Transport and Production of Vorticity in the Atmosphere

By J. VAN MIEGHEM, University of Brussels

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Abstract

The classical form of the vorticity equation is transformed into an equation of balance from which the vorticity transport and the rate of vorticity production are deduced. Transport and production are given for 1) an arbitrary component of the absolute vorticity, 2) the absolute vorticity vector, 3) the absolute vorticity flux through a surface, 4) the total absolute vorticity within a volume. Special consideration is given to the absolute vertical vorticity, the absolute vorticity component normal to the isentropic surfaces and the total absolute vertical vorticity of the atmosphere. The influence of vertical motions and of heating and cooling on the vorticity transport is discussed. The sign of the meridional vorticity transport is tentatively determined.

1. A convenient form of the vorticity equation for the study of the general circulation is the equation of balance of the absolute vorticity giving explicitly the local rate of change of the absolute vorticity per unit volume in terms of a vorticity transport and a vorticity production.

The classical form of the vorticity equation may be written, with the usual vector notation,

$$\frac{dQ_z}{dt} + Q_z \operatorname{div} \mathbf{v} = \mathbf{Q} \cdot \nabla v_z + (\nabla p \times \nabla \alpha)_z + \operatorname{curl}_z \mathbf{F}, \qquad (1.1)$$

where Q_z represents the component of the absolute vorticity $\mathbf{Q}$ along an *arbitrary* axis z, fixed with respect to the earth, $\mathbf{v}$ the wind velocity, p the atmospheric pressure, α the specific volume of the air, $\mathbf{F}$ the frictional force per unit of mass, ∇ the "del" operator and $\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$ the individual derivative along the air motion with respect to the earth. The suffix z designates the component of a vector along the z-axis.

Substituting in (1.1) the well known identities

$$\operatorname{div} \mathbf{Q} \equiv 0 \text{ and } \operatorname{div}(a\,\mathbf{A}) \equiv a \operatorname{div} \mathbf{A} + \mathbf{A} \cdot \nabla a,$$

where $\mathbf{A}$ is an arbitrary vector and a an arbitrary scalar quantity, the vorticity equation assumes the form of an equation of balance:

$$\boxed{\frac{\partial Q_z}{\partial t} + \operatorname{div}(Q_z \mathbf{v} - v_z \mathbf{Q}) = (\nabla p \times \nabla \alpha)_z + \operatorname{curl}_z \mathbf{F}} \qquad (1.2)$$

a form which is appropriate for the evaluation of local vorticity changes.

The equation of balance (1.2) is simply another expression for the circulation theorem of V. Bjerknes. Thus, the z-component Q_z of the absolute vorticity $\mathbf{Q}$ in unit volume, at rest with respect to the earth, varies in consequence of:

a) *a vorticity transport*

$$\mathbf{C} \equiv Q_z \mathbf{v} - v_z \mathbf{Q} \qquad (1.3)$$

across the boundary of the unit of volume;

b) *a vorticity production* within the unit volume at the rate

$$(\nabla p \times \nabla \alpha)_z + \text{curl}_z \mathbf{F} \qquad (1.4)$$

determined by the isobaric-isosteric solenoides and the vortex unit-tubes of the frictional force $\mathbf{F}$.

The term $Q_z \mathbf{v}$ of the transport $\mathbf{C}$ of the z-component Q_z of the absolute vorticity $\mathbf{Q}$ represents the convection with the wind velocity $\mathbf{v}$ of the instantaneous absolute rotation of the air about the z-axis, the non-convective term $-v_z \mathbf{Q}$, the contribution to the transport of the z-component of the absolute vorticity, of the instantaneous absolute rotation of the air moving in the z-direction.

The transport C_y of the z-component Q_z of the absolute vorticity $\mathbf{Q}$ in an *arbitrary* direction y is evidently given by

$$C_y \equiv Q_z v_y - v_z Q_y, \qquad (1.3')$$

where the suffix y designates the component of a vector in the y-direction. It should be noted that:

1) the transport in the y-direction of the z-component of the absolute vorticity is numerically equal, but of opposite sign, to the transport in the z-direction of the y-component of the absolute vorticity;

2) there is no transport in the z-direction of the z-component of the absolute vorticity.

Hence, the three-dimensional divergence in (1.2) reduces to a two-dimensional one (see eq. (2.2)).

2. Let us now suppose that the z-axis is no longer arbitrary but coincides with the upward pointing vertical z. The vertical component Q_z of the absolute vorticity $\mathbf{Q}$ is the vortex quantity most commonly considered in dynamic meteorology and is often called "the absolute vorticity".

The transport of the absolute vertical vorticity Q_z is defined by the horizontal vector, (1.3),

$$\mathbf{C} \equiv Q_z \mathbf{v} - v_z \mathbf{Q} \equiv Q_z \mathbf{v}_h - v_z \mathbf{Q}_h \equiv \mathbf{C}_h, \qquad (2.1)$$

where as usual the suffix h refers to a horizontal vector. Hence, the transport of the absolute vertical vorticity occurs along lines of the geopotential surfaces, $(C_z \equiv 0)$.

Therefore the equation of balance (1.2) of the absolute vertical vorticity Q_z assumes the simpler form:

$$\frac{\partial Q_z}{\partial t} + \text{div}_h (Q_z \mathbf{v}_h - v_z \mathbf{Q}_h) = $$
$$= (\nabla p \times \nabla \alpha)_z + \text{curl}_z \mathbf{F}, \qquad (2.2)$$

where div_h represents the "horizontal" divergence (two-dimensional divergence in the geopotential surfaces).

The convective part $Q_z \mathbf{v}_h$ of the vorticity transport $\mathbf{C}$ (or $\mathbf{C}_h$) results from the advection of the instantaneous absolute rotation of the air about the vertical axis z and the non-convective part $-v_z \mathbf{Q}_h$, from the instantaneous rotation of the air flowing vertically through the geopotential surfaces. The non convective transport $-v_z \mathbf{Q}_h$ represents the transformation or conversion of horizontal vorticity $\mathbf{Q}_h$ into vertical Q_z. This transport vanishes and hence, the absolute vertical vorticity Q_z is simply advected in the geopotential surfaces, when the air moves horizontally $(v_z \equiv 0)$.

If N represents the external normal to the earth's surface S $(v_N \equiv 0)$, the transport of absolute vertical vorticity across the earth's surface is determined by $-v_z Q_N$. This transport is zero at all points where the earth's surface S coincides with a geopotential surface $(v_z \equiv 0)$, and therefore exists only in mountain regions $(v_z \neq 0)$. The vorticity transport $-v_z Q_N$ across the earth's surface represents the mountain effect [see also eq. (3.1″)] on the redistribution in the atmosphere of absolute vertical vorticity. It is readily seen from (2.1) that

$$-v_z Q_N \equiv Q_z (\mathbf{v}_h \cdot \mathbf{N}) - v_z (\mathbf{Q}_h \cdot \mathbf{N}),$$

where $\mathbf{N}$ designates the unit-vector of the N-axis. Hence, the mountain-effect on the distribution of the absolute vertical vorticity consists of advection $Q_z(\mathbf{v}_h \cdot \mathbf{N}) = -Q_z \cdot \cdot (\mathbf{v} \cdot \mathbf{N})$ of absolute vertical vorticity and conversion $-v_z (\mathbf{Q}_h \cdot \mathbf{N})$ of horizontal into vertical absolute vorticity at the earth's surface. With increasing slope of the mountains the advection term decreases whilst the conversion term becomes predominant. At the limiting case of a vertical wall the transport of vertical absolute vorticity across the wall is due only to conversion of horizontal into vertical vorticity.

The rate of production of the absolute vertical *vorticity* Q_z determined by the number of isobaric-isosteric solenoides and vortex unit-tubes of the frictional force per unit area in the geopotential surfaces, may be given the following expressions:

$$\left.\begin{aligned}(\nabla p\times\nabla\alpha)_z+\operatorname{curl}_z\mathbf{F}\equiv\operatorname{curl}_z(\mathbf{F}-\alpha\nabla p)\equiv\\ \equiv\frac{\alpha}{\Theta}(\nabla p\times\nabla\Theta)_z+\operatorname{curl}_z\mathbf{F}\equiv\\ \equiv f\,\mathbf{u}_h\,\frac{\nabla_h\Theta}{\Theta}+\operatorname{curl}_z\mathbf{F},\end{aligned}\right\}\quad(2.3)$$

where Θ represents the potential temperature of the air, $\mathbf{u}_h$ the geostrophic wind velocity, f the Coriolis parameter and ∇_h the horizontal "del" operator.

In the surface layer of the subtropical high pressure belt, $\operatorname{curl}_z\mathbf{F}>0$ and in the surface layer of the subpolar low pressure belt, $\operatorname{curl}_z\mathbf{F}<0$. Hence, at the earth's surface, there is a source region of (cyclonic) vorticity in the subtropical latitudes and a sink in the subpolar latitudes, (S. PETTERSSEN, 1950). The other term of the rate of production (2.3) depends upon the "horizontal" baroclinicity and is in fact rather small outside the frontal zones.

3. Integrating the equation of balance (1.2) over a volume τ at rest with respect to the earth ($\delta t\equiv 0$), one finds the equation of balance of the total absolute vertical vorticity $\iiint_\tau Q_z\delta\tau$ within the volume τ,

$$\frac{\partial}{\partial t}\iiint_\tau Q_z\delta\tau+\iint_\sigma(Q_z v_n-v_z Q_n)\delta\sigma=\zeta,\quad(3.1)$$

where n is the external normal to the boundary σ of the volume τ and where the rate of production ζ of the total absolute vertical vorticity within the volume τ has the following expressions, (2.3),

$$\left.\begin{aligned}\zeta\equiv\iiint_\tau[\nabla p\times\nabla\alpha)_z+\operatorname{curl}_z\mathbf{F}]\delta\tau\equiv\\ \equiv\iiint_\tau\operatorname{curl}_z(\mathbf{F}-\alpha\nabla p(\delta\tau\equiv\\ \equiv\iint_\sigma[\mathbf{n}\times(\mathbf{F}-\alpha\nabla p)]_z\delta\sigma\equiv\\ \equiv\iint_\sigma[f\mathbf{u}_h\cdot\mathbf{n}+(\mathbf{n}\times\mathbf{F})_z]\delta\tau,\end{aligned}\right\}\quad(3.2)$$

$\mathbf{n}$ being the unit-vector of the n-axis. As the volume integral expressing the production ζ reduces, by virtue of Green's theorem, to a surface integral over the boundary of the volume, the sources of the absolute vertical vorticity at all the interior points of the volume τ do not contribute to the production of the total absolute vertical vorticity within the volume τ, and hence, the rate of change of the total absolute vertical vorticity within the volume τ of air, at rest with respect to the earth, depends only on the state of atmospheric motion at the boundary σ of the volume τ, (see also section 6).

If the volume τ is multiply connected, the boundary σ is the "complete" boundary of τ. Moreover we have implicitly assumed that the functions under the integration symbols in (3.1) and (3.2) satisfy the required analytical conditions.

Considering the whole atmosphere instead of a volume τ of air, we obtain the equation of balance of the total absolute vertical vorticity of the atmosphere

$$\left.\begin{aligned}\frac{\partial}{\partial t}\iiint_{\text{atm.}}Q_z\delta\tau+\iint_S\{v_z Q_N+\\ +[\mathbf{N}\times(\mathbf{F}-\alpha\nabla p)]_z\}\delta S=0,\end{aligned}\right\}\quad(3.1')$$

or

$$\left.\begin{aligned}\frac{\partial}{\partial t}\iiint_{\text{atm.}}Q_z\delta\tau+\iint_S[v_z Q_N+(\mathbf{N}\times\mathbf{F})_z+\\ +f\mathbf{u}_h\cdot\mathbf{N}]\delta S=0,\end{aligned}\right\}\quad(3.1'')$$

provided the horizontal components of the acting forces $\mathbf{F}$ and $-\alpha\nabla p$ vanish at infinity with sufficient rapidity except when the external boundary is a geopotential surface (see section 6). There are no reasons to believe that this requirement is not fulfilled in the atmosphere. The three terms in square brackets in (3.1″) are identically equal to zero at all points where the earth's surface coincides with a geopotential surface. In the ideal case of an earth's surface S coinciding everywhere with a geopotential surface (the mean sea-level, for instance), the total absolute vertical vorticity of the atmosphere is constant in time. This constant is zero, [see formula (3.3)].

Thus, the *instantaneous* rate of change of the total absolute vertical vorticity of the atmosphere results from a mountain effect only. As stationary conditions prevail in the long run, the time average of the surface integral in (3.1″) for a sufficiently long time interval must be equal to zero.

The influence of the earth's surface S on the total absolute vertical vorticity of the atmosphere may be shown in another way. Recalling that the absolute vorticity $\mathbf{Q}$ is equal to the sum of the earth vorticity $2\boldsymbol{\omega}$ and the relative vorticity curl $\mathbf{v}$ of the air, one finds

$$\iiint_{\text{atm.}} Q_z \delta\tau \equiv \iiint_{\text{atm.}} 2\omega \sin\varphi\, \delta\tau + \iiint_{\text{atm.}} \operatorname{curl}_z \mathbf{v}\, \delta\tau,$$

or, applying Green's theorem,

$$\iiint_{\text{atm}} Q_z \delta\tau = \iiint_{\text{atm.}} 2\omega \sin\varphi\, \delta\tau + \iint_S (\mathbf{v} \times \mathbf{N})_z \delta S \quad (3.3)$$

where ω is the constant angular speed of the earth and φ the latitude. If the space occupied by the atmosphere is symmetrical with respect to the earth's axis and the plane of the equator, the volume integral in the second member of (3.3) is identically equal to zero. This double symmetry not being exactly realized, the value of $\iiint_{\text{atm.}} 2\omega \sin \varphi\delta\tau$ depends upon the shape of the earth's surface and is therefore a constant. It should be noted that

$$\iiint_{\text{atm.}} 2\omega \sin\varphi\, \delta\tau = \iint_S \omega r \cos\varphi (\mathbf{i} \times \mathbf{N})_z \delta S,$$

where r is the distance from the earth's centre of an arbitrary point of the earth's surface S and $\mathbf{i}$ the unit-vector tangent to the latitude circle and pointing from west to east at the point considered.

The surface integral in (3.3) is identically equal to zero if the earth's surface is a geopotential surface. This again is not truly so, and the total absolute vertical vorticity therefore depends upon the slope of the earth's surface and of the air motion at the earth's surface in those regions where the earth's surface differs from a geopotential surface (mountain regions).

4. Integrating now the equation of balance (2.2) over a horizontal area Σ (an area of a given geopotential surface), at rest with respect to the earth, we obtain

$$\frac{\partial}{\partial t}\iint_\Sigma Q_z \delta\Sigma + \int_s (Q_z v_\nu - v_z Q_\nu)\delta s = \int_s (F_s \delta s - \alpha\delta p), \quad (4.1)$$

where ν is the external normal to the boundary s of Σ in the geopotential surface considered. The integration sense along the closed curve s is counterclockwise (right-handed screw rule). It is well known that $\int_s -\alpha\delta p$ represents the number of isobaric-isosteric solenoides embraced by the horizontal closed curve s and that, in the *northern hemisphere*, $\int_s F_s\, \delta s$ is positive [production of vertical (cyclonic) vorticity] when the wind circulation is anticyclonic and, negative [destruction of vertical (cyclonic) vorticity] when the wind circulation is cyclonic along s. When stationary conditions are realized, a production of absolute vorticity flux $\iint_\Sigma Q_z \delta\Sigma$ through the horizontal area Σ (or what amounts to the same thing, a production of absolute velocity circulation along the horizontal closed curve s, boundary of Σ) must be compensated by a horizontal vorticity outflow $\int_s (Q_z v_\nu - v_z Q_\nu)\delta s > 0$ across s and a destruction of vorticity flux through Σ, by a horizontal vorticity inflow across s.

5. Assuming the geostrophic hypothesis and using the thermal wind equation, the absolute horizontal vorticity $\mathbf{Q}_h$ may be given the following expression, valid as a first approximation,

$$\mathbf{Q}_h \cong 2\boldsymbol{\omega}_h - \frac{g}{f}\frac{\nabla_h \Theta}{\Theta} \cong -\frac{g}{f}\frac{\nabla_h\Theta}{\Theta}, \quad (5.1)$$

where g is the acceleration of gravity. Substituting (5.1) in (2.1) we find a useful approximate expression for the transport of absolute vertical vorticity, namely:

$$\mathbf{C} \equiv \mathbf{C}_h \cong Q_z \mathbf{v}_h + \frac{g}{f} v_z \frac{\nabla_h \Theta}{\Theta}. \quad (5.2)$$

In an air current $\mathbf{v}_h$, vertical vorticity is advected with the air, but at the same time, vertical vorticity is also transported approximately in the direction of the horizontal temperature ascendent (roughly to the south, on the scale of the general circulation) where the air ascents ($v_z > 0$) or in the direction of the horizontal temperature gradient (roughly to the north, on the scale of the general circulation) where the air descends ($v_z < 0$). Hence, when in a horizontal layer the horizontal temperature gradient $(-\nabla_h\Theta)$ converges (cold pool), vertical (cyclonic) vorticity may accumulate as a consequence of convergence in the horizontal transport of vorticity, if the cold air descends. Similarly, when in a horizontal layer the horizontal temperature ascendent $\nabla_h\Theta$ converges (warm pool), vertical (cyclonic) vorticity may accumulate as a consequence of convergence in the horizontal transport of vorticity, if the warm air ascents.

6. More generally, *the rate of production of the absolute vorticity* $\mathbf{Q}$ per unit of volume is defined by the vector

$$\nabla p \times \nabla\alpha + \operatorname{curl} \mathbf{F},$$

and *the transport* by the antisymmetrical tensor

$$\mathbf{Q}\mathbf{v} - \mathbf{v}\mathbf{Q},$$

the component of which along the n-axis, $\mathbf{Q}v_n - \mathbf{v}Q_n$, determines the transport of the absolute vorticity $\mathbf{Q}$ in the direction n, (VAN MIEGHEM, 1951).

Consequently, the vorticity budget of a given volume τ of air, at rest with respect to the earth $(\delta t \equiv 0)$, assumes the form

$$\frac{\partial}{\partial t}\iiint_\tau \mathbf{Q}\delta\tau + \iint_\sigma (\mathbf{Q}v_n - \mathbf{v}Q_n)\delta\tau = \Omega, \qquad (6.1)$$

where the production Ω of the total absolute vorticity $\iiint_\tau \mathbf{Q}\delta\tau$ within the volume τ may assume the following expressions:

$$\left.\begin{aligned}\Omega &\equiv \iiint_\tau [\nabla p \times \nabla\alpha + \operatorname{curl} \mathbf{F}]\delta\tau \equiv \\ &\equiv \iiint_\tau \operatorname{curl}(\mathbf{F} - \alpha\nabla p)\delta\tau = \\ &= \iint_\sigma \mathbf{n} \times (\mathbf{F} - \alpha\nabla p)\delta\sigma,\end{aligned}\right\} \qquad (6.2)$$

use being made of Green's theorem. The equation of balance (6.1) – with (6.2) – is similar to a vector equation obtained by C. TRUESDELL (1948). One obtains Truesdell's equation by replacing the wind velocity $\mathbf{v}$ in (6.1) by the absolute wind velocity, the vector $\mathbf{F} - \alpha\nabla p$ by the absolute acceleration and the rate of change $\frac{\partial}{\partial t}$ with respect to the earth by the rate of change with respect to the absolute frame. Truesdell's conclusions may very easily be extended to the case of relative motion with respect to the earth, in particular the conclusion recalled in section 3.

It should be noted that the gravitational force does not appear in (6.2), this force being expressed by the gradient of a potential function (geopotential).

Finally, considering the whole atmosphere instead of an arbitrary volume τ of air, one finds the vorticity budget of the atmosphere

$$\frac{\partial}{\partial t}\iiint_{\text{atm.}} \mathbf{Q}\delta\tau + \iint_S [\mathbf{v}_s Q_N + \mathbf{N} \times (\mathbf{F} - \alpha\nabla p)]\delta S = 0, \qquad (6.1')$$

where $\mathbf{v}_S$ designates the wind velocity $\mathbf{v}$ at the earth's surface S. Hence, the total absolute vorticity of the atmosphere is a function of the state of air motion at the earth's surface only. It should be noted that the vector $\mathbf{v}_S$ is tangent to S, $(v_N \equiv 0)$.

The equation of balance (6.1') is valid provided that one assumes that the normal component $\mathbf{Q}v_n - \mathbf{v}Q_n$ of the tensor $\mathbf{Q}\mathbf{v} - \mathbf{v}\mathbf{Q}$ and the tangential component of the vector $\mathbf{F} - \alpha\nabla p$ upon a sphere concentric with the earth vanish more rapidely than the reciprocal of the square of the radius of the sphere, when the radius tends to infinity. If this requirement is not fulfilled, there exists a contribution from infinity to the vorticity budget of the atmosphere. In order to avoid this unrealistic circumstance we have assumed implicitly that the above condition is satisfied ipso facto, [see also eq. (3.1')].

7. Using general coordinates in a frame moving arbitrarily with respect to the absolute frame (VAN MIEGHEM, 1951), it is possible to generalize the equation (6.1) to the case of a volume τ moving with the new relative frame $(\delta' t \equiv 0)$. The same equation of balance is valid provided that one replaces the wind velocity $\mathbf{v}$ by the air velocity $\mathbf{w}$ with respect to the new frame and $\frac{\partial}{\partial t}$ by the time derivative $\frac{\partial'}{\partial t}$ in the same new frame. One obtains

$$\left.\begin{aligned}\frac{\partial'}{\partial t}\iiint \mathbf{Q}\delta\tau + \iint_\sigma(\mathbf{Q}w_n - \mathbf{w}Q_n)\delta'\sigma &= \\ = \iint_\sigma \mathbf{n} \times (\mathbf{F} - \alpha\nabla p)\delta'\sigma.&\end{aligned}\right\} \qquad (7.1)$$

Similarly, the equation of balance (4.1) may be extended. For instance, supposing that one of the new coordinates is the potential temperature Θ, we find the equation of balance

$$\frac{\partial'}{\partial t}\iint_{\Sigma} Q_n \delta'\Sigma + \int_s (Q_n w_\nu - w_n Q_\nu)\,\delta' s = \int_s F_s \delta' s, \tag{7.2}$$

where Σ is an area of an isentropic surface (Θ = constant), s the complete boundary of Σ, n the upward pointing normal to the isentropic surface, ν the external normal to the curve s in the isentropic surface, $Q_n \equiv \frac{\mathbf{Q}\cdot\nabla\Theta}{|\nabla\Theta|}$ the absolute vorticity component normal to the isentropic surfaces and $\mathbf{w}$ the air velocity in a relative frame moving with these surfaces $\left(w_n \equiv \frac{\dot{\Theta}}{|\nabla\Theta|}, \dot{\Theta}\right.$ being the individual time derivative of Θ).

In (7.2) the term $-\alpha\,\delta p$ has dropped out, $\alpha\,\delta p$ being an exact differential on an isentropic surface.

The transport

$$\mathbf{\Gamma} \equiv Q_n \mathbf{w} - w_n \mathbf{Q} \tag{7.3}$$

of the absolute vorticity component normal to an isentropic surface is evidently isentropic ($\Gamma_n \equiv 0$).

Employing *longitude, latitude, potential temperature*, and *time* as independent eulerian variables, it may be shown that the *zonal* (x), *meridional* (y) and *vertical* (z) components of the vorticity flux $\mathbf{\Gamma}$ assume respectively the forms:

$$\left.\begin{aligned}\Gamma_x &\equiv Q_n v_x - w_n Q_x\\ \Gamma_y &\equiv Q_n v_y - w_n Q_y\end{aligned}\right\}\mathbf{\Gamma}_h \equiv Q_n \mathbf{v}_h - w_n \mathbf{Q}_h, \tag{7.3'}$$

and

$$\Gamma_z \equiv (\nabla_h \Theta \cdot \mathbf{\Gamma}_h) : (-\Theta_z),$$

where v_x, v_y and Q_x, Q_y are the usual zonal and meridional components of the wind velocity $\mathbf{v}$ and the absolute vorticity $\mathbf{Q}$ and where Θ_z stands for $\frac{\partial\Theta}{\partial z}$.

The slope of an isentropic surface being small (10^{-3} to 10^{-2}), the isentropic transport $\mathbf{\Gamma}$ of the absolute vorticity component normal to the isentropic surfaces may be identified, for all practical purposes, with the horizonta component $\mathbf{\Gamma}_h$ of $\mathbf{\Gamma}$.

If s is a closed curve in an isentropic surface, along which the transport of Q_n takes place, $\Gamma_\nu \equiv 0$; hence the instantaneous rate of change of the absolute vorticity flux $\iint_{\Sigma} Q_n \delta\Sigma$ through an isentropic area Σ (or what amounts to the same thing of the absolute velocity circulation along the isentropic closed curve s) is due to a frictional effect only. In an inviscid fluid, the absolute vorticity flux through an isentropic area is independent of time t provided that there is no isentropic absolute vorticity transport across the complete boundary of the isentropic area. When moreover the processes are adiabatic ($w_n \equiv 0$), the absolute vorticity flux through an isentropic area is independent of time t when the boundary of the area is a streamline of the isentropic flow. In this simplified case, the Q_n-vorticity is simply advected in the isentropic surfaces along the streamlines of the isentropic flow.

Let us now consider an isentropic surface intersecting the mean sea level along a curve s entirely situated in the region of the westerlies, then $\int_s F_s \delta' s < 0$, and if stationary conditions prevail, the vorticity destruction along s must be compensated by an isentropic vorticity inflow across s, corresponding roughly to a horizontal vorticity transport to the north. If on the other hand s is entirely situated in the region of the tropical easterlies, $\int_s F_s \delta' s > 0$, and if stationary conditions again prevail, the vorticity production along s must be compensated by an isentropic vorticity outflow across s, corresponding roughly to a horizontal vorticity transport to the south. Hence the subtropical high pressure belt at the earth's surface, which is a source region of (cyclonic) vorticity (see section 2), is also a region of horizontal divergence of (cyclonic) vorticity transport, stationary conditions being realized. Similarly the subpolar low pressure belt at the earth's surface, which is a sink region of (cyclonic) vorticity, is also a region of horizontal convergence of (cyclonic) vorticity transport, stationary conditions being realized, (E. T. EADY, 1950 and S. PETTERSSEN, 1950).

8. From the point of view of the general circulation, it is of interest to consider the transport C_y of the absolute vertical vorticity Q_z along the meridians (positive y-axis to the north),

$$C_y \equiv Q_z v_y - v_z Q_y \cong Q_z v_y + \frac{g}{f} \frac{v_z}{\Theta} \frac{\partial \Theta}{\partial y}. \quad (8.1)$$

Let us first examine the advective meridional transport $Q_z v_y$ in *middle latitudes*. Recalling that Q_z is generally positive in the *northern hemisphere*, this transport is in a northerly direction to the east ($v_y > 0$) and southerly to the west ($v_y < 0$) of the upper air troughs in the westerlies. On the average along a latitude circle the advective meridional transport of vertical vorticity will be towards the pole if the vertical vorticity is larger to the east than to the west of the troughs, but towards the equator if, on the contrary, the vertical vorticity is larger to the west than to the east of the troughs in the westerlies. Apparently the first alternative occurs most frequently in the troposphere.

It is much more difficult to state precisely the sign of the non advective mean meridional transport along a latitude circle. As a rule $\frac{\partial \Theta}{\partial y} < 0$, and presumably, on the average, $v_z > 0$ to the east of the troughs in the westerlies and $v_z < 0$ to the west. Hence, the advective and the non-advective transports have generally opposite signs, but the non-advective transport is at least of one order of magnitude (very often even two orders of magnitude) less than the advective transport.

The non-advective transport $\frac{g}{f} \frac{v_z}{\Theta} \frac{\partial \Theta}{\partial y}$ is in a southerly direction to the east and northerly to the west of the upper air troughs. On the average, the *non*-advective meridional transport of absolute vertical vorticity across a latitude circle will be towards the pole when the meridional poleward gradient of potential temperature is larger to the west than to the east of the upper air troughs, but towards the equator if, on the contrary, the meridional poleward gradient of potential temperature is larger to the east than to the west of the troughs.

Generally, in deepening upper air troughs, the baroclinicity is larger to the east than to the west of the troughs and hence, the zonal mean *non*-advective transport of vorticity across a latitude circle is presumably towards the equator.

9. In the same way that formula (2.1) is approximately given by formula (5.2), we may also approximate the formula (7.3'),

$$\mathbf{\Gamma}_h \cong Q_n \mathbf{v}_h + \frac{g}{f} \frac{w_n}{\Theta} \nabla_h \Theta, \qquad (9.1)$$

where $Q_n \cong Q_z$, except in regions of high baroclinicity, and obtain for the non-advective term in (9.1) results similar to those of section 5 regarding the non-advective term of (5.2).

In an air current $\mathbf{v}_h$, absolute vorticity normal to the isentropic surfaces is advected with the air, but, at the same time, also transported approximately in the direction of the horizontal temperature ascendent (roughly to the south, on the scale of the general circulation) where the air is heated ($w_n > 0$) or in the direction of the horizontal temperature gradient (roughly to the north, on the scale of the general circulation) where the air is cooled ($w_n < 0$). Hence, when in a horizontal layer the horizontal temperature gradient ($-\nabla_h \Theta$) converges (cold pool), (cyclonic) vorticity normal to the isentropic surfaces may accumulate as a consequence of convergence in the horizontal vorticity transport, if the cold air cools down. Similarly, when in a horizontal layer the horizontal temperature gradient diverges (warm pool), (cyclonic) vorticity normal to the isentropic surfaces may accumulate as a consequence of convergence in the horizontal vorticity transport, if the warm air warms up.

Finally, it should be noted that the velocity components v_y (meridional wind component) and w_n (heating or cooling) have generally opposite signs in the upper air troughs and ridges. Hence the non-advective meridional transport of absolute vorticity normal to the isentropic surfaces, due to heating and cooling, reinforces, as a rule, the advective meridional vorticity transport.

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