On the Theory of Magnetic Storms and Aurorae

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The ionized gas from the sun moves in the magnetic fields of the sun and the earth and in an electric field. The drift motion can be divided into three terms, one due to the crossed electric and magnetic fields, one due to the inhomogeneity in the magnetic field and one due to the inertia of the charged particles. The last term is of special interest for the motion of the ions.

Far from the earth the current is zero since the density of positive and negative charges are practically equal. In the region where the magnetic field from the sun and the earth are of the same order of magnitude the current due to drift in crossed fields and that due to inertia form a circular current round the earth. That has been discussed by H. Alfvén[1] but the formal proof is not given in his paper. His statement can be proved in the following manner.

I. Let us first discuss the ion current. Since we are specially interested in the drift due to inertia, and that drift is small except when $\eta \approx 1$,[1] we can neglect the drift due to inhomogeneity. H. Alfvén has given the positive ion current (ibid. § 8, eq. 3—6) and if we introduce $\xi = x/\lambda$, $\zeta = y/\lambda$, $p = -6x_\lambda/\lambda$ it can be rewritten in the following form

$$j_x^E = 0 \tag{1}$$

$$j_y^E = -en^+ v_0 \frac{\eta^3}{1+\eta^3} \tag{2}$$

$$j_x^i = -en^+ v_0 \frac{\eta^7 \zeta}{(1+\eta^3)^4} p \tag{3}$$

$$j_y^i = 0 \tag{4}$$

In the stationary state this current has to satisfy the divergence condition

$$\operatorname{div} \mathbf{j} = 0 \tag{5}$$

If we introduce $q = -en^+ v_0 \eta^3/(1+\eta^3)$ equations (1)—(4) and (5) give

$$\frac{\partial}{\partial \xi}\left(qp \frac{\eta^4 \zeta}{(1+\eta^3)^3}\right) + \frac{\partial q}{\partial \zeta} = 0 \tag{6}$$

We have thus to solve the equation

$$\frac{d\xi}{p\dfrac{\eta^4\zeta}{(1+\eta^3)^3}} = \frac{d\zeta}{1} = \frac{dq}{qp\xi\zeta\eta^2 \dfrac{4-5\eta^3}{(1+\eta^3)^4}} \tag{7}$$

This can be split into

$$\frac{d\xi}{d(\zeta^2)} = \frac{1}{2} p \frac{\eta^4}{(1+\eta^3)^3} \quad \text{and} \tag{7a}$$

$$\frac{dq}{d\zeta} = qp\xi\zeta\eta^2 \frac{4-5\eta^3}{(1+\eta^3)^4} \tag{7b}$$

H. Alfvén (§ 7) has shown that $p/6$ is of the order 0.05 for the ions. Let us thus put ξ = constant in the r.h.s. of (7 a) in order to get a first approximation.

$$\xi - \xi_\infty = -\frac{p}{6} \frac{1+2\eta^3}{(1+\eta^3)^2} \tag{8}$$

[1] For the notations we refer to H. ALFVÉN (1955).

We can rewrite (7 b) as

$$\frac{d(\ln q)}{d(\zeta^2)} = \frac{1}{2} p\xi\eta^2 \frac{4 - 5\eta^3}{(1 + \eta^3)^4} \tag{9}$$

For the ξ which appears explicitly we may introduce the value from (8) but in η we keep ξ constant, as before. We get

$$\ln q = C + p\xi_\infty \frac{\eta^4}{(1 + \eta^3)^3} - \frac{p^2}{6} Q(\eta) \tag{10}$$

where C is an arbitrary constant and Q is a function of η we do not specify. The general solution is now obtained when ξ_∞ is eliminated between (8) and (10) and C is exchanged for f which is an arbitrary function of the argument

$$\xi + \frac{p}{6} \frac{1 + 2\eta^3}{(1 + \eta^3)^2}$$

The boundary condition is that q is independent of ξ when $\zeta \to \infty$. That means that f must be a constant. We thus get

$$q = q_0 \exp\left(p\xi \frac{\eta^4}{(1 + \eta^3)^3} + \frac{p^2}{6} Q_1\right) \tag{11}$$

In the whole interval $0 < \eta < \infty$ is $|Q_1| < 0.1$ but the first factor in the exponent of (11) is smaller than $0.13\ p$. For the values of interest of p we can thus neglect the second term in the bracket besides the first one. Further the value of the exponent is below unity and hence is, if we introduce n^+ again,

$$n^+ \approx n_0 \frac{1 + \eta^3}{\eta^3} \left(1 + p\xi \frac{\eta^4}{(1 + \eta^3)^3}\right) \tag{12}$$

which is the same expression as given by Alfvén (1955), (§ 8 (14).

II. For the electrons we can neglect j^i but we have to take account of j^E and j^m. Then it is easy to see that

$$n^- = n_0 \left(1 + \eta^{-3}\right) \tag{13}$$

is an exact solution to (5) satisfying the boundary condition that n^- is independent of the coordinates at infinite distance.

III. If we now keep to the region where the solution to I is valid we get for the total current (ions + electrons)

$$j_x^E = 0 \tag{14}$$

$$j_y^E = e(n^- - n^+) v_0 \frac{\eta^3}{1 + \eta^3} = -e n_0 v_0 p\xi \frac{\eta^4}{(1 + \eta^3)^3} \tag{15}$$

$$\begin{aligned} j_x^i &= -(n^+m^+ + n^-m^-) \frac{ev_0}{m^+} p\zeta \frac{\eta^7}{(1 + \eta^3)^4} = \\ &= e n_0 v_0 p\zeta \frac{\eta^4}{(1 + \eta^3)^3} \left(1 + \frac{n^-m^-}{n^+m^+}\right) \cdot \\ &\quad \cdot \left(1 + p\xi \frac{\eta^4}{(1 + \eta^3)^3}\right) \end{aligned} \tag{16}$$

$$j_y^i = 0 \tag{17}$$

The second term in the last bracket of (16) is smaller than $0.13\ p$. If p is well below 1 we can, in the first approximation, neglect this second term. That means that we can put $n^- \approx n^+$ in the first bracket. Since $m^- \ll m^+$ also this bracket can be taken as 1 in the first approximation. Under these assumptions we find

$$j_y^E \zeta = -j_x^i \xi \quad \text{or} \quad j_y^E y = -j_x^i x$$

That means that the current due to drift and inertia together show circular symmetry—a conclusion at which H. Alfvén arrived in the mentioned paper.

REFERENCE

Alfvén, H., 1955: On the electric field theory of magnetic storms and aurorae, *Tellus* 7, p. 50.