On Conversions between Potential and Kinetic Energy in the Atmosphere

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From a consideration of the large-scale horizontal variations of individual pressure change and 500 *mb temperature in a mid-latitude sector* of the Northern Hemisphere, computations are made of the required mean conversion of potential energy into the kinetic energy of the horizontal wind systems. The order of magnitude of the estimate obtained is in agreement with that obtained by Brunt from considerations of the frictional dissipation of kinetic energy. In addition, the role of organized overturnings is investigated. It is indicated that overturnings in east-west vertical planes associated with the large-scale disturbances are of primary importance in effecting the release of potential energy.

1. Introduction

In recent years increasing attention has been given to basic questions relating to the manner in which the energy provided by solar heating is utilized in maintaining the large-scale atmospheric wind systems. In this connection it has been convenient to think in terms of a "cycle" which traces the transformation and flow of energy in the atmosphere. In such an energy cycle, conversions between the potential, internal, and kinetic forms play a vital role. Aspects of this conversion process have been discussed by many authors including EADY (1949), CHARNEY (1951), FJØRTOFT (1951), VAN MIEGHEM (1952, 1955), KUO (1954), LORENZ (1954), PHILLIPS (1954, 1955), PISHAROTY (1954), and STARR (1954).

Although the stages of the energy cycle which involve the partition of available potential energy and kinetic energy into eddy and zonal components have been measured by direct observations (see, for example, STARR and WHITE, 1954), little quantitative observational evidence has been obtained thus far concerning the conversion between potential and kinetic energy. This has been largely due to the difficulties in computing the field of vertical motion, or divergence, on which such measurements critically depend.

It is the purpose of this article to present some observational estimates of the conversion process based on data recently computed by means of a two-parameter model of atmospheric flow, for a grid network covering a portion of the North American continent.

2. The nature of the conversion process

In accordance with the ideas originally expressed by MARGULES (1903) and recently extended by LORENZ (1954), one may conceive of the primary effect of solar heating as being the creation of horizontal temperature differences which represent an "available" potential energy in the atmosphere. Through the action of vertical motions, under conditions closely approximating hydrostatic equilibrium, this available potential energy is released and converted largely into the kinetic energy of the horizontal wind (see KUO, 1954).

In order to obtain a mathematical expression which represents this energy conversion process we may proceed as follows. Taking the pressure, p, as the vertical coordinate and using the hydrostatic equation we may write the equation of motion for the horizontal wind in the form,

$$\frac{d\mathbf{v}}{dt}+f\mathbf{k}\times\mathbf{v}=-\nabla\phi-\mathbf{F}, \qquad (1)$$

where $\mathbf{v}$ is the horizontal vector wind, f is the Coriolis parameter, $\mathbf{k}$ is the unit vector in the vertical, $\phi=gz$ is the geopotential of an isobaric surface, ∇ is the two-dimensional del-operator in a pressure surface and $\mathbf{F}$ is the vector frictional force per unit mass. If we scalar-multiply this equation by $\mathbf{v}$, and expand the total derivative, we obtain the energy equation for the horizontal motions in the form,

$$\frac{\partial k}{\partial t}+\mathbf{v}\cdot\nabla k+\omega\frac{\partial k}{\partial p}=-\mathbf{v}\cdot\nabla\phi-D, \qquad (2)$$

where $k=\mathbf{v}^2/2$ is the kinetic energy of horizontal motion per unit mass, $D=\mathbf{v}\cdot\mathbf{F}$ is the rate of frictional dissipation of kinetic energy per unit mass, and $\omega=dp/dt$. With the use of the hydrostatic equation,

$$\frac{\partial\phi}{\partial p}=-\alpha, \qquad (3)$$

where $\alpha=1/\varrho$ is the specific volume, and the continuity equation,

$$\frac{\partial\omega}{\partial p}=-\nabla\cdot\mathbf{v}, \qquad (4)$$

we may rewrite equation (2) in the form,

$$\frac{\partial k}{\partial t}+\nabla\cdot(k+\phi)\mathbf{v}+\frac{\partial}{\partial p}(k+\phi)\omega=-\omega\alpha-D. \qquad (5)$$

Finally, if this equation is integrated over the entire mass of the atmosphere, M, we obtain,

$$\frac{\partial}{\partial t}\int_M k\,dm=-\int_M \omega\alpha\,dm-\int_M D\,dm, \qquad (6)$$

where $dm=g^{-1}\,dx\,dy\,dp$ (x is distance eastward and y is distance northward).

This equation states that for the entire mass of the atmosphere the kinetic energy of the horizontal wind can vary as a result of a frictional dissipation and an effect depending on the product of ω and the specific volume throughout the mass of the fluid.

That this latter effect actually represents a conversion of potential and internal energy may be demonstrated as follows. The first law of thermodynamics may be written in the form,

$$\frac{dQ}{dt}=c_p\frac{dT}{dt}-\omega\alpha \qquad (7)$$

where c_p is the specific heat at constant pressure, T is temperature, and dQ/dt is the rate of heat addition per unit mass (including the generation of heat by friction). With the use of (4), this equation becomes

$$c_p\frac{\partial T}{\partial t}+\nabla\cdot c_pT\mathbf{v}+\frac{\partial}{\partial p}c_pT\omega=\omega\alpha+\frac{dQ}{dt}. \qquad (8)$$

Integrating this equation over the entire atmosphere and recalling that for a vertical column of the atmosphere in hydrostatic equilibrium $c_pg^{-1}\int_0^{p_0}T\,dp$ is equal to the sum of the potential and internal energy of the column,[1] we find that the expression for the time-rate of change of total potential and internal energy, $\int_M(\Phi+I)\,dm$, takes the form,

$$\frac{\partial}{\partial t}\int_M(\Phi+I)dm=\int_M\omega\alpha\,dm+\int_M\frac{dQ}{dt}dm. \qquad (9)$$

The appearance of $\int_M\omega\alpha\,dm$ with opposite sign in both equations (6) and (9) indicates that this term represents a conversion between potential and kinetic energy, a loss in one representing a gain in the other.

By virtue of the strong relation between ω and the vertical motion it is clear that the effect represented by this term is associated with the process of rising of warm air masses and sinking of cold air masses, which has been connected with the conversion process since the time of Margules.

Since over a long period of time there is no observed change in the total kinetic energy of

[1] Owing to the fact that in a hydrostatic atmosphere the potential energy in a given column bears a fixed ratio to the internal energy, we shall henceforth include both of these forms under "potential" energy.

the horizontal wind, it follows from equation (6) that the conversion integral, $-\int \omega\alpha\, dm$, must be positive in the mean to balance the frictional dissipation. From another point of view, it has been noted by EADY (1949), PHILLIPS (1954), and KUO (1954) that the vertical transport of heat (of which this integral is a close measure) should, in the long-time mean, be positive, in order to maintain the observed stable stratification against radiational losses in the upper levels of the atmosphere.

It may be noted also, that by comparing the long-time averages of equations (6) and (9), the well-known result that the mean rate of heat addition equals the mean rate of frictional dissipation is obtained.

3. Measurement of the conversion term

An evaluation of the conversion term requires a knowledge of the spatial distribution of ω over the entire atmosphere. Although it is impossible at present to obtain such extensive data, computations of ω for limited regions of the atmosphere now being made in connection with numerical weather prediction experiments are becoming available. The reliability and coverage of these new computations are in many respects superior to those provided by other methods. It seems desirable therefore to make use of such data for the present problem.

In the process of carrying out a series of numerical forecasting experiments for each day of January 1953, values of the vertically-averaged individual pressure change were computed by the joint Geophysics Research Directorate—Air Weather Service numerical weather prediction project (THOMPSON and GATES, 1956). These computations were based upon a simple 2-parameter, quasi-geostrophic, baroclinic model developed by THOMPSON (1953). The results of these computations were kindly made available to the authors for the present study.

More specifically, the vertically-averaged individual pressure change, $\tilde{\omega}$, was computed by means of the adiabatic energy equation in the form,

$$\tilde{\omega} = -\frac{g}{f\tilde{\varkappa}}\left[\frac{\partial h}{\partial t} + \frac{g}{f}J(Z, h)\right] \qquad (10)$$

where $\tilde{(\ )} = 1/p_0 \int_0^{p_0} (\ )dp$, $\varkappa = RTf^{-1}\Theta^{-1}\partial\Theta/\partial p$ (Θ = potential temperature), and $J(Z, h)$ is the two-dimensional Jacobian of the 500 mb height, Z, and the thickness between 1,000 and 500 mb, h. $\tilde{\varkappa}$, which is a measure of the static stability through the depth of the atmosphere, was estimated from a set of observational data for the period 1—10 January, 1953.

The values of the tendency, $\partial h/\partial t$, computed from the thermal vorticity equation for the initial time $t = 0$, and the corresponding values of $J(Z, h)$ were used to evaluate $\tilde{\omega}$ at a network of 204 grid points covering a large part of the North American sector. In the solution of the thermal vorticity equation it is necessary to specify the thickness tendency along the boundaries of the region. This was approximated by assuming that the tendency at the initial time is proportional to the known 24-hour thickness change centered on time $t - 0$.

Since these numerical computations provide *vertically-averaged* values of ω only, it is not possible in the present study to consider the multi-level effects of the horizontal variations of α. Synoptic evidence suggests, however, that the general mid-tropospheric variations of α are of considerable importance in the conversion process. Accordingly, as a simple assumption the horizontal variations of α were estimated by measuring the field of temperature along the 500 mb surface. It is recognized that such an estimate does not necessarily give a representative measure of effects in the upper levels of the atmosphere where the horizontal field of α tends to be somewhat out-of-phase with that in the troposphere.

The 500 mb temperature fields and the corresponding $\tilde{\omega}$-fields were analyzed on separate charts, from which the values of temperature and $\tilde{\omega}$ were extracted at every 5 degree latitude-longitude intersection over the area between latitudes 35° N to 60° N and longitudes 70° W to 120° W. An example of the type of distribution of $\tilde{\omega}$ and temperature which was found is shown in Fig. 1.

It is reasonable to expect that, in the mean, the conditions in the area sampled in this study are similar in their essential features to other middle-latitude areas of the hemisphere. The geographical extent of this area is sufficiently

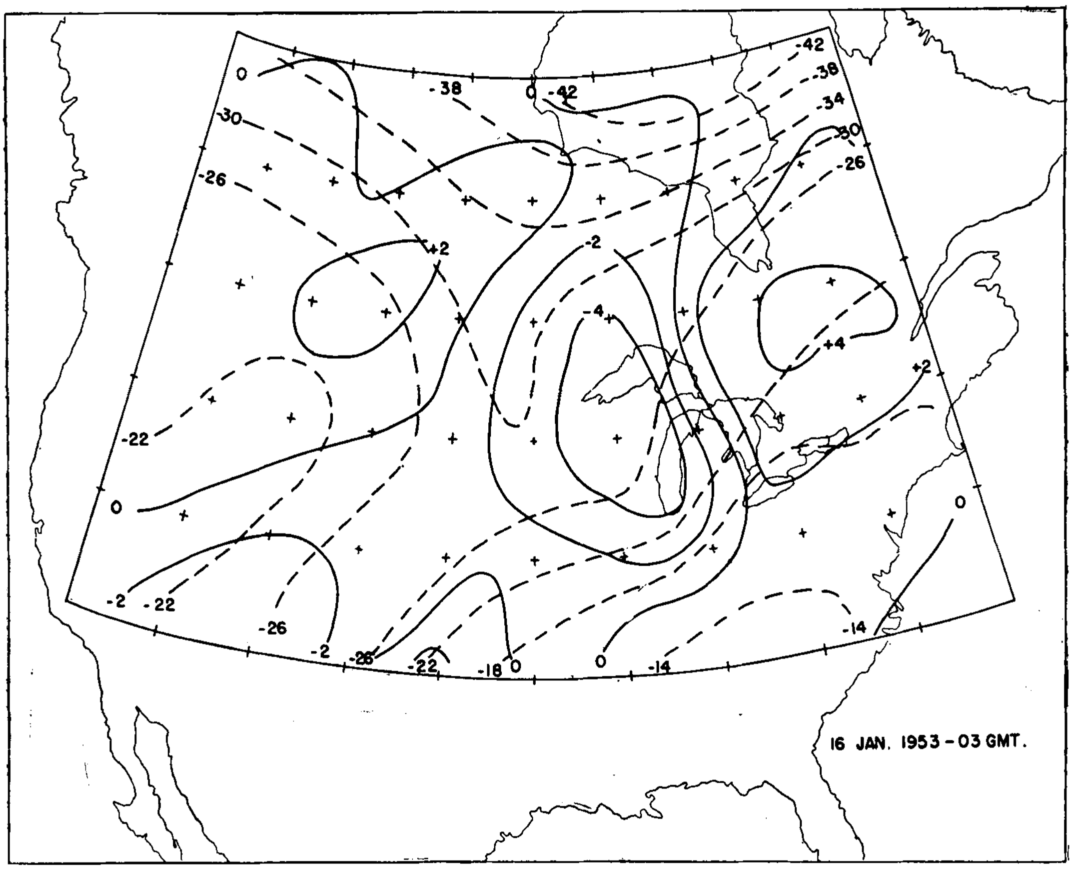

Fig. 1. Map for 16 January 1953. The values of $\tilde{\omega}$, converted to estimates of the vertical velocity (cm sec^{-1}) by the formula $w = -\varrho^{-1} g^{-1} \tilde{\omega}$, are represented by the solid lines. The dashed lines represent the 500 mb isotherms in degrees Centigrade.

large so that it always includes at least one synoptic-scale disturbance with associated temperature and $\tilde{\omega}$-fields.

In view of the approximations outlined above and the limited geographical area treated, the computations based on these data can be taken only as a rough measure of the actual conversion process.

4. Analysis of the data

As outlined in the preceding section, estimates of the horizontal variation of the individual pressure change and specific volume were obtained in this study. We henceforth denote these estimates by $\omega^\star(x, y)$ $(=\tilde{\omega})$ and $\alpha^\star(x, y)$ ($\sim$ 500 mb temperature) respectively.

We may write the time mean of the space average of the product $\omega^\star\alpha^\star$, in the form, $\overline{\{[\omega^\star\alpha^\star]\}}$, where the brackets, brace and bar represent averages with respect to x, y and t respectively. This quantity may be further resolved into components representing the contributions due to vertical overtunings in $x-p$ "planes", overturnings in the $y-p$ "plane", and the time variations of the space averaged individual pressure change and specific volume. Such a resolution is accomplished by the following expansion,

$$\overline{\{[\omega^\star\alpha^\star]\}} - \overline{\{[\omega^\star]\}}\;\overline{\{[\alpha^\star]\}} = \overline{\{[\omega^\star]''[\alpha^\star]''\}} + \overline{\{[\omega^{\star\prime}\alpha^{\star\prime}]\}} + \overline{\{[\omega^\star]\}'''\{[\alpha^\star]\}'''}, \qquad (11)$$

where the single, double and triple primes denote deviations from the x-, y- and t-means respectively.

It can be shown that, in the long-time mean, the integral of ω over the entire mass of the atmosphere must vanish. Hence, the second term on the left, which involves $\overline{\{[\omega^\star]\}}$, is identically zero if the entire atmosphere is considered. Since only a limited region of the atmosphere is treated in the present study this term can have a significant non-zero value. It is desireable, therefore, to take as a more representative measure of the *hemispheric* conversion process the left-hand side of equation (11) rather than $\overline{\{[\omega^\star \alpha^\star]\}}$ alone. If the vertical motions associated with the large-scale synoptic features in mid-latitudes are the important ones in effecting the required conversion,[1] and if the region sampled is somewhat representative of other regions of the hemisphere, one would expect this quantity to be negative in accordance with the theory of section 2.

The first term on the right-hand side of (11) represents the contribution resulting from the correlation of $[\omega^\star]$ and $[\alpha^\star]$ in the north-south direction, associated with overturnings in the $y-p$ "plane". The second term on the right depends on the correlation between $\omega^\star$ and $\alpha^\star$ along a latitude circle and results from overturnings in $x-p$ "planes". The third term depends on the time correlation between the space averages of $\omega^\star$ and $\alpha^\star$.

More meaningful interpretations of these expressions can be made in terms of the energy cycle of the general circulation conceived by LORENZ (1954). In accordance with his scheme, $y-p$ overturnings are associated with transformations between mean zonal kinetic energy and the "*zonal* available potential energy", while the $x-p$ overturnings represent a transformation between eddy kinetic energy and "*eddy* available potential energy".

[1] It is possible that phenomena of a smaller scale than the grid network used in this study, such as cumulus convection, may be of some importance in the global conversion process.

5. Results

Each of the terms in equation (11) were computed from the data, and their values are given in table 1.

If we consider that the mass of a column of the atmosphere of unit cross-sectional area is approximately 10^3 gm, then an estimate of the net rate of energy conversion for the entire depth of the atmosphere may be obtained from the first column. In accordance with the remarks of section 2, this quantity also represents an estimate of the mean rate of frictional dissipation of kinetic energy per unit area. In the present case the value obtained is 5.0×10^3 ergs cm^{-2} sec^{-1}, representing about two per cent of the effective solar radiation. Considering the approximations made in this study, this estimate agrees well with that obtained by BRUNT (1941) from other considerations. This result lends some support to the assumption implicit in this study that the gross-scale horizontal tropospheric variations of ω and α in mid-latitudes are of primary importance in the conversion process.

As previously noted, the total energy conversion is brought about by various kinds of organized atmospheric circulations. The processes represented by the second and third columns of table 1 are those associated with vertical overturnings in north-south and east-west vertical "planes" respectively. On the basis of these data it appears that in middle latitudes the $x-p$ overturnings consistently act to increase the kinetic energy, with $y-p$ overturnings acting in the opposite sense. Thus, in terms of the Lorenz energy cycle, the overturnings result in the conversion of eddy available potential energy into the kinetic energy of the large scale atmospheric disturbances while the $y-p$ overturnings give rise to an increase in the zonal available potential energy at the expense of the zonal kinetic energy.

A more detailed description of the $x-p$ overturning process is given in Fig. 2 where

Table 1. Rate of conversion between potential and kinetic energy based on data for 2—31 January, 1953. A minus sign indicates a conversion from potential to kinetic energy. Units in ergs gm^{-1} sec^{-1}.

$\overline{\{[\omega^\star\alpha^\star]\}} - \overline{\{[\omega^\star]\}}\,\overline{\{[\alpha^\star]\}}$	$\overline{\{[\omega^\star]''[\alpha^\star]''\}}$	$\overline{\{[\omega^{\star\prime}\alpha^{\star\prime}]\}}$	$\overline{\{[\omega^\star]\}'''\{[\alpha^\star]\}'''}$
— 5.0	+ 2.0	— 6.8	— 0.2

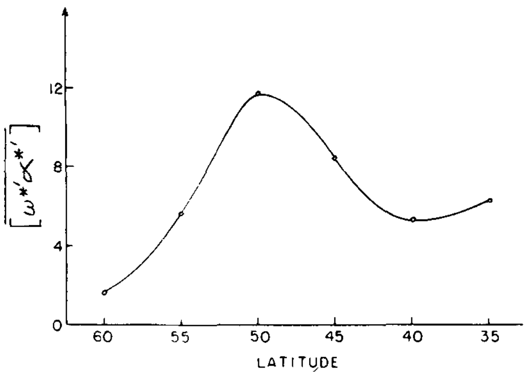

Fig. 2. Latitudinal variation of the mean rate of conversion of eddy available energy into eddy kinetic energy as measured by $\overline{[\alpha\star'\ \alpha\star']}$, in units of ergs $cm^{-2}\ sec^{-1} \times 10^3$.

the latitudinal variation of the mean rate of conversion by this process is shown. It can be seen that kinetic energy conversion from eddy available potential energy due to this process is most intense in the vicinity of 50° N. Interestingly, this is also the region where the horizontal poleward flux of sensible heat, which is a measure of the generation of eddy available potential energy, is a maximum (see, for example, STARR and WHITE, 1954).

The negative contribution of the $y-p$ overturnings result largely from the existence of an indirect mean meridional circulation over the latitudinal belt which has been sampled in this study. It should be noted that observational evidence provided by STARR and WHITE (1954), MINTZ and LANG (1955) and PALMEN (1955) suggest the existence of weak direct meridional circulations in tropical and polar regions. Such circulations convert zonal available potential energy into zonal kinetic energy, tending therefore to counteract the negative contribution from the middle-latitude indirect cell. One would not expect this effect to be appreciable, however, since observations have already revealed that the kinetic energy of the zonal wind is maintained primarily by a transfer of kinetic energy from the disturbances through the action of horizontal eddy stresses (e.g. KUO 1951).

The energy conversion associated with the term in the fourth column of table 1, which depends on the time variations of the space means of $\omega^\star$ and $\alpha^\star$, appears to be negligibly small.

6. Conclusions

From a consideration of synoptic-scale horizontal variations in a middle-latitude sector of the atmosphere, an estimate of the required mean conversion of potential to kinetic energy has been obtained which appears to be of the correct order of magnitude.

The empirical study has suggested that this total conversion is accomplished mainly by overturnings in east-west vertical planes, in general agreement with the proposals of STARR (1954) and the descriptions of the energy cycle of the general circulation proposed by LORENZ (1954) and PHILLIPS (1955).

As noted earlier, it is recognized that, due to the limitations in data, the results of this study can only be taken as a rough measure of the true global conversion process. When sufficient data become available it would, accordingly, be desirable to extend computations of the type presented here to the entire hemisphere on a multi-level basis.

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