NOTES

Note on Simple Assumptions Regarding the Baroclinic Structure of the Atmosphere

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(Manuscript received November 11, 1954)

1. Background

In a series of recent articles (STARR 1953, LORENZ 1953, PFEFFER and SALTZMAN 1954) 'local' angular momentum has been used as a basic physical parameter in the study of atmospheric flow. In the last of these references the following approximate dynamical equation was shown to be valid for regions of the atmosphere free of mountain barriers;

$$\frac{\partial M}{\partial t} = \tau, \tag{1}$$

where t is time, M is the total angular momentum within an arbitrarily-located, vertically-oriented, fixed cylindrical volume extending through the depth of the atmosphere, measured relative to the axis of the volume, and τ is the horizontal transport of this 'local' momentum across the fictitious walls of the volume.

LORENZ (1953) has shown that local momentum, M, represents a space-weighted average of the vorticity in a given region of the atmosphere. He has shown, also, that it is possible to make a distinction between displacement and intensification processes by resolving the local momentum transport, τ, into barotropic and baroclinic components, similar to the conventional resolution of the vorticity advection. In more concrete terms we may write,

$$\tau = \frac{p_0}{g} R^2 \int_0^{2\pi} \overline{C_T C_R}\, d\theta \tag{2}$$

or,

$$\tau = \frac{p_0}{g} R^2 \int_0^{2\pi} \overline{C_T} \cdot \overline{C_R}\, d\theta + \frac{p_0}{g} R^2 \int_0^{2\pi} \overline{C_T' C_R'}\, d\theta, \tag{3}$$

where p_0 is the surface pressure (assumed uniform), g is the acceleration of gravity, R is the radius of the cylindrical volume, C_T and C_R are the tangential (positive counter-clockwise) and radial (positive inward) components of the wind velocity respectively, θ is the polar coordinate, the bar denotes a vertical average throughout the atmosphere with respect to pressure, and the primes denote a departure from this average. In (3) the first integral

[1] The research reported in this paper has been sponsored by the Geophysics Research Directorate, Air Force Cambridge Research Center, under Contract No. AF 19(604)-1000.

represents the barotropic component of τ, while the second integral gives the baroclinic contribution resulting from the vertical variation of the horizontal wind field.

It is the purpose of this note to report some observed short-comings of the evaluation of the baroclinic term of (3) by means of an assumption regarding the vertical variation of the wind, as compared with a "three-dimensional" evaluation of the baroclinic contribution by means of data at five levels of the atmosphere. These results may have some bearing on numerical prediction attempts which aim to capture the effects of baroclinicity by similar assumptions.

2. Procedure

In the previous study (PFEFFER and SALTZMAN, 1954) the integral (2) was evaluated directly on a daily basis, using geostrophic winds, for the two-month period January—February, 1949, for a cylindrical volume located over the North Atlantic Ocean. Data at the surface and the 700, 500, 300, and 100 mb levels were employed. The 300, and 100 mb charts were kindly loaned by the U.C.L.A. General Circulation Project and are believed to be one of the best series of maps available at these levels. In addition, the barotropic part of (3) was evaluated for the same region and period, with 500 mb taken as the level of the mean wind.

As an extension of this study it was decided to test whether sufficiently accurate estimates of τ could be obtained by using an assumed wind profile to measure the baroclinic term of (3). Specifically, it was assumed that the tangential and radial components of the wind vary linearly with pressure, the rate of variation (or thermal wind per unit pressure difference) in the troposphere being determined by the 700—300 mb thickness in accordance with the geostrophic principle. Although it does not affect the results appreciably, for greater realism the wind was assumed to decrease above the 200 mb level in a simple linear fashion. These are similar to the assumptions used in the so-called $2^1/_2$-dimensional numerical prediction models.

By subtracting the barotropic component from the "three-dimensional" momentum transport it is possible to obtain a measure of the daily baroclinic contributions. It would be hoped that the baroclinic term computed on the basis of the assumed vertical variation of the wind compares favorably with these values. As it turns out, however, the correlation coefficient between these two quantities for the two-month period is disappointingly small (+.20) and the root mean square deviation is of precisely the same magnitude as the baroclinic term itself. It is recognized that these results may, in part, be attributed to inaccuracies in the estimation of the barotropic term using 500 mb data. It is most likely, however, that the results are mainly a consequence of the crudeness of the representation of the vertical variation of the momentum transport through the use of the assumed wind profile.

One may further inquire as to whether there is an improvement in the verification of the dynamical relationship (1) as a result of using the values of τ based on data at five levels of the atmosphere rather than the values based on the assumed wind profile. Verification of (1) may be measured by the correlation coefficient between the change of local momentum over 24 hours and the time-integrated momentum transport over the same period. As noted in reference 2, estimates of this time-integrated momentum transport over 24 hours, obtained by summing the values of τ at the beginning and end of the period, are far less satisfactory than those obtained when the intermediate (12-hourly) information is incorporated. Unfortunately, in the present case only 24-hourly data were available at upper levels for the "three-dimensional" measurement of the momentum transport and, for comparison purposes, it was necessary to use 24-hourly data in the case of the barotropic and "$2^1/_2$-dimensional" quantities also. Consequently, the correlation coefficients presented in Table 1 are all rather low and can be taken only as a very crude indication of the true relative differences in the verification of the three models[1]. Subject to this reservation, these coefficients suggest that the improvement obtained over the barotropic model by the

[1] When 12-hourly information is included in estimates of $\int_0^{24} \tau dt$ the +.29 correlation coefficient for the barotropic case becomes +.70 (see reference 2).

"2 $^1/_2$-dimensional" model is small compared with the improvement obtained when baroclinic effects are taken into account in a more elaborate manner by means of multi-level computations.

Table 1. Correlation coefficients between the time-integrated transport of local momentum and the simultaneous change of local momentum over a 24-hour period, using data at 24-hour intervals. Number of pairs: 58.

barotropic	+.29
"2 ½-dimensional" baroclinic	+.32
"3-dimensional" baroclinic	+.54

As would be expected, it is observed that the greatest improvement over the barotropic verification occurs on specific days when the baroclinic component of the local momentum transport is large. On these same days it is often found that the baroclinic term computed by means of the assumption fails to give an adequate measure of this baroclinicity.

These findings are, of course, of a limited nature and cannot be regarded as a conclusive test of the "2 $^1/_2$-dimensional" assumption. Strictly speaking, the results apply to a model which has angular momentum rather than vorticity as the primary physical parameter. The results may be of some general interest, however, in that they suggest possible difficulties in obtaining significant improvement over barotropic numerical forecasting procedures by incorporating simple assumptions regarding the baroclinic structure of the atmosphere.

Acknowledgements

The author wishes to express his appreciation to Professor V. P. Starr for his constant encouragement; to Fil. Lic. G. Arnason for reading the note critically; and, particularly, to Mr. R. L. Pfeffer for his helpful suggestions and stimulating discussions.

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