Model Experiments on Aurorae and Magnetic Storms

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Abstract

Model experiments on Alfvén's electric field theory of aurorae and magnetic storms have been performed. The transformation laws from nature to model are considered. It is concluded, that a proper scaling down of every detail in nature is not possible. Only the most significant features are scaled down properly. It is shown, that a mechanism like Alfvén's proposed auroral mechanism is very likely to be present in the model. In particular the conception seems to be experimentally confirmed, that space charge accumulated in the discharge region rather than destroy the whole pattern of motion, discharges along the magnetic field lines.

Introduction

In Alfvén's electric field theory of aurorae and magnetic storms (ALFVÉN, 1939, 1940, 1950) it is assumed, that a gaseous discharge due to an interplanetary electric field takes place in the surroundings of the earth during a storm. This discharge is affected by the terrestrial magnetic field and reaches the earth's atmosphere only in the auroral zones. The currents of the discharge are responsible for the magnetic disturbances observed during a storm.

Previously MALMFORS (1946) has made some experiments on this theory. In a cylindrical vacuum chamber (diam. 40 cm, length 75 cm), covered at the ends with glass plates, he arranged a magnetized steel sphere (diam. 10 cm) in the centre of the chamber and two insulated metal plates (60 × 25 cm) vertically on each side of the sphere separated by 24 cm. He found that a discharge between the two plates had many similarities with the discharge considered in the theory. Indeed, the results seemed so promising, that it was thought worth while to make a more detailed experimental study on a somewhat larger scale.

Experimental arrangement

A total view of the apparatus used is seen in fig. 1. The volume of the vacuum chamber is approximately 1.5 m³. A pressure of about

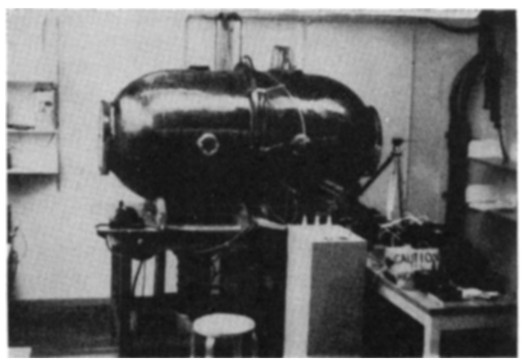


Fig. 1. Total view of the apparatus.

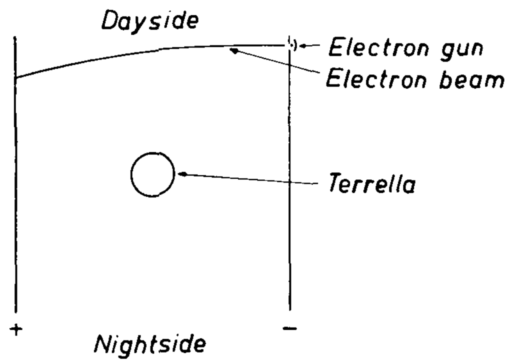


Fig. 2. Terrella and electrodes seen from above. The dipole direction of the terrella is perpendicular to the paper with the magnetic south pole above and the north pole below the paper.

10^{-4} mm Hg could be obtained in it at best. Fig. 2 is a sketch of the electrodes and the terrella, seen from above. They are arranged in exactly the same way as in Malmfors' experiment except that the dimensions are larger here. The electrodes are aluminum plates (60×60 cm). Only the terrella is of the same size as that of Malmfors, the diameter being 10.6 cm, but here a coil is used inside the terrella to produce the magnetic field, instead of Malmfors' permanently magnetized steel sphere. Thus we can vary the field and make it stronger than in Malmfors' experiments. The coil is so designed, that it can produce a rather good dipole field. The deviation from the exact dipole field is less than 5 % everywhere outside the terrella. Cooling by air is provided, because when 3,000 gauss at the poles is produced, the heating by the current amounts to 8 kW. An automatic switch is used to break the current, when the temperature of the windings reaches a certain value (150—200° C) in order to avoid damage. The temperature is determined by measuring the coil resistance.

Various probes and other electrodes for special investigations can if needed be placed in the discharge region.

Transformation from nature to model

Before describing the various experiments made in the model we must consider in some detail, how such experiments should be carried out in order to simulate a 'real' discharge of terrestrial dimensions in a proper manner. If the linear dimensions of a gaseous discharge are changed, certain similarity laws are valid (see COBINE, 1941, p. 209, ENGEL and STEENBECK, 1932, p. 95, or ALFVÉN, 1950, p. 39). In nature the following data are assumed to be valid (ALFVÉN, 1955).

Earth's diameter .... $D = 1.3 \cdot 10^7$ m,
velocity of the ionized beam from the sun .. $v = 2 \cdot 10^6$ m/s,
sun's magnetic field at the earth's orbit ..... $B_0 = 0.7 \cdot 10^{-5}$ gauss,
electric field of the discharge .... $E = 1.4 \cdot 10^{-3}$ V/m,
earth's magnetic field at poles .... $B_p = 0.6$ gauss.

According to the similarity laws all potentials should be unaltered, because ionization and excitation potentials are the same. Therefore, all electric fields are inversely proportional to the linear dimensions, and according to Maxwell's equations, the same is valid for the magnetic fields. Now, in the actual case, the linear dimensions are diminished by a factor $k = 10^8$. Hence, the magnetic field strength at the poles of the terrella ought to be of the order of 60 million gauss. In nature the potential over a length of the order of the earth's diameter is $V_D = D \cdot E = 18$ kV. The normal distance between the electrodes of the model discharge is 5.4 terrella diameters, and so the applied voltage should be of the order of 100 kV.

Since these strong magnetic fields are impossible to obtain, we will neglect the condition about the unchanged potentials and put the question: How are the magnetic and electric fields connected, if the electron and ion orbits are unaltered, assuming them to be undisturbed by collision processes?

First we note that we can confine ourselves to the non-relativistic case, because the energies of the particles are at most 100 keV.

An electron moving in a magnetic field B with the velocity v perpendicular to the magnetic field lines has an orbital radius of curvature

$$\varrho = \frac{mv}{eB} \tag{1}$$

Rationalized Giorgi-units are used throughout. If the kinetic energy $1/2\ mv^2$ is assumed to be proportional to the electric field E or the applied discharge voltage V_d, we find that if

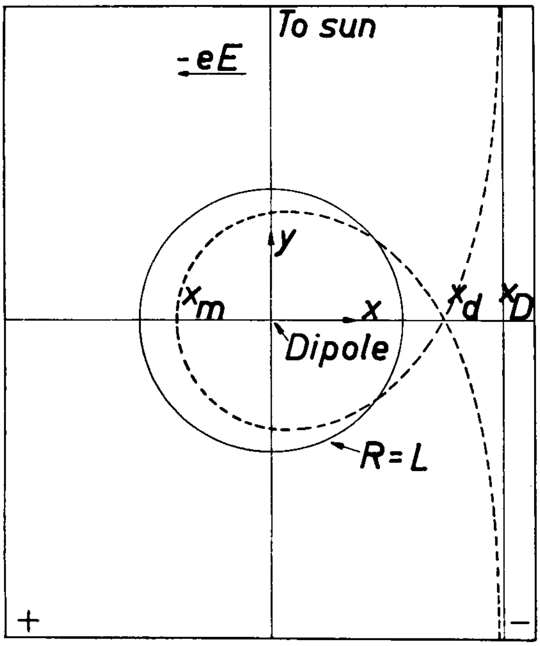


Fig. 3. Border line of the forbidden zone in the equatorial plane. Compare fig. 6.1, p. 180 in Cosmical Electrodynamics.

$x_D = 1.76\ L$, $x_d = 1.32\ L$, $x_m = -0.74\ L$.
$x_D - x_d = 0.44\ L$, $x_D - x_m = 2.50\ L$.

$$\frac{V_d}{B^2} = \text{constant} \tag{2}$$

the electron orbits remain unchanged. This will be illustrated with some examples, which are closely connected with the theory of aurorae and magnetic storms.

In a gaseous discharge two classes of electrons are usually present. Some electrons start from the cathode and acquire a high energy in the cathode fall. When they enter the plasma, their energy is of the same order as the whole discharge potential. We will call these electrons primary electrons. If the mean free path in the plasma is of the same order as the distance between cathode and anode, most of the primary electrons will reach the anode with only slightly diminished energy. A strong magnetic field perpendicular to the electric field may, however, prevent the primary electrons from penetrating deep into the plasma without a considerable loss of energy. The other class of electrons consists of secondary electrons formed in the plasma due to ionization by primaries, other secondaries or photons.

The primary electrons can then be considered as entering the magnetic field at the boundary between cathode fall and plasma with an initial energy equal to the cathode fall or in most cases approximately the discharge voltage. We see at once that (2) is applicable in this case. The secondaries on the other hand start in an electric field E and a magnetic field B, and we will assume their initial energy to be zero. Such an electron then starts moving in a trochoidal orbit with the mean energy perpendicular to the magnetic field expressed in electron volts

$$V'_\perp = \frac{m}{e}\left(\frac{E_\perp}{B}\right)^2 = 5.7 \cdot 10^{-12}\left(\frac{E_\perp}{B}\right)^2$$

where $E_\perp$ is the electric field component perpendicular to the magnetic field.

If we combine (1) and (3) we see that because $v \sim \sqrt{V'_\perp}$, we have

$$\varrho_{\text{sec}} \sim \frac{\sqrt{V'_\perp}}{B} \sim \frac{E_\perp}{B^2} \sim \frac{V_d}{B^2} \tag{4}$$

and (2) is applicable also in this case provided the initial energy is small compared with $eV_\perp$ as calculated from (3).

Further the so called Störmer-unit of length (STÖRMER, 1933) is unchanged if the particle energy eV is proportional to B^2. Störmer has defined a characteristic length for the particle orbits in a magnetic dipole field

$$C = k\sqrt{\frac{a}{B \cdot \varrho}} \tag{5}$$

where a is the magnetic dipole moment, $B\varrho$ is characteristic of the particle and k is a constant, which depends on the unit system. Now $B\varrho$ is proportional to the velocity of the particle and $B \sim a$, so that

$$C \sim \left(\frac{B}{V^{1/2}}\right)^{1/2} \tag{6}$$

Alfvén has calculated the electron orbits in the equatorial plane of the dipole field. He has introduced a characteristic length (see e.g. Cosmical Electrodynamics p. 179)

$$L = \sqrt[4]{\frac{\mu_0}{4\pi} \cdot \frac{\mu a}{eE}} \tag{7}$$

where a is the magnetic dipole moment, E is the electric field, $\mu_0 = 4\pi \cdot 10^{-7}$ henry/m is the permeability of vacuum and μ (not to be confused with μ_0) is the orbital magnetic moment of the particle

$$\mu = \frac{eV_\perp}{B} \tag{8}$$

which can be shown to be a constant of the motion. Here $eV_\perp$ is the kinetic energy of the particle referred to a system where the centre of curvature is at rest, i.e. where the drift velocity is zero. $V_\perp$ must not be confused with $V'_\perp$ of eq. (3). In the special case of eq. (3). $V_\perp = V'_\perp/2$. In most other cases the difference between $V_\perp$ and $V'_\perp$ is very small, although always $V_\perp \leq V'_\perp$. From equation (3) we see that

$$\mu \sim \frac{E^2}{B^3} \sim \frac{V_d{}^2}{B^3} \tag{9}$$

Thus we find

$$L^4 \sim \frac{V_d{}^2}{B^3} \cdot \frac{B}{V_d} = \frac{V_d}{B^2} \tag{10}$$

According to Malmfors (see MALMFORS, 1946 equation (7) or appendix II.) the latitude angle of the auroral zones should be a function of E/B^2.

Thus we see that we always arrive at the same connection between the applied voltage of the discharge and the magnetic field of the dipole

$$V_d = kB^2 \tag{11}$$

Particle energies in nature

The particle energies in the beam from the sun determine to a considerable extent the properties of the discharge in terrestrial regions. Therefore, we will consider these energies in this section. These considerations are of course very uncertain. Their purpose is only to permit us some familiarity with the problems of the discharge, rather than to obtain any definite results.

From the time difference between a solar flare and the commencement of a magnetic storm, the speed of the beam front from the sun can be calculated as $10^6 - 2 \cdot 10^6$ m/s. Now $2 \cdot 10^6$ m/s means an electron energy of 11 eV and a proton energy of 20 keV associated with the translation of the beam. These are lower limits, because the particles may travel in curved paths, e.g. trochoids. It seems very improbable, that the particles should go in approximately straight lines from sun to earth. It is worth while to note, that from the relation

$$eV = \frac{3}{2} kT, \tag{12}$$

one finds that 7,700° K is equivalent to 1 eV. This means that if the beam is ejected from the corona, where the temperature may be of the order of 10^6 °K, the electronic energies may be some 100 eV, which is significantly greater than the above mentioned value of 11 eV.

From the penetration depth of the auroral particles in the atmosphere Störmer and others have estimated the energy to 50—100 keV. Measurements made by MEINEL (1951) on the H_α-line found in auroral archs indicate that the impact velocity of the protons seems to be

$$v_H \geq 3{,}300 \text{ km/s}$$

or

$$W_H \geq 57 \text{ keV},$$

and the velocities perpendicular to the impact direction seem to be distributed between 0 and 1,000 km/s.

According to Alfvén's theory the particles penetrating the earth's upper atmosphere are accelerated in the electric field of the beam. If the border of the forbidden zone is 7 earth radii from the earth, the electric field must be of the order of 10 kV per earth radius or 10^{-3} V/m, which agrees well with the values calculated earlier in this paper.

From equation (7) we see that the currents of the discharge are determined to a great extent by the electron energy perpendicular to the magnetic field in the equatorial plane. We rewrite (7) in a somewhat different form

$$L^4 = \frac{V_\perp}{B} \cdot \frac{B_{eq} \cdot R^3}{E} \tag{7 a}$$

$eV_\perp$ = electron energy perpendicular to the magnetic field, referred to a zero drift velocity system.
B_{eq} = earth's magnetic field at equator on surface,
R = earth's radius,
E = electric field of the beam.

Further we may note, that $V_\perp/B$ is constant along the path of the electron.

We will now find a reasonable value for $V_\perp$

at the border of the forbidden region in the equatorial plane. This has approximately the distance L from the dipole. Thus we have

$$L^4 = \frac{V_\perp \cdot B_{eq} \cdot R^3}{B_{eq}\frac{R^3}{L^3}E} = \frac{V_\perp}{E} L^3$$

or

$$L \cdot E = V_\perp \tag{7 b}$$

at the distance L from the earth's dipole. This occurs at two points on the border of the forbidden region. In fact the energy $eV_\perp$ of the electrons varies along the border line according to the relation

$$V_\perp = E(x_D - x) \tag{7 c}$$

See fig. 3. Hence we find that

$$2.5\, LE > V_\perp > 0.44\, LE \tag{7 d}$$

This means that, according to the values given above, $V_\perp$ may be of the order of 100 kV. If L is assumed to be seven earth radii, the magnetic field at the border line is approximately 10^{-3} gauss, and as $V_\perp/B$ is constant, we find that at a magnetic field of 10^{-5} gauss, i.e. in the beam outside the earth's magnetic field, $V_\perp$ = 1,000 volts.

According to these strictly mathematical considerations these high energy electrons would have nearly all their kinetic energy perpendicular to the magnetic field. If we assume that μ = constant of the motion, also when the particles move along the field lines from the equatorial plane towards the auroral zones, the diamagnetic repulsion of the terrestrial magnetic field would be very strong (proportional to $V_\perp$) and prevent the particles from reaching the auroral zones. Therefore, one must assume that particles of all energies are present in the beam with a mean energy, which determines the dimensions of the forbidden zone and hence the auroral zones. It must also be assumed, that a statistical distribution of the velocities is maintained, mainly by plasma oscillations, everywhere in the discharge in spite of the strongly increasing $V_\perp$ in the earth's magnetic field. If the velocity distribution of the particles is isotropic, we are led to another constant of the motion than the μ of equation (8) as is shown in appendix I

$$\varkappa = \frac{V^{3/2}}{B} \tag{13}$$

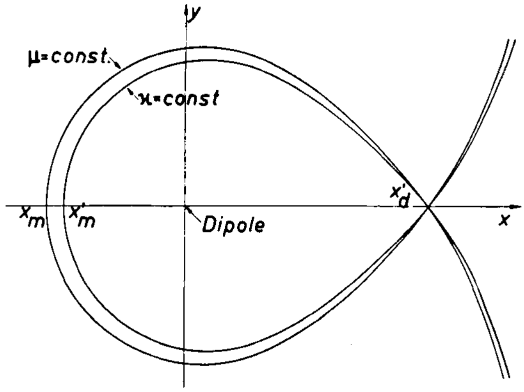


Fig. 4. Comparison between forbidden zones

a) assuming the orbital magnetic moment of the particles to be constant,

b) assuming isotropic velocity distribution everywhere $x'_d = 1.26\ L$, $x'_m = -0.63\ L$.

Here eV is the total energy (referred to a zero drift velocity system) and is therefore according to our assumptions equal to $3/2\, eV_\perp$. It is also shown in the appendix, that this leads to another form of the forbidden zone

$$x_0 - x = \frac{L^3}{R^2}$$
$$L^3 = \frac{1}{E}\left(\frac{\mu_0}{4\pi} a\varkappa\right)^{2/3} \tag{14}$$

instead of

$$x_0 - x = \frac{L^4}{R^3}$$
$$L^4 = \frac{\mu_0}{4\pi} \cdot \frac{a\mu}{eE} \tag{15}$$

for μ = constant. The forbidden zones in the two cases are compared in fig. 4. The difference is very small, the eccentricity being a little smaller for μ = constant.

Transformation of pressure from nature to model

As will be seen in the next section the pressure used in the experiments was about 10^{-3} mm Hg. This corresponds to a particle density of about 10^{13} particles/cm³, and according to the transformation laws, the particle density should be inversely proportional to the linear dimensions. Since the scale factor is 10^8, the above mentioned particle density in the experiments corresponds to 10^5 particles/cm^3 in nature. In reality the discharge takes place in regions with strongly varying pressures, the extremes of which are the ionosphere with 10^{12} or 10^{13} particles/cm^3 in the E-layer and the interplanetary space, where the density is probably much lower than 10^5 particles/cm^3. Therefore, the pressure used may be a good compromise between the two extremes.

Performed experiments

The various experiments performed will now be described, and after that we can apply the above considerations to the results obtained.

There are two kinds of discharges which have been studied. If the pressure was low ($< 10^{-3}$ mm) there was an invisible non-self-sustained discharge (see COBINE, 1941, Ch. VII), which could only be maintained by means of some ionizing device, such as an electron beam indicated in fig. 2. The mean free path of the high energy electrons was then of the same order as the distance between anode and cathode. If the pressure was higher ($> 2 \cdot 10^{-3}$ mm) it was possible to get a visible glow discharge without any extra ionizing device. If the visible discharge was used, the cathode voltage V_c was of the order of -2 kV with respect to the terrella (every voltage is given with respect to the terrella if not otherwise specified), the anode voltage $+20$ to 50 volts (it could not be higher because then the anode currents drawn from the plasma would be too high), and the currents were about 10—50 mA. In the case of the invisible discharge, however, V_c was usually -10 kV, $V_a = 3$—5 kV and the currents only 0.1—1 mA. It is probable, that the invisible discharge has more to do with the phenomena in nature than the visible one. This question will be considered later.

In figs. 5—11 photographs of the various discharges are shown. Fig. 5 shows the visible discharge with increasing magnetic field B. At zero magnetic field the electrons move in straight lines from the cathode to the terrella, which is covered by fluorescent material. Thus only the terrella side facing the cathode is bright. If a weak magnetic field (20 gauss at poles) is turned on, the electrons are deflected through a small angle, and the bright hemisphere of the terrella is turned over through the same angle to the dayside. With increasing magnetic field we get more complicated figures. At about 300 gauss, however, the equator becomes part of a forbidden zone, and 'auroral zones' begin to develop. With B still more increasing the auroral zones become narrower and move towards the poles as expected. With increasing B the discharge current increases, and hence the discharge voltage decreases due to the internal resistance of the rectifiers. In connection with this discharge another experiment was made, which may be interesting to note. A small non-inductive resistance (150 ohms) was coupled in series with the anode, and the noise voltage over this resistance was measured with an oscillograph, the amplifiers of which had a frequency band ranging from a low value up to 5 Mc/s. The result was, that no noise was observed for low magnetic fields up to about 425 gauss at the poles. At this field a noise suddenly appeared corresponding to a noise current with top value of about one or two μA. This noise was sensibly constant up to the highest magnetic fields. The anode current was 6.3 mA. It is probable that the noise level is much higher nearer the terrella, where space charge is accumulated, so that discharges along the magnetic field lines take place. The author hopes to be able to make more detailed investigations on this later.

In fig. 6 photos are shown of the 'invisible' discharge. As the electric field was much stronger than for the visible discharge, the magnetic field where the auroral zones begin to appear is much stronger, viz. 800 gauss.

According to MALMFORS (1946) the discharge starts at the singular point between the terrella and the cathode in the equatorial plane, where the drift velocity of the electrons is zero (the drift due to the crossed electric and magnetic fields opposes and equals that due to the inhomogeneity of the magnetic field). At this point the electrons spiral repeatedly around the same magnetic field line from the northern to the southern auroral zone thus causing an appreciable ionization. It should therefore be possible to switch off the discharge by a horizontal plate at this point. Such a plate made of insulating material was inserted, but it had no appreciable effect neither on the visible discharge nor on the invisible one. Then it was thought that the plate may be charged by

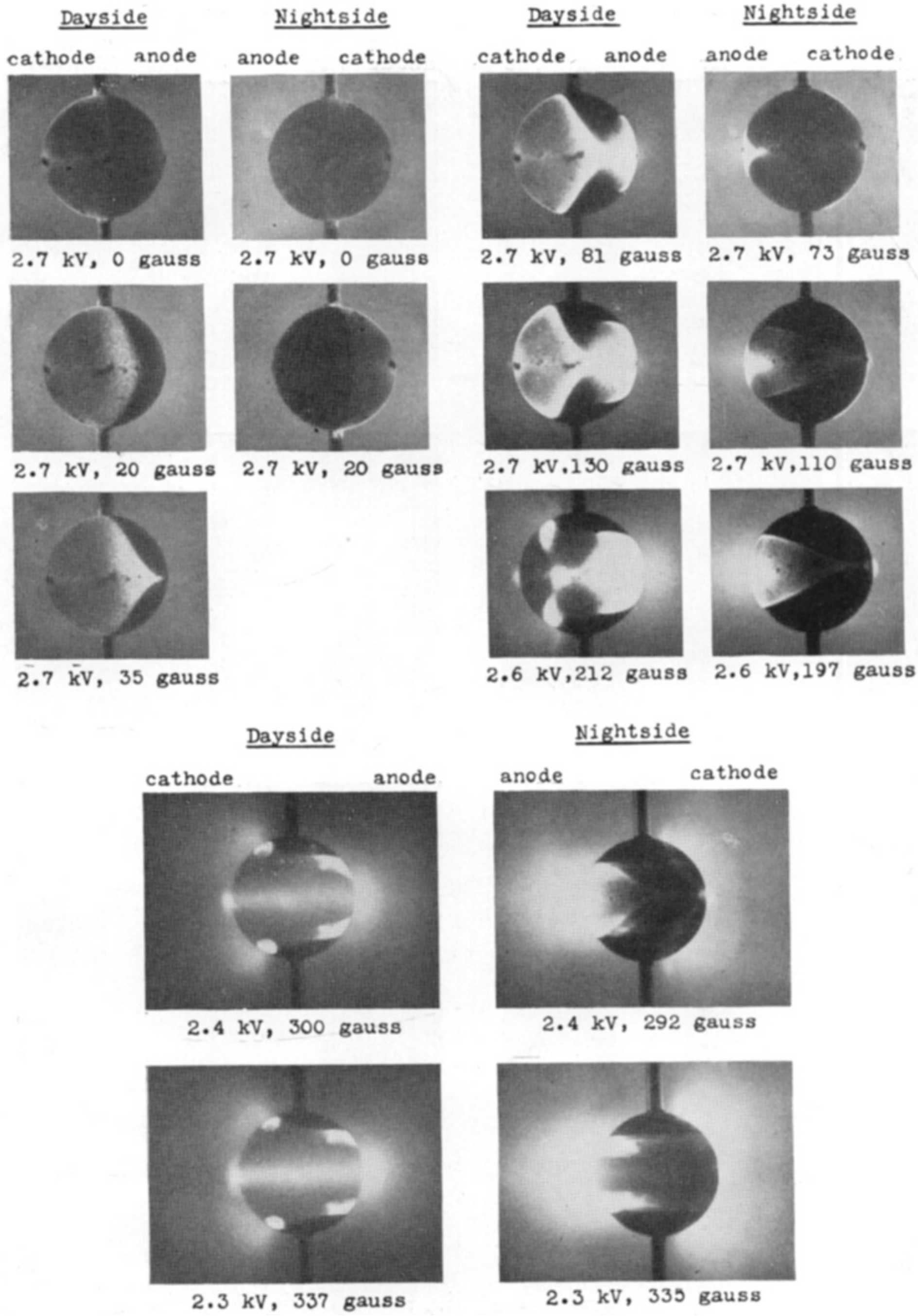


Fig. 5. Visible discharge. Below every picture the cathode voltage with respect to the terrella and the magnetic field at the poles of the surface of the terrella are given. Anode voltage was 30 volts. Pressure $3.5 \cdot 10^{-3}$ mm. Cathode current varying between 10 mA at 0 gauss and 22 mA at 1,350 gauss, anode current between 6.3 mA at 0 gauss and 5.5 mA at 1,350 gauss. The figure is continued on page 72 and 73.

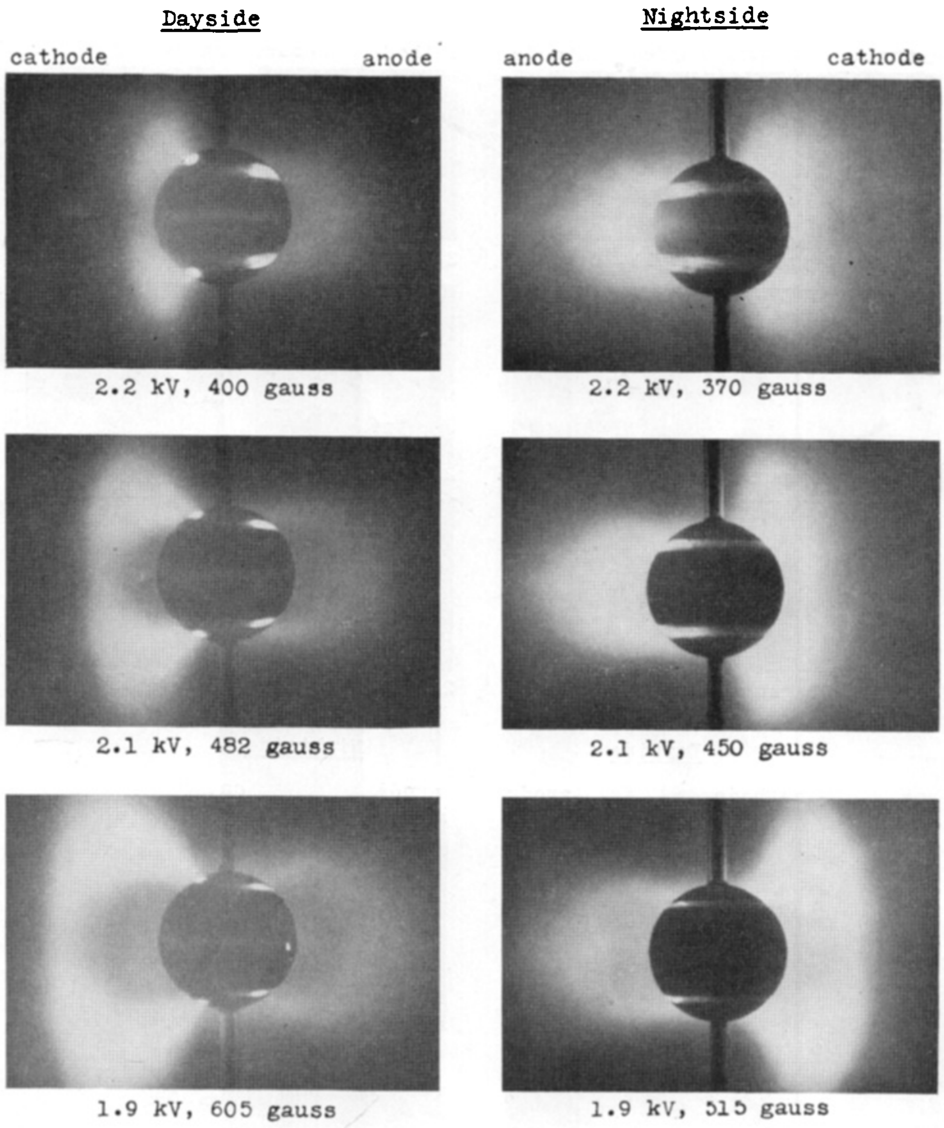


Fig 5. See text on page 71.

electrons to a negative potential thus reflecting the electrons on both sides, so that they could oscillate from the equator to the auroral zones. Therefore, the lower side of the plate was covered with a copper sheet, the potential of which could be varied. The effect of this is shown in figs. 7 and 8. The whole discharge is not switched off, but a part of the auroral zones is 'erased'. This is also sketched in fig. 12. The plate seems to have a greater effect in this respect on the visible than on the invisible discharge. One can, however, qualitatively understand that the singular point of zero drift velocity may have moved to another longitude because of the modification of the electric field caused by the plate. It seems as if the discharge

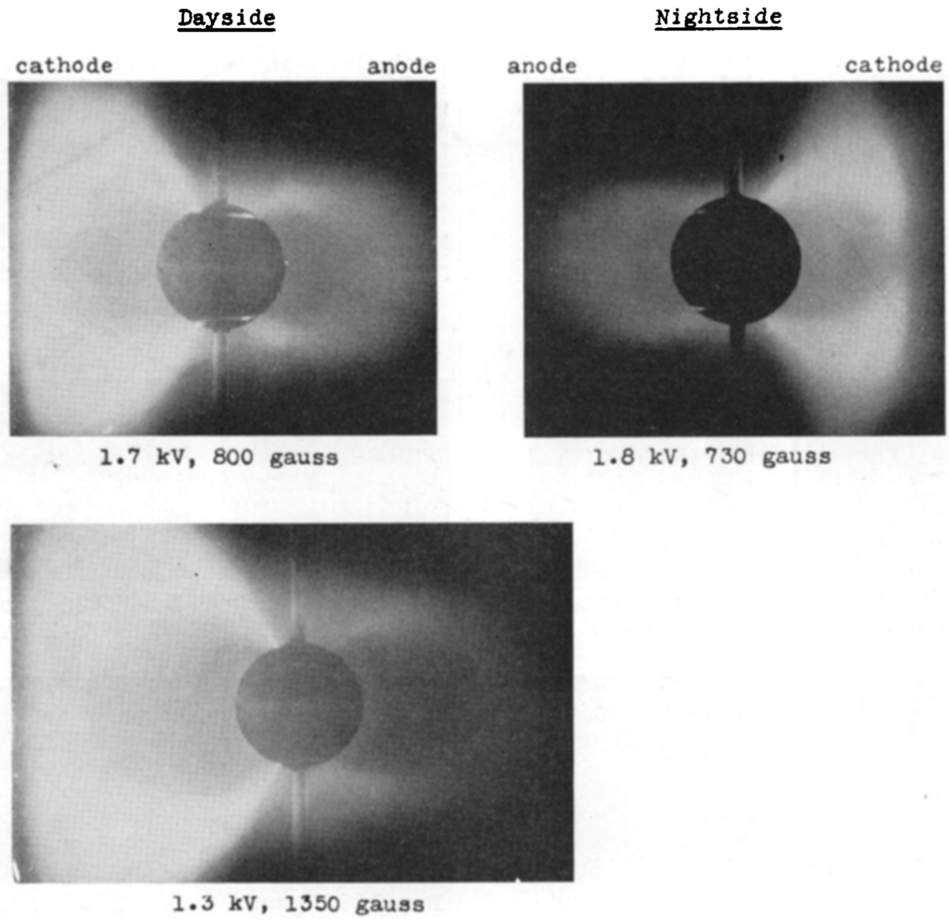


Fig 5. See text on page 71.

can always elude the investigator, but this experiment shows the great importance of the equatorial region for the generation of the auroral zones.

One may question if the two bright spots, one within each of the auroral zones on the cathode (morning) side of the terrella, have anything to do with the singular point of zero drift velocity, e.g. if they are the projection of this point on the terrella along the magnetic field lines. Fig. 9 shows that this is not the case, because the movable plate inserted between the cathode and the spot shadows the latter entirely. In the last photo of the figure a magnetic field line is indicated from the spot to the shadowing plate, showing that the electrons responsible for the spot move from the point, where a magnetic field line is perpendicular to the boundary between the plasma and the cathode fall. In the cathode fall the magnetic field may be too weak to deflect the electrons appreciably, but the plasma begins, where the electrons are caught by the field lines.

Fig. 10 shows ho the awuroral zones can be cut off by a plate in a meridional plane. Malmfors has also made this experiment, from which it is apparent that the auroral zones in some way are generated at the morning side

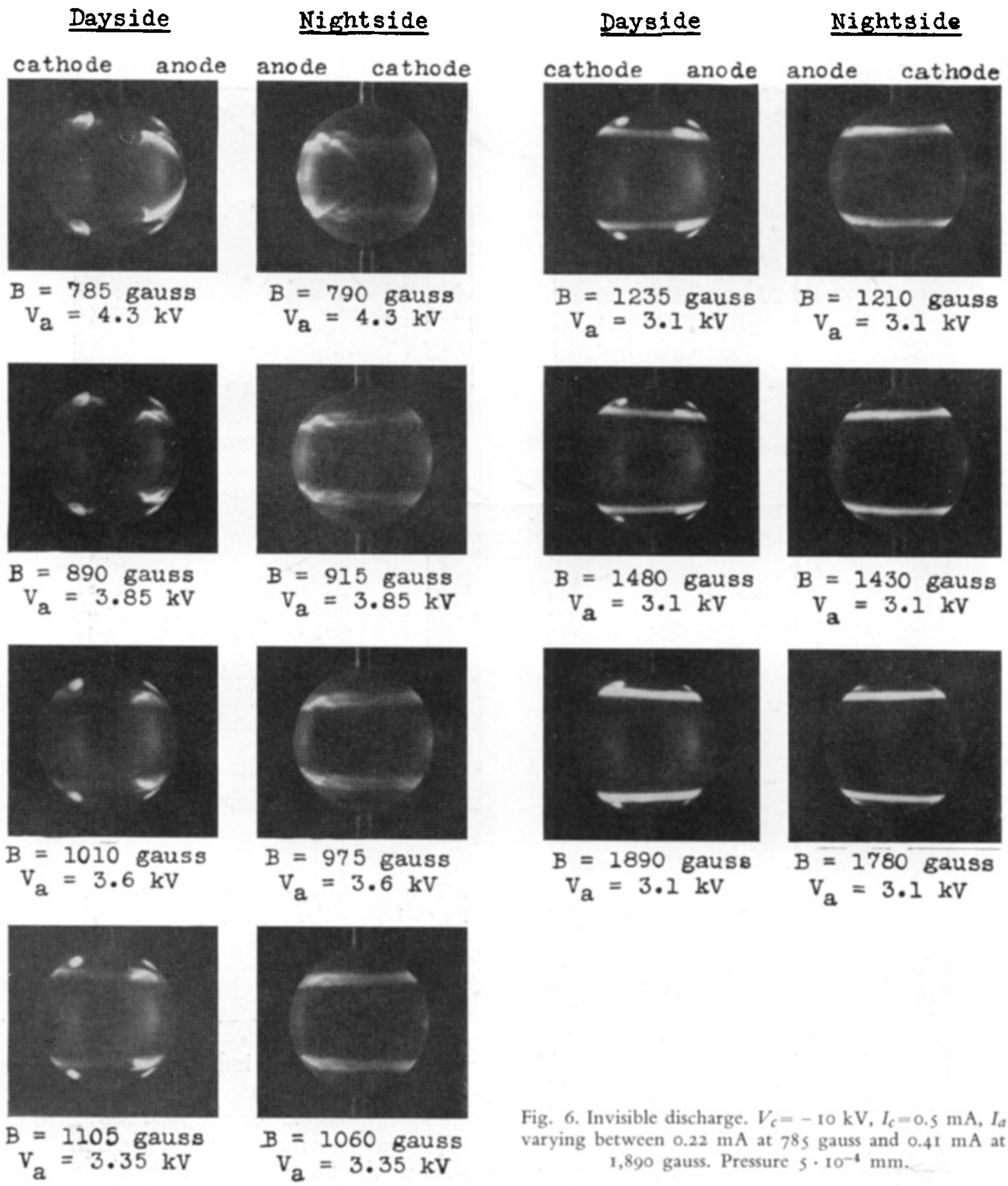


Fig. 6. Invisible discharge. $V_c = -10$ kV, $I_c = 0.5$ mA, I_a varying between 0.22 mA at 785 gauss and 0.41 mA at 1,890 gauss. Pressure $5 \cdot 10^{-4}$ mm.

of the terrella, even if the generation is not perhaps exactly of the same kind as Malmfors has proposed.

A plate of the form seen in fig. 11 was also used. The teeth on the lower edge of the plate made the auroral zone 'dashed'. It could easily be seen, that a tooth on the plate and the corresponding point on the auroral zone were situated in the same meridional plane as far as one could judge by eye.

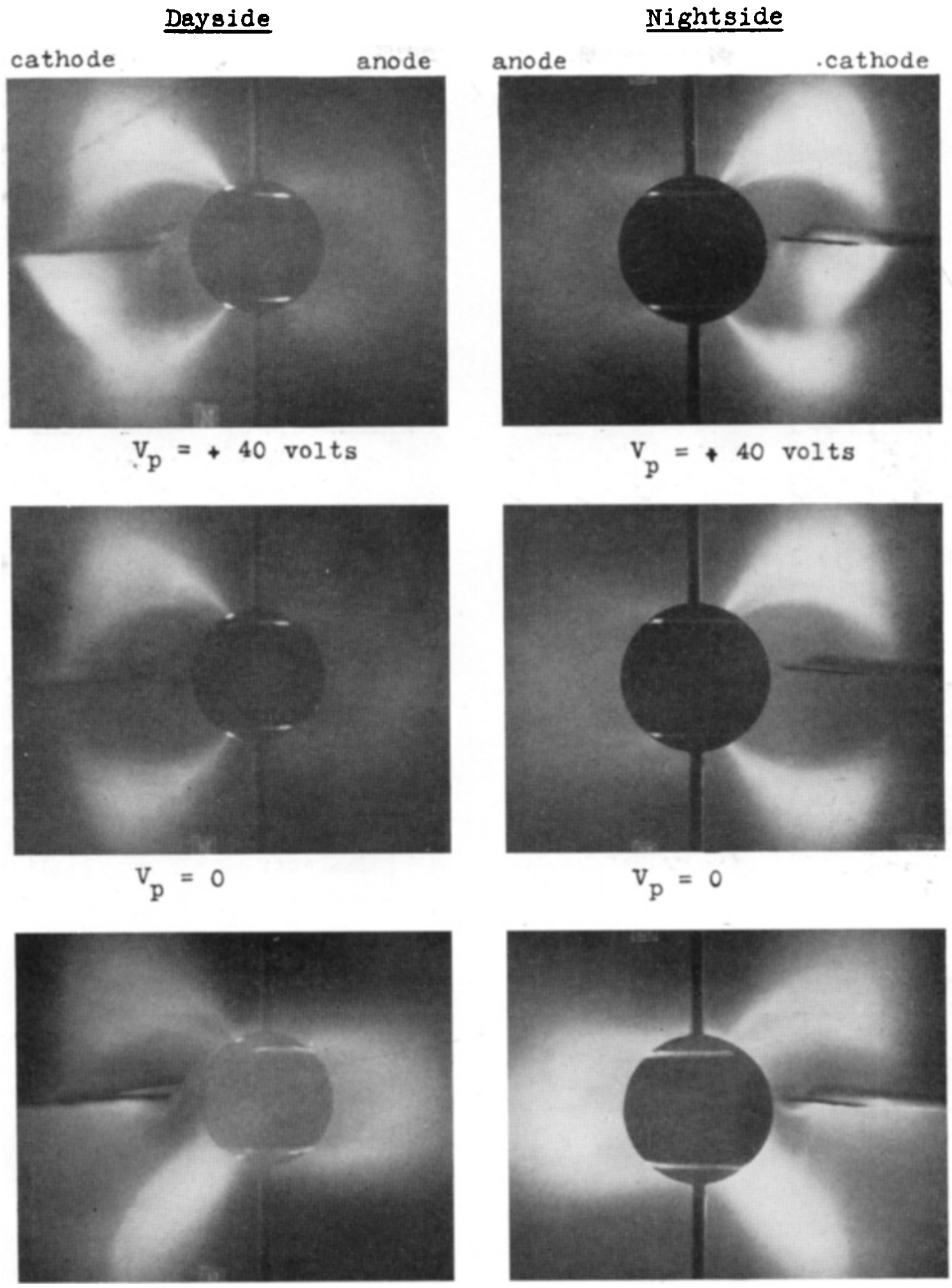


Fig. 7. Visible discharge. V_p = potential with respect to the terrella of the horizontal plate inserted on the cathode side. $V_c = -2$ kV, $V_a = +30$ V, $B = 1{,}090$ gauss, pressure $2.5 \cdot 10^{-3}$ mm.

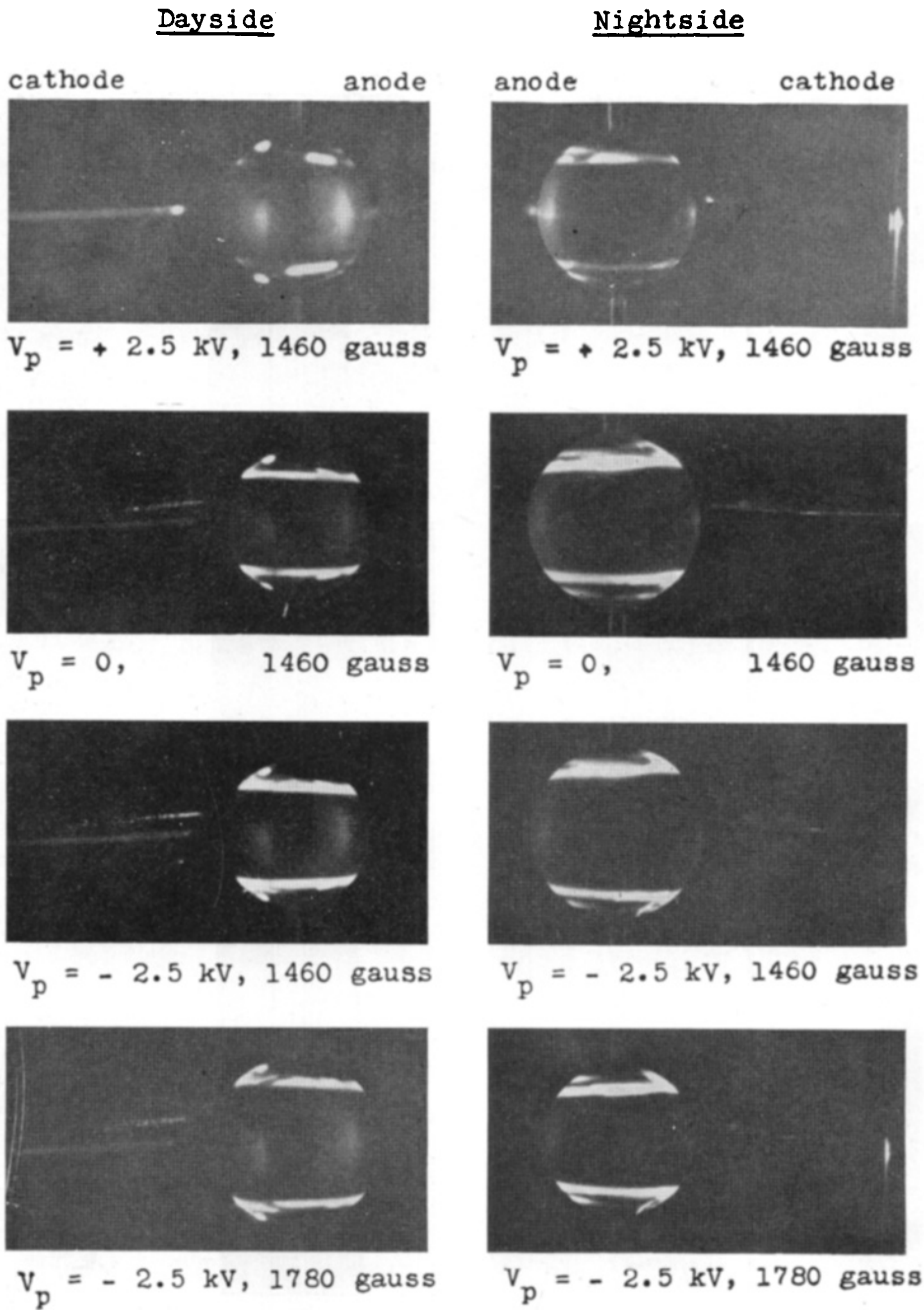


Fig. 8. Invisible discharge. $V_c = -10$ kV, $V_a = +5$ kV, pressure $5 \cdot 10^{-4}$ mm. Horizontal plate on the cathode side.

Calculation of forbidden zones according to Störmer and comparison with the experiments

One of the most important features of the discharges are the auroral zones and their latitude dependence on the magnetic and the electric field. STÖRMER (1933—37) has calculated the orbits of electrons moving from infinity into a magnetic dipole field and has found, that if the magnetic dipole field is strong enough or the electron energies small enough, there are forbidden zones resembling toroids around the dipole axis in the equatorial plane. It may be advantageous to calculate the size of these forbidden zones and to compare this with the forbidden latitudes on the terrella as seen on the photos. The reason for this is that there seem to be two possible explanations of the auroral rings on the terrella. The first is that electrons emitted from the cathode move in orbits very similar to those calculated by Störmer (properly modified by the electric field), thus causing the auroral zones. The second is that the particles impinging on the terrella are mainly formed by ionization in the discharge region and have then moved in orbits like those calculated by Alfvén. In the second case the energies of the impinging particles are lower than in the first case. We will therefore call the first kind of orbits high-energy orbits and the second kind low-energy orbits.

Störmer has defined a unit length C, the use of which makes all equations dimensionless.

$$C = \left(\frac{\mu_0}{4\pi}\cdot\frac{a}{B\varrho}\right)^{1/2} \qquad (16)$$

a = dipole moment,
$B\varrho$ is characteristic of the particle,
μ_0 = permeability of vacuum.

We confine ourselves to the non-relativistic case where $B\varrho$ is connected with the kinetic energy by

$$B\varrho = 3.37\cdot 10^{-6}\cdot V^{1/2} \qquad (16)$$

V is expressed in volts, ϱ in metres and B in Vs/m². All lengths are measured in C-units so that the distance

$$r = AC \qquad (18)$$

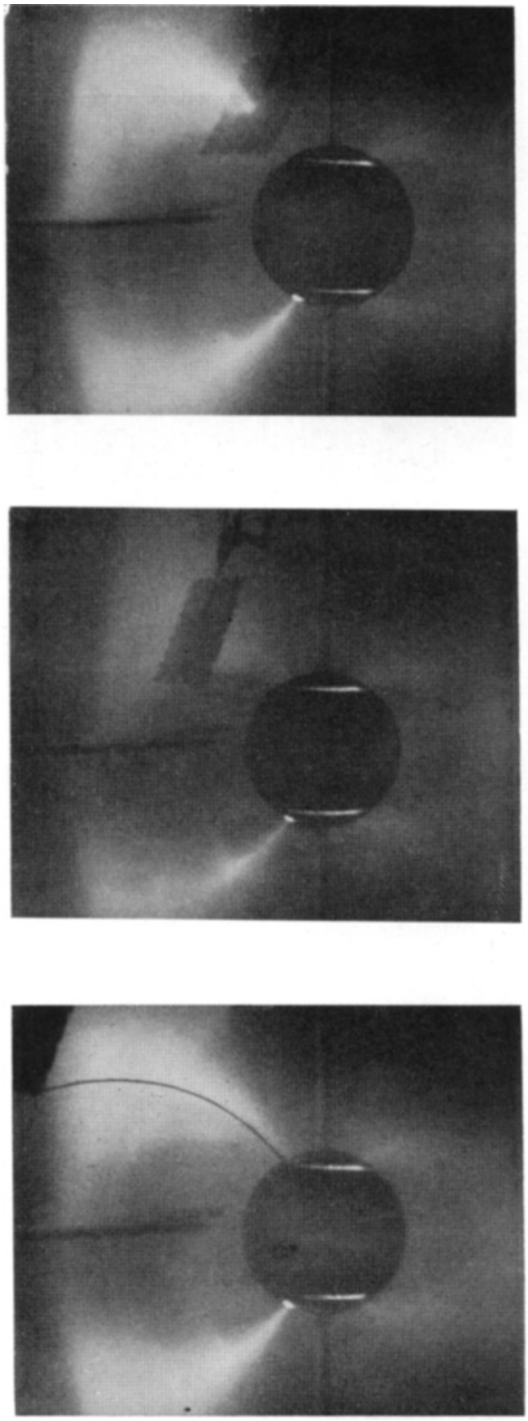
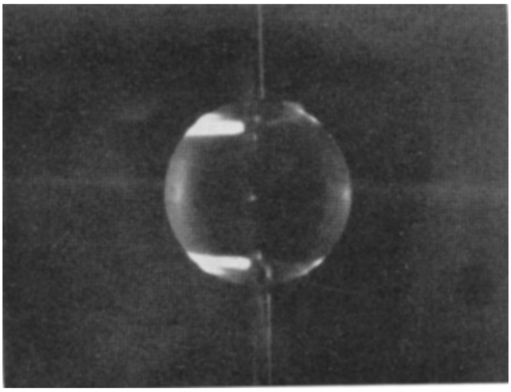
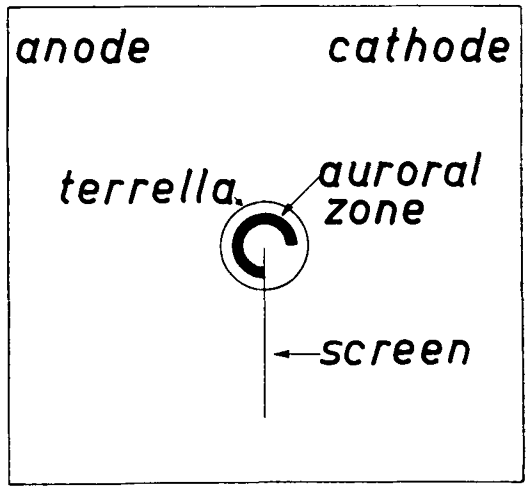


Fig. 9. Visible discharge. Dayside. Spot shadowed by movable plate. $V_c = -1.95$ kV, $V_a = +20$ V, $B = 1{,}110$ gauss, pressure $2.5\cdot 10^{-3}$ mm.





a b

Fig. 10 a. Invisible discharge seen from the nightside. Auroral zone cut off by a vertical plate (screen) in a meridional plane. $V_c = -10$ kV, $V_a = +3.2$ kV, $B = 1{,}900$ gauss, pressure $5 \cdot 10^{-4}$ mm. b. Sketch of the experimental arrangement seen from above.

If the dipole moment a is given in terms of the magnetic field B_p at the poles of the terrella, we find, that if r is the distance from the dipole to the particle, A and V are connected by

$$V = 0.594 \cdot 10^8 \cdot B_p{}^2 \cdot A^4 \qquad (19)$$

We use Störmer's theorem

$$\cos \omega = \frac{2\gamma}{A \cos \varphi} + \frac{\cos \varphi}{A^2} \qquad (20)$$

φ = magnetic latitude,

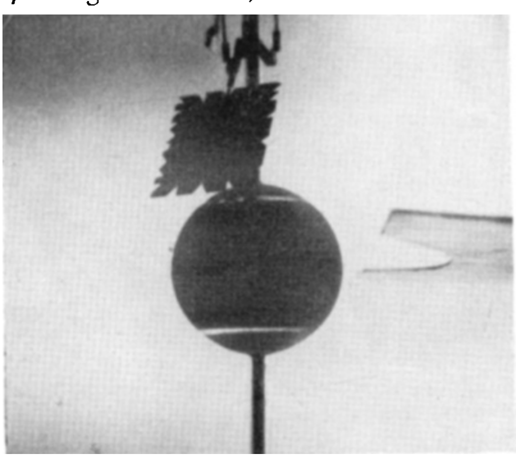


Fig. 11. Visible discharge. Nightside. 'Dashed' auroral zone. $V_c = -1.95$ kV, $V_a = +20$ V, $B = 1{,}110$ gauss, pressure $2.5 \cdot 10^{-3}$ mm.

ω = the angle between the impact direction o the particle and the east-west direction,

γ is a constant depending on the angular momentum of the particle with respect to the dipole axis at infinity.

It is easy to find that the boundary of the forbidden zone, i.e. the lowest latitude where aurora can appear, is defined by $\omega = 0$, $\cos \omega = 1$. Thus we can solve for $\cos \varphi$ in (20).

$$\cos \varphi = \frac{1}{2} A^2 \left(1 \pm \sqrt{1 - \frac{8\gamma}{A^3}}\right) \qquad (21)$$

The particles can reach the dipole only if $-1 \leqq \gamma \leqq 0$ according to Störmer, and as $\cos \varphi > 0$, we can only use the positive sign in (21). We assume that there are electrons with every γ-value between 0 and -1 present in the discharge. We are only interested in the lowest possible latitude φ_0, that is the highest possible value of $\cos \varphi$, which occurs if $-\gamma$ is as large as possible. Thus we must put $\gamma = -1$ which gives

$$\cos \varphi_0 = \frac{1}{2} A^2 \left(1 + \sqrt{1 + \frac{8}{A^3}}\right) \qquad (22)$$

We see from this that the equator is forbidden only if the A-value corresponding to the radius of the terrella is less than $\sqrt{2} - 1 = 0.414$.

By using (19) and (22) we can calculate φ_0 as a function of V and B_p. If the electrons from the cathode are accelerated in a relatively short cathode fall to an energy corresponding to the potential difference between cathode and terrella and then move only under the influence of the magnetic dipole field, then the lowest latitude of aurorae as seen from the photos should depend on V_c and B_p in the same way as φ_0 on V and B_p. This may in particular be the case in the visible discharge but not in the invisible one, where there is by no means a 'relatively' short cathode fall. It is also implied that no collisions and space charges appreciably affect the general behaviour of the discharge excepting their responsibility for the cathode fall.

Particularly in the invisible discharge the electrons do not immediately acquire an energy corresponding to V_c, but they are rather accelerated by the electric field through the whole path to the terrella. It is very difficult to calculate the forbidden zones if an arbitrary electric field is superposed on the magnetic field. STÖRMER (1931), however, has calculated the forbidden zones in the case of a spherically symmetric Coulomb field from the dipole. It is most advantageous to use his equation (l.c., p. 39, eq. 9)

$$c^2\left(1+\frac{c_1}{c^2}-\frac{b}{c^2}\cdot\frac{1}{r}\right)^2 \geq \left(\frac{c_2}{R}-a\frac{R}{r^3}\right)^2+c^2 \tag{23}$$

r = distance from dipole to particle,
$R = r\cos\varphi$ = equatorial distance,
c = velocity of light,
c_1 = kinetic energy at infinity divided by the rest mass of the particle.
a and b are defined by

$$a=-\frac{e\mu_0 M}{4\pi m_0},\quad b=\frac{eV_0 r_0}{m_0} \tag{24}$$

μ_0 = permeability of vacuum,
e = electronic charge in Coulombs,
m_0 = electronic rest mass in kg,
M = magnetic dipole moment in Am^2,
V_0 = potential in volts relative to infinity at r_0 metres from the dipole.
c_2 is a constant corresponding to γ in equation (20).

With a similar treatment of these equations as of (20) we get

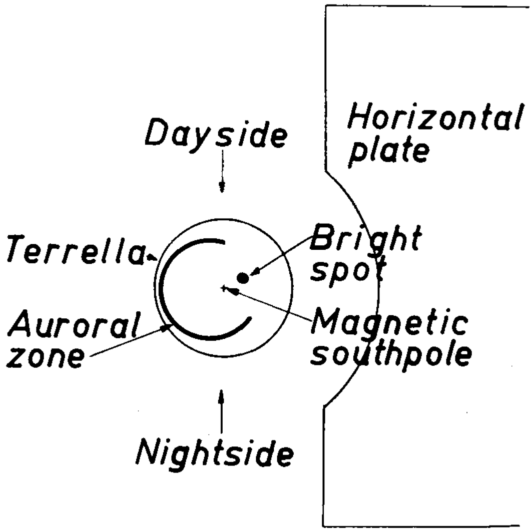


Fig. 12. Sketch of northern auroral zone seen from above. A horizontal plate is inserted between the terrella and the cathode so that a part of the auroral zone is 'erased'. Compare fig. 7.

$$\cos\varphi_0=\frac{1}{2}Z_0^{3/2}\left(1+\sqrt{1+\frac{3\sqrt[3]{16}}{Z_0^2}}\right) \tag{25}$$

$$Z_0=r_0\left(-\frac{2b}{a^2}\right)^{1/3}=0.256\cdot 10^{-2}\left(\frac{V_0}{B_p^2}\right)^{1/3} \tag{26}$$

where r_0 = terrella radius. It has been assumed that the kinetic energy at infinity ($c_1 m_0$) is zero. This result can be applied to our experiment if it is assumed, that V_0 is the potential difference between terrella and cathode and that the cathode is at infinity and the electrons start from the cathode with zero velocity. It may be of interest to compare this with the experimental results, although the electric field is not at all of the Coulomb type. The most important differences are, that in the experiments we have an anode, and the equipotential surfaces are not spheres, and thus they produce a radial component of $E\times B$, which means that the electrons can drift perpendicular to the magnetic field lines far closer to the dipole than if $E\times B$ had no radial component.

The results of these calculations are compared in figs. 13 and 14 with the experimental results obtained from the photos in figs. 5 and 6 respectively. It is to be expected that the experimental values should lie below the theoretical curve $\varphi_0(B_p)$ according to equations (25) and (26), where an electric Coulomb field

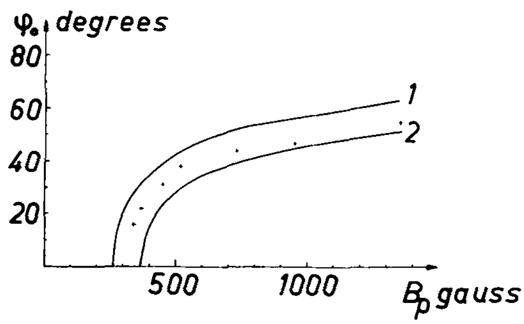


Fig. 13. The lowest allowed latitude φ_0 as a function of the magnetic field according to Störmer compared with the experimental values obtained from the visible discharge in fig. 5.

Curve 1: An electric Coulomb field is taken into account.
Curve 2: Neglecting the electric field.
Experimental values are indicated by +.

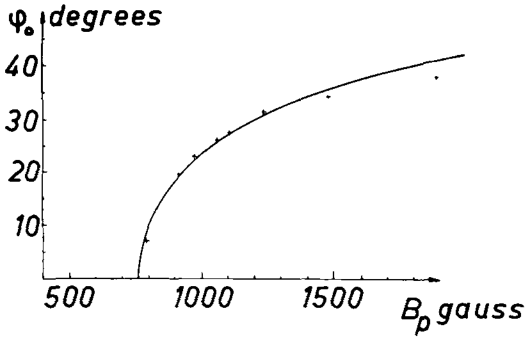


Fig. 14. The same as in fig. 13 but for the invisible discharge in fig. 6. No electric field is taken into account. Experimental values are indicated by +.

is also taken into consideration. This means that the electrons can be expected to reach lower latitudes than this theory permits because of the radial component of $E \times B$ in the experiment, and also because of low energy plasma electrons and ions moving according to Alfvén's theory. The latter will be considered in the next section.

It is seen from figs. 13 and 14 that our expectations are realized. In the case of the invisible discharge the experimental values for higher magnetic fields are even below the theoretical curve obtained from (22) without electric field. The curve calculated from (22) lies always below the one calculated from (25).

Alfvén's forbidden zones compared with the experiments

As already pointed out the auroral zones seen on the terrella may also be due to low energy plasma electrons moving according to Alfvén's calculations. In the case of an invisible discharge, however, one may not be allowed to speak of a real plasma, but still there are secondary electrons produced by ionization, perhaps mainly by the electron beam (see fig. 2), which may determine the latitudes of the auroral zones. In order to see if this is quantitatively possible, equation (7) or the last of equations (14) can be used. If we proceed in the same way as when we obtained (7 b), we find that the energy of the electrons responsible for the forbidden zone is

$$eV = e\frac{LE}{k^n} \qquad (27)$$

at the distance kL from the dipole. The exponent n is 3 using (7) and 2 using (14). If (7) is used, V is the 'perpendicular' energy,

We first apply (27) to the visible discharge. assuming that the electrons and ions responsible for the auroral zones are formed by ionization mainly at a distance kL from the dipole. Since the terrella is situated in the plasma, the E-value prevailing there must be used. Assuming the anode voltage V_a relative to the terrella to be an approximative measure of the electric field in the plasma, we get

$V_a \approx 30$ volts,
$d \approx 30$ cm = distance from terrella to anode,
$E = V_a/d = 1$ V/cm.

L may be seen from the photos to be about 15 cm or 3 terrella radii and k can be of the order of 1 to 5. From (27) we then see that eV can be of the order of a few electron volts, which seems to be a quite reasonable energy for electrons formed by ionization in a plasma.

In the invisible discharge we cannot assume the starting energy of the secondary electrons to be only a few volts, because we now have so strong an electric field, that the energies are determined by equation (3), suitably modified for $V = V_{\perp}$ instead of $V'_{\perp}$

$$V = 2.85 \cdot 10^{-12} \left(\frac{E}{B}\right)^2 \qquad (28)$$

A typical value of the discharge voltage $V_a - V_c$ is 14 kV, so that $E = 23$ kV/m, and if B is say 50 gauss $= 5 \cdot 10^{-3}$ Vs/m², we get $V = 60$

volts. Thus the best we can do is to combine (27) and (28) and solve for k. This can be done if we use the law of the dipole field

$$B = B_p \frac{R^3}{2r^3}, \tag{29}$$

where B_p is the field at the poles of the terrella, and R is the radius of the terrella. Putting $r = kL$ we obtain

$$k^{6+n} = 8.8 \cdot 10^{10} \frac{B_p^2 \cdot R}{E\left(\frac{L}{R}\right)^5} \tag{30}$$

If we take for example $B_p = 0.2$ Vs/m^2 = = 2,000 gauss, we see from fig. 14 that φ_0 is about 38° experimentally, and the correspondnig magnetic field line cuts the equatorial plane at a radius of 1.6 terrella radii. According to Alfvén's theory this point corresponds to 0.74 L from the dipole if $n = 3$ and 0.63 L if $n = 2$. Therefore, we get

for $n = 3$: $L/R = 1.6/0.74 = 2.2$, $k = 1.76$,
for $n = 2$: $L/R = 1.6/0.63 = 2.5$, $k = 1.74$.
These values are quite reasonable.

It is not possible to obtain a relation between B_p and the latitude of the auroral zones from equation (30), since we do not know how k varies with L. However, MALMFORS (1946) has obtained such a relation based on Alfvén's theory. See also appendix II equation (7).

$$\cos \varphi_a = \left(285 \frac{ER^5}{a^2}\right)^{1/16}, \tag{31}$$

φ_a = maximum latitude angle of the auroral zones,
E = electric field,
R = terrella radius,
a = magnetic dipole moment.

Putting $E = 23$ kV/m and $R = 5.3$ cm as before and using the relation between a and B_p we obtain

$$\cos \varphi_a = \frac{1.47}{\sqrt[8]{B_p}}, \tag{32}$$

where B_p is measured in gauss.

In fig. 15 the experimental results obtained from the photos in fig. 6 are compared with equation (32). It should be pointed out, that this equation is only valid, if there are real auroral zones with the equatorial regions of the terrella in the forbidden space, because

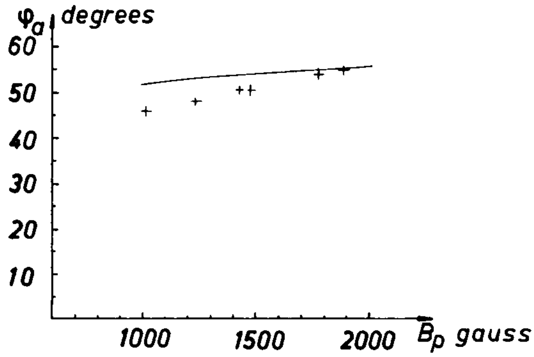


Fig. 15. The maximum latitude of the auroral zones according to Malmfors compared with the experimental values obtained from the invisible discharge in fig. 6. Experimental values are indicated by +.

the electric field theory is based on the assumption that the radius of curvature of the particles in the magnetic field is much smaller than the terrella radius. Therefore, the curves in fig. 15 are drawn only for $B_p > 1{,}000$ gauss. It is seen, that the experimentally obtained curve approaches satisfactorily the theoretical curve at high magnetic fields.

The angle φ_a is always greater than φ_0, the minimum latitude of the auroral zones, which we have considered in the preceding section.

The eccentricity of the auroral zones and Störmer's trajectories.

It may be seen from the photos, that the auroral zones are generally eccentric in qualitative agreement with Alfvén's theory. In this section we will examine this eccentricity in some detail. In the two preceding sections we have considered two possible explanations of the auroral latitudes and the forbidden zones. It is apparent, that from these considerations only, we cannot decide whether an Alfvén-mechanism or electrons moving in high-energy orbits are responsible for the auroral zones. With high-energy orbits we here, as before, mean trajectories described by high energy electrons coming directly from the cathode, and where space charge effects have been neglected. In the Alfvén-mechanism on the other hand, space charges are essential, because the auroral zones occur where space charge accumulated in the equatorial plane

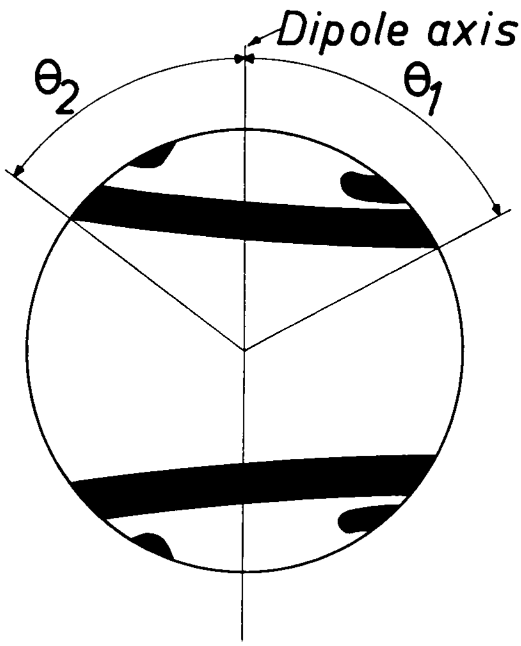


Fig. 16. Definition of the eccentricity of the auroral zones.

discharges along the magnetic field lines, and the energy of the particles may also be much smaller than in the case of high-energy orbits. As we shall see, the eccentricity strongly favours the Alfvén-mechanism, and there is also another reason against high-energy orbits.

We define the eccentricity ε as

$$\varepsilon = \frac{\Theta_1 - \Theta_2}{\Theta_1} \tag{33}$$

The angles Θ_1 and Θ_2 are defined in fig. 16. It may be noticed that $\Theta_1 + \varphi_0 = 1/2\,\pi$. In this definition it has been assumed, that the border of the forbidden region in the equatorial plane according to the theory defines the lower edge of the auroral rings, but it may more probably define the middle of the rings. There are, however, often bright spots within the rings. We do not know whether they belong to the rings or not, and so it is difficult to find the middle of the rings. The lower edge is more well defined but seems to give something like 10 % lower values of ε than the middle of the rings. According to Alfvén's theory the eccentricity should be about 0.26 or 0.31 (because of the two possible ways of calculating the forbidden zones). The only value which relates to nature is obtained by interpreting the auroral curve from the measurements of the azimuth of auroral arcs. This gives $\varepsilon = 0.48$ (see Cosmical Electrodynamics p. 186 fig. 6.7), but this is a very uncertain value, and not much weight should be attached to it. From the invisible discharge we get a considerably smaller eccentricity. From fig. 17 it may be seen that at $B_p = 1{,}000$ gauss the eccentricity is about 0.13, and at increasing field it goes down to about 0.10. This discrepancy may be due to the geometry of the electric field. In the theory it is assumed to be homogeneous, but in the experiment it is strongly modified by the terrella. It has also been observed, that the eccentricity strongly depends on the terrella potential relative to anode and cathode, if the potential difference between anode and cathode is fixed. We can say that $\varepsilon = \varepsilon(V_a)$ with $V_a - V_c = V_d = \text{constant}$. The dependence is such that ε increases with V_a. The theory corresponds to $V_a = 1/2\ V_d$, but the curve in fig. 17 is drawn for $V_a = 0.29\ V_d$. The experiment therefore gives greater eccentricities than those in fig. 17. More recent experiments indicate that the eccentricity at $V_a = 1/2\ V_d$ is about 0.20. Details of these experiments will be given in a later communication.

In nature the electric field is modified by the earth's ionosphere, as it is in the experiment by the terrella. The above mentioned value of ε in nature is probably too high. This is indicated by magnetic storm measurements and will be considered in a later communication.

There is also another very important question about the eccentricity: Is it possible to obtain eccentric auroral zones from Störmer trajectories, and can they be continuous around the whole earth or terrella? A detailed study of Störmer's papers reveals that this is not the case. We can get a good picture of this by examination of his paper in Astroph. Norv. I. 4, second part of Pl. XI. The orbits are there shown in the R—z-plane, the dipole pointing in the z-direction. If we follow the different orbits, we see that for higher values of $\gamma_1 = -\gamma$, the electrons make an increasing number of oscillations through the equatorial plane and reach lower latitudes of the terrella. This is exactly in accordance with our previous calculations (see the discussion before equation (22)). The increasing number of oscillations means that the electrons move through an increasing meridional angle. This may be seen

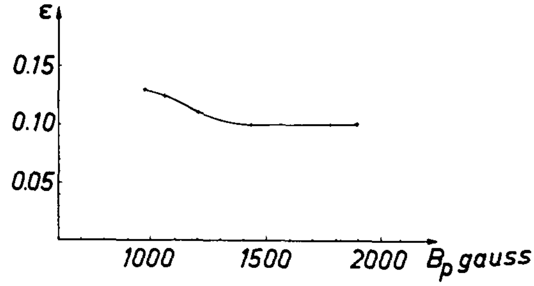


Fig. 17. The eccentricity of the auroral zones in the invisible discharge in fig. 6.

in Astroph. Norv. II. 1, Pl. III and the accompanying table on pp. 68—69 in the same paper. In fact, it is possible to construct from Störmer orbits an auroral zone, which is like a spiral starting from a high latitude on the cathode side and asymptotically attaining a latitude circle from above. This has been done by Störmer and is shown in fig. 18 with subsequent γ-values indicated. The observed auroral zone on the terrella may therefore be in agreement with Störmer's theory on the dayside, where its latitude is decreasing from left to right, but not on the nightside, where it is increasing from left to right. The spiral also shows, that it is not possible to obtain auroral zones which are continuous everywhere, especially not on the cathode side. In the experiment they are continuous at least in the case of invisible discharge, although we have also noted a bright spot within the rings on the cathode side, which probably is due to Störmer orbits (see fig. 9).

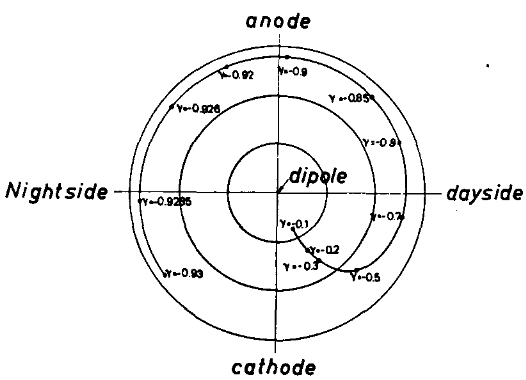


Fig. 18. 'Auroral spiral' constructed from Störmer orbits. See STÖRMER: Corpuscular theory of the Aurora Borealis, *Terr. Magn. and Atmosph. El*, **22**, fig. 4, p. 32.

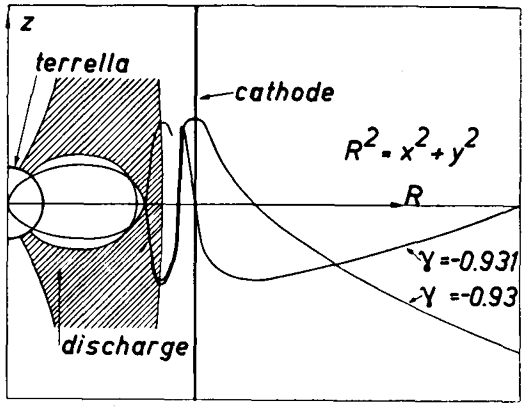


Fig. 19. Comparison of Störmer orbits in the *R—z*-plane and the visible discharge in the last photo in fig. 5. The Störmer orbits are taken from his paper in Astroph. Norv. I, 4, Pl. XI.

In fig. 19 some Störmer orbits from Astroph. Norv. I. 4, Pl. XI are compared with the discharge in fig. 5, the last dayside photo. It is very clearly seen, that there is insufficient space in the discharge for the electrons to oscillate through the equatorial plane so many times, that they can reach the nightside of the terrella (compare the γ-values in fig. 19 with those in fig. 18).

From this we may conclude, that the general behaviour of the discharge is not governed by high-energy orbits, although some subsidiary features, such as the bright spots on the cathode side, probably are due to such orbits.

It is apparent, that Alfvén's conception has been confirmed by these experiments, that space charge accumulated in the equatorial plane, rather than destroy the whole pattern of motion, immediately discharges along the

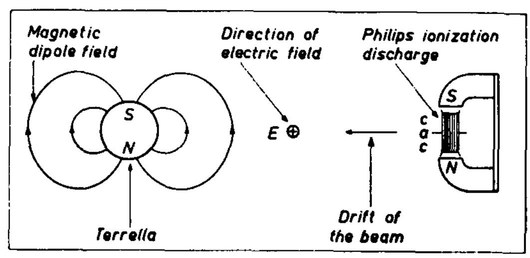


Fig. 20. Arrangement for suggested experiments where the neutral beam from the sun is also simulated. 'N' and 'S' refer to magnetic north and south poles. At the 'Philips' discharge, 'c' and 'a' mean cathode and anode respectively.

field lines. One should bear in mind, that experiments like these can never be a proper scale model of nature, but they can show in a convincing way, if a certain proposed mechanism, such as Alfvén's 'auroral mechanism', is likely to exist or not.

Suggestions for further experiments

As already pointed out it is impossible to scale down the real auroral discharge to laboratory dimensions in an entirely satisfactory way. In this section we will discuss one possibility of improving the experimental arrangement, so that we can be more sure of closer similarity to the real conditions in nature, as suggested by the electric field theory.

The neutral beam from the sun constitutes the most important difference between nature and experiment. Therefore, experiments arranged as indicated in fig. 20 may be more like nature. In 'Philips-type' discharges electrons and ions are produced, and they drift towards the terrella because of the crossed electric and magnetic fields. As will be seen, this arrangement makes it possible to work with lower electric field and without the usual type of discharge between the plates. The only purpose of the electric field is then to guide the drift of the particles, so that the beam itself can produce space charges, which may cause discharges along the magnetic field lines, just as in the theory.

We have pointed out in the beginning of this paper, that according to the correct transformation laws we should have
$V_d \approx 100$ kV,
$B_p \approx 60$ million gauss $= 6{,}000$ Vs/m^2.
Using (11), however, we can get the same electron orbits (assuming the kinetic energies to be proportional to the applied voltage), if we use another magnetic field B' and another voltage V'_d, which are connected by

$$\frac{V'_d}{10^5} = \left(\frac{B'}{6{,}000}\right)^2 \tag{34}$$

If $B' = 0.2$ Vs/m$^2 = 2{,}000$ gauss, we get $V'_d = 10^{-4}$ volts. As 10^{-4} volts corresponds to an electron temperature of $1°$ K, this is impossible to obtain in any experiment. The only way to get any similarity with nature is to confine ourselves to a correct value of L according to (14) or (15)

$$L^3 = \frac{1}{E}\left(\frac{\mu_0}{4\pi} a\varkappa\right)^{2/3}, \tag{35}$$

$$L^4 = \frac{\mu_0}{4\pi}\frac{a\mu}{eE} \tag{36}$$

If we want $L = 4r_0$ (r_0 = terrella radius) at $B_p = 2{,}000$ gauss, and if we use

$$\frac{\mu_0}{4\pi} a = \frac{1}{2} B_p \cdot r_0^3 \tag{37}$$

$$E = \frac{V_d}{5.4\, r_0}, \tag{38}$$

we get

$$\varkappa^{2/3} = 56\, V_d \tag{39}$$

or

$$\frac{\mu}{e} = 480\, V_d \tag{40}$$

respectively. If the magnetic field in the 'Philips' discharges is say 300 gauss the electronic energies in them must be

$$eV = e \cdot 56 \cdot V_d (3 \cdot 10^{-2})^{2/3} \approx 6\, eV_d, \tag{41}$$

or

$$eV_\perp = e \cdot 480\, V_d \cdot 3 \cdot 10^{-2} \approx 14\, eV_d. \tag{42}$$

In the 'Philips' discharges $V_\perp$ can be at most some hundred volts, and so V_d becomes less than 100 volts. This low value of V_d cannot cause a usual type of discharge between the plates.

Appendix I. Calculation of forbidden zone assuming the electronic velocities to be isotropically distributed everywhere.

According to the perturbation method for calculation of electron orbits (see Alfvén: Cosmical electrodynamics, Ch. II) we have

$$\mu = \frac{eV_\perp}{B} = \text{constant of the motion} \tag{1}$$

In a discharge collisions and plasma oscillations frequently occur, and they may to a certain extent maintain an isotropic distribution of the kinetic energies, which would not be possible if (1) is strictly valid. We may therefore assume, that an electron first moves a short distance, where (1) is valid. The magnetic field changes with a small value dB during this motion, and according to (1) $V_\perp$ changes to $V_\perp + dV_\perp$, so that

$$\frac{V_\perp}{B} = \frac{V_\perp + dV_\perp}{B + dB} \tag{2}$$

Then collisions and plasma oscillations redistribute this energy change so that the isotropic distribution is maintained.

$$\frac{\frac{2}{3}V}{B} = \frac{\frac{2}{3}V + Vd}{B + dB} \tag{3}$$

where $2/3\ V = V_\perp$ but $dV = dV_\perp$ of equation (2), since $dV_\parallel = 0$ there. Integrating (3) we get the new constant of the motion

$$\varkappa = \frac{V^{3/2}}{B} \tag{4}$$

Using the same notation as earlier in this paper, e. g. in fig. 3, we have

$$V = V_0 + E(x_D - x), \tag{5}$$

$$B = B_0 + \frac{b}{(x^2 + y^2)^{3/2}} \tag{6}$$

with

$$b = \frac{\mu_0}{4\pi} a \tag{6a}$$

B_0 is the solar magnetic field and eV_0 is the electronic energy far away from the earth. Together with (4) this gives

$$\varkappa \left(B_0 + \frac{b}{R^3}\right) = \left(V_0 + E(x_D - x)\right)^{3/2} \tag{7}$$

Since B_0 and V_0 are very small compared to the other quantities, we can neglect them here. They affect only the orbits far away from the dipole, where they are of little interest. Then we get

$$\frac{\varkappa b}{R^3} = \left(E(x_D - x)\right)^{3/2}, \tag{7 a}$$

which is equivalent to

$$x_D - x = \frac{L^3}{R^2}, \tag{8}$$

$$L^3 = \frac{1}{E}(b\varkappa)^{2/3} \tag{9}$$

From this we get one of the curves in fig. 4 which cuts the x-axis at

$$x'_d = \sqrt[3]{2}\,L = 1.26\,L,$$

$$x'_m = -\frac{1}{2}\sqrt[3]{2}\,L = -0.63\,L.$$

At infinity we get (still neglecting V_0 and B_0)

$$x = x_D = \frac{3}{2}\sqrt[3]{2}\,L = 1.89\,L.$$

The y-axis is cut at

$$y = \pm\frac{\sqrt[3]{2}}{\sqrt{3}} = \pm 0.73\,L.$$

Finally we must estimate the error made by neglecting V_0 and B_0. By putting

$$\varkappa B_0 = V_0^{3/2} \tag{10}$$

into (7) we get

$$x_D - x = \frac{1}{R^2}\left(\frac{L}{\phi}\right)^3 \tag{11}$$

with

$$\phi^3 = (1 + \delta)\left[1 - \left(\frac{\delta}{1 + \delta}\right)^{3/2}\right]^{2/3}, \tag{12}$$

$$\delta = \frac{V_0}{E(x_D - x)} \tag{13}$$

Supposing $B_0 = 0.7 \cdot 10^{-5}$ gauss, $E = 1.4 \cdot 10^{-3}$ V/m and $L = 7r_0$, where r_0 = earth's radius, we get

$$\frac{V_0}{E} = 3.85 \cdot 10^5\,m = 0.86 \cdot 10^{-2}\,L. \tag{14}$$

If we use this value in (13) and put $x_D = 1.89\ L$, we find that $\phi = 1.026$ at $x = 1.8\ L$. Since $\phi > 1$ and steadily increasing with increasing x for every $x < x_D$, the error made by neglecting V_0 and B_0 is negligible.

Appendix II. Calculation of the auroral latitudes according to Malmfors.

Earlier in this paper frequent reference has been made to Malmfors' experiments. Equation (31) is taken from his paper. Since it is out of print and somewhat inaccessible, it seems desirable to derive that equation here.

Electrons are assumed to oscillate along magnetic field lines from the northern to the southern hemisphere of the earth. If the amplitude is large enough, the electrons hit the atmosphere and are absorbed there, thus causing aurora. If r_0 is the distance from the dipole to the equatorial point cut by the field line, we get the equation of this field line

$$r = r_0 \cos^2\varphi, \tag{1}$$

and the magnetic field strength is

$$B = \frac{\mu_0 a}{4\pi r^3}\sqrt{1+3\sin^2\varphi} = \frac{\mu_0 a}{4\pi r^3}\cdot m \tag{2}$$

Only the morning side of the earth (region between terrella and cathode) is considered, because the discharge is assumed to be generated there (at the singular point of zero drift velocity). The electric field E V/m is then situated in the same meridional plane as the magnetic field line. If $r = r_1$, $\varphi = \varphi_1$ is the turning point of the electron, the 'perpendicular' energy there is

$$eV_1 = eV_0 + eE(r_0 - r_1 \cos\varphi_1), \tag{3}$$

where eV_0 is the 'perpendicular' energy in the equatorial plane. Since φ_1 is rather near 90°, $r_1 < r_0$ and V_0 is small, we can simplify (3) to

$$V_1 \approx Er_0 \tag{3a}$$

We also have

$$\frac{V_1}{B_1} = \frac{V_0}{B_0}. \tag{4}$$

According to (28) in the beginning of this paper V_0 is determined by

$$V_0 = 2.85\cdot 10^{-12}\frac{E^2}{B^2}. \tag{5}$$

Malmfors has used 5.7 instead of 2.85, becaues he has not considered the difference between $V_\perp$ and $V'_\perp$ discussed after equation (8) earlier in this paper. Combining (2), (3 a), (4) and (5) we arrive at

$$m^2\cdot r_1^3 = 285\frac{E}{a^2}r_0^8. \tag{6}$$

If the auroral zones are due to electrons with $r_1 \leq R$ = earth's radius, and if φ_a is the maximum latitude angle of the auroral zones, we get with (1) and (6)

$$m^2\cdot\cos^{16}\varphi_a = 285\frac{ER^5}{a^2} \tag{7}$$

Since m is of the order of unity, this equation is equivalent to the earlier equation (31).

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