Charge generation in Thunderstorms by collision of ice crystals with graupel, falling through a vertical electric field

By D. MÜLLER-HILLEBRAND

Stockholm

(Manuscript received May 12, 1954)

Abstract

When an ice-crystal impinges on the lower surface of a particle of hail or soft hail part of the charge induced by the electric field is transmitted to the ice-crystal. The relaxation time for the exchange of charge varies from a few milliseconds to about 10^{-6} seconds and is sufficiently short to enable this gain of charge. In a positive field the soft hail will therefore get a negative and the ice-crystal a positive charge. In an electric field E the final charge of a sphere with a radius R is $e_1 = -2.12\, ER^2$. At temperatures between $-18°$ and $-26°$ the number of ice crystals is sufficient to allow a time constant of about 5 minutes for the charging process. The charge per droplet and per g of water derived from this process are in good agreement with measurements of Ross Gunn. The growth of precipitate and the formation of strong electric fields can be brought into correspondence with the timing and development of thunderstorms.

Introduction

During the years 1937—1941 G. C. SIMPSON and collaborators [1] published the results of experimental investigations on the distribution of electricity in thunderclouds and the associated electric fields. This initiated a new era in our understanding of the origin of electricity in thunderclouds. The observations made it clear that electrical charge coincides with certain regions of temperature. Negative charge is centered in regions of temperatures below freezing point. This region of negative charge is marked by the 0° to $-20°$ isotherms. The positive charge is about 2—3 kms higher. It is roughly centered at the $-10°$ to $-45°$ isotherms. A third and smaller region of prevailing positive charge frequently, but not always, appears during thunderstorms between the $+10°$ and 0° isotherms. The free charge in this region is considerably smaller averaging about 1/5 of those in the other two regions. The measurements of Simpson have subsequently been extended by others. According to E. I. WORKMANN and E. S. REYNOLDS the centre of negative charge is between the 0° and $-25°$ isotherms, but mostly at $-15°$ [2, p. 139].

The generation of electric space charge and electric fields is always associated with precipitation phenomena about which the following is known: It appears that a first precipitate can be formed in either the liquid or the solid state. Larger drops originate inside the cloud in a temperature environment above the 0° isotherm. According to BYERS [2, p. 63] the examination of 66 radar-echos shows that larger drops appear—in the average—660 m above the 0° isotherm (i.e. at an altitude of about 5,600 m), but in many cases only 300 m above the 0° isotherm, i.e. at temperatures of $-4.0°$ to $-2.5°$. According to WORKMANN and REYNOLDS [2, p. 147] the first radar-echo is associated with the $-21°$ to $-24°$ isotherms, where it appears to be reflected by ice crystals.

When the radar cloud reaches the $-28°$ to $-29°$ isotherm a noticeable downward movement of the top of the cloud begins simultaneously with precipitation resulting in the separation of electric charge [2, p. 139]. The radar-cloud of a specific thunderstorm-cell will sink with approximately the same velocity with which it grew (2.5—20 m/s). This marks the beginning of electrical discharges lasting 25—40 minutes. According to Byers, however, the first lightning-discharge is observed when the radar cloud passes the $-20°$ or an even lower isotherm, 10—15 minutes after the appearance of the first radar-echo. The investigation of 35 cases has shown that radar-clouds were growing at a mean rate of 5.4 m/s, 90 cm/s less than that of the visible clouds. At a height of 6,000 m a down-drift velocity of 90 cm/s is reached by drops of about 100 μ radius or by soft rime of about 600 μ radius which are responsible for radar-echos from the top of the cloud. 16 minutes after the first radar-echo has occurred and a mean cloud height of 10,500 m ($-36°$ isotherm) has been reached the average down-drift-velocity of the top of the cloud is 3.6 m/s. The downward current of air is propagated from the lower part of the cloud to the top in connection with the formation of precipitation. It therefore commences a short time after the first radar-echo has appeared. 10—15 minutes later, when the first lightning discharge occurs the $-20°$—$-28°$ isotherm is reached. The most frequently found width of that area in which the air flows downward is 1,500 m (with a maximum of 7,000 m). At an altitude of 7,800 m the mean down-drift-velocity is 6.7 m/s with a maximum of 24 m/s (mean of 60 measurements) [3 p. 42].

Simpson's measurements by means of soundings have given evidence on the field strength within the clouds: In wide regions it seldom exceeds some 100 V/cm. Measurements made by using aeroplanes [3 p. 80] yielded values up to a little above 500 V/cm. Ross Gunn obtained a maximum gradient of 1,600 V/cm at an altitude of 3,900 m, shortly before the plane was hit by lightning. This value refers to the unperturbed field [2 p. 199]. (The field was of course disturbed by the aeroplane, the actual measured value being 3,400 V/cm [2 p. 199].)

The positive and the negative charge are by no means completely separated. According to ROSS GUNN [2 p. 200] in the region from about $+14.7°$ to $-10°$ both positive and negative drops are mixed. But there are also cells extending over 2 km containing uniform negative charge. In these mixed regions the effective density of space charge is considerably smaller than the space charge density resulting from the charge on single drops. Positive space charge prevails in the region between 6,000 m and 8,500 m where precipitation with positive charge has been exclusively identified.

The electric processes connected with the formation of ice crystals have been investigated by W. FINDEISEN, E. WALL, and R. ROSSMANN [4], H. LUEDER [5], E. J. WORKMANN and S. E. REYNOLDS [2]. They have been critically examined by B. M. MASON [6 a]. During the formation of a rime layer a negative charge is acquired by the ice. This process is capable of generating the required quantity of charge (B. I. MASON [6 b]). It must be supposed that *several electrical processes will occur simultaneously* in a thundercloud. We shall now investigate the process taking place when rime and ice crystals come into contact and are again separated in an electric field. In the event of such a contact between rime and an ice crystal the crystal will slip off the ice grain after touching it for a short moment. At the same time the ice crystal removes part of the charge induced on the "bottom" of the ice grain by the electric field. In a positive field the small crystal will become positively charged, consequently leaving the bigger ice grain negatively charged. ELSTER and GEITEL discussed this mechanism for drops in 1913 [7]. In this manner a rain drop was to convey part of its induced charge to a cloud droplet. An air cushion between the drop and the droplet was to prevent coalescence of the two particles, and permit the positively charged droplet to recoil from the drop. The experimental investigation of T. E. W. SCHUMANN [8] proves, however, that the process of collision between two liquid drops is different: Small droplets having a radius of less than 180 μ will coalesce with the raindrop. Larger droplets will travel through the raindrop and then disintegrate taking with them the charge induced on the top. The process then ends with a charge of opposite sign to that assumed in Geitel's and Elster's theory. The raindrop will receive a positive charge. Thus the raindrop weakens the existing positive field. Consequently this mechanism does not intensify the field and will itself slowly die out.

We assume, therefore, that the crystals slip off the soft hail in the form of small plates of varying thickness, as dendrites or as small frozen droplets. This will not—or not always—be the case with soft rime, at least not while the rime is still developing a conical habit through little supercooled droplets loosely depositing on a star-shaped crystal (spec. weight about 0.1). Such forms of soft rime are found as well as soft hail, and later on we shall see that this is particularly significant.

Relaxation Time of the Charge-Exchange Process of Ice

If charge is to flow from an ice grain to a small crystal of ice the latter must have a certain conductivity. The relaxation time of the exchange process must be shorter than the time of contact. This time is given by:

$$t_R = \frac{\varepsilon}{4\,\pi \cdot 9 \cdot 10^{11} \cdot \varkappa} \tag{1}$$

Now neither the dielectric constant ε nor the conductivity $\varkappa$ both of which vary with temperature, are defined values. Water always contains some impurities. This holds even for normally distilled water of a conductivity of about 1×10^{-6} $(\Omega\ \mathrm{cm})^{-1}$ because of absorption of ammonia and carbondioxyde from the air. The dissolved substances are not "tightly" frozen. The ice is permeated by a network of very thin channels. In respect to its dielectric constant and its conductivity it behaves as an inhomogeneous dielectric. Both properties depend strongly on the frequency. Table I shows some data taken from an investigation of C. P. SMITH and C. P. HITCHCOCK [9]. The figures refer to ice obtained from water having a conductivity of 2×10^{-6} $(\Omega\ \mathrm{cm})^{-1}$, at the applied frequencies given in the left-hand column.

The relaxation time obtained by this method ranges from 10^{-3} to 10^{-6} s within the ranges of temperature and frequency given above.

The relaxation time may also be obtained from other experimental data. It is assumed that at the instant of contact between ice grain and ice crystal the electric phenomena in the dielectric are similar to those at high frequencies. The dielectric constant will be determined by the relation $\varepsilon = 3.40 + 0.04\,(T+10)$ (T in deg. Cent.). This empirical relation was found by H. BROMMELS [10] to hold for temperatures down to $-18°$ (°C), and the corresponding lowest value of 3 for the dielectric constant will be used later on. The D.C. conductivity of ice like that of other ionic conductors, e.g. quartz, calcite, rock salt etc. is given by

$$\log \varkappa = A - \frac{B}{T_a} \tag{2}$$

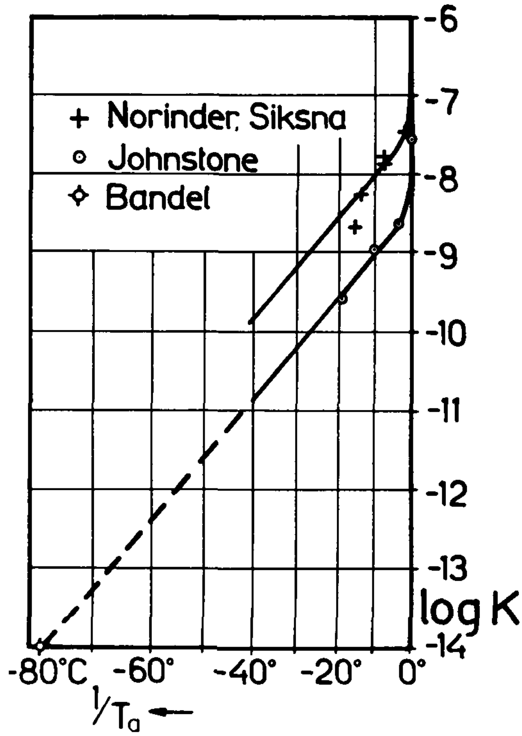


Fig. 1. Relation between conductivity of ice and temperature.

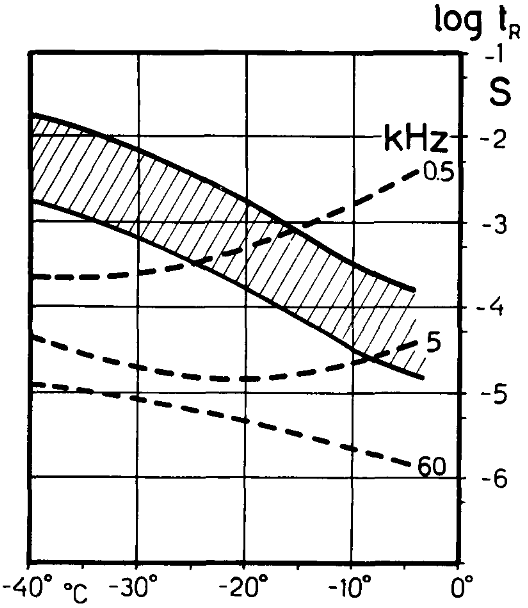


Fig. 2. Relation between relaxation time of exchange of charge and temperature.

with the two constants A and B and the absolute temperature T_a. The constants A and B which determine the magnitude of the conductivity depend on the number of foreign ions absorbed by the water. Carefully distilled water has a conductivity of $1 \div 2 \times 10^{-6}$ $(\Omega\ \mathrm{cm})^{-1}$ while the conductivity of rain water is greater by about one order of magnitude. For temperatures ranging from 0° to −19° C the values of the conductivity of ice are taken from JOHNSTONE [11], and for −78° that value obtained by BANDEL at LOEB's laboratory, namely 10^{-14} $(\Omega\ \mathrm{cm})^{-1}$ (fig. 1). These data refer to ice prepared from distilled water and are minimum values. Then the conductivity is given by

$$\log \varkappa = 5.35 - \frac{3.78 \times 10^3}{T_a} \qquad (2\,a)$$

This leads to a lower limiting value, because there are always very small quantities of sodium, magnesium, and chlorine dissolved in the ice crystal, particularly if they originated directly from supercooled droplets. Although no measurements are known, H. KÖHLER's investigation [13] carried on for a number of years suggest the following: melted soft rime shows a chlorine content of $(0.067 \div 56.3) \times 10^{-6}$ g per cm³ of water with a certain frequency distribution having a maximum at 3.5×10^{-6} g per cm³. This soft rime originates from a rather loose freezing (specific gravity 0.01 to 0.1) of small and very small droplets with a radius of 8.8 μ as the most frequent value. This value leads to 1.85×10^{-14} g of salt per droplet, this being the most frequent condensation nucleus, and is responsible for a greater conductivity as that given by (2 a). The values published by H. Köhler may be assumed to hold also at 7,000 m altitude, because when air is driven up vertically from the ground it will carry with it condensation nuclei from ground level. The measurements of H. NORINDER and R. SIKSNA [14] give further support to the assumption that the conductivity of ice crystals—i.e. soft rime, snow, soft hail, and hailstone—is greater than that given by (2 a) (provided the crystals do not originate from sublimation on water-insoluble nuclei). These measurements, obtained with compact snow, are shown in fig. 1.

Table I

	frequency kcycles	temperature °C			
		−50°	−30°	−10°	−5°
dielectric constant ε	60	3.02	3.04	3.33	3.71
	5	3.21	4.06	24.4	40.2
	0.5	4.65	31.3	73.6	73.8
conductivity $\varkappa$ $\times 10^8$ $(\Omega\ \mathrm{cm})^{-1}$	60	1.37	3.33	15.6	23.7
	5	0.29	1.84	9.0	9.86
	0.5	0.21	1.18	0.4	0.2
relaxation time t_R (s)	60	$1.94 \cdot 10^{-5}$	$0.81 \cdot 10^{-5}$	$1.89 \cdot 10^{-6}$	$1.38 \cdot 10^{-6}$
	5	$9.7 \cdot 10^{-5}$	$1.95 \cdot 10^{-5}$	$2.4 \cdot 10^{-5}$	$3.62 \cdot 10^{-5}$
	0.5	$1.95 \cdot 10^{-4}$	$2.32 \cdot 10^{-4}$	$1.63 \cdot 10^{-3}$	$3.25 \cdot 10^{-3}$

An approximate value for the conductivity of rime and hail will henceforth be assumed having an upper limit one order of magnitude higher than that given by (2 a), i.e.

$$\log \varkappa = 6.35 - \frac{3.78 \times 10^3}{T_a} \qquad (2\text{ b})$$

In this way the relaxation-time assumes the values shown in fig. 2 (shaded area): At temperatures between $-10°$ C and $-20°$ C the relaxation-time varies between 1.6×10^{-3} seconds (upper value, ice-crystals originating from sublimation) and 3×10^{-5} seconds (lower value, rime). At temperatures below $-40°$ C which are most frequently encountered at altitudes above 12 km the relaxation-time for ice-crystals is about 1.8×10^{-2} seconds. Therefore ice-crystals and ice-grains may be justifiable regarded as being "conductive" within that range of temperature in which the electricity of thunderstorms probably originates.

Electrification of ice by contact in an electric field

The following arrangement was investigated: A particle of rime is polarized by a vertical electric field through which it falls. Let it overtake small crystals of a shape shown, for instance, in the micrographs of H. Köhler [13 a] or V. Schaefer [15]. For a first approximation we consider the particle of rime (fig. 3) replaced by a conductive sphere of greater radius and the crystal by one of smaller radius. The greater sphere of radius R_1 be charged in proportion to the gradient thus

$$e_1 = \beta \times E R_1^2 \qquad (3)$$

where β denotes the "specific" charge. An infinitesimal change of charge is

$$de_1 = d\beta \times E R_1^2 \qquad (3\text{ a})$$

The charge transmitted by contact from the greater sphere to the smaller one of radius R_0 is proportional to the radial field strength at the point of contact:

$$e_2 = \gamma \times E_r R_0^2 \qquad (4)$$

where γ denotes a numerical factor which, when the ratio $\frac{R_0}{R_1} \sim \frac{1}{100}$ is equal to $\frac{\pi^2}{6} = 1.65$

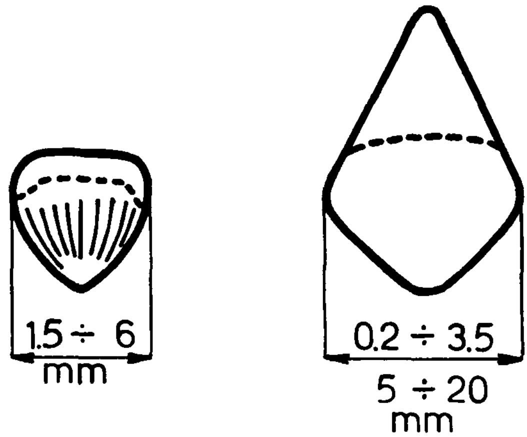


Fig. 3. Soft rime and soft hail.

and may be determined for other ratios from fig. 4.

At the surface of the drop polarized in the electric field E the radial field strength E_r at an angle ϑ is

$$E_r = E\,(3 \cos \vartheta + \beta) \qquad (5)$$

The maximum field strength is obtained for $\vartheta = 0$, i.e. for the centre of the circle representing the projection of the sphere on a horizontal plane. The probability of a collision between a small sphere and the bigger one is regarded as constant all over the circle representing the projection of the bigger sphere. Then the mean angle at which a collision occurs is $\vartheta = 45°$, and the mean field strength becomes 2.12 E. Thus at the point of contact with the sphere of the specific charge β the mean field strength is

$$E_r = E\,(2.12 + \beta) \qquad (5\text{ a})$$

Now, considering a large number of contacts during each of which a very small proportion of the charge on the big sphere is transmitted to the small sphere, the loss of charge per contact may be derived from (3 a) as

$$\frac{de_1}{dn} = -\frac{d\beta}{dn} E R_1^2 \qquad (5\text{ b})$$

The loss of charge on one hand is balanced by a gain of charge by the small sphere identical with (4). Combining this result with (5 a)

$$\frac{d\beta}{2.12 + \beta} = -\left(\frac{R_0}{R_1}\right)^2 \gamma\, dn \qquad (6)$$

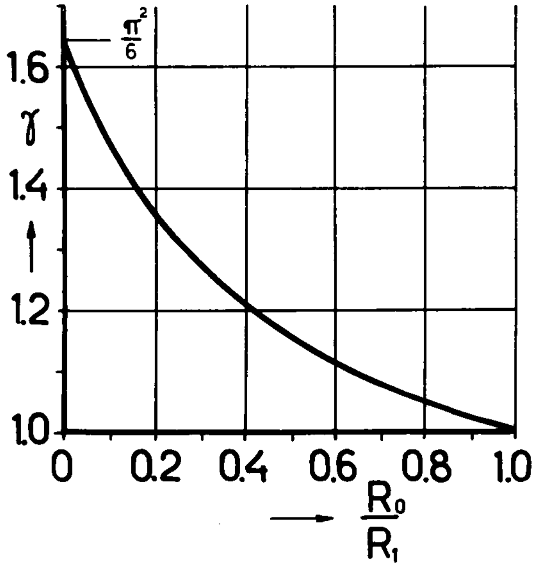


Fig. 4. The numerical value γ for contact between two spheres of radius R_0 and R_1.

Assuming the drop has an initial specific charge equal to zero ($n = 0$)

$$\beta = 2.12 \left(e^{-\gamma \left(\frac{R_0}{R_1}\right)^2 n} - 1 \right) \qquad (6\text{ a})$$

The specific charge tends towards the limit:

$$e_1 = -2.12\, E R_1^2 \qquad (6\text{ b})$$

this limiting value being reached after a large number of contacts. The number of contacts (n) is equal to the number of crystals in the path of the falling particle of rime and within a certain interception area. This area is somewhat smaller than the area $R_1^2\pi$ of the projected sphere (factor of proportionality $= \varphi$), because, as already mentioned above, some of the smaller crystals will be caused to deviate from their path by the rime.

Taking into account the aerodynamical deviation of the small sphere when flying towards the bigger one, the following procedure may be adopted in accordance with ALBRECHT [16] and SELL [17]. The deviation in a current of air is determined by a numerical factor whose physical meaning is determined by the ratio of two numbers, namely the path over which the small sphere is decelerated divided by the radius of the big sphere:

$$\delta = \frac{s \times R_0^2}{9\, g \eta} \left(\frac{v_{10}}{R_1} \right) \qquad (7)$$

where R_0 = radius of the "small sphere"
s = specific gravity of the small sphere
v_{10} = relative velocity of the small sphere with respect to the big sphere with radius R_1

$g = 981\ \frac{\text{cm}}{\text{sec}^2}$

η = viscosity of air $= 1.61 \times 10^{-7}\ \frac{g \times \text{sec}}{\text{cm}}$ at $-20°$ C

The relative interception-area φ of a sphere was investigated by Sell. φ denotes the ratio of a distant circular area from which the small particles will just reach the sphere, to the cross-section of the latter. Sell's results may very approximately be interpreted by

$$\varphi = \tanh 0.43\, \delta \qquad (8)$$

e.g. δ will have a value of 4 for a crystal (sphere) of 20 μ radius and S.G. $= 0.9$, travelling at a relative velocity of 1.6 m/s against soft hail of 1 mm radius. (Only very small particles for which δ is less than 0.1—i.e. particles of less than 2.3 μ radius in the above example—will deviate and not touch the sphere.) The relative interception-area is given by $\varphi = 0.94$. 94 % of the particles travelling in a cylinder of the same cross-section as the sphere will collide with it. Thus the 20 μ-crystals will fly towards the sphere on a nearly rectilinear path.

Precipitate of radius exceeding 1 mm, falls more and more slowly than in proportion to its size, in other words, because $\frac{v_{10}}{R_1}$ in (7) diminishes with growing R_1 the value δ and therefore the relative interception-area will become smaller. For hail with a radius of 10 mm δ is 2.3, and the interception-area for small particles of 20 μ radius becomes 0.76.

In connection with these aerodynamical discussions the mean angle of collision could now be investigated more closely for the particles hitting the big sphere. This would lead to a more accurate determination of the numerical factor given as 2.12 in (5 a). The probability of a collision is not uniform over the projection of the sphere. On the one hand the particles will be deviated in their flight from the central region of the projection and on the other there will no longer be any collision at the fringe areas. The angular limits are therefore not 0° and 90° but somewhat closer together, for instance ALBRECHT's investigations [16] allow their determination in the case of a cylinder, and Albrecht gives the diameter of that section of the cylinder with which the small spheres just collide. For the very small spheres of 2.3 μ radius in the example the limiting angle is approximately 40° for $\delta = 0.1$ and 76° for $\delta = 2$ under the above conditions. This would therefore have to be considered in investigating the collision of very small particles.

The rate of electrification

The rate of charging according to (6 a) depends on the number of ice crystals (small spheres) impinging on the particle of rime (large sphere). Let the number of crystals per cm³ in the interception-area $\varphi \pi R_1^2$ be N and let the relative distance of fall be $v_{10} \times t$ in time t. Then the number of ice crystals in impinging on the particle of rime equals the number of contacts n, where

$$n = N\varphi R_1^2 \pi v_{10} t \qquad (9)$$

Combining (9) and (6 a), the specific charge β of the rime-particle is

$$\beta = 2.12 \left(e^{-\frac{t}{T}} - 1\right) \qquad (10)$$

where the time constant T of the charging process is given by

$$T = \frac{1}{(\pi \cdot \gamma \cdot \varphi) \times NR_0^2 \times v_{10}} \qquad (11)$$

The numerical constants γ (eq. 4) and φ (eq. 8), and relative velocity v_{10} depend on the size of the rime.

The time constant may be illustrated by a numerical example. Let the radius of the ice crystal be $R_0 = 50\ \mu$ (that is equal to 5×10^{-3} cm), the number N per cm³ be 0.2, and the radius of the rime be 2 mm. Thus according to fig. 4 $\frac{R_0}{R_1}$ becomes 0.025 and γ becomes 1.6. According to (8) φ is 0.95, and as the relative velocity v_{10} is about 230 cm/s,

$$T = \frac{1}{(\pi \times 1.6 \times 0.95)\, 0.2 \times 5^2 \times 10^{-6} \times 230} = 190 \text{ sec}$$

i.e. the time constant is therefore 3 min 10 sec. For crystals (spheres) which exceed 20 μ in radius the product $\gamma\varphi$ is about 1.5 and therefore the time constant is given approximately by

$$T \sim \frac{0.21}{NR_0^2 \times v_{10}} \qquad (11\text{ a})$$

on the assumption that the number N of ice crystals per cm³ is invariant with time. This is to be expected if the rime corn remains in its position, in other words, if its terminal velocity when falling, is equal and opposite to the velocity of the convection current of the air. If this were not the case, a conventional time-constant would not suffice to describe this process.

The relative distance travelled by the ice crystals with respect to the rime-particle in a time equal to that given by time constant T shall be called the specific relative rise H. Let the relative distance travelled in time T by the first of the crystals impinging on the rime be h. Then (eq. 10) gives the specific charge

$$\beta = 2.12 \left(e^{-\frac{h}{H}} - 1\right) \qquad (10\text{ a})$$

where

$$H = \frac{1}{(\pi \cdot \gamma \cdot \varphi) \times NR_0^2} \qquad (13)$$

Consequently the specific relative rise depends primarily on the number of ice crystals per unit volume and the square of their size. Probable or possible values must now be assigned to these as no measurements are known.

The number of ice crystals per cm³ that can form under certain atmospheric conditions depends largely on the temperature, but is in no way fixed by it. At the same temperature there may be a variation of several orders of magnitude depending on the origin of the air. From numerous investigations on the number of ice crystals per unit volume the measurements of AUFM KAMPE and WEICKMANN [2] and V. I. SCHAEFER's researches [18] will be used hereafter. In artificially produced clouds ice crystals form only sporadically (some crystals per m³) at temperatures

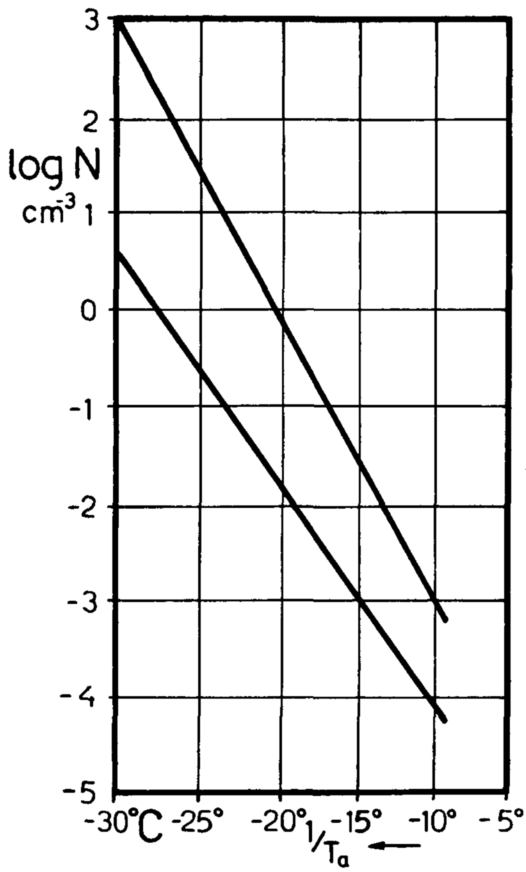


Fig. 5. Relation between number of ice crystals and temperature.

between $-3°$ and $-10°$. Occasionally only supercooled water droplets and an absence of ice crystals may be observed during laboratory tests on cloud air down to $-18°$. In the temperature-region from $-10°$ to $-25°$ the number of crystals increases by two orders of magnitude for every decrease of $-5°$; i.e. if the temperature decreases from $-10°$ to —25° the number of crystals increases by about 5 orders of magnitude.

Fig. 5 shows the data which will be used in the following.

The size of ice crystals will now be investigated in relation to their determined conditions, e.g. suppose an ice crystal originated at $-10°$ or $-15°$ and travelled at a fixed vertical velocity, say 5 m/s, through cold regions of known air-velocity till at a definite time it crosses, for instance, the $-25°$ or $-30°$ isotherm. Further simplifying assumptions must be made. Firstly let the forming ice crystal be spherical growing from zero-radius. On the other hand ice crystals can originate by freezing of supercooled droplets; the spheres of radius R_0 which may therefore be present at the beginning of the process, will be neglected. Further assumptions are as follows:

1) Diffusion will be considered on stationary spheres. If the radius does not exceed 100 μ the calculated diffusion is not influenced by the motion of the sphere.
2) The temperature of crystals is assumed to be equal to the temperature of the surrounding air. The influence of condensation heat on the temperature will be disregarded. The decrease of temperature of an ice crystal traversing a temperature field of constant gradient in a convection current of 5—15 m/s is invariant with time. In the numerical example given below this temperature decrease is one degree for between 10—30 seconds.
3) The vapour-pressure of the supercooled droplets determines the vapour-pressure in the cloud. The known difference between vapour-pressures of water in solid and liquid states is the effective difference of vapour pressure between ice crystal and the surrounding air.

The value of R_0 necessary for eqs (11) and (13) will be too great as a result of this simplification. Because the number N of crystals per unit volume varies at a given temperature between extremely wide limits, as seen above, there will in any case be certain limits for the wanted value NR_0^3. This makes the discussion of the results of the investigation possible. Finally it has to be considered that the radius R_0 of an ice crystal can change discontinuously namely if an ice crystal collides with a supercooled droplet. Droplets with a radius from 60 μ to 100 μ are to be found in cumulonimbus before reaching the 0° isotherm [2, p. 67]. If they are supercooled to a temperature of $-10°$ they solidify within 1 to 2 seconds after the absorption of an ice crystal, which can easily be calculated.

The increase of matter by diffusion on a sphere with radius R_0 is

$$\frac{dm}{dt} = 4\pi D \frac{M \triangle p R_0}{R T_a} \tag{14}$$

where

D = diffusion-coefficient. For water vapour in air at 16° C = 289 K° $D = 0.285$ cm²/s, D being approximately proportional to the absolute temperature

M = molecular weight (18 g/mol)

$\triangle p$ = difference of vapour pressure (kg/cm²)

R = gas constant (84.8 kg cm/mol degree)

T_a = absolute temperature (°K). If the above value of D is used, $T_a = 289°$ K and the expression becomes independent of absolute temperature.

The difference of vapour pressures $\triangle p$ can be calculated exactly. Table II gives values published by M. Robitzsch [19]. Because these values are unwieldy for an integration they will be replaced by an empirical relationship

$$\triangle p = 4.0 \times 10^{-5}\, T_-^{1.28}\, e^{-0.1068\, T_-}\ (\text{kg/cm}^2) \qquad (15)$$

where T_- denotes degrees below zero (− ° C). Table II shows a comparison between exact values and those calculated empirically according to (15), and they show reasonable agreement with each other.

Table II

T_- −°C	difference of vapour pressures $p\triangle$		
	exact	calculated by (15)	
	mm merc.	mm merc.	kg/cm²
3	0.104	0.087	1.19×10^{-4}
6	0.163	0.153	2.08
12	0.198	0.198	2.69
18	0.175	0.176	2.40
24	0.134	0.132	1.80
30	0.092	0.094	1.275
36	0.061	0.062	0.85

In eq. 14 mass (m) is replaced by volume × specific weight s, and the element of time dt by a temperature increment dT_- because the ice crystal is carried through a temperature field of gradient h (deg/cm) with a velocity v. From (15)

$$R_0 dR_0 = A \cdot T_-^{1.28}\, e^{-0.1068\, T_-}\, dT_- \qquad (16)$$

where the constant

$$A = \frac{DM \cdot 4 \times 10^{-5}}{sRT_a hv} \qquad (16\text{a})$$

For $h = 6.67 \times 10^{-5}$ deg/cm and $v = 500$ cm/sec A becomes equal to 0.278×10^{-6}. If the crystal originates at T_{-1} its size at temperature T_{-2} is given by

$$R_0^2 = 2A \int_{T_{-1}}^{T_{-2}} f(T_-)\, dT_- \qquad (17)$$

The integral may be expanded into an infinite series

$$\int f(T_-)dT_- = 0.1068^{-2.28} \cdot z^{2.28} \left[\frac{1}{2.28} - \frac{z}{3.28} + \frac{z^2}{2!4.28} - \frac{z^3}{3!5.28} + \cdots\right] \qquad (18)$$

where $z = 0.1068\ T_-$. Some numerical values of the integral are given in table III.

Table III

T_- °C	5	10	15	20	25	30
$\int_0^{T_-}$	12.0	41.0	74.0	104	128	149

Figure (6) gives the radius R_0 of a spherical ice crystal originating at the temperature indi-

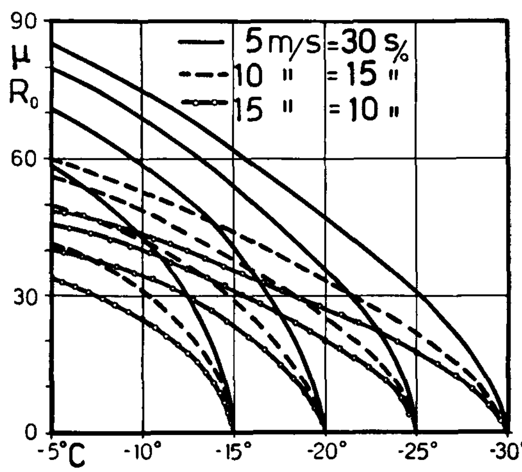


Fig. 6. Relationship between the size of ice crystals reaching the −15, −20, −25, and −30° C isotherms and their temperature of formation.

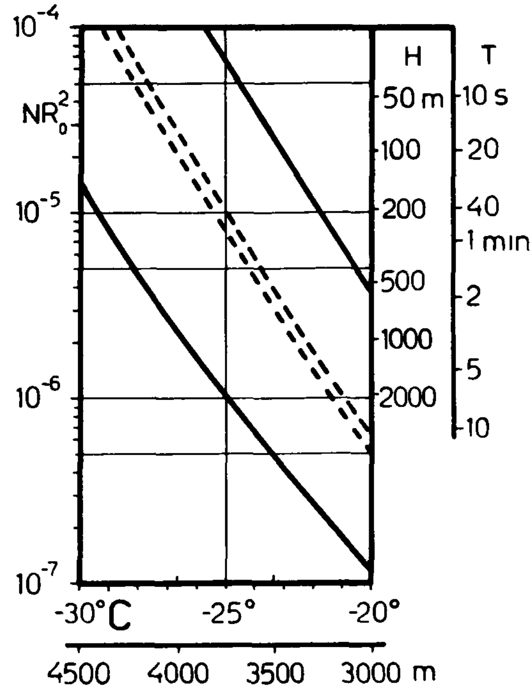


Fig. 7. Relation between temperature and (number of ice crystals) × (radius)².

cated by the abscissa; and which crosses the $-15°$, $-20°$, $-25°$, and $-30°$ isotherms at the three given velocities. Thus an ice crystal originating at $-15°$ has a radius R_0 of 40.9 μ, 28.9 μ, and 23.7 μ at respective convection velocities of 5, 10, and 15 m/sec on reaching the $-20°$ isotherm; in consequence the time interval is 150, 75, and 50 seconds. The temperature gradient $h = 6.67 \times 10^{-5}$ deg/cm corresponds to the decrease of temperature with height in thunderclouds at temperatures below $-10°$. This value considers the entraining of the air surrounding the cloud and is consequently a bit greater than the decrease of temperature corresponding an adiabatic curve of damp air.

From R_0^2 and the number N of ice crystals depending on the temperature (figure 5)

$$\int_0^{T_-} R_0^2 dN$$

is obtained by graphical integration as a function of temperature. These values of NR_0^2 are shown in figure 7. They hold for a vertical velocity of 5 m/s and the greatest as well as the smallest number N of ice crystals. These values are inversely proportional to the velocity of the crystals carried through the temperature field, and halved at 10 m/sec.

The size distribution of the ice crystals according to figure 8 was calculated at $-28°$ C and 16.6 crystals per cm³. The radii are tabulated in steps of 5 μ. The number ΔN and the ice content per cm³ is indicated respectively. The total ice content is 0.686 g/cm³ and this is small compared to the water content of supercooled droplets. The assumption that the vapour pressure of the supercooled droplets determines the rate of growth is reasonably substantiated. Figure 7 can now be interpreted as follows: If a particle of soft hail is floating, i.e. if its terminal velocity when falling is equal and opposite to the vertical velocity of convection, it passes an air current carrying N crystals per cm³ with a surface of $|R_0^2|$. In this case the appropriate values of the specific relative rise H and the time constant T are given at the right hand ordinate. The mean values of NR_0^2 at $-22°$ C give a time constant of 5 minutes. These values give some idea of the speed with which electrification can take place. Moreover the heights above the 0° isotherm are given on the abscissa for which a temperature gradient of 6.67×10^{-5} deg/cm is appropriate.

Discussion

The results so far obtained will now be checked against some measurements compiled by Ross Gunn [20]:

1) The maximum charge in a drop is $\pm(20 \div 30)$ ESU per gram (measurements at ground level)

2) The mean charge of a drop is approximately proportional to $\pm 2.8\ R^2$. The highest charge is about $\pm 10\ R^2$, where R is the radius of the liquid drop (measured at ground level)

3) The charge per drop is about ± 0.27 ESU at 2,300 m altitude ($+10.3°$ C), -0.135 ESU at 3,800 m altitude ($+2.4°$ C), and $+0.052$ or -0.062 ESU at 6,600 m altitude ($-9.9°$ C).

4) Only occasionally does the field gradient exceed 500 $\frac{V}{cm}$ in large cloud regions. The maximum field strength of 9 thunderstorms had a mean of 1,300 $\frac{V}{cm}$.

According to (6 b) the final charge of rime is:

$$e_1 = -2.12\, E R_1^2$$

where R_1 denotes the radius of the rime. If we replace R_1 by the radius R_2 of the liquid drop of equal weight, then $R_1^3 \times s = R_2^3$. Accordingly the charge per gram of water is

$$e_1' = -\frac{3}{4\pi} \times \frac{2.12}{s^{2/3}} \times \frac{E}{R_2} \qquad (19)$$

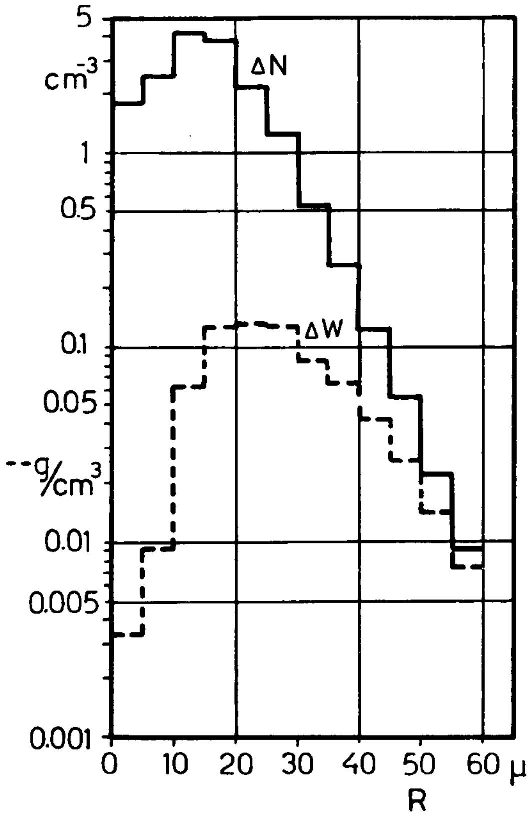


Fig. 8. Size-distribution of ice crystals and their water content on reaching the $-28°$ C isotherm in a convection current. Number of crystals per cm³

$N = 16.6$

$NR_0^2 = 68.9\ 10^{-6}\ cm^{-1}$

water content $w = 0.686$ g/cm³.

In table IV charge per gram as well as the charge in terms of (drop radii)² are compiled for various field gradients. The figures are based on a mean drop radius of 0.8 mm which may be taken as representative for thunderstorm rain near the earth's surface.

Table IV

field gradient $\frac{V}{cm}$	Mean drop radius = 0.8 mm			
	soft hail (s=0.6) -charge in ESU's		soft rime (s=0.125) charge in ESU's	
	per gram	$\times R^2$	per gram	$\times R^2$
350	10.5	3.48	29.5	9.90
500	14.8	4.98	42	14.15
1,000	29.6	9.96	84	28.3

The figures given by Gunn: 30 ESU/gram and 10 R^2 per drop are confirmed for soft hail at 1,000 $\frac{V}{cm}$ and for soft rime at 350 $\frac{V}{cm}$.

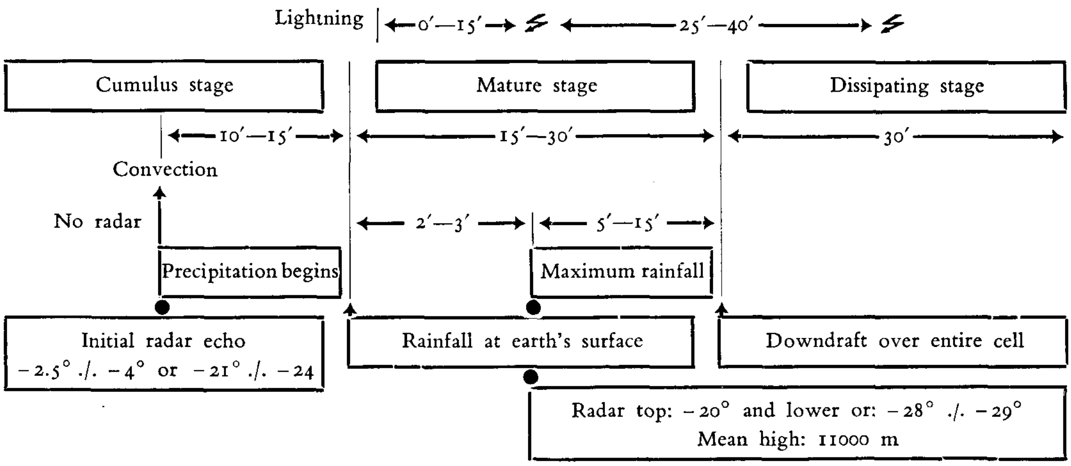
The charge per drop for which Gunn gives a maximum figure of ± 0.27 ESU is used in table V to compute the drop radius. This value refers to very heavy rain.

Table V

field gradient $\frac{V}{cm}$	drop-charge −0.27 ESU Rain drops originating from					
	soft hail (s=0.6)			soft rime (s=0.125)		
	radius (mm)		weight mg	radius (mm)		weight mg
	solid	liquid		solid	liquid	
350	3.31	2.80	92	3.31	1.66	19.1
500	2.77	2.34	53.7	2.77	1.385	11.15
1,000	1.96	1.65	19.0	1.96	0.98	3.96

Droplet weights of 10—20 mg are rare but peculiar to regions of very heavy rain. Thus Gunn's experimental results fit in with the calculations as far as negative charge is concerned. The following process applies to positively charged drops. Small positively charged ice crystals recoiling from rime after contact will finally cling to rime, e.g. to a growing particle of soft rime, thereby conveying their positive charge to the soft rime. The ice crystals are then frozen in by supercooled water droplets. A small percentage of charged ice crystals in the soft rime "skele-

Table VI



ton" is sufficient to give the soft rime the same charge held by soft hail. Thus the charges will finally be distributed in such a manner, that positively charged rime will exist beside negatively charged rime in a layer following on the first negatively charged layer, i.e.:

1. layer: negatively charged soft hail or small smooth hailstones,
2. layer: negatively and positively charged rime with a net charge approximately nil when taken over a wider space, but possibly having different mean rates of fall.
3. layer: positively charged ice crystals.

It will now be attempted to incorporate this process in the chronological development of thunderstorms that can be derived from the results of investigations mentioned above.

Single turret thunderstorm

To the extent table VI is based on the statements of Byers, it is valid for a temperature of 20° C and a pressure of 900 mb at the bottom of the cloud. The 0° isotherm lies within the cloud at 4,900 m. Rain reaches the ground 10—15 minutes after the origination of the radar-cloud. No rain can reach the ground as long as the updraft continues, neither can soft rime and soft hail—which are characteristic of thunderstorms according to J. Küttner [21]—appear at the altitude of the 0° isotherm. A downcurrent is a prerequisite to precipitate reaching the earth, at least in that part of the cloud from which the precipitate originates.

A crystal—whose growth by condensation was shown in figure 6—grows from approximately 60 μ radius taking up supercooled droplets, considerably faster than by condensation. It takes a relatively long time, though, before soft hail will descend to the ground in a convection current of 5 m/s. F. H. Ludlam [22] has investigated the growth of crystals partly by condensation and partly by coagulation. To simplify matters it will be assumed that precipitate of 60 μ radius will grow by coagulation with droplets of at least 15 μ mean radius, and in this connection the process of condensation may be neglected.

Let unit volume contain w grams of water as small droplets whose rate of fall may be neglected compared with that of the growing precipitate. Then

$$4\, s_1 dR_1 = w \cdot \varphi \cdot v_1 dt \tag{20}$$

Under these assumptions the relative interception-area according to (8) is practically 1 for water droplets and less than 1 for hail and soft hail. Thus

$$t_{01} = 4 \int_{R_{10}}^{R_{11}} \frac{s_1}{w \varphi v_1}\, dR_1 \tag{21}$$

This leads to the time interval t_{01} in which the precipitate of spec. weight $= s_1$ grows from radius R_{10} to R_{11}. The rate of fall v_1 of the precipitate can be expressed by Schmidt's formula:

$$v_1 = \frac{10^6}{\frac{A}{R_1^2} + \frac{B}{R_1^{1/2}}} \qquad (22)$$

The constants A and B, valid for an altitude of 6,000 m, are given in table VI.

Table VI

Hail: The values are valid for irregular forms. For regular forms reduce B with a factor 0.7.

	A	B	s_1
drops..........	0.682	370	1.0
rough hail......	0.576	781	0.83
soft hail........	0.317	1,350	0.60

The time t_{01} is given for an interval, over which w, s_1 and φ may be regarded as invariant, by

$$t_{01} = \frac{4 s_1}{w\varphi}\left[-\frac{A}{R_1} + 2\,BR_1^{1/2}\right]_{R_{10}}^{R_{11}} \times 10^{-6} \qquad (23)$$

During the time t_{10} the growing precipitate rises through a height S:

$$S = vt_{01} - \int_0^{t_1} v_1 dt = vt_{01} - \frac{4 s_1}{w\varphi}\,(R_{11} - R_{10}) \qquad (24)$$

where v denotes the velocity of the convection current. As a numerical example, the rise is related in figure 9 to the time for droplets, hail, and soft hail growing by coagulation in an atmosphere containing 5×10^{-6} gram water per cm³ as small droplets. Rise and time are reversely proportional to the water-content. The weights of precipitate in figure 9 are indicated alongside the curves. A water droplet rises about 500 meters from where its size corresponded to 60 μ radius. Then it falls and, on exceeding a weight of 100 mg, it may disintegrate and initiate a new process with a multitude of droplets, dropping only slowly below its origin as long as the convection current persists. Within the first 15 minutes taken from its beginning as 60 μ precipitate neither hail nor soft rime drop below their place of origin, but this is contradicted by experience. Therefore the downdrift of the air must start soon after the first

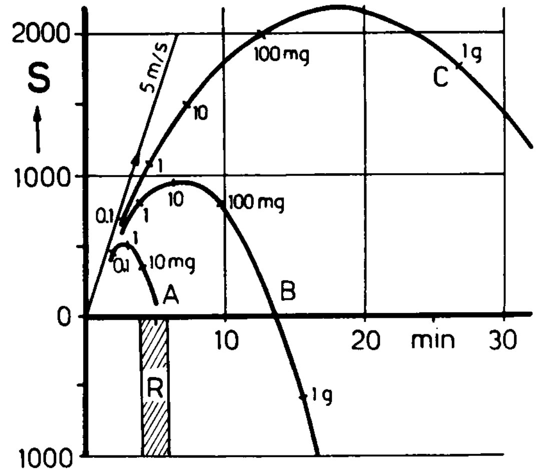


Fig. 9. Height to which precipitate rises while growing by coagulation in a convection-current. With

$S = 0$, $R_1 = 60\,\mu$, $G_1 = 0.82 \cdot 10^{-3}$ mg.

(A droplets, B hail, C soft hail, irregular form).

$w = 5 \times 10^{-6}$ g/cm³

Weights are denoted by numbers, for C very approximate.

R = beginning of radar sight.

strong precipitate is formed. According to scheme p. 12 this might take place after 5 minutes. The downdrift of air is propagated to the upper regions, so that a soft hailstone originating in the convection current will hardly exceed an altitude of 1,500 m above its origin. The timing would then be as follows: A soft hailstone originating at $-5°$ C and growing in the vertical air current by condensation will have a radius of nearly 60 μ when it reaches the $-15°$ C isotherm and its weight has increased to 10,000 times this initial value, when it reaehes the $-25°$ C isotherm, 1,500 m above its "coagulation-origin". The size of the precipitate and number per unit volume determine the possibility of radar soundings. It is assumed that the radar cloud appears 4—6 minutes after the beginning of the 60 μ precipitate. This fixes the rise at 300 to 600 m for droplets, and at about 1,200 m for soft hail. According to the above scheme these would be the limits for the beginning of the "cumulus stage".

During the next 10—15 minutes the downward movement of the air containing supercooled water droplets and ice crystals will increase more and more. Even now contact between ice crystals and soft hail takes place in the same way as described above, but the effective area originally lying between the $-18°$ and $-26°$ isotherms will move down to higher isotherms. The generation of charge will cease when the ice crystals have disappeared. This limit is given by their sublimation when the vapour pressure of the down-drifting air in the dissipating stage sinks below the vapour pressure of the ice by entrainment with the surrounding dry air.

Elsewhere [23] it was shown that in a region where charge is generated the electric field grows logarithmically, i.e. at first slowly and then very rapidly, according to a process intensifying with increasing field strength. If applied to the above case this calculation leads to plausible assumptions for the appearance of a strong field within a short time. Assuming, for instance, that soft hail of 2.3 m/sec velocity drops within about 3.5 minutes through a 500 m thick layer of uniform space-charge, thereby increasing this charge, then the field will be increased to 1,500 times its initial value, if the rate of precipitation is 1.25 mm water per minute and the time constant is 5 minutes. At altitudes in which charge is generated the earth's electric field is very weak: this is intensified immediately on the formation of a first heavy precipitate, because the small cloud-droplets have a slight negative charge leading to the formation of an initial field of about 1 V/cm within approximately 3 minutes. [23] Consequently a strong field could be built up within 6.5 minutes. This is in good agreement with the statement of BYERS [2, p. 62], who had observed no electric field when first flying through a thundercloud, but 8 minutes later he found a strong positive field associated with heavy precipitation.

In this connection a possibility may be pointed out to account for the third and smaller region of positive space charge, mentioned above, which is between the $+10°$ and $0°$ isotherms. This positive space charge does not always appear, and it is confined to a relatively small region. Its effectiveness may be roughly represented by a sphere of 500 m radius [1]. The space charge is relatively dense. According to G. C. Simpson's soundings the mean density is 2.4×10^{-5} ESU/cm^3. According to Ross GUNN [20], it is 15.5×10^{-5} ESU/cm^3. Ross Gunn who obtained this figure by using an aeroplane gives about 0.5 ESU per drop on the $+10.3°$ level. Negatively charged hail having passed a region of negative charge will then enter a region with a negative field gradient. At temperatures above $0°$ hail develops a moist and slippery surface. This makes it possible for small water-droplets—e.g. having a radius of 15 μ—to slip off the corn-shaped bottom of the hailstone thus leaving it again. Assuming that the hailstone does possess a conical shape, these considerations may be transformed for a spherical shape. Water droplets will then leave the sphere at an angle ϑ to the vertical axis, the final charge of the hailstone will be

$$e_2 = 3 \cos \vartheta \, ER^2 \tag{25}$$

i.e. it will equal 0.52 when $\vartheta = 80°$. The time constant of this process is very small. Assuming small water-droplets amounting to some tenths of a gram per m^3 its order of magnitude will be 10 sec which is sufficient to discharge the hailstone and re-charge it with the opposite sign. This would increase the existing negative field. These considerations can, however, not be extended by calculations without further experimental investigations.

REFERENCES

1. G. C. SIMPSON and F. I. SCRASE. Distribution of electricity in thunderclouds. *Proc. Roy. Soc.* London A **161,** (1937) p. 309.
   G. C. SIMPSON and G. D. ROBINSON. A **177** (1941) p. 281.
   G. D. ROBINSON. *Quart. J. R. Met. Soc.* **67** (1941) p. 332.
2. H. R. BYERS. *Thunderstorm Electricity,* Chicago Press, 1953.
3. H. R. BYERS, R. R. BRAHAM. *The Thunderstorm.* Washington 1949.
4. H. ISRAËL. *Das Gewitter.* Akad. Verl. Ges. Leipzig. 1950.
5. H. LUEDER. Vergraupelungs-Elektrisierung als eine Ursache der Gewitter-Elektrizität. *Z. ang. Phys.* **3** (1951) 288.
6 a. B. J. MASON. A critical Examination of Theories of charge generation in thunderstorms. *Tellus* **5** (1953) 446.
   b. On the generation of charge associated with graupel formation in thunderstorms. *Quart. J. R. Met. Soc.* **79** (1953) 501.
7. I. ELSTER and H. GEITEL. Zur Influenztheorie der Niederschlagselektrizität. *Phys. Z.* **14** (1913) 1287.
8. T. E. W. SCHUMANN. On Elster and Geitels theory of the electrification of rain drops. *Phys. Rev.* II **26** (1925) 105.
9. C. P. SMITH, C. S. HITCHCOCK. Dipole rotation in crystalline solids. *Journ. Am. Chem. Soc.* **54** (1932) p. 4631.
10. H. BROMMELS. Bestimmung der Dielektrizitätskonstanten und des elektr. Leitvermögens von Eis. *Soc. Scient. Fenn. Comm. Phys. Math.* I **19,** 1922.
11. I. H. L. JOHNSTONE. Internal. Crit. tables Vol. VI p. 152. *Proc. Trans. Nova Scot. Inst. of Science* **13** (1912) 126.
12. H. W. BANDEL. Corona from ice points. *Jour Appl. Phys.* **22** (1951) 984.
13 a. H. KÖHLER. Untersuchungen über die Elemente des Nebels und der Wolken. *Medd. Stat. Met.-Hydr. Anst.* **2** (1925) 1. (Stockholm).
   b. An experimental investigation on sea water nuclei. *Nova acta reg. soc. scient. Upsaliensis* IV. **12** No. 6 (1941) Uppsala.
14. H. NORINDER and R. SIKSNA. On the electricity of snow. *Tellus* **5** (1953) 260.
15. V. I. SCHAEFER. Experimental Meteorology. *Zeitschr. f. angew. Math. u. Physik ZAMP* **1** (1950) 1953.
16. F. ALBRECHT. Theoretische Untersuchungen über die Ablagerung von Staub. . . *Phys. Z.* **32** (1931) 48.
17. W. SELL. Staubausscheidung an einfachen Körpern und in Luftfiltern. *V. D. I. Forschungsheft* 347 (1931).
18. V. SCHAEFER. The formation of ice crystals in the laboratory and the atmosphere. *Chem. Rev.* 44 (1949) 291.
19. HANN-SÜHRING. *Lehrbuch der Meteorologie.* 5. Aufl. (1943) Leipzig. Verlag W. Keller.
20. ROSS GUNN. The free electrical charge of thunderstorm rain and its relation to droplet size. *J. geophys. Res.* **54** (1949) 57.
   The free electrical charge on precipitation inside on active thunderstorm. *J. geophys. Res.* **55** (1950) 171.
21. J. KÜTTNER. The electrical and meteorological conditions inside thunderclouds. *J. Met.* **7** (1950) 322. See also (4).
22. F. H. LUDLAM. The production of showers by the growth of ice particles. *Quart. J. R. Met. Soc.* **78** (1952) 543.
23. D. MÜLLER-HILLEBRAND. Limitations of the Wilson-Thunderstorm Theory. Ark. f. Geofysik *2* (Nr 11, 1954) 227.