Magneto-hydrodynamic Waves in the Ionosphere and their Application to Giant Pulsations

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Abstract

In magneto-hydrodynamics conservation of energy leads to the adiabatic relation, $du = pd\varrho/\varrho^2$, between the internal energy u per unit mass, the pressure p, and the density ϱ. The relation is valid for arbitrary amplitudes in a non-dissipative medium. The Hall current modifies the properties of magneto-hydrodynamic waves in an ionized gas. The compressive modes and the Alfvén mode earlier discussed by van de Hulst are shown to be coupled by means of the Hall current. The dissipation of slightly damped waves is expressed in terms of the effective conductivity $\sigma_3 = \sigma_1 + \sigma_2^2/\sigma_1$ earlier introduced by Cowling. The theory is applied to magneto-hydrodynamic oscillations in the layers of the ionosphere. The possible periods of the waves are consistent with the periods of "giant pulsations" which have been observed as a type of perturbation in the earth's magnetic field in the auroral zone. The results indicate that magneto-hydrodynamic waves in the E-, F_1-, and F_2-layers may explain the existence of giant pulsations as well as the occurrence of rapid vibrations in the terrestrial magnetic field.

Introduction

A type of perturbation in the earth's magnetic field was first observed on the records of Haldde Observatory and described by BIRKELAND (1911). The perturbation occurred as regular sinusoidal oscillations of considerable amplitude. Later on, similar series of oscillations were recorded in Abisko and reported by ROLF (1931) who described them as "giant micropulsations". A survey of data from giant pulsations at Bossekop, Tromsø, Bodø and Sodankylä during the period 1929—1941 has been given by HARANG (1941). Registrations of the pulsations are at present being performed at the observatories of Kiruna, Abisko, and Lovö (AMBOLT, 1954). The data obtained by Ambolt for the period 1951—1953 show essentially the same features as those given by Harang. The periods of giant pulsations are of the order of 100 seconds and durations between 20 and 240 minutes have been observed. According to Harang the pulsations seem to occur most frequently and with the greatest amplitude on records from points in or near the auroral zone.

Harang suggests that the pulsations may be produced by processes within the ionosphere. In this paper will be examined the possibilities of explaining the oscillations as magneto-hydrodynamic wave phenomena. Motions in the ionosphere, perpendicular to the earth's magnetic field, may give rise to induced electric currents which interact with the magnetic field in the form of an electromagnetic force. If the dissipation is small enough the motion will be governed by this force and its interaction with the inertia force, and magneto-hydrodynamic waves are generated. The application of magneto-hydrodynamic wave phenomena to the problem of giant pulsations leads to the following two questions:
(i) Do the observed periods correspond to possible modes of oscillation in the ionospheric layers?
(ii) Is the damping of the waves small enough to make oscillations possible?

The investigations presented here will start with a discussion on the laws of conservation of momentum and energy for an ionized gas in a magnetic field. Since the particle density is relatively low in large parts of the ionosphere the influence of the Hall current has to be considered. The phase velocities and the damping lengths for plane waves are deduced in Sec. 2 and the results applied to the ionosphere in Sections 3 and 4.

1. Conservation of momentum and energy

The propagation of magneto-hydrodynamic waves in a compressible conductor was first discussed by Herlofson (1950), Hoffmann and Teller (1950) and van de Hulst (1951). The first and the last authors have used the expression

$$dp = C^2\, d\varrho \tag{1}$$

between the changes in pressure p and density ϱ, C being the velocity of ordinary aerodynamic sound waves. However, no proof seems to have been given that this relation should also be valid in magneto-hydrodynamics. Hoffman and Teller show that the adiabatic condition

$$du = p\, d\varrho/\varrho^2 \tag{2}$$

for the change in the internal (thermal) energy u per unit mass is valid for plane, non-dissipative magneto-hydrodynamic waves of small amplitudes. Relation (1) follows from eq. (2), which is readily seen in the case of a polytropic gas for which

$$\Theta = p/R\varrho;\quad u = c_v \Theta \tag{3}$$

where R is the universal gas constant divided by the mean molecular weight, Θ the temperature and c_v the specific heat at constant volume. If c_p is the specific heat at constant pressure the adiabatic condition (2) leads to

$$C = (\gamma p/\varrho)^{1/2};\quad \gamma = c_p/c_v, \tag{4}$$

and p becomes a function of ϱ only, as stated by eq. (1).

We are now going to investigate the limitations of the assumption expressed by equation (1). Let us consider an ionized gas moving with velocities much less than that of light. The density and thermal energy of the charged particles are supposed to be so small that it is not necessary to discuss the molecular contributions to the electric and magnetic fields. The absolute dielectric constant and the absolute permeability are then approximately equal to the values ε_0 and μ_0 in vacuum, and Maxwell's equations for the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$, given in MKSA-units, are:

$$\text{curl } \mathbf{B} = \mu_0\, \mathbf{i} + \mu_0\, \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{5}$$

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{6}$$

and

$$\text{div } \mathbf{E} = \varrho_E/\varepsilon_0;\quad \text{div } \mathbf{B} = 0. \tag{7}$$

ϱ_E is the charge density and $\mathbf{i}$ the current density. Following Stratton (1941) we multiply eq. (5) vectorially with $\mathbf{B}/\mu_0$ and eq. (6) vectorially with $\varepsilon_0 \mathbf{E}$. The sum of the obtained equations becomes:

$$\varrho_E \mathbf{E} + \mathbf{i} \times \mathbf{B} = -\varepsilon_0 \frac{\partial}{\partial t}(\mathbf{E} \times \mathbf{B}) + \text{div } \mathbf{S}, \tag{8}$$

where $\mathbf{S}$ is the usual electromagnetic stress tensor with nine components of the form $\varepsilon_0 \mathrm{E}_\alpha \mathrm{E}_\beta + \mathrm{B}_\alpha \mathrm{B}_\beta/\mu_0$. Eq. (8) is the law of conservation of momentum of the electromagnetic field. The left hand side may be interpreted as an increase per unit time of the mechanical momentum of electric charges and currents. The increase is supplied by the electromagnetic field. Scalar multiplication of eq. (5) with $\mathbf{E}/\mu_0$ and of eq. (6) with $\mathbf{B}/\mu_0$ gives after summing up the obtained expressions:

$$\mathbf{E}\cdot\mathbf{i} = -\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0 \mathbf{E}^2 + \frac{1}{2}\mathbf{B}^2/\mu_0\right) - \text{div}(\mathbf{E} \times \mathbf{B}/\mu_0), \tag{9}$$

which is the law of conservation of energy of the electromagnetic field. The left hand side is the total energy per unit time and unit volume given off by the electromagnetic field and transformed into other forms of energy.

The mechanical conditions governing the motion of the gas are the laws expressing conservation of matter,

$$\text{div } \mathbf{v} = -\frac{1}{\varrho}\frac{d\varrho}{dt} \tag{10}$$

and conservation of momentum,

$$\varrho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{f}, \tag{11}$$

where $\mathbf{v}$ is the gas velocity and $\mathbf{f}$ represents all forces acting on the gas in addition to the pressure force. Viscous forces are assumed to be negligible. With the thermal energy u per unit mass of charged and uncharged particles and with the kinetic energy $\frac{1}{2}\mathbf{v}^2$ per unit mass the law of conservation of energy becomes

$$\frac{d}{dt}\left(u+\frac{1}{2}\mathbf{v}^2\right)=\frac{1}{\varrho}\left[-p\operatorname{div}\mathbf{v}-(\mathbf{v}\cdot\nabla)p+w\right]. \tag{12}$$

The increase in thermal and kinetic energy is supplied by the right hand side of eq. (12) which is the sum of the compression- and acceleration work performed by the pressure force per unit time and mass together with the input of energy, w/ϱ, per unit time and mass from other sources.

Now, let us assume that the gas is isolated from the surroundings, i.e., no external forces are acting and no energy, e.g., in the form of heat, is supplied from the outside. The left hand side of eq. (8) is the only force which couples the electromagnetic field with the hydrodynamic. Further, the total energy given off by the electromagnetic field has to be converted into thermal and kinetic energy. Consequently, equations (8), (9), (11) and (12) imply that

$$\mathbf{f}=\varrho_E\mathbf{E}+\mathbf{i}\times\mathbf{B};\quad w=\mathbf{E}\cdot\mathbf{i}, \tag{13}$$

and the laws of conservation of the total momentum and the total energy become

$$\varrho\frac{d\mathbf{v}}{dt}=-\nabla p+\varrho_E\mathbf{E}+\mathbf{i}\times\mathbf{B}, \tag{14}$$

and

$$\varrho\frac{d}{dt}\left(u+\frac{1}{2}\mathbf{v}^2\right)=-p\operatorname{div}\mathbf{v}-(\mathbf{v}\cdot\nabla)p+\mathbf{E}\cdot\mathbf{i}. \tag{15}$$

These relations follow in the non-relativistic limit from the theory on the total energy-momentum tensor (see, e.g., MØLLER, 1952).

We are now going to rewrite the energy theorem (15) in a more condensed form. For this purpose the relation between $\mathbf{E}$ and $\mathbf{i}$ will be given in terms of the electrical conductivity. For non-relativistic velocities the quantities $\mathbf{E}'$, $\mathbf{B}'$ and $\mathbf{i}'$ in a coordinate system following matter are related to the corresponding quantities $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{i}$ in a system "at rest" by

$$\mathbf{E}'=\mathbf{E}+\mathbf{v}\times\mathbf{B},\quad \mathbf{B}'=\mathbf{B}, \tag{16}$$

and

$$\mathbf{i}'=\mathbf{i}-\varrho_E\mathbf{v}. \tag{17}$$

The field $\mathbf{E}'$ is divided into components parallel and perpendicular to $\mathbf{B}'$:

$$\mathbf{E}'=(\mathbf{E}'\cdot\hat{\mathbf{B}})\,\hat{\mathbf{B}}+\hat{\mathbf{B}}\times(\mathbf{E}'\times\hat{\mathbf{B}}), \tag{18}$$

where $\hat{\mathbf{B}}$ is the unit vector along $\mathbf{B}$. The current density is

$$\mathbf{i}'=\sigma_0(\mathbf{E}'\cdot\hat{\mathbf{B}})\,\hat{\mathbf{B}}+\sigma_1(\hat{\mathbf{B}}\times\mathbf{E}')\times\hat{\mathbf{B}}+\sigma_2(\hat{\mathbf{B}}\times\mathbf{E}'), \tag{19}$$

where the parallel conductivity σ_0, the cross conductivity σ_1 and the Hall conductivity σ_2 are given by

$$\sigma_0=Ne^2(\tau_e/m_e+\tau_i/m_i), \tag{20}$$

$$\sigma_1=Ne^2[\tau_e/m_e(1+\omega_e^2\tau_e^2)+\tau_i/m_i(1+\omega_i^2\tau_i^2)], \tag{21}$$

and

$$\sigma_2=Ne^2[\omega_e\tau_e^2/m_e(1+\omega_e^2\tau_e^2)-\omega_i\tau_i^2/m_i(1+\omega_i^2\tau_i^2)]. \tag{22}$$

N is the density of electrons which has been assumed to be approximately equal to the density of ions. e, τ_e, m_e and $\omega_e=eB/m_e$ are the charge, collision time, mass and gyro-frequency of the electrons and τ_i, m_i and ω_i the corresponding mean values for the ions (cf CHAPMAN and COWLING, 1939 and COWLING, 1945).

Substitution of expressions (16) and (17) into eq. (19) gives, after rearranging terms and using well-known vector identities:

$$\mathbf{i}=\varrho_E\mathbf{v}+\sigma_1(\mathbf{E}+\mathbf{v}\times\mathbf{B})+(\sigma_0-\sigma_1)(\mathbf{E}\cdot\hat{\mathbf{B}})\hat{\mathbf{B}}+\sigma_2[\hat{\mathbf{B}}\times(\mathbf{E}+\mathbf{v}\times\mathbf{B})], \tag{23}$$

or

$$\mathbf{E}=-\mathbf{v}\times\mathbf{B}+\mathbf{E}^\star, \tag{24}$$

where

$$\mathbf{E}^\star=(\mathbf{i}-\varrho_E\mathbf{v})/\sigma_1+(1-\sigma_0/\sigma_1)(\mathbf{E}\cdot\hat{\mathbf{B}})\hat{\mathbf{B}}+[(\mathbf{E}+\mathbf{v}\times\mathbf{B})\times\hat{\mathbf{B}}]\,\sigma_2/\sigma_1. \tag{25}$$

From these results we deduce

$$\mathbf{E}\cdot\mathbf{i}=(-\mathbf{v}\times\mathbf{B}+\mathbf{E}^\star)\cdot\mathbf{i}=(\varrho_E\mathbf{E}+\mathbf{i}\times\mathbf{B})\cdot\mathbf{v}+(\mathbf{i}-\varrho_E\mathbf{v})\cdot\mathbf{E}^\star, \tag{26}$$

since $(\mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} = 0$. The expression obtained is substituted into the energy theorem (15). The first term of the right hand side of eq. (15) is rewritten by the help of the equation of continuity (10) and

$$\varrho \frac{du}{dt} + \varrho \mathbf{v} \frac{d\mathbf{v}}{dt} = \frac{p}{\varrho}\frac{d\varrho}{dt} - (\mathbf{v} \cdot \nabla)\, p + \\ + (\varrho_E \mathbf{E} + \mathbf{i} \times \mathbf{B}) \cdot \mathbf{v} + \mathbf{E}^\star \cdot (\mathbf{i} - \varrho_E \mathbf{v}). \quad (27)$$

Hence

$$\varrho \frac{du}{dt} - \frac{p}{\varrho}\frac{d\varrho}{dt} - \mathbf{E}^\star \cdot (\mathbf{i} - \varrho_E \mathbf{v}) = -\mathbf{v} \cdot \\ \cdot \left(\varrho \frac{d\mathbf{v}}{dt} + \nabla p - \varrho_E \mathbf{E} - \mathbf{i} \times \mathbf{B}\right) = 0, \quad (28)$$

because the bracket of the right hand side of the last equation vanishes according to the law of conservation of momentum (14). Two special cases of eq. (28) will be discussed:

(i) The gas is assumed to be ionized to a high degree and the density is made so high that the parallel conductivity becomes very large. We have $\sigma_0 \to \infty$, $N\tau_e \to \infty$, $N\tau_i \to \infty$, $N \to \infty$ and $\tau_e \to 0$, $\tau_i \to 0$ simultaneously. Then $\sigma_1 \to \to \sigma_0 \to \infty$ and $\sigma_2/\sigma_1 \to 0$. This gives $\mathbf{E}^\star \to 0$ and the law of conservation of energy (28) reduces to the adiabatic condition (2).

(ii) For small amplitudes pressure and density deviate slightly from their static values and the first two terms in eq. (28) are linear. The quadratic term containing $\mathbf{E}^\star$ can be neglected and eq. (2) is still valid.

The results show that relation (1) is correct for non-linear, non-dissipative motions as well as for motions of small amplitudes, damped by Joule losses. For large amplitudes the Joule heat introduces a dissipation term into the energy equation (28) and the pressure is no longer a function of density only.

Scalar multiplication with $\mathbf{v}$ of the law (14) gives a relation which may be transformed by the help of eq. (9) and (23) into a form containing the kinetic and electromagnetic energies. Such expressions have been derived by Walén (1944) and Baños (1955) and may form a convenient starting point for many discussions. However, since such forms are derived from the law of conservation of momentum they do not give any information about the conservation of the total energy.

2. Plane waves

The effective electric conductivity in a gas of low density, such as the ionosphere, depends on the special problem being considered. For a binary gas, where only one type of charged particles carries the main part of the electric current, Cowling (1933) has shown that the effective conductivity becomes equal to the parallel conductivity when the Hall current is prevented from flowing. If the contributions to the currents made by the ions are included the result will be modified. Examples are given by Hirono (1950, 1952) and Baker and Martyn (1953) in treating the problem of the electro jet. The motion of ionized gas in combined fields of force has been discussed by Piddington (1954). Two special types of wave phenomena in the ionosphere have recently been subject to investigations by Hines (1953, 1955) and Ilse Lucas (1954). Concerning the conductivity of ionized gases the reader is also referred to the papers by Cowling (1945, 1953) and Chapman and Cowling (1939).

a. *Approximations*

Before we start to work out a theory on plane waves in detail some simplifying approximations will be made. The amplitudes of the magnetic oscillations which have been recorded range between $b_s = 2 \times 10^{-9}$ Vs/m² and 3×10^{-8} Vs/m² (1 gauss $= 10^{-4}$ Vs/m²) for the perturbation field b_s at the earth's surface (Harang, 1941 and Ambolt, 1954). The field is assumed to be produced by a current system in the ionosphere. A rough estimation of the perturbation field b in the ionosphere is made with Biot-Savart's law. If the values given in table 1 of altitude and thickness of the ionospheric layers are used, we get $b \approx 10\, b_s$. The actual amplitude is probably larger for oscillations in the upper layers due to a screening action caused by underlying parts of the ionosphere. The strength of the geomagnetic field in the auroral regions is $B_0 = 6 \times 10^{-5}$ Vs/m². We have $b \approx 10\ b_s \leq 3 \times 10^{-7}$ Vs/m², and the assumption of small amplitudes seems to be justified.

Relativistic effects and the influence of the displacement current are neglected. This is seen from the magnitude of the propagation velocities obtained in table 2.

The ratio between the Coriolis force and the electrodynamic force acting on a plane wave is given by LEHNERT (1954):

$$\chi = \Omega/\varkappa V, \tag{29}$$

where Ω is the angular velocity, $\varkappa$ the wave number and $V = B_0/(\mu_0 \varrho)^{1/2}$ the Alfvén velocity. For the ionosphere we have $\Omega = 10^{-5}$ sec.$^{-1}$ and $V \geq 5.5$ m/sec. Wave lengths at most of the order of the thickness of the ionospheric layers correspond to $\chi \leq 0.03$. Thus, the Coriolis force can be neglected in the first approximation.

The ratio between the gravitation force and the electromagnetic force per unit volume is of the order

$$f_g/f_m = g\,\varrho/|\mathbf{i} \times \mathbf{B}| \approx g\mu_0 \varrho\lambda/bB_0 = B_0 U g t_0/bV^2, \tag{30}$$

where $\lambda = Ut_0$ is the wave length, U the phase velocity and t_0 the period of oscillation. The smallest values of t_0 are about 60 seconds and the smallest value of U/V^2 is 5×10^{-5} sec./m according to table 2. This gives $f_g/f_m \geq 8$ which implies that the gravitation force cannot generally be neglected and agrees with estimations carried out by HINES (1955). We will limit our investigations to horizontal motions when the theory of this section is applied to the ionosphere.

The relative importance of the kinematic viscosity ν and the electromagnetic "viscosity", $1/\mu_0\sigma_3$, is estimated from

$$\mu_0 \sigma_3 \nu \approx \frac{1}{3}\,\bar{u} l \mu_0 \sigma_3, \tag{31}$$

where $\bar{u}$ is the thermal velocity, l the mean free path and σ_3 is given in table 1. With $\sigma_3 \leq 10^{-3}\,\Omega^{-1} \cdot \text{m}^{-1}$, $\bar{u} \approx 10^3$ m/sec. and $l \leq 3 \times 10^3$ m we obtain $\mu_0\sigma_3\nu \leq 10^{-3}$. Consequently, the viscosity can be neglected with good approximation.

The thermal energy density of the charged particles does not exceed the order of $NkT \approx 10^{-8}$ newton m/m^3 and the magnetic energy density is $B_0^2/2\mu_0 \approx 10^{-3}$ n m/m^3. Thus, we do not have to discuss the influence which the thermal motion may have on the total magnetic field.[1]

Finally, the density of negatively charged particles is assumed to be approximately equal to the density of positive particles so that the convection current can be neglected besides the conduction current and the electrostatic force besides the electrodynamic.

b. *The relation between wave number and frequency*

A coordinate system is chosen with the z-axis along the wave normal of a plane wave which moves in a homogeneous and constant magnetic field,

$$\mathbf{B}_0 = (0, B_y, B_z). \tag{32}$$

The induced magnetic field of the wave is $\mathbf{b}$ and the static values of pressure and density are p_0 and ϱ_0. The field quantities $\mathbf{E}$, $\mathbf{b}$, $\mathbf{i}$, $\mathbf{v}$, $p - p_0$ and $\varrho - \varrho_0$ are assumed to vary as exp $[j(\omega t - \varkappa z)]$ in the wave and are propagated with the frequency ω and the wave number $\varkappa$. Differential operations are represented by $\partial/\partial z = -j\varkappa$ and $\partial/\partial t = j\omega$, where $j = (-1)^{1/2}$. The approximations mentioned in section 2.a are introduced and equations (14), (1) and (4) give

$$\frac{\partial \mathbf{v}}{\partial t} = (\text{curl}\,\mathbf{b}) \times \mathbf{B}_0/\mu_0\varrho_0 - \frac{C^2}{\varrho_0}\nabla\varrho. \tag{33}$$

We introduce the notations

$$U = \omega/\varkappa, \quad \mathbf{V} = \mathbf{B}_0/(\mu_0\varrho_0)^{1/2}, \tag{34}$$

and obtain from equations (10), (5), (6), and (33):

$$\varrho = \varrho_0\, v_z/U, \tag{35}$$

$$E_x = Ub_y, \quad E_y = -Ub_x, \tag{36}$$

and

$$v_x = -\frac{V_z}{U}\,b_x/(\mu_0\varrho_0)^{1/2}, \quad v_y = -\frac{V_z}{U}\,b_y/(\mu_0\varrho_0)^{1/2},$$

$$v_z = \frac{UV_y}{U^2 - C^2}\,b_y/(\mu_0\varrho_0)^{1/2}. \tag{37}$$

With

$$\eta_{0,1,2} = \mu_0\sigma_{0,1,2}/\omega = \mu_0\sigma_{0,1,2}/\varkappa U \tag{38}$$

[1] I am indebted to Dr. E. Åström for pointing this out to me.

equations (5) and (23) are combined to

$$(b_y, -b_x, 0) = -j\eta_1 U(E_x + v_y B_z - v_z B_y, E_y - v_x B_z, E_z + v_x B_y) - $$
$$-j(\eta_0 - \eta_1)\frac{U}{B^2}(E_y B_y + E_z B_z)(0, B_y, B_z) -$$
$$-j\eta_2 \frac{U}{B}(E_z B_y - E_y B_z + v_x B^2, E_x B_z + v_y B_z^2 - - v_z B_y B_z, -E_x B_y - v_y B_y B_z + v_z B_y^2). \quad (39)$$

Expressions (36), (37), and (34) are substituted into the three equations (39) which become

$$\begin{vmatrix} j\eta_2\hat{V}_z(U^2 - V^2) & j\eta_1 L + 1 & j\eta_2\hat{V}_y \\ 1 + j\eta_1\hat{V}_z^2(U^2 - V^2) + j\eta_0\hat{V}_y^2 U^2 & -j\eta_2\hat{V}_z L & -j(\eta_0 - \eta_1)\hat{V}_y\hat{V}_z \\ -j\eta_1\hat{V}_y\hat{V}_z(U^2 - V^2) + j\eta_0\hat{V}_y\hat{V}_z U^2 & j\eta_2\hat{V}_y L & -j\eta_0\hat{V}_z^2 - j\eta_1\hat{V}_y^2 \end{vmatrix} \cdot \begin{vmatrix} b_x \\ b_y \\ UE_z \end{vmatrix} = 0, \quad (40)$$

where we have introduced

$$\hat{V}_y = V_y/V, \quad \hat{V}_z = V_z/V, \quad (41)$$

and

$$L = L(U) = $$
$$= [U^4 - (C^2 + V^2)U^2 + C^2 V_z^2]/(U^2 - C^2). \quad (42)$$

Non-trivial solutions of equations (40) are obtained when the determinant vanishes. With

$$T = T(U) = U^2 - V_z^2, \quad (43)$$

and after some straightforward deductions, this condition reduces to

$$[(\eta_1^2 + \eta_2^2)L - j\eta_1](\eta_0 T - j\hat{V}_y^2) -$$
$$-j\eta_0\hat{V}_z^2(\eta_1 L - j) = 0, \quad (44)$$

or

$$LT - jL[\hat{V}_y^2/\eta_0 + \eta_1\hat{V}_z^2/(\eta_1^2 + \eta_2^2)] -$$
$$-jT\eta_1/(\eta_1^2 + \eta_2^2) -$$
$$-(\eta_1\hat{V}_y^2 + \eta_0\hat{V}_z^2)/\eta_0(\eta_1^2 + \eta_2^2) = 0. \quad (45)$$

The frequency ω may be considered as given and the complex phase velocity U is the quantity to be determined.

When the density becomes large and the collision times small expressions (20), (21), and (22) indicate that $\sigma_1 \to \sigma_0 \equiv \sigma$ and $\sigma_2 \to 0$. Equation (44) reduces to

$$(\eta L - j) \cdot (\eta T - j) = 0, \quad (46)$$

when $\eta \equiv \eta_0$ is assumed to differ from zero. The system (40) is then divided into two sets:

Set I. The first set corresponds to

$$\eta L - j = 0 \quad (47)$$

and applies to the fields b_y, E_x, v_y, and v_z, as seen from eq. (40), (36), and (37). The particle velocity may have a component v_z in the direction of the wave normal and the solution may contain longitudinal waves.

Set II. The second set is given by

$$\eta T - j = 0 \quad (48)$$

and contains the fields b_x, E_y, $E_z = V_y V_z b_x/U$, and v_x. The induced magnetic field and the particle velocity are both perpendicular to the wave normal and the waves are transverse Alfvén waves.

Equations (47) and (48) are identical with those obtained by VAN DE HULST (1951). However, when the density decreases a modification of these results is introduced by the Hall current, as shown by eq. (40) and (44). The Hall current generated by one type of wave interacts with the other type. In such a case a coupling arises between the longitudinal and transverse modes which no longer can be separated into two independent wave systems.

c. *Slightly damped waves*

Only slightly damped waves are of practical interest in the following investigations which aim at a well defined type of oscillations. The complex "phase velocity" is written as

$$U = P + jQ, \quad (49)$$

P and Q being its real and imaginary parts and $Q/P \ll 1$ for slightly damped solutions. The field quantities vary as

$$\exp[j\omega(t - z/U)] \approx \exp[j\omega(t - z/P) - \omega Q z/P^2], \quad (50)$$

corresponding to a phase velocity approximately equal to P and a damping distance

$$z_0 \approx P^2/\omega Q. \qquad (51)$$

The parallel and cross conductivities are large as well as η_0 and η_1. Infinite values of η_0 and η_1 lead to van de Hulst's two sets of undamped waves described by eq. (45) in the reduced form

$$L \cdot T = 0, \qquad (52)$$

where the coupling caused by the Hall current has disappeared. Consequently, the coupling is small for large values of η_0 and η_1 and we may distinguish between two loosely coupled sets of waves with essentially the same features as those studied by van de Hulst. This is also confirmed by the discussion which follows.

Introducing the notation (49) into eq. (42) and (43) we get in first order:

$$L(U) \approx L(P) + 2jPQG(P), \qquad (53)$$

where

$$G(P) = [2P^2 - C^2 - V^2 - L(P)]/(P^2 - C^2), \qquad (54)$$

and

$$T(U) \approx T(P) + 2jPQ. \qquad (55)$$

Hence

$$L(U) \cdot T(U) \approx L(P) \cdot T(P) + 2jPQ \cdot \cdot [G(P)T(P) + L(P)]. \qquad (56)$$

Expressions (53), (55) and (56) are substituted into eq. (45) and real and imaginary parts are separated;

$$L(P)T(P) + 2PQG(P)[\hat{V}_y^2/\eta_0 + \eta_1\hat{V}_z^2/(\eta_1^2+\eta_2^2)] + PQ\eta_1/(\eta_1^2+\eta_2^2) - (\eta_1\hat{V}_y^2 + \eta_0\hat{V}_z^2)/\eta_0(\eta_1^2+\eta_2^2) = 0 \qquad (57)$$

and

$$2PQ[G(P)T(P) + L(P)] - L(P)[\hat{V}_y^2/\eta_0 + \eta_1\hat{V}_z^2/(\eta_1^2+\eta_2^2)] - T(P)\eta_1(\eta_1^2+\eta_2^2) = 0. \qquad (58)$$

As far as orders of magnitude are concerned eq. (57) and (58) may be written as

$$L(P)T(P) + QG(P)O(1/N) + O(1/N^2) = 0 \qquad (59)$$

and

$$Q[G(P)T(P) + L(P)] + L(P) \cdot O(1/N) + T(P)O(1/N) = 0, \qquad (60)$$

where $O(1/N)$ stands for terms of the order $1/N$, N being the density of charged particles. For slightly damped waves $L(P) \cdot T(P)$ is close to zero whereas $G(P)$ differs from zero according to eq. (54). Eq. (59) and (60) are equivalent to

$$L(P) \cdot T(P) = O(1/N^2); \quad Q = O(1/N). \qquad (61)$$

We distinguish between two sets of solutions giving the results (47) and (48) in the limit of high densities:

Set I. For the longitudinal set the phase velocity P is given by eq. (42) with $L(P) \approx 0$, which has the solution

$$P \approx \pm \left\{\frac{1}{2}(C^2 + V^2) \pm \pm \frac{1}{2}\left[(C^2+V^2)^2 - 4C^2V_z^2\right]^{1/2}\right\}^{1/2}. \qquad (62)$$

Q is obtained from eq. (58) and (61):

$$Q \approx \eta_1 (P^2 - C^2)/2(\eta_1^2 + \eta_2^2)P(2P^2 - C^2 - V^2). \qquad (63)$$

From $L(P) = 0$ follows that

$$(P^2 - C^2)(P^2 - V^2) \geq 0, \qquad (64)$$

which shows that $Q > 0$ when $P > 0$ and $Q < 0$ when $P < 0$ according to eq. (63) and the waves are damped as it ought to be for solutions of the form (50). The damping distance is deduced from eq. (51), (63) and (38):

$$z_0 = 2\mu_0\sigma_{eff}|P^3|(2P^2 - C^2 - V^2)/\omega^2(P^2 - C^2), \qquad (65)$$

where the effective conductivity is

$$\sigma_{eff} = \sigma_3 = (\sigma_1^2 + \sigma_2^2)/\sigma_1 \qquad (66)$$

and P stands for the solutions (62).

Set II. For the transverse set $T(P) \approx 0$ and the phase velocity becomes

$$P \approx \pm V_z. \qquad (67)$$

Further, from eq. (58)

$$Q \approx [\hat{V}_y^2/\eta_0 + \eta_1\hat{V}_z^2/(\eta_1^2+\eta_2^2)]/2P. \qquad (68)$$

The damping distance is

$$z_0 = 2\mu_0\sigma_{eff}V_z^3/\omega^2, \qquad (69)$$

with the effective conductivity

$$\sigma_{eff} = \sigma_0/[1 - \hat{V}_z^2(1 - \sigma_0/\sigma_3)], \tag{70}$$

where σ_3 is defined by eq. (66).

The terrestrial magnetic field is nearly vertical in the auroral zones. Two solutions with horizontal velocity vectors will be discussed in the following, namely, the longitudinal wave of set I propagating in horizontal direction and the Alfvén wave of set II propagating in vertical direction. The phase velocity and damping distance are

$$|P| = (C^2 + V^2)^{1/2}$$

and $$z_0 = 2\mu_0\sigma_3(C^2 + V^2)^{3/2}/\omega^2 V^2 \tag{71}$$

for the former type and

$$|P| = V \text{ and } z_0 = 2\mu_0\sigma_3 V^3/\omega^2 \tag{72}$$

for the latter. These special cases have earlier been discussed by PIDDINGTON (1954 b).

d. *Free oscillations*

Instead of assuming the frequency ω to be given we may consider an initial value problem where a harmonic distribution in space with the wave number $\varkappa$ is given at time $t = 0$. We assume the behaviour at later times to be given by the form exp $[j(\omega t - \varkappa z)]$. The frequency is now a complex quantity the imaginary part of which corresponds to a decrease in amplitude with time. The formal deductions are identical with those of the preceding sections. The F_1- and F_2-regions are of special importance and here the approximations $C^2 \ll V^2$, $\sigma_3 \approx \sigma_1$ and $\sigma_2 \ll \sigma_1$ are valid according to table 1. Since U must be of the same order as V we put $U^2 \gg C^2$ and obtain for the horziontal, longitudinal wave ($V_z = 0$):

$$U^2 - V^2 \approx j\omega/\mu_0\sigma_1 \tag{73}$$

according to eq. (44), (42) and (38). The same result applies to the vertical Alfvén wave ($V_y = 0$). Eq. (73) is solved for the frequency:

$$\omega = \frac{1}{2} j\varkappa^2/\mu_0\sigma_1 \pm \frac{1}{2}\{-(\varkappa^2/\mu_0\sigma_1)^2 + 4\varkappa^2 V^2\}^{1/2}. \tag{74}$$

If $\varkappa/\mu_0\sigma_1 \geq 2V$ the solutions are aperiodic (LEHNERT, 1955), i.e., the damping is too large to permit free oscillations. Equality corresponds to the wave length $\lambda_0 = 2\pi/\varkappa_0$ where the oscillations first set in:

$$\lambda_0 = 4\pi\mu_0\sigma_1 V/\varkappa_0^2 = 2\pi \cdot 2\mu_0\sigma_1 V^3/|\omega_0|^2. \tag{75}$$

Comparing this result with eq. (71) and (72) we see that free oscillations may start, roughly speaking, when 2π times the damping distance is of the order of the wave length λ.

3. Ionospheric data

The preceding results will now be applied to the ionosphere. Data for the ionosphere are taken from the works by RAWER (1953), KUIPER (1954) and NICOLET (1953). The most important ions are O^+, O_2^+, N^+, and N_2^+. For ions we choose the mean values $\omega_i \approx 200$ sec. for the gyrofrequency and $\tau_i \approx \tau_e \cdot (m_i/2m_e)^{1/2} \approx$ $\approx 120\ \tau_e$ for the collision time. The electron density is put approximately equal to the ion density and the ratio between the specific heats $\gamma \approx 1.5$. The values of N given in table 1 refer to the daytime. At night the density of charged particles is considerably lower in the D- and E-layers, whereas it stays nearly constant in the F_1- and F_2-layers. The value $B_0 = 6 \times 10^{-5}$ Vs/m^2 is used for the earth's magnetic field. We observe that the conductivity σ_3 in table 1 has a peak in the E-layer, as pointed out by BAKER and MARTYN (1953). The wave lengths $\lambda = |P|\, t_0$ and damping distances z_0 of the longitudinal wave and the Alfvén wave given by eq. (71) and (72) are shown in table 2.

4. Giant pulsations

Observations have so far indicated the following main features of giant pulsations:

a. The pulsations are regular and have a period range between 60 and 300 seconds and a duration between 10 and 30 periods. The growth of the amplitude is continuous as well as its decrease. The maximum amplitude occurs about half way between the beginning and the end of the pulsation.

b. Oscillations have been observed almost exclusively in the auroral zone and occur with the highest frequency during disturbed years with maximum solar activity. They are less frequent at Lovö (near Stockholm) than at Kiruna and Abisko and are rare at Rude Skov (Copenhagen). No pulsations have been registered at Godhavn and Thule to the north of the auroral zone (OHLSEN, 1954).

c. Pulsations, probably due to the same source, have been observed simultaneously at different places as far as 500 km and 1,000 km apart

Table 1. Ionospheric data.

Layer	D	E	F_1	F_2
Mean altitude h (m)	7×10^4	10^5	2×10^5	3×10^5
Mean thickness d (m)	3×10^4	5×10^4	10^5	2×10^5
Temperature Θ (°K)	220	240	800	1,180
Pressure p (n/m²)	6.1	5.4×10^{-2}	3.9×10^{-5}	2.3×10^{-6}
Density ϱ (kg/m³)	9.7×10^{-5}	8.6×10^{-7}	1.7×10^{-10}	7×10^{-12}
Sound velocity C (m/s)	310	306	586	635
Alfvén velocity V (m/s)	5.5	58	4,100	2×10^4
Electron density N (m⁻³)	10^9	1.5×10^{11}	2.5×10^{11}	10^{12}
Collision frequency ν_e (sec.⁻¹)	10^7	3×10^4	3×10^3	10^3
Collision time τ_e (sec.)	10^{-7}	3×10^{-5}	3×10^{-4}	10^{-3}
Parallel conductivity σ_0 (Ω^{-1} m⁻¹)	2.8×10^{-6}	0.13	2.1	28
Cross conductivity σ_1 (Ω^{-1} m⁻¹)	1.4×10^{-6}	1.4×10^{-6}	1.3×10^{-4}	1.6×10^{-4}
Hall conductivity σ_2 (Ω^{-1} m⁻¹)	1.3×10^{-6}	2.7×10^{-4}	1.3×10^{-5}	4.6×10^{-6}
Effective conductivity σ_3 (Ω^{-1} m⁻¹)	2.6×10^{-6}	5.2×10^{-2}	1.3×10^{-4}	1.6×10^{-4}

Table 2. Wave lengths and damping distances for magneto-hydrodynamic waves in the ionosphere. Pairs of data correspond to the range of periods $t_0 = 60 \ldots 300$ seconds.

Layer	D	E	F_1	F_2
Mean thickness d (m)	3×10^4	5×10^4	10^5	2×10^5
Longitudinal wave				
Phase velocity $\|P\|$ (m/s)	310	311	4,240	2×10^4
Wave length λ (m)	$2\times10^4\ldots$ $\ldots10^5$	$2\times10^4\ldots$ $\ldots10^5$	$2.5\times10^5\ldots$ $\ldots13\times10^5$	$12\times10^5\ldots$ $\ldots60\times10^5$
Damping distance z_0 (m)	$57\ldots$ $\ldots1.4\times10^3$	$9.6\times10^3\ldots$ $\ldots2.4\times10^5$	$2.1\times10^3\ldots$ $\ldots5.1\times10^4$	$3\times10^5\ldots$ $\ldots72\times10^5$
$2\pi z_0/\lambda$	$0.02\ldots$ $\ldots0.08$	$3\ldots$ $\ldots15$	$0.05\ldots$ $\ldots0.25$	$1.6\ldots$ $\ldots7.5$
Alfvén wawe				
Phase velocity $\|P\|$ (m/s)	5.5	58	4,100	2×10^4
Wave length λ (m)	$3.3\times10^2\ldots$ $\ldots16\times10^2$	$3.5\times10^3\ldots$ $\ldots2\times10^4$	$2.5\times10^5\ldots$ $\ldots13\times10^5$	$12\times10^5\ldots$ $\ldots60\times10^5$
Damping distance z_0 (m)	$10^{-7}\ldots$ $\ldots25\times10^{-7}$	$2.3\ldots$ $\ldots58$	$2.1\times10^3\ldots$ $\ldots5.1\times10^4$	$3\times10^5\ldots$ $\ldots72\times10^5$
$2\pi z_0/\lambda$	$2\times10^{-9}\ldots$ $\ldots10^{-8}$	$0.004\ldots$ $\ldots0.018$	$0.05\ldots$ $\ldots0.25$	$1.6\ldots$ $\ldots7.5$

(Bodø—Sodankylä and Abisko—Lovö). The phase differences are moderate between pairs of stations such as Tromsø—Bossekop, Tromsø—Bodø and Kiruna—Abisko. This may indicate that the characteristic linear dimensions of the source are of the same order as the distances between the stations, i.e., at least of the order 100 km. The variation of the disturbance components with latitude leads HARANG (1941) to suggest a current system directed along the auroral zone.

d. Very few pulsations have been observed between 13^h and 21^h. The greatest number occurs $2\frac{1}{2}$ hours after the diurnal maximum of terrestrial and magnetic storminess, i.e., around 3^h in the morning. However, giant pulsations have been recorded during the bright time as well as during the dark time of the day. HARANG (1941) describes 17 pulsations during the bright time and 80 during the dark and corresponding numbers observed by AMBOLT (1954) in the period 1951—1953 are 11 and 33 respectively.

e. There exist observations where a correlation has been found between the pulsations and the properties of the ionosphere. During a pulsation on December 28, 1938 in Tromsø the radio echo from the ionosphere showed periodically varying properties with exactly the same period as the pulsation.

f. More or less regular vibrations in the earth's magnetic field with periods of the order of a second have been observed in rapid magnetic registrations. The vibrations occur in the auroral zone, simultaneously at different stations.

The discussion on the possibility of magneto-hydrodynamic oscillations with the observed periods will be based on the data given in table 2. An exact formulation of the boundary conditions with a description of all possible modes of oscillation and of the generating mechanism which supplies the energy is not within the scope of this investigation. We may only suggest that oscillations may be generated by an agency with a frequency which drifts through a resonance of the ionosphere. This would offer a plausible explanation of the observed continuous growth and decrease of the amplitude of giant pulsations. Further, the required energy is easily estimated. The magnetic energy of a pulsation with the amplitude $b = 3 \times 10^{-7}$ Vs/m^2 is about 3×10^9 n m within a region of the size of $1000 \times 1000 \times 100$km^3. This corresponds to a heating by 500°C within a volume of $35 \times 35 \times 35$ km^3 at an altitude of 200 km. Thus, the pulsations may be formed by energetically possible processes within the ionosphere.

We will concentrate on two cases of horizontal motion; the longitudinal wave (71) propagating in horizontal direction and the Alfvén wave (72) propagating in vertical direction. For the former the largest imaginable wave length is about for times the breadth of the auroral zone. This would correspond to a standing wave across the zone with a wave length $\lambda \leq 4{,}000$ km and with induced currents directed along the zone. For the latter we may suggest a vertical standing wave between the "surfaces" of an ionospheric layer. If the pulsation is generated by some agency situated near one surface and oscillating close to the resonant frequency of the layer the thickness d will roughly be a quarter of a wave length. Consequently, we put $\lambda \leq 4d$ for the Alfvén wave. The range of wave lengths in table 2 show that the periods of the longitudinal mode are consistent with those of giant pulsations in all layers except in the F_2-layer for periods near $t_0 = 300$ sec., where λ may exceed four times the breadth of the auroral zone. The border between the F_1- and F_2-regions is not well defined and we may possibly treat the two layers as a whole when considering vertical Alfvén waves. Concerning the periods such waves would be possible in all layers exept for periods near 300 sec. where $\lambda/4$ exceeds the thickness of the F-region.

However, a limitation is introduced by the dissipation, even if we assume the effective conductivity in the auroral zone to be 10 times larger than its normal value outside the zone. From table 2 is seen that the ratio between damping distance and wave length is small enough to exclude magneto-hydrodynamic oscillations in the D-layer for both the considered modes and in the E-layer for the Alfvén mode. In the F-layers, on the other hand, the ratio sometimes comes close to the values where oscillations may be expected. It is not unimaginable that the damping under normal ionospheric circumstances is too large to permit oscillations, but that a slight increase in the electrical conductivity in the auroral zones would make such oscillations possible for both modes in the F_1- and F_2-regions and for the longitudinal mode in the E-region in the daytime. Rawer (1953) expects a considerable increase in electron density in the auroral zones, at least in the E-layer. An enhancement of electron density caused by the auroral discharge would explain why the pulsations are limited to the auroral zone and would exclude other types of oscillations not governed by electrodynamic forces.

Finally, the rapid vibrations with periods of the order of 1 second correspond to $2\pi z_0/\lambda \approx \approx 0.04$ for the longitudinal wave in the E- and F_2-regions and for the Alfvén wave in the F_2-region. The wave lengths are 3×10^2, 2×10^4 and 2×10^4 meters, respectively. Thus, we cannot exclude the possibility that the vibrations are caused by magneto-hydrodynamic waves in the auroral zone.

5. Conclusions

The preceding investigations show that magneto-hydrodynamic oscillations may possibly take place in the ionosphere with the period range observed for giant pulsations.

The assumption of an electric conductivity which is somewhat higher within the auroral zone than in its surroundings is likely to make oscillations possible within the zone andexclude oscillations outside of it. The pulsations will then occur in the F_1- and F_2-regions in the daytime as well as at night and in the E-region in the daytime. Their correlation with terrestrial and magnetic storminess may be caused partly by an enhancement in electrical conductivity, partly by an encrease in the number and strength of the sources which may generate giant pulsations.

Probably, the possibility cannot be excluded that rapid vibrations sometimes observed in the earth's magnetic field have periods and damping distances consistent with magneto-hydrodynamic oscillations in the auroral zones,

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