The Equatorial Undercurrent in the Light of the Vorticity Equation[1]

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(Manuscript received 23 July 1954)

Abstract

The Equatorial Undercurrent, recently observed in the central Pacific Ocean by Cromwell, Montgomery, and Stroup, is a narrow eastward current occupying the lower part of the surface layer at the equator. In the present study the Undercurrent, treated as a permanent feature of the oceanic circulation, is found to be consistent with the conservation of absolute cyclonic vorticity in the equatorward flow comprising one segment of the meridional circulation. The continental barriers permit the formation of the longitudinal pressure-gradient component essential for the existence of the Undercurrent.

CROMWELL, MONTGOMERY, and STROUP (1954), in a preliminary account of drogue observations in the central Pacific Ocean, have reported the discovery of the Equatorial Undercurrent. Although this current appears to be a normal feature of the oceanic circulation, further study is required to test this tentative conclusion. The Undercurrent was observed in the zone from the equator to latitude 1° N and at depths from 50 m to 150 m. The velocity was eastward and exceeded 1 knot at the axis. As the bottom of the surface layer or top of the thermocline was at about 100 m, the Undercurrent was found both in the lower part of the surface layer and in the upper part of the thermocline. Previous meridional sections of serial oceanographic observations in this region have been published by CROMWELL (1953), JERLOV (1953), and MONTGOMERY (1954).

[1] This study was sponsored by the Office of Naval Research. U. S. Department of the Navy, under contract with Brown University.

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In the present paper the Equatorial Undercurrent is found to agree with a simple application of Rossby's (1936) vorticity equation, that for a horizontal layer of fluid displaced frictionlessly

$$\frac{d}{dt}\frac{f+\zeta}{D} = 0, \qquad (1)$$

where D is the thickness of the layer, ζ is the vertical component of vorticity in the cyclonic sense—or simply *cyclonic vorticity*—relative to the earth's crust, and f is the Coriolis parameter, equal to $2\omega \sin\varphi$, where ω is the angular speed of the earth's rotation and φ is latitude. The absolute cyclonic vorticity is $f+\zeta$. By using cyclonic vorticity and the convention that latitude and Coriolis parameter are positive on both sides of the equator, the discussion can be so worded as to apply in either hemisphere. Fluid crossing the equator forms an exception to (1), for ζ reverses sign.

Let x be distance eastward and y be distance from the equator. Let u and v be the eastward and poleward components of velocity. Then, to a good approximation in low latitudes,

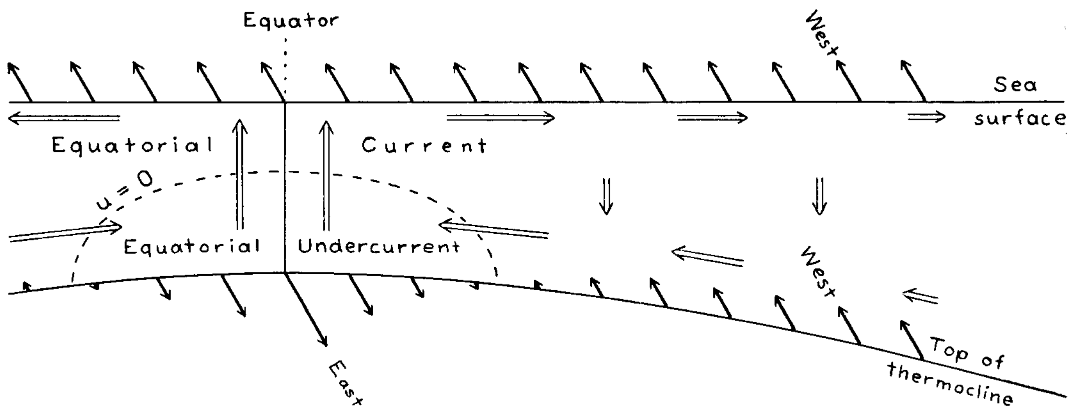

Fig. 1. Meridional section showing idealized currents in surface layer (about 100 m deep) within about 3° of equator, reader looking west. Arrows with double shafts give flux components in the plane of the section. Longitudinal component of velocity at top and bottom of layer is given by diagonal arrows in perspective. Vertical exaggeration 10^3.

$$\zeta = \frac{\delta v}{\delta x} - \frac{\delta u}{\delta y}.$$

One important assumption in the present discussion is that longitudinal gradients are small compared with meridional gradients. In the Pacific Ocean the system of equatorial currents at the surface parallels the equator for some 130 degrees of longitude, so this assumption is strongly supported. As a consequence, the cyclonic vorticity reduces to simply

$$\zeta = -\frac{\delta u}{\delta y} = -\frac{1}{a}\frac{\delta u}{\delta \varphi}, \qquad (2)$$

where a is the earth's radius. This assumption is not to be construed as making conditions longitudinally uniform. The thermocline's upward slope to the east, though small compared with the meridional slope, is an essential characteristic, made possible by the continental barriers. The upward slope of the thermocline is accompanied by a downward slope of the sea surface, and in the surface layer $\partial p/\partial x < 0$, where p is pressure. The longitudinal component of pressure gradient was considered by MONTGOMERY and PALMÉN (1940) to be essential for the existence of the Equatorial Countercurrent and is found in the present study to be essential for the existence of the Equatorial Undercurrent.

The second assumption is that within the surface layer there is a meridional circulation basically like that described by CROMWELL (1953). Easterly winds everywhere within 5 degrees of the equator both drive the west current (South Equatorial Current) and produce drift currents that transport water away from the equator. The poleward transport of the drift current at first increases with distance from the equator, as the deflection due to the earth's rotation becomes effective. Outside an equatorial zone the earth's rotation is fully effective; the transport of the drift current is directed normal to the wind, and the unit-length transport equals τ_0/f, where τ_0 is wind stress, and thus decreases with increasing latitude. At some such latitude as 1° (CROMWELL, 1953, Fig. 7), therefore, there as a maximum poleward transport of the drift current.

As another segment of the meridional circulation, water flows equatorward in the part of the surface layer below the drift current. There is upward flow at the equator, downward flow poleward of latitude 1°.

The actual winds and currents are not symmetrical about the equator, and southeast trades prevail at the equator. As far as the basic features of the Undercurrent are concerned, however, an adequate model seems to consist of a uniform east wind and an ocean symmetrical about the equator. This model is sketched in Fig. 1. The surface layer in this model contains a closed meridional circulation on each side of the equator.

The longitudinal component of surface current is represented as uniform, the relative cyclonic vorticity being zero. The justification for this simplification is that current charts depict the South Equatorial Current as comparatively broad and uniform; for the entire zone from 5° S to 5° N the surface current is west with maximum speed less than 1.5 knots. The actual lateral shear is small, therefore, compared with what it would be in such a zone if the absolute cyclonic vorticity were uniform (as will be seen from Fig. 2).

If a water layer moves frictionlessly from an initial state f_0, ζ_0, D_0 to a new state f, ζ, D, the vorticity equation gives the relation

$$\frac{f+\zeta}{D} = \frac{f_0+\zeta_0}{D_0}. \quad (3)$$

For water sinking from the surface in the model described, $\zeta_0 = 0$. If this water flows to the equator, the properties become ζ_E, D_E, where

$$\frac{\zeta_E}{f_0} = \frac{D_E}{D_0}. \quad (4)$$

Hence, the water arrives at the equator with positive cyclonic vorticity, regardless of the ratio D_E/D_0.

If the thickness D is assumed constant during displacement and if all the water is assumed to have started from the initial state f_0, $\zeta_0 = 0$, u_0, the exact distribution of zonal component of velocity is easily found by integration of

$$f + \zeta = f_0 \quad (5)$$

to be

$$u - u_0 = 2a\omega[(\varphi_0 - \varphi)\sin\varphi_0 + \cos\varphi_0 - \cos\varphi]. \quad (6)$$

The result is given in terms of the speed of the earth's crust at the equator, $a\omega = 465$ m/s. The minimum eastward component is u_0 (which is negative in the model).

For small latitudes the last equation may be written in the approximate and simpler form

$$u - u_0 = a\omega(\varphi - \varphi_0)^2 \quad (7)$$

(accurate within one per cent in a zone from the equator to latitude $11^1/_2°$), which is drawn in Fig. 2. This convenient graph is valid for any value of u_0 and for any small value of φ_0.

Suppose surface water in a west current (South Equatorial Current) of, say, 0.5 knot with no lateral shear sinks and flows frictionlessly and without change in thickness to the equator. To reach the equator as the east Undercurrent of, say, 1.5 knots, a change of 2 knots, Fig. 2 shows that the water must have come from latitude 2.7°. If frictional losses are allowed for, the average latitude of origin is greater.

The Undercurrent and Cromwell's meridional circulation therefore fit in with the conservation of absolute cyclonic vorticity in the equatorward flow. It is not sufficient to consider the vorticity equation alone, however, for the gain of eastward momentum by the water flowing toward the equator must also be explained. The source of this eastward momentum is to be found in the longitudinal component of pressure gradient, already mentioned. The existence of the longitudinal component of pressure gradient (other than locally) is made possible by the presence of the continental barriers. The conclusion is reached that the Equatorial Undercurrent could not be set up by meridional circulation in a uniform zonal canal encircling the earth.

The longitudinal component of pressure gradient can be expressed in terms of the longitudinal slope of isobaric surfaces. In the lower part of the surface layer in the Pacific Ocean, MONTGOMERY and PALMÉN (1940, Table IV) found this slope to be about 3×10^{-8}, which is roughly substantiated by JERLOV (1953, Fig. 4).

The equatorward component of velocity in the water moving from latitude φ_0 (say, 3°) to the equator can now be calculated. The longitudinal component of the momentum equation is

$$\frac{du}{dt} = fv - \alpha\frac{\delta p}{\delta x}, \quad (8)$$

where α is specific volume. As

$$fv - \frac{du}{dt} = fv - v\frac{\delta u}{\delta y} = (f+\zeta)\,v = f_0 v,$$

(8) can be written in the form

$$v = \frac{\alpha}{f_0}\frac{\delta p}{\delta x} = \frac{g}{f_0}\left(\frac{dz}{dx}\right)_{\varphi,p} \approx \frac{g}{2\omega\varphi_0}\left(\frac{dz}{dx}\right)_{\varphi,p}, \quad (9)$$

where g is gravity. As $\partial p/\partial x$ is presumably uniform within the zone from the equator to φ_0, v is also uniform. Furthermore, v is seen to be inversely proportional to φ_0. For $\varphi_0 = 3°$ ($f_0 = 7.6 \times 10^{-6}\ s^{-1}$) and a slope $(dz/dx)_{\varphi,p} =$

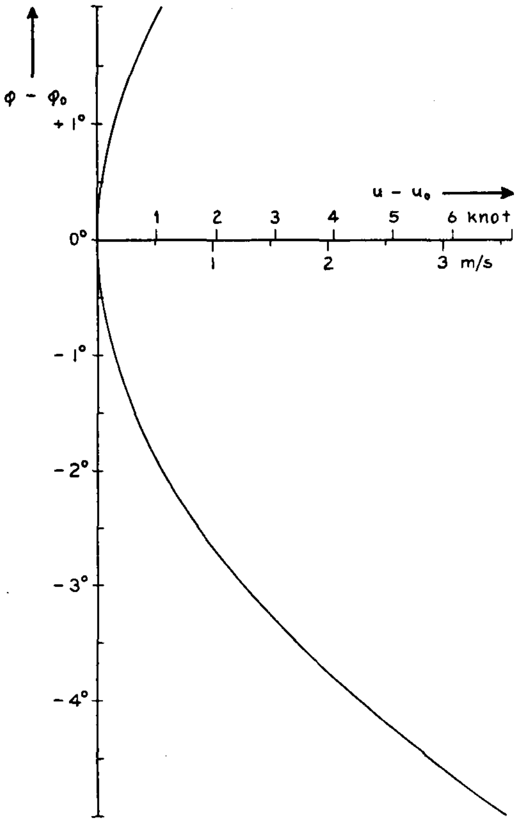

Fig. 2. Eastward speed in zonal flow near equator when the absolute cyclonic vorticity is uniform. The minimum speed (u_0) and the latitude of minimum speed (φ_0) are arbitrary.

$= -3 \times 10^{-8}$, the equatorward component of velocity is calculated to be

$-v = 4$ cm/s $= 2$ nautical miles per day.

The time required for the water to travel from latitude φ_0 to the equator is

$$-\frac{a\varphi_0}{v} = -\frac{2a\omega\varphi_0^2}{g}\left(\frac{dz}{dx}\right)^{-1}_{\varphi,p} \qquad (10)$$

and, therefore, is proportional to the square of φ_0. For the values already assumed, this time is

$$-a\varphi_0/v = 87 \times 10^5 s = 100 \text{ days}.$$

There is another noteworthy restriction of the Undercurrent. As is well known, most oceanic and atmospheric flow is quasi-geostrophic. Over most of the earth, one current does not underly an opposite current within the same homogeneous layer. An undercurrent opposite to the surface current, both in the surface layer, can occur only very close to the equator. Off a coast facing poleward at a moderate latitude, east winds would be expected, therefore, to produce upwelling but no undercurrent.

In summary, the Equatorial Undercurrent is found to be consistent with the equatorward flow in the lower part of the surface layer close to the equator if this flow is approximately frictionless so that absolute cyclonic vorticity is conserved. Continental barriers, which permit a longitudinal component of pressure gradient, are essential for the extensive development of the Undercurrent. In general, an undercurrent within the surface layer can occur only in a narrow equatorial zone. The theoretical latitude of origin of the equatorward flow, calculated from the observed axial speed of the Undercurrent, is about 3°, which may be regarded as a minimum estimate. With the additional use of the observed longitudinal component of pressure gradient, the theoretical equatorward component of velocity is calculated to be 4 cm/s, and the time of travel from 3° to the equator is calculated to be 100 days.

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