: in particular the wind maximum and its location in the vertical.

Rewriting equation (9) in combination with (11), we have

$$\frac{d}{dz}\left(\frac{\partial u'}{\partial x}\right) = F(z) + \frac{gs}{\bar{u}^2} w' \qquad (13)$$

In the subcloud layer $s \cong 0$, $F(z)$ is positive, and the divergence increases rapidly upward. As the cloud base is reached stabilization rather suddenly occurs. The vertical velocity is negative and so a large term abruptly appears on the right to be subtracted from $F(z)$ which at this level is changing only very slowly with elevation. This means that the divergence must start decreasing upward instead of continuing to increase. This point is brought out clearly in Figure 5 b which shows $d/dz(\partial u'/\partial x)$ calculated from the observational curve of Figure 5 a, compared with $d/dz(\partial u'/\partial x)$ computed from the model. The sharp "knee" in the observationally-deduced curve is strikingly reproduced by the theory. It would seem that the average appearance of the trade-wind maximum near cloud base is no coincidence and, barring large changes in the forcing function,

$F(z)$, it would be tied there, at the level where the static stability rather suddenly becomes positive.

The latter conclusion is not sensitive to small changes in the amplitude and exact height dependence of $F(z)$ as a whole. The lower part of the solid curve in Figure 5 b may be approximated by other functions and the main features of the flow field, such as subsidence and wind maximum, derived within the same observational accuracy. The chosen $F(z)$, however, is composed of two subtractive terms and small changes in each individually lead to rather large alterations in their difference. An example is illustrated in Section X, following a discussion relating the components of $F(z)$ to physical processes.

VII. The Distribution of Shearing Stress

The total forcing function $F(z)$ for the differential equation (9) was seen earlier to be physically related to the heating and shearing stresses as follows

$$F(z) = F_1(z) - F_2(z) = \frac{1}{\bar{\varrho}\bar{u}}\frac{\partial^2 \tau_{xz}}{\partial z^2} - \frac{gV}{c_p\bar{u}^2\bar{T}} \quad (14)$$

In solving the differential equation for w', however, this relation was temporarily ignored and a functional form of $F(z)$ was deduced from observations independent of the above. It was found to be well represented by the difference between two exponentials, as written out in equation (12). The second of these exponentials A_2e^{-z/a_2} is within computational accuracy the same as the heating term $Pe^{-z/h}$ of equation (6 a) evaluated previously in section III. Thus when we make the association between $gV/c_p\bar{u}^2\bar{T}$ in (14) with the A_2e^{-z/a_2} in (12), (14) becomes a differential equation for τ_{xz}, namely

$$F_1(z) = \frac{1}{\bar{\varrho}\bar{u}}\frac{\partial^2 \tau_{xz}}{\partial z^2} = A_1e^{-z/a_1} =$$
$$= 1.74 \cdot 10^{-10}\, e^{-z/1.715} \quad (15)$$

Since $\bar{\varrho}$ varies rather little percentually through the height of the section, in most cases integration of this equation may be approximated by two simple quadratures. Two boundary conditions on τ_{xz} are, however, required. We will assume here that the shearing stress (or vertical transport of horizontal momentum) is zero

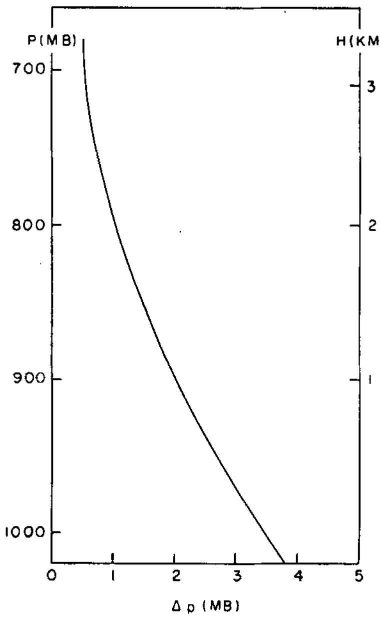

Fig. 7. Theoretical height distribution of downstream pressure drop, $\triangle p$, in mb. $\triangle p$ is the difference between the pressure at the upstream end and that at the downstream end, or over a horizontal distance of 2 500 km along the flow. Observations show a surface pressure drop of just under 5 mb and little or no pressure drop at 3 km (see Fig. 1).

at the height of the trade-wind maximum, a premise well justified by the work of SHEPPARD, CHARNOCK, and FRANCIS (1956), and we will take a value of $\tau_{xz} = 1.0$ dyne/cm^2 at the lower boundary. The latter is consistent with the values used in I for the drag of the sea surface on the lowest air. Under these conditions, τ_{xz}, the shearing stress, and $1/\bar{\varrho}\ \partial\tau_{xz}/\partial z$, the frictional force per unit mass are specified as functions of height. Introducing the latter into equation (1 b) the downstream pressure gradient is also determined. The results are given in Table 2 and Figure 7.

The next-to-last column tests the shearing stress by comparing it to the mean (observed) wind profile. It gives the eddy momentum exchange coefficient, defined as the ratio between the calculated shearing stress and the wind shear at the level considered. Beside it in the last column is shown the eddy coefficient which was required in I to balance the momentum budget. The agreement is within observational and computational error. The results in the two middle columns are also

Table 2. The Stress Distribution and Its Consequences

Height	Calculated τ_{xz}	Calculated $\partial\tau_{xz}/\partial z$	$\Delta p = p_{\text{upstream}} - p_{\text{downstream}}$	μ(from τ_{calc} and du/dz)	μ from momentum budget
m	dyne/cm^2	dyne/cm^2 per 100 m	mb	c.g.s.	c.g.s.
0	1.0	—.149	3.8	330	
500	0.4	—.111	2.8	270	370
900	0	—.088	2.2	indeterminate	
1,500	—0.4	—.062	1.5	330	630
2,000	—0.7	—.046	1.1	700	680
2,500	—0.9	—.035	0.8	600	
3,000	—1.1	—.026	0.6	700	570

close to observational accuracy. The height derivative of shearing stress, $\partial\tau_{xz}/\partial z$, has a distribution similar to that described by SHEPPARD (1954). It falls off with height in a manner consistent with the hypothesis that the frictional drag is brought about by the same processes which distribute the heating. Due to the small contribution of $\bar{u}\ \partial u'/\partial x$ in equation (1 b), the downstream pressure drop is to all intents and purposes proportional to $\partial\tau_{xz}/\partial z$. It also decays exponentially and has nearly vanished at 3 km.

Although the model says nothing explicitly about the cross-stream pressure gradient (which is given by the y-equation of motion), the disappearance of the large downstream pressure gradient toward the top of the section means that the isobars are rotating with height into parallel orientation with the air trajectory, as observed. This conclusion has been arrived at without any *a priori* assumptions concerning the pressure field or of any simple relation between it and the wind field.

VIII. The Relation between Heating, Shearing Stress, and Convection

So far in this paper the relation between the small scales of motion in the trades and the heating and friction distributions has only been stated as an hypothesis. In the case of the shearing stresses, the hypothesis has received recent observational test in the western Atlantic trade-wind region (SHEPPARD, CHARNOCK, and FRANCIS, 1956; BUNKER, 1955). The shearing stresses contributed by turbulent and convective motions ranging in size from small eddies up to convection cells (10—50 km across) have been obtained and compared with mean wind profiles and the total stresses calculated therefrom. The agreement is sufficiently good to conclude that in the low-level easterlies the vertical transport of horizontal momentum is mainly achieved by scales of motion not larger than convection cells. The percentage contributed by the larger-scale systems apparently becomes greater with height, perhaps discontinuously rapidly through the trade inversion. The increased role played by synoptic systems (present in the trades with less regularity than smaller convective motions) in maintaining the momentum transports aloft may be related to the upward diminution of wind steadiness, as will become clearer in Sections IX—XI.

The production of the heating function V is somewhat more complex. The downstream rise in potential temperature represents a small net difference between opposing processes. Radiation is a heat sink throughout the free atmosphere. In the subcloud layer, the net warming is due to a slight excess of eddy flux convergence over outgoing radiation. In the cloud layer, although the potential temperature increases strongly upward, the direction of the eddy heat flux is not definitely known. In any case it probably plays a small role in the heat balance, which the evidence in I indicates is primarily achieved between a large precipitation source and radiation sink. Connecting the net warming throughout the moist layer to convective processes, therefore, implicitly assumes that the radiational heat sink varies relatively little.

IX. Fluctuations about the Steady State

The pronounced steadiness of the flow has been emphasized as one of the significant features of the trades. The over-all dynamic stability of a Hadley-type meridional cell in the low latitudes of a heated rotating system has been suggested by experimental (FULTZ, 1949) and theoretical (KUO, 1954) investigations of simplified planetary analogs. The present treatment of a limited portion of the real Hadley cell proposes consideration of another kind of stability, which may not be unrelated. The results of the preceding sections indicate a coupling between the small-scale motions occurring within a section along the easterlies and the modifications in the large-scale flow observed between the upstream and downstream ends. It has been shown that the transports effected by the small-scale systems impose a constraint upon the large-scale flow, or alternatively, that the large-scale flow must adjust itself to permit the development of those small scales which transport heat and momentum in the manner indirectly specified here. The response of either scale of motion to an alteration in the other is significant in determining the nature and amplitude of the system's fluctuations about its average condition. In the westerlies, disturbances of certain critical sizes are able, by drawing on stored energy, not only to grow in a self-accelerative fashion, but in so doing often to upset the entire structure of the flow, which thereby fluctuates so widely that an average picture has only very restricted meaning as a "steady state". In the easterlies, by contrast, the average and the daily picture bear a closer resemblance to one another. The infrequency of intense disturbances in the low-level trades, and the reliable presence of trade cumulus clouds suggests that the coupling between small- and large-scale processes is stable and perhaps contributes to the over-all dynamic stability of the meridional circulation.

In the framework of the present model, therefore, we might inquire what happens, for example, if the heating is weakened, presumably through a diminution of convective activity. A stable coupling of the type suggested would be substantiated if we could show that a weakening of the heat source would lead to decreased subsidence. It is well known that subsidence in the trades is one of the most effective brakes upon convection, and even a slight release of this brake would lead to renewed outbreaks of convective activity.

Any such approach, however, patently represents a drastic oversimplification, and its mathematical formulation must be preceded by several reservations. The present theoretical treatment cannot be interpreted to state that the heating within this limited section and the flow within it are cause and effect in a straightforward manner; it merely prescribes relations between the two which must, on the average, be satisfied. To deal with the complete, time-dependent behavior of the trades, the writer believes that much larger circulation branches must be considered.

The present theory can, on the other hand, tell us what other steady or quasi-steady configurations are possible and self-consistent for this section. If such configurations may be interpreted as describing the slow fluctuations about the mean seasonal picture discussed in the foregoing paragraphs, we may profitably inquire whether a given one of these exhibits a structure likely to restore the seasonal mean or to lead to even greater departures from it.

Re-examining equation (9) and temporarily ignoring variations in $gs/\bar{u}^2$, we see that the flow may be restored in two ways, either by a stable interaction between the forcing function as a whole and the motion field, or by readjustment between the two right-hand terms maintaining their difference the same. In both categories, only a few of the possible rearrangements will give a consistent relation between the resulting wind profile and stress distribution; the remainder are therefore excluded. Due to our inadequate knowledge of the dependence of the two right-hand terms both upon one another and upon external influences, only one illustrative calculation is worked out here.

X. An Example with Decreased Heating

Rainfall in the trade stream is the least reliable of its properties. Even casual observers have noted that a skyful of trade cumuli will on some days produce plentiful showers from clouds of all sizes, while on other apparently similar days with an apparently similar cloud distribution, no drop of rain appears. We have seen that the net non-adiabatic heating in the cloud layer is probably controlled by the

Table 3. Results of a 10 % Reduction in Heating

Height	w'	divergence · 10^6	calc. τ_{xz}	calc. $\frac{\partial \tau_{xz}}{\partial z}$	Δp
m	m/day	sec^{-1}	$dyne/cm^2$	$dyne/cm^2$ per 100 m	mb
0	0	−1.1	2.2	−.19	4.8
300	+22	−0.6	1.7	−.17	4.2
650	+23	−0.2	1.2	−.14	3.5
1,000	+16	+0.6	0.7	−.12	3.0
1,500	−23	+1.2			
1,600			0	−.10	2.6
2,000	−65	+1.1	−0.4	−.08	2.0
3,000	−105	+0.3	−1.2	−.07	1.8

amount of precipitation. The evidence in I suggests that precipitation heating in the cloud layer may also be of importance indirectly in the net warming of the air below cloud base (if the eddy heat transports are, in fact, down the potential temperature gradient). It is not impossible, therefore, that we might have significant changes in the V term in equation (9) while the frictional stresses produced by the convective and turbulent elements remain more or less the same. Thus we have chosen as the example case one in which just the net heating is reduced.

The situation is now considered where the amplitude of the heating function is cut by 10 % without altering its height dependence.

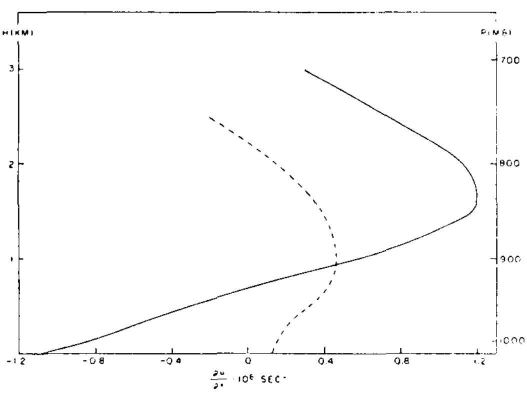

Fig. 8. Solid curve gives the theoretical divergence profile resulting when the heating is reduced by 10 % relative to that used for the average situation. The dashed curve reproduces the theoretical divergence curve for the average situation as a comparison (same as dashed curve of Fig. 5 a). Note that according to the model the wind profiles in the two cases would have shapes similar to the divergence profiles.

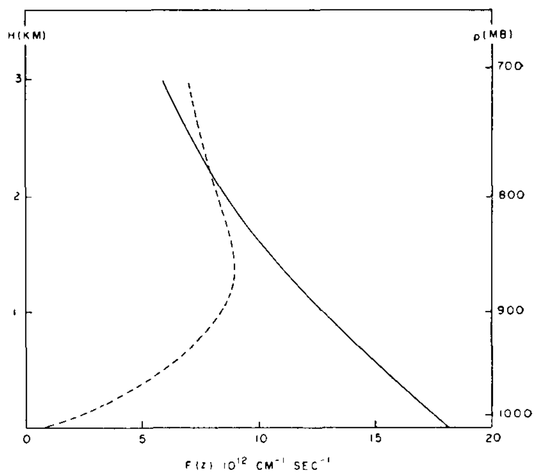

Fig. 9. Solid curve is the new forcing function $F(z)$ appearing (with a negative sign) on the right side of the differential equation for w' when the heating amplitude is reduced by 10 %. The dashed curve is the $F(z)$ used in the model of the mean flow with the heating distribution approximating that observed.

The only change in the corresponding stress term will be that its level of exponential decay is now taken as 1.6 instead of 1.7 km. This slight change is necessary to give consistent results throughout and would presumably allow for a small reduction of mean cloudiness. The solution of equation (9) is now carried through as previously, with the same stabilities in the two layers and the same boundary conditions on w'. The results are presented in Table 3 and Figures 8 and 9.

Since the wind (divergence) maximum now occurs at 1,600 m, the boundary conditions used here on τ_{xz} were that it be zero there and 2.2 dynes/cm^2 at the bottom surface. That choice of the lower boundary condition makes a self-consistent model because the surface wind now decreases downstream 2.8 m/sec (in the 2,500 km), while that at 1,600 m increases 3 m/sec, giving a positive wind shear of about 3.5 m/sec per km in the lower part of the section. The calculated τ_{xz} distribution in Table 3 requires an average eddy momentum exchange coefficient in the subcloud layer of about 500 c.g.s., or a substantial increase over the mean situation. This is plausible in view of the intensified eddying anticipated in consequence of replacing the former subsidence by comparable ascent in the low levels. The rate of wind shear up to the wind maximum and the height of the wind maximum are both at the upper limit of average daily values observed by Sheppard et al. in the western Atlantic trade. Thus Table 3 represents a realistic but rather extreme situation.

Although the heating has only been reduced slightly, a comparison of the new total forcing function (Figure 9) with that used previously shows that its average value has been altered by a factor of more than two in the lowest km. Apparently this is near the permissible limit of changes in the total forcing function still giving a flow field in the observed range. The new situation, however, does seem to contain restoring influences. Table 3 shows that sinking has been replaced by mean ascent up to 1 km and that the subsidence is much weaker than normal through most of the cloud layer. Weakening the descent in the cloud layer favors increased cloudiness and precipitation, while ascent in the subcloud layer aids upward transport of sensible heat, both directly and by destabilizing the already nearly neutral lapse rate. Recent observational studies (MALKUS, 1955 b) suggest that the processes in the tropical subcloud layer are critically sensitive to slight amounts of subsidence or its removal. In view of this it seems likely that the system would often restore its heating and return to normal even before departing so far as the conditions of Table 3.

XI. Concluding Remarks

Within the moist convective layer the trade stream seems to have at its disposal the mechanisms to adjust: it is able to meet the imposed constraints by mutual rearrangement between heating, stress, and motion field in such a way that the motion field is allowed to fluctuate little from the mean. Above the convective layer, these mechanisms are no longer available and we are not surprised that the steadiness vanishes.

Nevertheless, the reader familiar with the trade-wind system may be disturbed by the egregious unreliability of its precipitation. It was seen that only a 10 % reduction in heating produced, in one test situation, flow patterns in the outer range of those observed. Although the variation percentage-wise in the weekly or monthly rainfall along a 2,500 km oceanic section cannot even be estimated today, we may safely guess that it often exceeds 10 %. In conclusion of this work, therefore, we must briefly examine the consequences of larger variations in heating and inquire whether any plausible shearing stress field can conceivably maintain the flow for short periods. Thus the extreme limiting question to ask of our model is whether the total forcing function (and so the flow) can temporarily remain the same as given in Section VI, without any heating at all. This is readily tested by setting V equal to zero in equation (14) and solving for τ_{xz} under the boundary conditions originally used. An observationally possible distribution of τ_{xz} arises, provided that larger eddy coefficients are occasionally permitted near the section top than those required to balance the average budgets. We find, however, that the downstream pressure drop decreases much more slowly with height than before. Even allowing for the height variation of density in equation (14), Δp is about 3 mb at the bottom and 2 mb at the top of the section. Simple hydrostatic reasoning makes this inevitable, for if the lower column is warmed less, it can contribute proportionally less to a pressure fall along the bottom surface. Concomitantly, $\frac{\partial \tau_{xz}}{\partial z}$ shows $^2/_3$ of its surface value at 3 km. SHEPPARD (1954) has found that such situations are sometimes observed in the trades. It would be highly desirable to determine observationally whether occasions of abnormally slow vertical decay of frictional forces and pressure drop are associated with greatly diminished downstream warming, as the model suggests. This and other valuable tests of the relations between stress, motion field, and heating could be carried out were a series of weather ships or other sounding devices again placed along a trade-wind air trajectory.

Acknowledgments

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