On the Origin of Cosmic Radiation

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Abstract

The "local" theory of the origin of Cosmic Radiation is developed. According to the theory the radiation (possibly with exception of the highest energies) is generated in interplanetary space around the sun and trapped in an interstellar magnetic field. The acceleration takes place in electromagnetic fields in the sun's environment which are produced by the solar activity and especially associated with magnetic storms.

A theory of the origin of Cosmic Radiation should at the same time be a theory of the extra-terrestrial variations in the radiation. The present theory incorporates the magnetic storm variation, the diurnal variation and the long time (solar cycle) variation.

The acceleration process of the theory is efficient for protons and especially for for heavy nuclei but is inefficient for electrons. This explains the absence of electrons in the primary Cosmic Radiation. The latitude "knee" is accounted for as a direct effect of the acceleration process.

1. Introduction

Since the discovery of the Cosmic Radiation the general picture has changed drastically several times. The primary radiation was at first believed to be a γ radiation, later a β radiation, until it was proved to consist merely of protons. Recently even heavy nuclei have been found in the primary radiation.

Even the views concerning the isotropy and constancy of cosmic radiation have been revised from time to time. The first measurements indicated strong fluctuations but these were soon found to be due to experimental errors. During a long time cosmic radiation was regarded to be almost completely isotropic and constant in time. Recently this picture has been changed again and the study of the time variations of cosmic radiation is attracting more interest. FORBUSH's ionization chamber measurements, which have continued during more than 15 years, show large variations in the yearly means of the total intensity[1]. THAMBYAHPILLAI and ELLIOT, and SARABHAI and KANE[2] have demonstrated that the phase and the amplitude of the diurnal variation changes considerably with the solar activity. Further, during magnetic storms large fluctuations are observed which cannot be due to terrestrial causes[3]. The neutron counter technique developed by SIMPSON allows a study of the low energy part of the spectrum, which varies very much with time[4]. High altitude measurements by LORD and SCHEIN, by SWETNICK, NEUBURG, and KORFF, and by YNGVE and especially by NEHER indicate variations in the primary radiation[5].

The changing views concerning the properties of the primary radiation have been

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reflected in the theories of the origin of cosmic radiation. As long as it was generally agreed that the time constancy was the most remarkable feature, it was natural to consider the radiation as a universal, or at least a galactic, phenomenon. As we now realize that the radiation is variable and that the variations are closely connected with the sun, it becomes natural to look for the origin of cosmic radiation in our close environment.

The measurements by SIMPSON and his collaborators[4] demonstrate beyond a doubt that at least a considerable part of the low energy radiation must be of local origin. On the other hand, TELLER-RICHTMYER's suggestion[6] that most of the cosmic radiation, perhaps all, is of local origin, seems not to have been generally accepted, although no decisive arguments against this view have been brought forward. The aim of the present paper is to show that the local origin theory deserves to be taken seriously.

The discussion of the universal and galactic theories have demonstrated that these theories encounter several difficulties. During the last few years FERMI's theory[7] has been in the centre of the discussion. Objections against this theory have been forwarded by UNSÖLD[8]. Introducing values into Fermi's formula for the energy spectrum which from astrophysical point of view are most probable, Unsöld finds that the theoretical value of the exponent should be about 200 instead of the empirical value 1.5. Further, the theory does not explain the abundance of heavy nuclei and cannot account for the low-energy cut-off. These objections seem not to have been overcome altogether by the recent development of the theory by FAN[9], MORRISON, OLBERT and ROSSI[10], and by FERMI[11].

It seems not to be generally observed that even if these difficulties could be successfully overcome – which does not seem likely at present – only half of the problem of the primary radiation would have been solved, because we must also have a theory of the variations. Such a theory will probably be as difficult as a theory of the origin.

The local theory has a different position in this respect, because it attempts to account both for the variations and the origin. Hence, an acceptable local theory would solve the two main problems at the same time. The study of the variations gives us much experimental material by which a local theory can be checked.

2. Acceleration mechanisms

It is most likely that cosmic radiation is generated by acceleration of charged particles in electromagnetic fields in space or in stellar atmospheres. The first mechanism of this kind was proposed by SWANN[12] who pointed out that a betatron acceleration may take place in the growing magnetic field of a sunspot (the "cygnotron"). Other electromagnetic accelerating mechanisms have later been proposed by EHMERT[13], by MENZEL and SALISBURY[14] and by BAGGE, BIERMANN and SCHLÜTER[15].

In this paper we shall study a mechanism of a somewhat similar kind by which charged particles are accelerated by electromagnetic fields in the neighborhood of stars. For the production of fields of the required type it is essential that we have at least two magnetized bodies, moving in relation to each other. In the first attempt to develop a theory along these lines the two bodies were represented by the two components of a double star, which were assumed to possess magnetic fields[16]. It was shown that by their relative motion electric fields could be produced which are powerful enough to accelerate particles up to the energies found in cosmic radiation. Two alternative processes were discussed. In the first one charged particles moving in trochoidal orbits are accelerated by an induced electric field, a process which is equivalent to the betatron effect. In the second alternative the motion of charged particles parallel to the magnetic field was considered and it was shown that under certain conditions accelerating electric fields could be produced.

Mechanisms of these kinds can explain the high energies in cosmic radiation, but it was found more difficult to account for the intensity.

Electromagnetic generators could obviously not generate sufficient cosmic radiation to fill up the whole space, so cosmic radiation could not be a "universal" phenomenon. The assumption of a galactic magnetic field had to be introduced, which should be strong enough to trap all the radiation generated near the stars in our galaxy, but even so, it was difficult to account for the observed intensity without very far-stretched assumption. In fact, according to any "galactic theory", the energy of cosmic radiation within our galaxy must be of the same order as the kinetic energy of the galaxy[6], and it is difficult to believe that generators near the stars could give as much power as that. On the other hand, one of the successful results of the theory was a prediction of the existence of heavy nuclei in cosmic radiation[17]. This result was confirmed by observation about a decade later.

Thus the theory explained some of the properties of cosmic radiation, but the main difficulty was the intensity. This problem came in different light when TELLER and RICHTMYER[6] pointed out that cosmic radiation may be a "local" phenomenon, confined by trapping magnetic fields to the neighbourhood of the stars. If in this way cosmic radiation fills up only small volumes (of the order of 10^{17} cm) around the stars, the total energy goes down by a factor 10^6 or more and the intensity problem is no longer serious. However, as the radiation we observe must be generated in the neighbourhood of our sun, the initial theory must be modified because the sun is no double star. If we still accept the basic idea, i.e. that cosmic radiation is generated by electromagnetic processes due to the relative motion of magnetized bodies, we must consider the sun and a gas cloud moving in relation to it, instead of the two components of a double star. With this as a background a theory of the generation of cosmic radiation was developed in which the energy source is located in the storm-producing beams emitted from the sun[18]. Of the two alternative processes mentioned in the summary of the double star theory only the first one was considered. As recent experimental and theoretical results support the local theory, it seems worth while to discuss the whole problem again.

3. Magnetic storms

As stated in § 1 a theory of the origin of cosmic radiation should at the same time be a theory of the variations in the primary radiation. Of the different types of variations, the storm variation is of special importance because it gives us information in a rather direct way about the electromagnetic fields in our surroundings.

Occurrences on the sun, such as solar flares, are frequently followed one day later by magnetic storms and aurorae on the earth. Also when a sunspot crosses the solar meridian, a similar disturbance sometimes occurs on the earth after about one day. This has led to the assumption that a beam of ionized gas is ejected from the sun with such a velocity

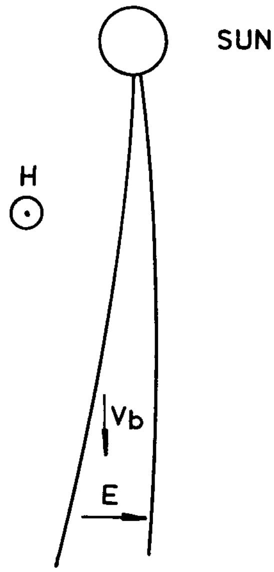

Fig. 1. Storm-producing beam, emitted from the sun (Equatorial plane).

($v_b = 2 \cdot 10^8$ cm sec.$^{-1}$) that the time of travel from the sun to the earth is approximately one day (Fig. 1). This storm-producing beam has never been observed directly, probably because its density is very low. Further, in order to produce a magnetic storm on the earth, the beam must be directed towards the earth and a beam sent out from the sun towards us should be difficult to observe. Gas ejected from the solar limb perpendicular to the sun—earth line has frequently been observed. Usually, the ejection velocity is smaller than the velocity which a storm-producing beam should have, but in many cases velocities of the same order of magnitude have been measured.

The concept of a storm-producing beam consisting of ionized rarified gas was introduced by Schuster. CHAPMAN and FERRARO[19] have studied the properties of such a beam under the assumption that the magnetic field within the beam is zero. When the beam reaches the neighbourhood of the earth the terrestrial magnetic field is disturbed by the beam, and a magnetic storm should occur. This theory, which has been further developed by MARTYN[20], has not been able to account for the phenomena observed during a magnetic storm, and it seems legitimate to conclude that a magnetic storm cannot be produced by a beam of this type. Nor does it seem possible to account for the magnetic storm variation of cosmic rays in this way.

As it is very difficult to find another way by which a disturbance in the sun could be propagated to the earth in about one day, it seems necessary to preserve the concept of a storm-producing beam consisting of ionized rarified gas. The beam originates at the solar surface, where, no doubt, considerable magnetic fields exist. Hence, it is natural to assume that the ionized gas of the beam initially is permeated by a magnetic field. If this is the case, the conductivity of the beam is no doubt so large that the magnetic field is "frozen" into the beam, so that the beam carries a magnetic field with it as far out as to the earth. A beam with a magnetic field H in it has properties which are fundamentally different from Chapman-Ferraro's beam. Due to the motion with the velocity v_b the beam becomes electrically polarized, the electric field being

$$\vec{E} = -\frac{1}{c}\vec{v}_b \times \vec{H} \qquad (3.1)$$

This electric field seems to be the most important property of the storm-producing beam, and there is good hope that the essential features of magnetic storms and aurorae can be explained as effects of this field[21]. The difficulties of Chapman-Ferraro's theory seem to be due to the fact that they have neglected the electric field of the beam. The cosmic ray decrease, associated with magnetic storms, seems also to require the existence of such an electric field.

4. Storm variations of cosmic radiation

In connection with magnetic storms the cosmic radiation frequently decreases by a few per cent as shown by FORBUSH and others[22]. A study of the storm variations of cosmic rays at different latitudes shows that the variations cannot be caused by a change in the earth's magnetic field. This is evident from the fact that storm variations are observed at all latitudes, even above the latitude knee (FORBUSH)[22]. Recently, SINGER[23] has observed storm decreases even near the magnetic pole. Simpson has shown that the variations of the low energy radiation observed with neutron counters are of extra-terrestrial origin[24].

As the storm variations cannot be produced by any terrestrial phenomenon, it is necessary to assume that cosmic rays are affected in interplanetary space already before they reach the earth. Only an electric field (or a time variation in a magnetic field, which of course is equivalent to an electric field) could produce a change in the cosmic ray intensity. As the cause of magnetic storms seems to be the electric field of the beam, it is reasonable to assume that the storm effect in cosmic rays is a direct effect of this field. A theory along these lines[25] has recently been developed by BRUNBERG and DATTNER[26] and seems to explain the main features of the cosmic ray storm effect.

From the change in cosmic radiation, it is possible to estimate the field and the voltage difference of a storm-producing beam. If the average electric fields is E_b, and the breadth of the beam, in the direction of E_b, is b, the voltage difference is

$$V_b = E_b b \qquad (4.1)$$

Consider cosmic ray particles with energy V_0. When they pass the beam, which we depict as an electric double layer, their energies change by V_b. Further, the space density changes proportional to V_b/V_0. The result is that the measured intensity I changes by

$$\Delta I = kIV_b/V_0 \qquad (4.2)$$

where k is a constant, which depends on the measuring device, and probably has a value of about 3.*

If the average energy of cosmic radiation is $3 \cdot 10^{10}$ ev a change of 1 % means $V_b = 10^8$

* A recent calculation by S. LUNDQUIST[27] indicates a somewhat higher value.

volt and a change of 10 % means $V=10^9$ volt. The breadth b of the beam may be estimated from the duration of a magnetic storm. If a storm lasts about two days, as it often does, this is an indication that the beam is so broad that the earth is situated within it during two days. As the synodic period of the sun is 27 days and the earth's orbital radius is $1.5 \cdot 10^{13}$ cm this gives

$$b = \frac{2}{27} \cdot 2\pi\, 1.5 \cdot 10^{13} = 0.7 \cdot 10^{13} \text{ cm}$$

This means that we have $E = 10^{-5}$ to 10^{-4} volt/cm, values which are acceptable in the theory of magnetic storms. With $v_b = 2 \cdot 10^8$ cm sec^{-1} we find $H = 10^{-5}$ to 10^{-4} gauss. It is not necessary that the radius of curvature is larger than the breadth of the beam, because the particle may cross the beam moving in a trochoid.*

In the absence of electric currents outside the sun, the magnetic field in interplanetary space outside the sun derives from the sun. At some distance from the sun this field will be approximately a dipole field. The beam disturbs this field very much. It carries "frozen-in" field from the neighbourhood of the sun outwards. This means that currents flow at the borders of the beam. A possible structure of the field in interplanetary space when disturbed by the beam is seen in Fig. 2. If the beam is switched off the field will after some time resume a dipole character.

5. The interplanetary magnetic field

Magnetic fields in interplanetary space may derive both from the magnetic fields of the celestial bodies and from currents in interplanetary space. It is usually assumed that the field in interplanetary space derives from the solar dipole field (except in small regions near the planets), which tacitly implies that the currents in interplanetary space are negligible.

This assumption seems not to be justified. Instead it is likely that the magnetic field deriving from the sun is violently disturbed by currents in interplanetary space. These currents produce large local irregariulties and make the field vary with time. They also seem to produce a large-scale effect so that outwards from the sun the average field decreases much slower than a dipole field. It is therefore misleading to refer to the magnetic field in interplanetary space as "the solar dipole field" and in the following we shall call it "the interplanetary field". The reasons for assuming that the field deviates very much from a dipole field are the following, of which 1) and 2) refer to the general shape of the field and 3) and 4) to the local irregularities.

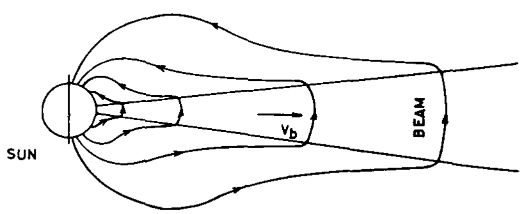

Fig. 2. Simplified model of the interplanetary magnetic field, consisting of the solar field which is very much disturbed by the storm-producing beams. (Meridional plane.)

1. Eclipse photographs of the corona show in the polar regions a structure which may be reconcilable with a magnetic dipole field, in which matter streams along the lines of force. However, in the equatorial region long "streamers" are observed which go out from the sun. Together with the fact that matter often is seen to be ejected from the equatorial regions this shows that there often are outward streams of ionized gas near the equatorial plane. As the magnetic field is "frozen" into the gas, which is a good conductor, an original dipole field will be disturbed very much. The outward motion will in general tend to make the average field decrease slower than a dipole field.
2. Cosmic ray evidence speaks for a much higher field near the earth's orbit than is reconcilable with a dipole field. In § 4 we have estimated the field in the beam to 10^{-5}—10^{-4} gauss. A dipole field with this strength at the earth's distance would be

* Compare. Swann, *Phys. Rev.* **93**, 905, 1954, and H. Alfvén, *Phys. Rev.* **94**, 1,082, 1954. See also Fig. 7.

100—1000 gauss at the sun's surface, which is excluded. Even outside the beam the field probably exceeds 10^{-5} gauss according to BRUNBERG and DATTNER[26].

3. The solar corona seems to be highly turbulent. This must also hold for the magnetic fields, because of magneto-hydrodynamic effects. It is reasonable that the turbulence is not confined to the neighborhood of the sun but spreads out into interplanetary space. The fine-structure of the corona found by radio-astronomy observations by RYLE and collaborators[28], and the "dark prominences" studied by ÖHMAN[29], give further indication of strong local disturbances, which probably are connected with electromagnetic effects.

4. Magnetic storms and aurorae are caused by disturbances originating from the sun and transmitted through the interplanetary space. The irregular and rapidly varying character of these phenomena may be a reflection of the electromagnetic state in interplanetary space.

In the following we shall at first discuss the large-scale variations of the interplanetary field which to a considerable part is produced by the storm-producing beams. The local disturbances of the field which are due to the turbulence will later be found to produce a diffusion of Cosmic Rays which is of essential importance for the generating mechanism we are going to study.

6. Changes in the interplanetary magnetic field

A storm-producing beam carries magnetic flux with it. The flux $\frac{d\varphi}{dt}$ transported per unit time across the earth's orbit by a beam is

$$\left(\frac{d\varphi}{dt}\right)_b = -H_b b v_b = -cV_b \qquad (6.1)$$

where the negative sign means that flux is transported outwards. With $V_b = 10^8$ volt $= \frac{1}{3} \cdot 10^6$ esu we obtain $d\varphi/dt = 10^{16}$ gauss cm^2 sec^{-1} and $V_b = 10^9$ volt gives $d\varphi/dt = 10^{17}$ gauss cm^2 sec^{-1}.

Table 1. Properties of storm-producing beam near the Earth

$\frac{\Delta I}{I}$	V_b (volt)	E volt/cm	H gauss	$\frac{d\varphi}{dt}$ $\frac{\text{gauss cm}^2}{\text{sec}}$
1 %	10^8	$1.4 \cdot 10^{-5}$	$0.7 \cdot 10^{-5}$	10^{16}
10 %	10^9	$1.4 \cdot 10^{-4}$	$0.7 \cdot 10^{-4}$	10^{17}

$b = 0.7 \cdot 10^{13}$ cm $\qquad v_b = 0.0067$ c

From the intensity drop ΔI of cosmic radiation the voltage V_b over the beam can be calculated. This gives the field E and H in the beam and the flux transport $d\varphi/dt$

If we consider the general magnetic field of the sun as produced by a homogeneously magnetized body with a polar field of say $H_p = 10$ gauss*, its total flux is $\varphi = \pi R_\odot^2 H_p = 1.5 \cdot 10^{23}$ gauss cm^2. Hence, a beam with $V = 10^8$ volt would be able to empty the whole flux in $1.5 \cdot 10^{23}/10^{16} = 1.5 \cdot 10^7$ sec, which is only half a year, and a beam with $V = 10^9$ volt would produce the same effect in less than one month. The latter case is certainly exceptional, but beams with $V = 10^8$ volt (corresponding to changes in cosmic ray intensity of about 1 %) are quite normal. Several beams may be sent out in different directions at the same time.

From this follows:

1. The storm-producing beams cause a drastic redistribution of the magnetic field in the sun's environment.
2. Magnetic flux must be transported inwards, back towards the sun, in some way.

This conclusion seems inevitable because the storm decrease of cosmic rays can only be accounted for by the existence of an electric field in space and such an electric field means necessarily a change in the magnetic flux.

If the beam is ejected during a very long time so that a stationary state is reached, the back flow of flux will take place outside the beam. A simple model of this is the following (Fig. 3). There is only one beam and this is supposed to be radial; the effect of the solar rotation is neglected. Outside the beam there is a radial flow with velocity v_a in towards the sun; as earlier we consider the conditions

* This is not in conflict with the Zeeman-effect measurements. Compare H. ALFVÉN, Arkiv f. Fysik 4, Nr 24, 1952.

near the equatorial plane. The magnetic field at the distance R is $H_a(R)$. The electric field is

$$E_a = \frac{v_a}{c} H_a \tag{6.2}$$

The voltage over the backflow is

$$V_a = (2\pi R - b) E_a \tag{6.3}$$

and the flux transport

$$\left(\frac{d\varphi}{dt}\right)_a = H_a (2\pi R - b) v_a = cV_a \tag{6.4}$$

If the same flux is transported inwards as outwards we have

$$\left(\frac{d\varphi}{dt}\right)_a + \left(\frac{d\varphi}{dt}\right)_b = 0 \tag{6.5}$$

and

$$V_a = V_b \tag{6.6}$$

The magnetic field derives in part from the sun, in part from currents at the borders of the beam, and – in case the fields H_a and H_b do not decrease as R^{-3} – in part from currents perpendicular to R.

If at a certain time the interplanetary field derives only from the sun, so that it is a dipole field, and the beam is switched on, it takes some time before this stationary state is reached. During this time we have

$$V_a < V_b \tag{6.7}$$

As a measure of the time which is needed to reach a stationary state we could take the transit time of magneto-hydrodynamic waves. This should be of the order $\tau = R/V = RH^{-1}(4\pi\varrho)^{1/2}$ where R is the distance, V the magneto-hydrodynamic velocity, and ϱ the mass density. As we are especially interested to study the flux transport across the earth's orbit, we put $R \sim 10^{13}$ cm and $H \sim 10^{-5}$ gauss which is a reasonable value for the field near the earth's orbit (BRUNBERG and DATTNER)[23]. The density in the outer corona is 10^{-19} g cm^{-3} and in interstellar space 10^{-24} g cm^{-3}. Hence, it is reasonable to use an intermediate value for ϱ, say $\varrho = 10^{-21}$ g cm^{-3}. This gives $\tau = 10^8$sec $=$ $= 3$ years, a value which obviously could be in error by a factor of 10. Near the sun the stationary state is reached much quicker.

When the beam is switched off it takes a time of the same order before the magnetic field has reached equilibrium, which with no beam on, would be a dipole field. (It is doubtful whether this state is ever reached.) During this resetting time we have

$$V_a > V_b$$

It should be observed that the model of stationary flow can be valid only within certain limits. Inwards it can extend only to a radius $R_{\min} > R_\odot$, because near the sun we

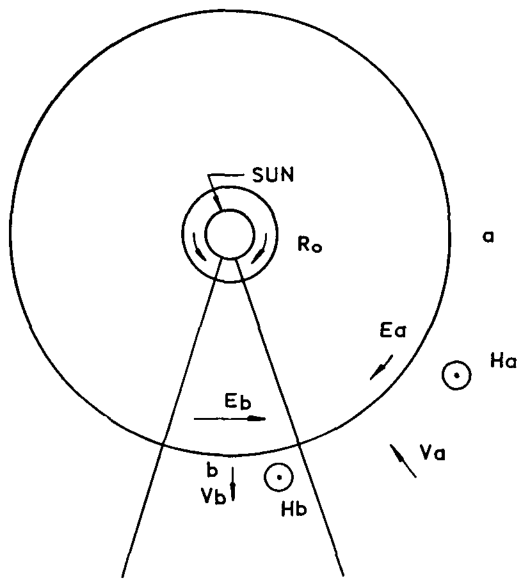

Fig. 3. Simplified model of storm-producing beam, (Equatorial plane). Within the beam with breadth b there is a rapid outflow with velocity v_b which produces an electric field E_b. Outside the beam there is a slow inflow with the velocity v_a, producing the field E_a.

must have a tangential flow. There must also be an outer limit $R_{\max}$ outside which stationary conditions have not been reached. Further, the model is applicable only at a certain distance from the equatorial plane. We assume that it holds approximately up to a latitude angle φ. This means that our model is limited by a double cone with the angle $\frac{\pi}{2} - \varphi$ around the solar axis.

The stationary model is not exactly the same as used by Brunberg and Dattner for computing the storm effect. Their model represents the non-stationary conditions immediately after switching on the beam, because they assume $E_a = 0$. As the influence of E_a on most of the particle orbits they consider is small, their results remain qualitatively the same.

7. Acceleration

In order to explain the generation of cosmic radiation many different acceleration mechanisms, have been proposed. It is reasonable that cosmic radiation is a result of some very general magneto-hydrodynamic phenomena in space which transfers energy from kinetic or magnetic energy into cosmic ray energy. We shall here study a general type of acceleration of particles in a changing magnetic field.

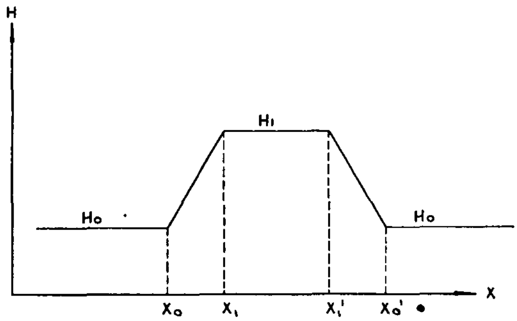

Fig. 4. Accelerator model. A compressible gas with high conductivity flows in the x-direction. Due to change in the flow velocity the magnetic field changes from H_0 to H_1 and back again to H_0.

Besides this effect which may be called a *betatron effect*, it is also necessary to consider the *diffusion* of particles between regions of different magnetic field strength. These two processes, both of a very general character, are the only ones we need to discuss. We shall study them with a model still simpler than that of § 6, and later apply the results to the conditions in interplanetary space.

Let us assume that we have a flow of ionized gas in the x-direction with a variable velocity v. The conductivity of the gas is infinite and there is a frozen-in magnetic field H parallel to the z-axis. The gas has an infinite extension in the y-direction. In the z-direction it is confined between two perfectly reflecting plane surfaces. The velocity v of the gas decreases slowly from an initial high value v_0, to a low value v_1 and increases again to the first value v_0. As the density δ of the gas varies as

$$\delta = \delta_0 v_0 / v \tag{7.1}$$

and as the magnetic field is proportional to δ we have

$$H = H_0 v_0 / v \tag{7.2}$$

The variation in H is shown in fig. 4. The subscript $_0$ denotes the values to the left of x_0.

We assume that the gas contains high energy charged particles which do not interact with the molecules of the gas. Due to the magnetic field the particles move in trochoidal orbits. Suppose that their momentum p has the component $p_\perp$ in the x—y plane.

From the equations of motion we obtain[31]

$$2\frac{dp_\perp}{p_\perp} = \frac{dH}{H} \tag{7.3}$$

which gives

$$\frac{p_\perp^2}{H} = \text{const.} \tag{7.4}$$

Suppose that the initial velocity v_0 of the particle is small compared to c. If the velocity vector has a random direction we have in average

$$\left.\begin{aligned} v_0^\perp &= \sqrt{\frac{2}{3}}\, v_0 \\ v_0^{\parallel} &= \sqrt{\frac{1}{3}}\, v_0 \end{aligned}\right\} \tag{7.5}$$

When the field increases from H_0 to H_1 we have

$$\left.\begin{aligned} p_1^\perp &= \sqrt{\frac{2}{3}}\, m_0 v_0 \left(\frac{H_1}{H_0}\right)^{1/2} \\ p_1^{\parallel} &= \sqrt{\frac{1}{3}}\, m_0 v_0 \end{aligned}\right\} \tag{7.6}$$

If we consider large values of H_1/H_0 we have $p_1^{\parallel} \ll p_1^\perp$, and $p_1 \approx p_1^\perp$. As the kinetic energy W is $W = m_0 c^2 \left\{ \sqrt{1 + \frac{p^2}{m_0^2 c^4}} - 1 \right\}$ we have

$$W = m_0 c^2 \left\{ \sqrt{1 + \frac{2}{3}\left(\frac{v_0}{c}\right)^2 \frac{H_1}{H_0}} - 1 \right\} \tag{7.7}$$

Seen from a coordinate system which takes part in the motion the increase in energy could be regarded as produced by the betatron action of the increasing magnetic field. Seen in a fixed coordinate system it can be considered as an effect of the electric field

$$\vec{E} = -\frac{1}{c}\vec{v} \times \vec{H} \tag{7.8}$$

which because of (7.2) is a constant field in the y-direction. As long as the magnetic field H is constant, the particles drift parallel to the x-axis, in a trochoidal orbit due to the combined action of the electric and magnetic field (Fig. 5). The drift velocity $v = cE/H$ equals the flow velocity. As soon as $\frac{dH}{dx} \neq 0$ a drift component parallel to the y-axis is produced. This drift is parallel to the electric field, which means that the energy of the particle changes. It is easily shown that this change in momentum is the same as obtained by considering the betatron effect.

In the double star theory[16] and in the first papers on the present theory[18] the acceleration was presented in the latter way.

The result of this process is that when the particles move from H_0 to H_1, their momentum increases, but when they continue from H_1 to H_0', their momentum goes down again to the initial value. No net momentum gain is obtained.

8. Disturbed magnetic field

So far we have considered the ideal case when the magnetic field is mathematically perfect. In order to approach the real case this assumption is the first one we must drop, because due to the long time the particles spend in the field, even small inhomogeneities are of importance. In order to account for the effect of inhomogeneities of the magnetic field in a simple way, let us assume that a charged particle at intervals τ_i passes a region of disturbed field. The passage does not change the absolute value of the momentum vector, but it changes its direction. The disturbance of the momentum vector could be described as

(1) a change in the angle with the z-axis, and

(2) a change of the angle between the x-axis and the projection of the vector on the x—y plane. The former effect means a *redistribution* of the momentum between $p^{\perp}$ and $p^{\parallel}$. We account for this in a simplified way by assuming that after the disturbance we have $p_{\perp}^2 = \frac{2}{3}p^2$; $p_{\parallel}^2 = \frac{1}{3}p^2$.

The second effect means that in the x—y-plane the centre of curvature is displaced. We account for this by assuming that each disturbance displaces the centre of curvature a distance ϱ in a random direction. The result is a *diffusion*.

In order to discuss the effect of the redistribution let τ_0 be the time it takes for the gas to move from x_0 to x_1, τ_1 the time from x_1 to x_1', and τ_0' the time from x_1' to x_0'. Let us first suppose that

(a) $\tau_1 \gg \tau_i \gg \tau_0$ and τ_0'. The particle will gain momentum according to (7.4) between

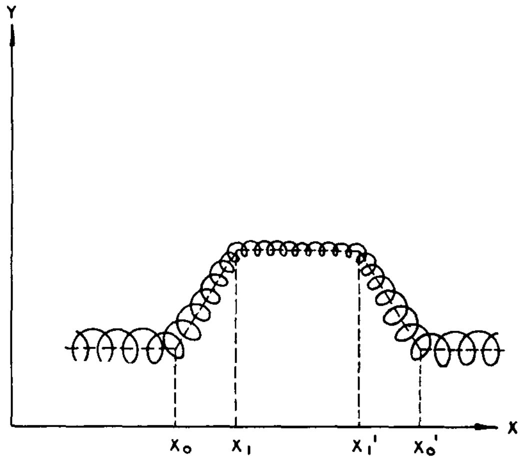

Fig. 5. Path of a high energy charged particle in a flow according to Fig. 5.

x_0 and x_1. Between x_1 and x_1' the gain in $p_{\perp}$ will be equally distributed over the three degrees of freedom, so that part of it is transferred to $p_{\parallel}$. When the particle moves from x_1' it losses momentum again, but this refers only to $p_{\perp}$. Hence the momentum stored as $p_{\parallel}$ is not lost and the result of the whole process is a net gain. – If instead

(b) τ_0 and $\tau_0' \gg \tau_i$, the gain of momentum will be "adiabatic". Of the gain in momentum a part will constantly be transferred to $p_{\parallel}$, so that in average we have

$$\left.\begin{aligned} p_{\perp}^2 &= \frac{2}{3}p^2 \\ p_{\parallel}^2 &= \frac{1}{3}p^2 \end{aligned}\right\} \qquad (8.1)$$

This gives from (7.3)

$$3\frac{dp}{p} = \frac{dH}{H} \qquad (8.2)$$

or

$$\frac{p^3}{H} = \text{const} \tag{8.3}$$

Instead of (7.6) we have

$$p_1 = m_0 v_0 \left(\frac{H_1}{H_0}\right)^3 \tag{8.4}$$

and

$$W = m_0 c^2 \left\{ \sqrt{1 + \left(\frac{v_0}{c}\right)^2 \left(\frac{H_1}{H_0}\right)^{2/3}} - 1 \right\} \tag{8.5}$$

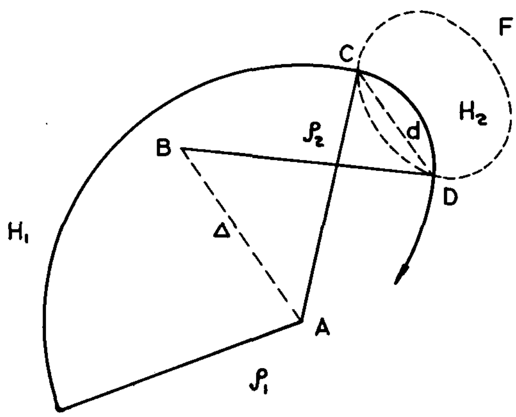

Fig. 6. Diffusion caused by field disturbance.

(c) If either τ_0 or τ_0' is smaller than τ_i momentum is gained as in (a) but the gain is somewhat smaller.

The second effect mentioned above means simply a *diffusion* perpendicular to the magnetic field. Seen in a coordinate system which takes part in the flow, it is obvious that a diffusion can take place without change in momentum. In order to check this result we shall treat the problem also in a fixed coordinate system (Fig. 6). The magnetic field is H_2 everywhere except in a certain region CDF where it is $H_3\,(>H_2)$. A particle moves in a circle with radius of curvature ϱ_2 with the centre located in A. At C it crosses the border to H_3. The radius of curvature changes to ϱ_3 and the centre is displaced to B. When the particle has reached D it leaves H_3 so that the radius of curvature is ϱ_2. The centre of the circle is now A'.

Denoting the distance AA' by Δ and CD by d we have

$$\frac{\Delta}{d} = \frac{\varrho_2 - \varrho_3}{\varrho_3} = \frac{H_3 - H_2}{H_2} \tag{8.6}$$

Inside H_3 there is an electric field

$$\vec{E}_3 = -\frac{1}{c}\vec{v} \times \vec{H}_3 \tag{8.7}$$

and outside the field is

$$\vec{E}_2 = -\frac{1}{c}\vec{v} \times \vec{H}_2 \tag{8.8}$$

The field is non-potential. The result of the disturbance is that the trochoidal orbit of the particle is displaced in the x-direction and in the y-direction. The displacement in the x-direction which is perpendicular to the electric field cannot produce any change in energy. The displacement in the y-direction changes the energy by

$$\Delta W = -e\left[\vec{E}_2\left(\vec{\Delta} + \vec{d}\right) - \vec{E}_3 \vec{d}\right] \tag{8.9}$$

which because of (8.6), (8.7), and (8.8) gives

$$\Delta W = 0 \tag{8.10}$$

It should be observed that according to (8.6) the displacement of the centre of curvature is independent of the energy of the particle. Hence particles with different energies passing the same distance d in a disturbed region, will have their orbits displaced the same distance. However, as $d \leq 2\,\varrho$, the distance d can be larger for particles with high energy, which means that high energy particles diffuse more easily.

Without diffusion the density of the particles would be proportional to v^{-1} or to H. Hence the density in H_1 would be much higher than in H_0. The diffusion tends to equalize the densities, so that in general the particles are displaced from H_1 to H_0 and to H_0'.

If the velocity of diffusion in the *x-direction* is larger than the flow velocity v a high energy particle in H_1 can leave this region and move to H_0 without loss of energy. If the diffusion velocity is smaller than v, particles will not be able to move from H_1 to H_0, but they will stay longer in the region $x_0 x_1$ and hence gain more energy. Further, they will pass $x_1 x_0'$ more rapidly and hence lose less energy. If the diffusion velocity is of the same order as the flow velocity particles may stay a prolonged time in the region x_0x_1 and gain much energy.

At the same time low energy particles from H_0 and H_0' will eventually diffuse in H_1. If they later are transported out to H_0' the net result will be a loss of energy. As the diffusion of low energy particles is slower than of high energy particles, this effect will in general not be very important.

If to left of x_0 we feed in particles with energy W_0, some of these particles will follow the flow and be accelerated up to W_1 and then diffuse back to x_0. Other particles will not reach the full energy before they diffuse back. The result is that near x_0 we will find particles with all energies between W_0 and W_1. There will also be particles with still higher energies which have drifted to the right and diffused back several times.

9. Application to interplanetary field

We shall now apply the results from the simple accelerator model of §§ 7—8 to the conditions in interplanetary space. We start with the stationary flow discussed in § 6, and confine the discussion to motions near the equatorial plane. (Fig. 7.) A particle at *a* where the magnetic field is rather weak will follow the flow inwards to a stronger field. This corresponds to the motion from x_0 to x_1 in the accelerator model. Later it may enter the beam, by crossing the border between the beam and the surrounding region. Then it will follow the beam outwards again, a process corresponding to the motion from x_0' to x_1' in the accelerator model. The distance corresponding to x_1x_1' is small.

In the same way as in the model the increase in energy can be depicted either as due to the betatron action of the increasing magnetic field, or to the trochoidal motion in the combined electric and magnetic field. The latter treatment makes it easy to construct the path, if we know H and E_a. As the flow is stationary we have a potential field and find

$$W = W_0 + e(V - V_0)$$

where W is the energy of the particle and V the potential at an arbitrary point, W_0 and V_0 denoting the same quantities for the starting point. Combined with (6.4) or (7.2) or this gives us the trochoidal path of the particle.

Fig. 7 shows the orbit if the particle is confined to the equatorial plane. The path is a result of the drift due to the inhomogeneous magnetic field which tends to make the particle drift around the sun at constant distance, and the drift due to the electric field, which tends to make the particle drift radially towards the sun.

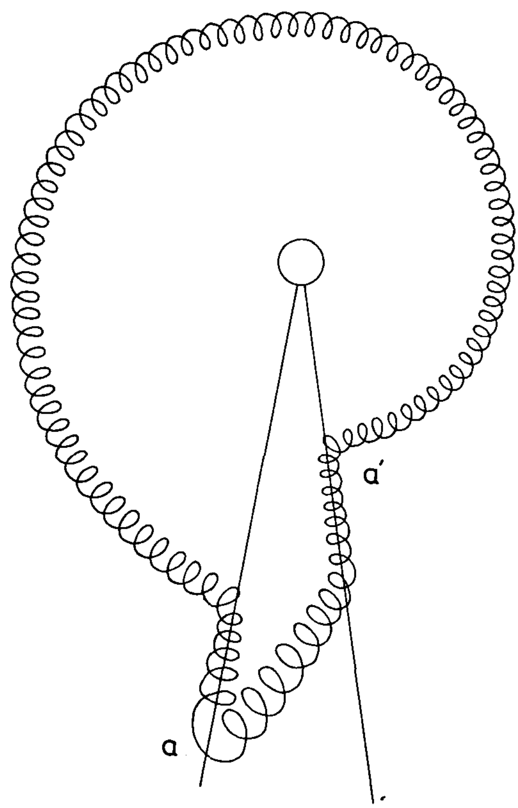

Fig. 7. Path of a high energy particle in the model of Fig. 4. Stationary state $V_a = V_b$.

The energy which a particle can reach in the stationary model of § 6 is limited in two different ways. As we have a potential field the maximum energy gain is given by the maximum potential difference which is V_a. If a particle starts at *a* with an energy W_a it gains energy and approaches the sun until at a' it has reached the energy $W_a' = W_a + eV_a$. It enters the beam and is carried out again, losing energy. If both processes are either adiabatic or non-adiabatic no energy is gained and the particle returns to its starting point and may continue to move in the same orbit.

If the conditions are non-stationary the particle does not come back to the same point after one turn. If $V_a < V_b$ which is the case if the beam has just been switched on or is increasing in intensity, the particle will lose $e(V_b - V_a)$ for every turn. On the other hand, if $V_a > V_b$ characterizing the conditions when the beam just has been switched off or is decreasing in intensity, the energy of the particle will continue to increase. In this case the whole solar field is resetting, and the particles are carried inwards with the general flow.

Both in the stationary case and non-stationary case an upper limit to the gain of energy is set by the ratio of the magnetic field at the starting point H_0 and the strongest field H_1 the particle reaches. The particle may start far away from the sun in a region where the field may be 10^{-5} gauss or 10^{-6} gauss. When it comes very close to the sun the field may be as high as 1 gauss or even 10 gauss. Hence the field ratio may be as high as $H_1/H_0 = 10^6$. In many cases it may only be brought to the region of the outer corona where $H = 10^{-1}$ or 10^{-2} gauss, giving $H_1/H_0 = 10^4$.

In order to reach very high energies the particles should start the process in a low magnetic field with as high initial energy as possible. Hence, we should look for particles with reasonably high velocities far out from the sun. Without introducing any new assumption we expect to find particles with velocities of the same order as the beam velocity ($v_b = 2 \cdot 10^8$ cm). There are probably regions near the border between the beam and the interplanetary matter at rest, where the relative motion produces an injection of particles. In this way beam particles with the velocity v_b are shot into the surrounding regions, and particles from these regions are injected into the beam with a relative velocity v_b.

In the beam electrons and ions move with the same velocity, so the different particles are injected with the same initial velocity v_i. We put $v_i = v_b = 0.0067\,c$ and use this for the calculation of the maximum energy with different values of H_1/H_0. Depending upon whether we assume the process to obey (8.5) ("adiabatic") or (7.7) ("non-adiabatic") we obtain:

Table 2

$\frac{H_1}{H_0}$	Acceleration according to (7.7)	Acceleration according to (8.5)
10^4	$W = 0.14\ m_0c^2$	$W = 0.010\ m_0c^2$
	Electrons 0.00007 GeV	0.000005 GeV
	Protons 0.13 »	0.009 »
	O nuclei 2.1 »	0.15 »
10^6	$W = 4.5\ m_0c^2$	$W = 0.20\ m_0c^2$
	Electrons 0.0023 GeV	0.00010 GeV
	Protons 4.2 »	0.19 »
	O nuclei 67 »	3.1 »

The energy values shown are obtained if the particles start at H_0 and are carried with the flow to H_1. After the acceleration the particles may diffuse out without loss of energy and eventually start gaining energy again. The absolute value, which in some cases is in the lower energy range of the Cosmic Radiation, is not very interesting, because by multiple processes much higher values can be reached as we shall see later. The relative values for electrons, protons, and heavy nuclei are very important, because they demonstrate that the process we consider, *favours heavy nuclei and strongly discriminates against electrons.* This is only in part due to the assumption that the injection takes place with constant velocity. Even if the particles were injected with the same kinetic energy there would be a similar effect although less pronounced. This is a consequence of the fact that the acceleration is more effective in the non-relativistic then in the relativistic energy range.

It is of interest to observe that the charge of the particles does not enter explicitly into the formulae for the energy. The energy per nucleon is constant. Even charged dust grains should be accelerated, provided the magnetic field is strong enough to keep them trapped.

10. The injection problem

In FERMI's theory[7] the injection constitutes a serious difficulty. In the low energy range protons and especially heavy nuclei pick up energy so slowly that their energy loss in interstellar matter is not covered. We shall investigate the same problem for our case.

The whole acceleration is supposed to take place in a time which is of the order of a month or a year, say 10^7 sec. This means that the magnetic field doubles its value in about 10^6 sec. During this time a particle moving with $v = 2 \cdot 10^8$ cm/sec has a path of $2 \cdot 10^{14}$ cm. The mass density in interplanetary space may be estimated to 10^{-21} g cm^{-3}. This means that when the particle doubles its energy due to electromagnetic acceleration, it passes $2 \cdot 10^{-7}$ g cm^{-2} of matter. An oxygen nucleus with $v = 0.0067\, c$ can pass 10^{-4} g cm^{-2} before it is absorbed. Hence, the particle certainly gains much more energy than it loses.

Due to the swift acceleration there is no injection difficulty in our case.

The presence of heavy nuclei in Cosmic Radiation is a strong argument in favor of a swift acceleration process. As a process can be expected to be swift only near the stars, it favours the local hypothesis.

11. Multiple processes

The processes we have discussed will accelerate particles up to high energies. In the case of a stationary model the energy gain is limited by V_a, which is of the order 10^8—10^9 volt according to Table 1. In the non-stationary state with $V_a > V_b$ there is no limitation of this kind. In both cases the values given in Table 2 set an upper limit. As summary it may be stated that particles injected by the beam be accelerated up to energies of the order 10^8—10^9 volt in one process of the indicated kind. If the solar magnetic field were undisturbed the accelerated particles would be completely trapped and particles with these energies could be found only close to the sun. Small local disturbances permit a diffusion, but due to the small radius of curvature, this will be rather slow. Only violent disturbances of the solar field will allow them to diffuse out, say to the earth or still further out. For example, a storm-producing beam will disturb the field in its neighbourhood so that there are lines of force running from regions with strong field near the sun far out to regions with low field. (See Fig. 2.) Particles which normally are trapped in a strong field near the sun will be able to move along such lines out to regions with low field without losing energy. (Besides this effect there will also be particles trapped in the beam and transported out, but as within the beam the magnetic field is decreasing with time, the particles will lose energy.) Of course, some particles will also move inwards in a disturbed region, but as in the flow model the density increases with the magnetic field this effect will not be so important.

In this way particles, which have followed the flow inwards and gained energy in the increasing magnetic field, will be able to move out some distance without losing the energy they have gained.

Much higher energies can be reached by such multiple processes. If by a disturbance of this kind particles accelerated to 10^8 or 10^9 e volt are injected in a low magnetic field H_0' and follow the flow to a field H_1', they will be accelerated again. The process may be repeated if they are taken from the field H_1' and without loss of energy are injected in a field H_0'', which later increases to H_1''. The energy gained by this triple process can be calculated from (7.7) or (8.5) if H_1/H_0 is substituted by

$$\frac{H_1}{H_0}, \frac{H_1'}{H_0'}, \frac{H_1''}{H_0''}$$

The energy which can be reached in this way is only limited by the condition that the magnetic field must be able to keep the particle in a trapped orbit. Compare § 17.

If the acceleration takes place during a time when $V_a < V_b$ so that the dominating motion is directed outwards, the acceleration outside the beam is less efficient than the deceleration in the beam, and the chance of a multiple acceleration up to high energies is small. If $V_a > V_b$, the motion inwards dominates. Particles which have been accelerated and been brought outwards by a disturbance will be taken inwards again by the flow. Multiple accelerations are quite normal processes. In the stationary case $V_a = V_b^1$ the acceleration will in general dominate as we shall see in 12. Also in this case, multiple processes are probable.

As long as ϱ is small the particle cannot leave the acceleration process unless it is captured within the beam. If it is brought out to a weaker field by a disturbance in the field, it will be brought inwards again by the flow. This condition is changed when the energy of a particle, by one process or iterated processes, has increased so much that ϱ becomes comparable to R. Then it is no longer compelled to follow the inward flow to the same degree as earlier, because its diffusion velocity increases. Hence, it can move outwards not only due to very large disturbances. Even more "normal" disturbances of the solar field will make it possible for it to diffuse outwards. In this way a considerable emission of high energy particles takes place. The condition $\varrho \sim R$ is satisfied, for example, if $\varrho \approx R = 10^{13}$ cm and $H = 10^{-5}$, when $V = 3 \cdot 10^{10}$ e volt. Already when ϱ is sometimes smaller than R, the diffusion is considerable, so that the cosmic ray generator emits particles well below the mentioned energy.

It is of interest to estimate the order of magnitude of the low energy limit for particles emitted by the cosmic ray accelerator. Let us suppose that a particle moving with the velocity u at the solar distance R encounters a disturbed region after having travelled a distance γR, where γ is a constant. Hence, the time τ_c between two disturbances is

$$\tau_c = \gamma R / u \qquad (11.1)$$

Each disturbance displaces the orbit a distance ϱ in an arbitrary direction. In order to escape from the generator the path must be displaced a distance of the order R which requires $(R/\varrho)^2$ disturbances. The momentum p is

$$p = \frac{eZ}{c} H\varrho \qquad (11.2)$$

Hence, the time τ_d needed to diffuse out is

$$\tau_d = \left(\frac{R}{\varrho}\right)^2 \tau_c = \frac{\gamma e^2 Z^2 R^3 H^2}{c^2 u p^2} \qquad (11.3)$$

If during this time the inwards flow which has the velocity

$$v_a = c \frac{V_a}{\alpha R H} \qquad (11.4)$$

(where α is the angle of the inflew) has carried the particle inwards a comparable distance the particle cannot escape. Hence if we put

$$v_a \tau_d = R \qquad (11.5)$$

we obtain an approximate limit to the momentum of the particles which can escape. From (11.3) we obtain

$$\frac{cV_a}{\alpha R H} \frac{\gamma e^2 Z^2 R^3 H^2}{c^2 u p^2} = R \qquad (11.6)$$

or

$$p = \frac{Ze}{c} \left[\gamma \frac{HRV_a c}{\alpha u}\right]^{1/2} \qquad (11.7)$$

Putting $\alpha \approx 2\pi$; $u = c$; $V_b = 3 \cdot 10^8$ volt $= 10^6$ e.s.u.; $H = 10^{-5}$ gauss, $R = 10^{13}$ cm referring to the conditions near the earth's orbit, we obtain

$$\frac{cp}{Ze} = 0.4 \cdot 10^7 \sqrt{\gamma} \qquad (11.8)$$

or

$$p = 1.2 \cdot 10^9 \sqrt{\gamma} \text{ e volt } Z/c \qquad (11.9)$$

We do not know γ. In order to make $p = 0.5 \cdot 10^9$ e volt, which is the empirical value for the low energy cut-off, we should have $\gamma = 0.2$. This means that the path of a particle is likely to be disturbed considerably after a travel over a distance $0.2\ R$. This seems to be acceptable. The radius of curvature is $\varrho = 2 \cdot 10^{11}$ cm $= 0.02\ R$.

12. Production of cosmic ray energy

Consider the flow studied in § 6. We treat the conditions in the equatorial plane and assume that these are valid up to a latitude angle φ. A beam with an angular breadth $\beta = \frac{b}{R}$ is emitted with a velocity

$$v_b = c \frac{V_b}{\beta R H_b} \qquad (12.1)$$

We assume that V_b and β are constant and that

$$H_b = H_b^{\male} (R_{\male}/R)^{n_b} \qquad (12.2)$$

$H_b^{\male}$ is the field at the earth's orbit $R_{\male}$ and n_b is a constant. This gives

$$v_b = \frac{cV_b}{\beta H_b^{\male} R_{\male}^{n_b}} R^{n_b - 1} \qquad (12.3)$$

Outside the beam there is an inflow with the breadth $\alpha R = 2\pi R - b$. In this region H obeys the formula

$$H_a = H_a^{\male} (R_{\male}/R)^{n_a} \qquad (12.4)$$

and

$$v_a = \frac{cV_a}{\alpha H_a^{\male} R_{\male}^{n_a}} R^{n_a - 1} \qquad (12.5)$$

If a particle with energy W moves in a changing magnetic field its energy increases so that

$$\frac{dW}{Wdt} = A \frac{dH}{Hdt} \qquad (12.6)$$

where A is a factor which depends on the energy.

According to (7.7) and (8.5) we have $1/3 < A < 1$. In the following approximate calculations we shall put $A = 1/2$. If the total cosmic ray energy per unit volume is w we have

$$\frac{dw}{w dt} = \frac{1}{2}\frac{dH}{H dt} = \frac{v}{2\,H}\frac{dH}{dR} \tag{12.7}$$

For the beam this gives

$$\left(\frac{dw}{dt}\right)_b = -\frac{w}{2}\frac{n_b}{R}\frac{c V_b R^{n_b-1}}{\beta H_b^{\oplus} R_{\oplus}^{n_b}} \tag{12.8}$$

and a similar expression outside the beam. From this we can calculate the total energy production P within the distance $R_{\oplus}$ from the sun. We obtain

$$P = \int_{R_0}^{R_{\oplus}} \left(\frac{dw}{dt}\right)_a \cdot 2\,\varphi R^2 \alpha\, dR - \int_{R_0}^{R_{\oplus}} \left(\frac{dw}{dt}\right)_b 2\,\varphi R^2 \beta\, dR \tag{12.9}$$

or

$$P = c\varphi\left[\frac{V_a n_a}{H_a^{\oplus} R_{\oplus}^{n_a}}\int_{R_0}^{R_{\oplus}} w R^{n_a} dR - \frac{V_b n_b}{H_b^{\oplus} R_{\oplus}^{n_b}}\int_{R_0}^{R_{\oplus}} w R^{n_b} dR\right] \tag{12.10}$$

R_0 is the inner limit of the acceleration process. As $R_0 \ll R_{\oplus}$ this is not important. In order to obtain the whole increase in energy inside $R_{\oplus}$ we shall add the transport $\frac{dU'}{dt}$ of particles through the surface $R_{\oplus}$.

$$\frac{dU'}{dt} = 4\,\pi\varphi R_{\oplus}^2\left[\alpha\,(wv)_a - \beta\,(wv)_b\right] \tag{12.11}$$

It is assumed that no energy is transported through the limiting cones. In stationary state the energy increase must be compensated by an energy outflow

$$\frac{dU''}{dt} = \frac{dU'}{dt} + P \tag{12.12}$$

This is effected by outward diffusion of cosmic ray particles. The net outflow of cosmic ray energy through $R_{\oplus}$ is $\frac{dU''}{dt} - \frac{dU'}{dt} = P$

As the storing capacity of the volume inside $R_{\oplus}$ is negligible in this connection, a unit surface perpendicular to $R_{\oplus}$ near the earth's orbit is hit by the same number of particles from outside as from inside. The particles coming from the inside, have been accelerated by the process we have considered and hence have higher energy than those coming from the outside. In this way the produced energy P is emitted. If the energy density outside $R_{\oplus}$ is w, the energy flux from outside through the unit surface is wc and through the whole surface we consider

$$\frac{dU_0}{dt} = 4\,\pi\, R_{\oplus}^2 \varphi\, wc \tag{12.13}$$

In a similar way the outflow is

$$\frac{dU_1}{dt} = 4\,\pi\, R_{\oplus}^2 \varphi w_1 c \tag{12.14}$$

where w_1 is the average energy density inside $R_{\oplus}$. As

$$\frac{dU_1}{dt} = \frac{dU_0}{dt} + P \tag{12.15}$$

we find from (13), (14), and (15)

$$w_1 - w = \frac{1}{4\,\pi\, R_{\oplus}^2 \varphi c} \cdot P$$

13. Diurnal variation

If a measuring instrument shows the intensity I when receiving radiation with average energy w, it gives the reading $I + \Delta I$ for w_1. If we put

$$\Delta I = k' I \Delta w / w \tag{13.1}$$

with $\Delta w = w_1 - w$, the constant k' is as in § 4 between 1 and 10, and probably about 3.

An instrument which rotates with the earth will show a diurnal variation with amplitude D_R and a maximum at 12^h, given by

$$2\,D_R = \frac{\Delta I}{I} \tag{13.2}$$

Hence we have

$$D_R = \frac{k'}{8\,\pi\, R_{\oplus}^2 \varphi c} \cdot \frac{P}{w} \tag{13.3}$$

In order to calculate the diurnal variation it remains only to estimate how w waries. In the energy range of most of the cosmic rays, the radius of curvature ϱ is rather large so that the diffusion is relatively rapid. This means that w cannot vary very much. Even if it increases somewhat when we approach the sun, it is probably legitimate to put it approximately constant. This gives from (12.10)

$$P = c\varphi w R_\delta \left[\frac{n_a}{n_a+1}\frac{V_a}{H_a^\delta} - \frac{n_b}{n_b+1}\frac{V_b}{H_b^\delta}\right] \quad (13.4)$$

and

$$D_R = \frac{k'}{8\pi R_\delta}\left[\frac{n_a}{n_a+1}\frac{V_a}{H_a^\delta} - \frac{n_b}{n_b+1}\frac{V_b}{H_b^\delta}\right] \quad (13.5)$$

In order to discuss this formula let us assume that there is no solar activity during a long time, so that $V_a = V_b = 0$. If then a beam is switched on, we have $V_b > 0$ but V_a is still zero. Hence there is a deceleration of cosmic rays: P and D_R are negative and there is an inflow of radiation across the earth's orbit. Let us suppose that V_b remains constant. After some time — of the order of a year — V_a has increased to the same values as V_b and we have stationary conditions. With $V_a = V_b$ we have

$$D_R = \frac{k' V_b}{8\pi R_\delta}\left[\frac{n_a}{n_a+1}\frac{1}{H_a^\delta} - \frac{n_b}{n_b+1}\frac{1}{H_b}\right] \quad (13.6)$$

According to § 5 the value of n_b is about $n_b = 1$. According to estimates by Brunberg and Dattner we may put $n_a = 2.5$. This gives

$$D_R = \frac{k' V_b}{8\pi R_\delta}\left[\frac{0.7}{H_a^\delta} - \frac{0.5}{H_b^\delta}\right] \quad (13.7)$$

The magnetic fields are probably of the order $H_a = 10^{-5}$ gauss and $H_b = 3 \cdot 10^{-5}$ gauss. Further, we may put $V_b = 3 \cdot 10^8$ volt, and $k' = 3$. This gives

$$D_R = 4 \cdot 10^{-4} \quad (13.8)$$

i.e. a 12^h component of the diurnal variation of 0.04 %.

If now the beam is switched off so that $V_b = 0$ the last term disappears, so that D_R increases. The magnetic field resettles with an inward motion of magnetic flux. This means that both H_a and V_a decreases. Depending on which of them decreases most, the value of D_R may either go up or down.

It is of interest to note that in the stationary state $V_a = V_b$ there is still an energy production if $n_a/(n_a+1)\,H_a^\delta > n_b/(n_b+1)\,H_b^\delta$. This is not in conflict with the fact that a particle which drift around the sun a whole turn in a closed orbit does not increase its energy, because the drift velocity is smaller inside the beam. In order to have the same Cosmic Ray density outside and inside the beam there must be particles which do not enter the beam. Experimental evidence speaks rather for a lower density inside the beam.

14. Comparison with observations

The results we have obtained could be compared with observations of the diurnal variation and the variation of the total intensity.

Thambyahpillai and Elliot[2] have pointed out that the phase of the diurnal variation varies in a systematic way with the solar cycle. This variation has further been discussed by Sarabhai and Kane[3]. Using the results of their terrella experiment Brunberg and Dattner[26] have shown that if reduced for the deflection in the terrestrial field the maximum of the diurnal variation occurs at 18^h or during some years a few hours earlier. Their interpretation is that the diurnal variation consists of a "*tangential*" component with a maximum at 18^h and a "*radial*" component with maximum at 12^h. The tangential component is due to an excess of particles coming in parallel to the earth's orbit, which possibly is an effect of the solar rotation in which most of the Cosmic Rays in our environment may take part. We are here more interested in the radial component D_R because it shows us how much cosmic ray energy is streaming out across the earth's orbit. This has been calculated theoretically in § 13 so a comparison between observations and theory is possible.

The variation of D_R with time can best be followed from the long-time registrations with ionization chambers by Forbush. Fig. 8 shows his results for Huancayo, Cheltenham, Christchurch, and Godhavn plotted in harmonic dials.

Before we use these values we must correct them for the deflection of cosmic rays in the terrestrial magnetic field. If we know the energy of the variable component, the time angle correction Ψ_E can be found from Brunberg and Dattner's curves. The energy is not known with certainty but could be expected to be between 20 and 40 GeV. Table 3 shows the corrections for the zenith direction. It is evident that we do not make a large error if we use 30° for Godhavn and 40° for Cheltenham and Christchurch. The value for Huancayo is uncertain.

Table 3

Station	Geom. Lat.	Correction Ψ_E for zenith			
		20 GeV	30 GeV	40 GeV	Adopted
Godhavn	79°.8	30°	31°	30°	30°
Cheltenham	50°.1	44°	40°	35°	40°
Christchurch	— 48°.0	42°	40°	35°	40°
Huancayo	— 0°.6	110°	64°	45°	90°

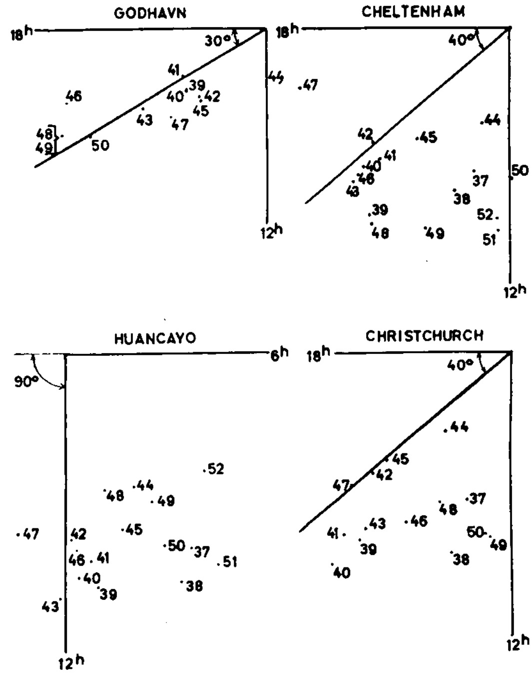

Fig. 8. Harmonic dials of diurnal variation in Godhavn, Cheltenham, Huancayo, and Christchurch. Yearly means 1938—1952. (FORBUSH.)

An ionization chamber measures rays from all directions although the atmospheric absorption makes it most sensitive to rays from zenith. BRUNBERG and DATTNER[26] have recently estimated the integrated effect and calculated the average value of Ψ_E. They find that at Godhavn the diurnal variation should be a complex phenomenon because rays from very different primary directions reach the ionization chamber. For Cheltenham and Christchurch they obtain values of Ψ_E which are about 5° higher than those adopted in Table 3.

This indicates that the corrections are most reliable for Cheltenham and Christchurch. For Huancayo the value is uncertain. As we shall see a good agreement between all those three stations is obtained if we adopt the correction 90° for Huancayo. This corresponds to an energy of 25 GeV for the zenith direction.

In each of the harmonic dials a line making the angle Ψ_E with the 18^h axis is drawn. The distance of the dots from this line gives the 12^h component D_R of the diurnal variation. The values of D_R obtained in this way are plotted in Fig. 10. The agreement between Cheltenham, Christchurch and Huancayo is good. The absolute values for Huancayo are uncertain. The variations in D_R at Godhavn are small and not very well correlated with the variations at the other stations, probably for the reasons given above. The average for Cheltenham, Christchurch and Huancayo is shown in Fig. 9. The position of the zero line is uncertain due to possible errors in Ψ_E. An error of 10° means a shift in the zero line of 0.025 %. Further, atmospheric effects may influence the diurnal variation.

The value of D_R reaches a maximum in 1944 when the sunspot cycle has a minimum, and a minimum in 1947, coinciding with the sunspot maximum. It increases to high values in 1952 when we approach next minimum. There is a sunspot maximum at 1938, but D_R does not go down to a low value until 1939, and it stays at this low value during some years.

The same variation can be traced in BRUNBERG and DATTNER's diagram of the phase angle.[26]

Thus the general tendency seems to be that an increased solar activity gives a decrease in D_R. This is in a qualitative agreement with the theoretical results of § 13.

From (13.7) a value of $D_R = 0.04$ % was estimated for the stationary state. This is well reconcilable with the average of the D_R curve in Fig. 9, but this agreement is not very convincing, because there is considerable uncertainty regarding the zero value. It is seen from (13.5) that the variations in D_R should be of a comparable order of magnitude. The observational values show a difference of 0.16 % between the highest value in 1952 and the lowest value in 1947. Taking into account the many uncertain factors the quantitative agreement is encouraging.

15. Variation of the total intensity

According to FORBUSH[28] the total intensity I of Cosmic Radiation varies with the solar cycle by as much as 4 %. The variation at Huancayo, Cheltenham, and Godhavn is shown in Fig. 9. (There is no reason to exclude Godhavn as in the case of the diurnal variation.) The variation seems to have a negative correlation with the sunspot curve. The amplitude has no systematic dependence on the latitude, so it is likely that it is not only the low-energy part of the spectrum which varies.

The observed variation is of decisive importance to the question about the trapping field. As obviously the intensity in the whole galaxy cannot change by 4 % in three years, the intensity variations must be confined to a relatively small volume around the solar system. Consequently we must assume that there is some sort of screening, most likely produced by a "trapping field". The intensity we measure characterizes the conditions inside the screen.

A more difficult question is how perfectly the trapping field screens. There are two alternatives:

1. There is so much leakage in the trapping field that averaged over a very long time, there can only be a small difference in intensity. This means that in average the intensity in e.g. the whole galaxy should be about the same as here. The consequence is that besides the local generation which we have considered here, there must also be a galactic generation, and further, we must assume a galactic trapping field, which prevents the radiation in the galaxy from leaking out into intergalactic space.

2. The leakage in the trapping field may be so small that a large difference in intensity can be established. This means that we are not concerned with the intensity outside the trapping field. This intensity may be very low. We need not assume any appreciable galactic generation, and it is not necessary to postulate a galactic trapping field.

Although at present there is no decisive argument in either direction, the latter alternative seems more attractive because we do not need so many *ad hoc* assumptions.

Variations in the total intensity are expected from the point of view of the theory of § 12. The efficiency of the generating mechanism varies with the solar activity as discussed in § 13. How this should affect the total intensity depends on what assumptions we make about the diffusion of Cosmic Rays through the trapping field and its storing capacity. The observational result is that the total intensity shows a negative correlation with the sunspot activity. This means that the intensity near the earth may be a direct measure of the efficiency of the energy production, and should vary according to (13.4).

Forbush's curves of I are somewhat similar to the curves of D_R. Especially the rapid rise from 1944 to 1947 is the same. The similarity during the first years of the registrations is less convincing.

16. Total energy production

The total energy produced within the earth's orbit can be found from (13.3):

$$P = \frac{8\pi}{k'} R_\delta^2 \varphi c w D_R \tag{15.1}$$

using essentially only empirical data. We put $\varphi = 1$ and $k' = 3$. As an average for D_R we may put $D_R = 4 \cdot 10^{-4}$. This gives

$$P = 2 \cdot 10^{34}\, w$$

With $w = 10^{-12}$ erg cm^{-3} we find $P = 2 \cdot 10^{22}$ erg sec^{-1}.

It is of interest to compare this with the kinetic energy of the beam. If near the earth its cross-section is $S = 10^{25}$ cm^2 and the density is ϱ g cm^{-3}, the outward transport of energy is

$$\frac{dU_b}{dt} = S\frac{\varrho v_b^3}{2} = 4 \cdot 10^{49} \varrho$$

Hence even if the density were as low as $\varrho = 10^{-24}$ g cm^{-3} corresponding to 1 particle/cm^3 the beam energy would be 2,000 times as large as the cosmic ray production. If the half-life of cosmic rays in interstellar space is say $T = 10^6$ years $= 0.3 \cdot 10^{14}$ sec, this energy production is enough to fill a volume of the order

$$\Phi = \frac{T}{w} P = 10^{48}\ \text{cm}^3$$

which has a diameter of 10^{16} cm. The beam has probably only dissipated a small fraction of its energy inside the earth's orbit.

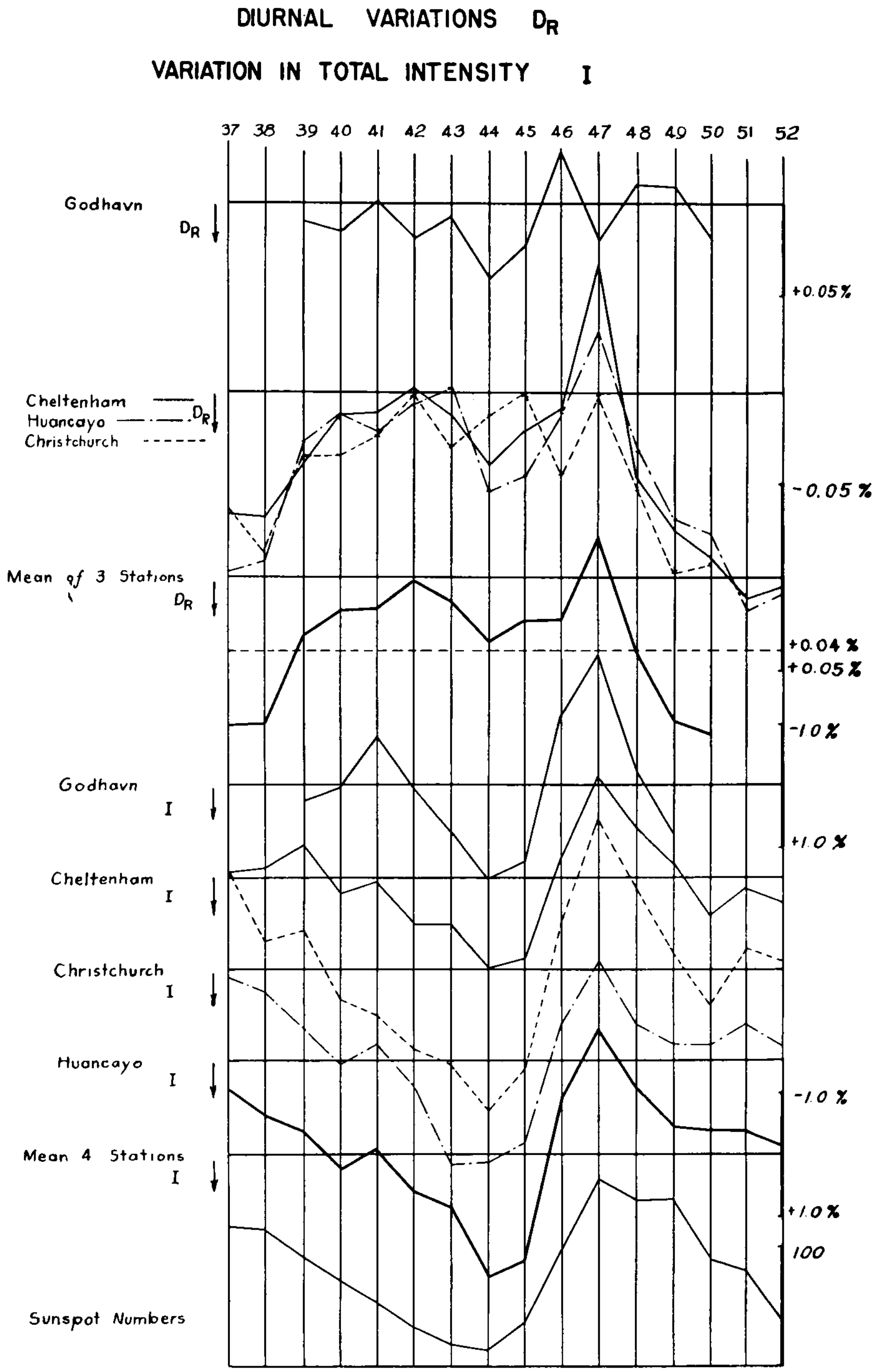

Fig. 9. Radial component D_R of the diurnal variation, in Godhavn, Cheltenham, Huancayo and, Christchurch, obtained from Fig. 8 after correction for the terrestrial magnetic field.
Mean of the three latter stations. Departure from average intensity I at the four stations. Mean of these stations. Sunspot numbers.

Processes of similar kind may go on outside the earth's orbit so the total energy production may be enough to fill up a much larger volume.

The number of particles which must be injected is

$$\frac{dn}{dt} = \frac{1}{eV_0} P$$

where eV_0 is the average energy per particle. As the number of particles per unit volume is $n = w/eV_0$ we have

$$\frac{dn}{dt} = 2 \cdot 10^{34}\, n$$

With $n = 10^{-10}$ cm^{-3}, we obtain

$$= 2 \cdot 10^{24} \text{ particles/sec.}$$

As the injection takes place with $v = 2 \cdot 10^8$ cm, an injection cross-section of only $S = 10^{16}$ cm^2 is enough even if we assume a density as low as 1 particle cm^{-3}. Hence there seems to be no difficulty of injecting a sufficient number of particles.

17. Upper energy limit

The present theory has been developed in order to account for the properties of that part of the Cosmic Ray spectrum which is measured by ionization chambers and counters, i.e. 10^{10}—10^{11} e volt. The neutron counter measurements, which cover the lower part of the spectrum, seem to fit into the picture rather well, although much more work in this field is desired. Concerning the high energy part of the spectrum ($> 10^{11}$ e volt), we know at present very little about its properties, and the different types of variations have not been studied. Hence it is believed that the time is not yet ripe for a detailed discussion of the highest energies. In this paragraph we shall at first show to what energies it is possible to extend the present theory and then make some speculations about possible ways to account for the highest energies.

A local theory cannot be expected to account for energies above a certain limit. This limit may be given either

(1) by the accelerating mechanism; or

(2) by the trapping field which we must assume in order to make the radiation isotropic.

The accelerating mechanism can produce particles up to the maximum momentum which can be kept in trochoidal orbits close to the sun. In the case of a dipole field near the sun the maximum momentum p of a trochoidal path in the equatorial plane is given by the Störmer orbit of a particle which just does not hit the sun. For this we have

$$p = \frac{Ze H_\odot R_\odot}{c} (3 - 2\sqrt{2}) \qquad (17.1)$$

where Ze is the charge of the particle and $H_\odot$ the field at the sun's equator. Putting $H_\odot = 5$ gauss we find

$$p = 1.8 \cdot 10^{13} \text{ e volt } Z/c \qquad (17.2)$$

In a dipole field there may be more complicated orbits which give somewhat higher values. Further, if the field is not a dipole field it is possible that still higher values could be obtained, but the above figure is likely to give the order of magnitude.

The corresponding value of the rigidity is

$$H\varrho = 6 \cdot 10^{10} \text{ gauss cm.} \qquad (17.3)$$

This means that a trapping field of 10^{-5} gauss could keep the particles within a distance of some 10^{16} cm.

Multiple charged heavy nuclei with $Z = 30$ or somewhat higher brings us up to a maximum energy of the order of 10^{15} e volt, but higher energies could not be accounted for by a local theory. Although at present we know too little about the energies above 10^{15} e volt to be able to discuss the matter very seriously, the following speculations about the energies 10^{15}—10^{17} e volt may be of interest.

1. The most energetic particles may derive from other stars which generate Cosmic Rays in a similar way but possess much stronger magnetic fields. Some of the particles may leak out and reach us.

2. As Cosmic Rays may travel some million years in space before they are absorbed, some of the particles which we observe may have been generated very long ago. It is possible that the sun has had a much stronger magnetic field during some period the last million years. If so, very energetic particles may have been generated. In order to keep them trapped we need a trapping field of for example 10^{-4} gauss extending 10^{17} cm (giving $p = 3 \cdot 10^{15}$ e volt Z/c, or with $Z = 30$, $p = 10^{17}$ e volt/c).

3. The existence of primary particles with very high energies is proved only by measuring the total energy of extended air showers. It seems at present not to be possible to exclude that such showers originate from multiple charged dust grains which are accelerated by the mechanism we have discussed. Possible values would be: charge 10^4 e, mass 10^6 nucleons, total energy 10^{17} e volt, energy/nucleon 10^{11} e volt, $H\varrho = 3 \cdot 10^{10}$ gauss cm.

18. The trapping field

Both the galactic and local theories must postulate the existence of a trapping field in order to prevent the cosmic radiation from filling up the whole universe with the same intensity. In both cases the properties of the trapping field are *ad hoc* assumptions which at present cannot be checked in any other way. The problem has been treated by UNSÖLD[8] and BIERMANN and SCHLÜTER[30].

The local theory requires a trapping field which is stronger and more "insulating" than in the galactic theories.

If the magnetic field in our environment is produced only by *local* currents it may easily be insulating over large regions. In order to discuss a very simple model, let us assume that the field is produced by a current in a circular loop. Suppose that charged particles having a radius of curvature which is small compared to the size of the loop are produced in the field. They are free to move along the lines of force. Due to the inhomogeneity of the field they will have a superimposed drift in circles around the axis. From a small region near the axis they could leak out to infinity but they will be trapped in a large portion of the field.

As at some distance the field from a loop is a dipole field the problem is related to Störmer's problem. It is well known that in a dipole field there are regions from which there are no trajectories out to infinity.

Hence if the trapping field is caused by a system of local currents, it may very well be insulating. However, in the case of the interplanetary field we have seen that a diffusion of charged particles is caused by small-scale irregularities in the field. The same kind of disturbances may cause a diffusion out of the trapping field. It is reasonable that the disturbances which are caused by the solar activity, are much more important near the sun. As the trapping field is far from any source of disturbance, it is possible that the small-scale irregularities have been smoothed out, so that the diffusion is small.

If besides the field from a local current system there are also fields from other sources, the combined field has a complicated structure and leakage is more probable. Hence, the necessity of assuming an insulating trapping field probably sets an upper limit to a galactic field in our neighbourhood.

In the earlier discussion[18] of the local theory it was pointed out that a trapping field may be produced by magneto-hydrodynamic effects of the motion of the solar system in relation to interstellar matter. Another possibility is that it may be generated by the storm-producing beams. When after the ejection from the sun the beams pass the earth's orbit, they have certainly much energy left and it is possible that far outside the earth's orbit this energy is used to build up a trapping field.

As a summary of this brief discussion we may state that the assumption of a trapping field which makes the radiation isotropic is not in conflict with any known facts. On the other hand we know so little about it that at present it is hardly worth while to try to make a detailed theory of it.

19. Summary of the accelerating mechanism

The generating process we have studied takes place in interplanetary space around the sun, to a large extent at solar distances comparable to the earth's distance. The process has been found to have the following properties.

1. *Absence of electrons.*

   The accelerated particles have an energy which is proportional to their rest mass. This means that the process is inefficient for electrons but favourable for protons and still more so for heavy nuclei.

   This explains the absence of electrons in cosmic radiation, and the fact that heavy nuclei constitute an important part of the radiation.

2. *Latitude knee.*

   The diffusion is slow below a certain momentum, which is of the order 10^9 e VZ/c. Accelerated particles below this limit are found only close to the sun. Above this limit the diffusion becomes more rapid and cosmic ray particles diffuse outwards. This seems to account for the low energy cut-off in the cosmic radiation energy spectrum and also for the large variability in the lowest energy range.

3. *Intensity variation with the sunspot cycle.*

The acceleration process is a result of the solar activity, and must be expected to vary with the solar cycle. At the beginning of the sunspot cycle when the solar activity is strong the outflow dominates and the decelaration may be as strong as the acceleration or even stronger, resulting in a decrease in cosmic ray intensity. At the end of a sunspot cycle the inflow dominates and the acceleration becomes more efficient so that the intensity becomes large. This is in agreement with Forbush's curves, which show intensity variations of 4 %.

4. *The diurnal variation.*

The diurnal variation of the primary radiation seems to consist of two effects. One of these is due to an excess of particles coming in from the 18^h direction. Superimposed on this is the radial flow of cosmic ray energy which gives a component D_R with a maximum at 12^h. This component is a measure of the energy produced by the accelerating process inside the earth's orbit. From Forbush's curves it is evident that it varies in roughly the same way as the total intensity, and is correlated with the solar activity.

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