The Cellular Solution of the Dynamical Equations and the Actual Perturbation of the Westerlies up to 16 km at Valentia

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Abstract

The cellular solution of the linearized hydrodynamical equations requires the existence of nodal surfaces for the perturbation of air density, pressure and velocity components. When a constant, which appears in the formulae giving the heights of the nodal surfaces, is determined from the observed value of one of them, namely, the first nodal surface of the vertical velocity, all the other heights of nodal surfaces agree within reasonable limits with the heights determined from actual observations by several investigators using very different methods. More than 400 upper wind ascents reaching at least 16 km made by radio wind or radar methods at Valentia Observatory, have been classified according to the sign of the change of pressure at 8 km during the 12 hours prior to the ascent, and average values of the meridional and zonal components of the wind have been computed. The difference at each level between the corresponding average components of the wind when the pressure has increased and when the pressure has decreased gives a good approximation to double the value of the horizontal perturbation of the Westerlies. Applied to this case, the cellular solution requires that the zonal component of the perturbation so determined should be zero at all levels while the meridional component should have nodal surfaces at 4 and 12 km. The observations show that the zonal component is the one having nodal surfaces at the required levels while the meridional component has a maximum value at about 9 km. Taking into account that the equations refer to a single-layer atmosphere and neglect the meridional gradient of temperature, the disagreement between observation and theory appears to be less significant than the confirmation of the existence of nodal surfaces which cannot be explained by the generally accepted exponential solution of the dynamical equations.

1. In the wave theory of atmospheric perturbations, the solution of the linearised dynamical equations leads to the integration of an equation of the second order with constant coefficients. Neglecting the case when the auxiliary equation of the second degree has only one root, the solution may be either exponential, when the roots are real, or cellular, when the roots are imaginary. In the simplest case of an isothermal atmosphere in equilibrium on a flat, stationary earth, the full discussion of the different types of solutions, and therefore of the different possible modes of motion, was given by V. BJERKNES et alia (1934, page 361 and following) who studied them by means of the Lagrangian form of the dynamical equations. The nature of the solution depends on the wave length and period of the waves, the type of transformation the air undergoes, and the height of the homogeneous atmosphere.

In a previous paper the present author (1944) called attention to the fact that for waves of period of the order of one day and greater and wavelength of a thousand kilometres and longer, i.e. for periods and wave lengths of the kind observed in large-scale motion of the actual atmosphere, the type of solution that obtains is of the cell type. In the case of motion in two dimensions the solution of the Eulerian equations is:

$$u = C_0 e^{\frac{z}{2H}} \frac{\mu\gamma g}{\nu^2 - \gamma g H \mu^2} \left[\left(\frac{1}{2} - \frac{1}{\gamma} \right) \sin \Theta z + \right.$$
$$\left. + H\Theta \cos \Theta z \right] \sin (\mu x - \nu t) \qquad (1\text{ a})$$

$$w = C_0 e^{\frac{z}{2H}} \sin \Theta z \cos (\mu x - \nu t) \qquad (1\text{ b})$$

$$p = C_0 e^{-\frac{z}{2H}} \frac{\gamma g \nu \bar{\varrho}_0}{\nu^2 - \gamma g H \mu^2} \left[\left(\frac{1}{2} - \frac{1}{\gamma} \right) \sin \Theta z + \right.$$
$$\left. + \Theta H \cos \Theta z \right] \sin (\mu x - \nu t) \qquad (1\text{ c})$$

$$\varrho = C_0 e^{-\frac{z}{2H}} \frac{\nu \bar{\varrho}_0}{(\nu^2 - \gamma g H \mu^2) H} \left[\left\{ \frac{\mu^2}{\nu^2} g H (\gamma - 1) - \right. \right.$$
$$\left. \left. - \frac{1}{2} \right\} \sin \Theta z + \Theta H \cos \Theta z \right] \sin (\mu x - \nu t) \qquad (1\text{ d})$$

where u and w are respectively the zonal and vertical components of the air velocity with their usual signs; p and ϱ are the perturbation of the pressure and air density; $\bar{\varrho}_0$ the undisturbed surface air density; $\mu = \frac{2\pi}{\lambda}$ and $\nu = \frac{2\pi}{\tau}$ where λ and τ are the wave length and period of the waves, respectively; γ is the ratio of specific heats for air; H is the height of the homogeneous atmosphere; x, z, t, and g have the usual meanings, and

$$\Theta = \sqrt{\frac{\mu^2}{\nu^2} \frac{g}{H} \frac{\gamma - 1}{\gamma} + \frac{\nu^2}{\gamma g H} - \frac{1}{4H^2} - \mu^2}$$

2. The interest of the cell-type solution above is due to the fact that equations (1) show the existence in the atmosphere of nodal and antinodal horizontal surfaces at heights which are dependent on the internal parameters of the atmosphere and on the wave length and period of the motion. The existence of certain nodal (and antinodal) surfaces in the atmosphere has been a well established fact for nearly half a century; evidence of the existence of other nodal surfaces has been accumulated in the last ten years. The importance of this solution is enhanced by the fact that if the value of the constant Θ is determined by means of the height of one of the nodal surfaces, the heights of the others can be computed from the equations (1) and the values so computed agree reasonably well with the heights determined experimentally. Thus —

(a) From equation (1 b) it is deduced that the amplitude of the vertical component of motion is zero for values $z_w = \frac{n\pi}{\Theta}$ where $n = 0, 1, 2 \ldots$ For $n = 0$ this formula re-states the boundary condition.

Nearly forty years ago SHAW (1914) from three days' sounding-balloon observations in Ireland and Great Britain found that the air moves horizontally at a height estimated to be about 8 km. PALMÉN (1932), from a consideration of potential temperatures, deduced that in cases of polar air filling the troposphere the air moved horizontally at about 8 km. The AUTHOR (1943 b, 1952) has shown that if the effects of horizontal advection are neglected it is possible to compute the vertical displacement of the air at different levels from average values of the local variations of pressure and temperature. The method was applied to observations made with radiosondes in 1939 at Thorshavn, and led to the result that there were two surfaces of horizontal motion, one at $z = 8.91$ km, the other at about 16 km. The effect of horizontal advection, which had been neglected, was not expected to be very large especially at the 8—9 km level. The same method applied to radiosonde observations at Sault Ste. Marie and to Valentia gave for the height of the first surface of horizontal motion average values of about 9 km and 9.5 km respectively. FAUST (1952) using radiosonde data at different pressure-levels found that horizontal motion obtains at the level of 240—280 mbs.

From the height of the lowest layer of horizontal motion found at Thorshavn we find

$$\Theta = 3.53 \cdot 10^{-6} \text{ c.g.s.}$$

and therefore the height of the second layer at which horizontal motion should be found is 17.82 km. The estimated value at Thorshavn was about 16 km.

(b) It is also deduced from (1 b) that the vertical component of motion reaches maximum values of opposite signs at 4.46 and 13.37 km.

The vertical displacements of the tropopause and the cloud formation and subsidence in the troposphere support to some extent these results.

(c) From equation (1 d), and giving to the constants the values appropriate to Thorshavn, the amplitude of the perturbation of the air density is zero at heights given by

$$z_\varrho = 8.20 + 8.91\ n\ \text{km}\ (n = 0,\ 1,\ 2\ldots) \quad (2)$$

Therefore, the first or tropospheric layer would occur at 8.20 km.

WAGNER (1910) was the first to give evidence of the existence of an isopycnic layer at about 8 km from sounding-balloon observations over Germany. SEN (1924) computed the air density in July at different levels from the normal surface isobars and isotherms, assuming the lapse rate to be the same over land and over sea at all latitudes and heights, and found that the air density at 8 km was practically constant all over the world. DOPORTO (1943 a) defined the tropospheric, or first isopycnic layer, as the level at which the standard deviation of air density at the different levels from the respective average is a minimum and found that the Thorshavn data utilised gave a very well defined minimum at 7.80 km. The same results were obtained for Sault Ste. Marie (DOPORTO, 1943 b) and Valentia (DOPORTO and MORGAN, 1947), etc.

On the basis of a discussion of the statical equation the existence of a stratospheric isopycnic layer at an estimated height of 25 km was suggested by DOPORTO (1943 a). This isopycnic layer, found at about 24 km by GOLDIE (1947) corresponds to the value obtained from equation (2) for $n = 2$, namely 26.02 km.

(d) From equation (1 c), it is deduced tha the amplitude of the pressure perturbation should be zero at heights given by

$$z_p = 4.24 + 8.91\ n\ \text{km}\ (n = 0,\ 1.2\ldots)$$

DINES (1919) anticipated the existence of a layer of constant pressure at 18 or 20 km. For $n = 2$ the above formula gives $z_p = 22.06$ km.

(e) It is also deduced from equation (1 c) that the variability of pressure should have maxima at the surface and at 8.70 km.

Observations show that the interdiurnal variation of pressure is maxima at the surface and about 8 km (HANN-SÜRING, 1938, page 273).

(f) Equation (1 a) gives for the heights where the amplitude of the horizontal component of the perturbed motion vanishes

$$z_u = 4.24 + 8.91\ n\ \text{km}\ (n = 0,\ 1,\ 2\ldots)$$

which is the same as in (c).

At the heights given by this formula, namely 4.24 for $n = 0$ and 13.15 km for $n = 1$, horizontal divergence should be zero with divergence of opposite signs above and below them. Such a distribution of divergence was found with nodal surfaces at about 4 and 12 km for Saul, Ste. Marie and 7 and 11 km for Thorshavn (DOPORTO, 1943 b) and Valentia (DOPORTO, 1952). FAUST (1952) found that the levels of zero divergence occur to about 500 and 180 mb. In recent literature to which the author has had access it is assumed, on the basis of general considerations, that horizontal divergence is zero at about the 600 mb level (4 km) and 12 km.

3. A similar structure for the arrangement of nodal surfaces was found by the AUTHOR (1944) for the case of an atmosphere in equilibrium with constant lapse rate in the case of a flat, non-rotating earth. The equations giving u, w, p, and ϱ contain, instead of sines and cosines, Bessel functions of a function of z; and by taking four roots, besides the root zero, in the case of the equation giving w, and making the fourth root coincide with the surface and the zero-root with the top of the atmosphere, values for the heights of the nodal surfaces of all variables were obtained which differ but little from the values given in (a) to (f) above.

In the same paper the present author discussed the cases of (i) isothermal atmosphere moving with uniform horizontal velocity over flat non-rotating earth; (ii) atmosphere with constant lapse rate in a similar case; (iii) isothermal atmosphere in equilibrium on a flat, rotating earth; (iv) atmosphere with constant lapse rate in a similar case; (v) isothermal atmosphere moving with uniform velocity over flat rotating earth and (vi) atmosphere with constant lapse rate in a similar case. The solutions differ from those discussed above in that (1) the rotation of the earth does not allow for cellular motion except for periods shorter than half the pendulum day; (2) the rotation of the earth introduces a separation of the nodal surfaces of pressure and of the horizontal components of the perturbed velocity but for wave-lengths of 1,000 km or greater and for periods shorter, but of the order of the half-pendulum day, the distance between the nodal surfaces for the different elements is negligible; and (3) the solutions in the case of an atmosphere with uniform horizontal zonal motion, referred to axes moving with the undisturbed air, are the same as those for the corresponding case of an atmosphere in equilibrium. This last result makes it possible for an observer stationary on the ground to register cellular motion of an apparent period greater than the half-pendulum day if the waves move, with reference to axes fixed in the unperturbed air, in a direction opposite to the movement of the axes relative to the observer.

There are several other aspects of the cellular solution of the dynamical equations which may provide explanations to other observed facts (DOPORTO, 1944), such as the variation of the tropopause height between 8 and 17 km, etc. but there are also some serious difficulties to overcome before the cellular solution can account for all the dynamical aspects of actual atmospheric perturbations.

4. For the purpose of this discussion the cellular solution for the pressure and wind perturbations may be represented approximately by the following equations:

$$u = A \cos \Theta z \sin (\mu x - \nu t) \quad (3\text{ a})$$
$$v = -B \cos \Theta z \cos (\mu x - \nu t) \quad (3\text{ b})$$
$$w = C \sin \Theta z \cos (\mu x - \nu t) \quad (3\text{ c})$$
$$p = D \cos \Theta z \sin (\mu x - \nu t) \quad (3\text{ d})$$

where A, B, C, and D may be considered as positive constants. (The complete solution is given in equations (58) of DOPORTO, 1944.)

Equation (3 d) shows that for $\Theta z = \frac{\pi}{2}$, i.e. at about 4 and 12 km the perturbed pressure should be zero, and that if an anticyclonic situation obtains at the surface there should be a cyclone above it with maximum amplitude at 8 km, the situation reverting to anticyclonic between 12 and 20 km with maximum amplitude at 16 km. This, of course, is not found in the atmosphere. If it is remembered that to reduce the difficulties of the mathematical problem the meridional gradient of temperature has been taken as zero; and that the sloping tropopause has been ignored and the atmosphere treated as a single layer, either isothermal or with lapse rate, the non-existence of the nodal surfaces at 4 and 12 km required by the equation is qualitatively explained. From a consideration of the values of the constants in the complete formula the author indicated that the amplitude of the perturbed pressure should be of the order of a few millibars only. If this is correct, the theoretical opposition of phases of the perturbed pressure due to the dynamics of the motion within the stratum 4—12 km and above and below it would be masked by the statical change of pressure due to the meridional displacements of the atmosphere as a whole, which at the surface is generally greater than 10 mb.

Another difficulty arises from the signs of the coefficients A, B, and D of equations (3). In effect, the motion of the air in throughs and ridges of pressure (or around cyclones and anticyclones), resulting from the combination of signs of A, B, and D in eqs. (3) is contrary to that observed in the actual atmosphere. The horizontal divergence as computed by DOPORTO (1944, 1952) and FAUST (1952), although in agreement with the cellular arrangement, is of opposite sign to that deduced from equations (3).

5. Equations (3 a) and (3 b) indicate that about 4 and 12 km the components u and v of the perturbation change their sign. This is a requirement of the cellular motion which, like those regarding the distribution of divergence and vertical velocity, does not appear in the exponential solution of the differential equations. It has been considered of interest to investigate whether the wind observations in the free atmosphere are in agreement with the theoretical variation with height of the zonal and meridional components of the perturbation of the westerlies.

For this purpose the upper winds observed at Valentia were the only homogeneous mass of data available over a sufficiently long period to give averages which might be of statistical significance.

The upper wind observations utilised in this investigation were carried out with Rawin equipment No. SCR-658 on loan from the United States between 1/10/46 and 30/6/48 and from 1/7/48 up to 30/9/50 with British radar equipment A.A. No. 3 MK. 2.

The standard daily schedule of observations with the Rawin equipment was two ascents, launched at 10.45 and 22.45 G.M.T. up to 1/1/48 and at 02.30 and 14,30 since that date. However, due to shortage of hydrogen and transmitters there were many breaks in the schedule which could only be maintained by producing hydrogen locally and using transmitters made by the staff of the Observatory. Shortage of hydrogen forced this schedule to be reduced, except in a few cases, to one ascent per day launched at 14.30 G.M.T. as from 30/6/48 when the Radar equipment came into operation. When, due to weather conditions it was considered impracticable to make a combined radiosonde and radiowind launching, an assumed rate of ascent of about 1,400 m/m determined experimentally was used for the computation of the wind.

The computations were in all cases made graphically and the measured wind corresponds to averages of overlapping layers of the order of 800 metres.

All the ascents made between 1/10/46 and 30/9/50, i.e. during four years, were scrutinized and those not reaching 16 km rejected. Also rejected were those ascents which gave winds considered doubtfull by the observer at the time of the computation. These doubtful observations are due mainly to ground reflection of the electromagnetic waves, which occur when the angle of elevation is small and therefore are more frequent with strong winds and for the upper levels of the ascents. The minimum angle of elevation at which reliable observations can be made is never less than 12° for radiowind and 6° for radarwind. The presence of hills up to 300 ms high towards the ESE in the neighbourhood of the observatory raises this minimum angle of elevation in that direction to 20° for radiowind and 12° for radarwind. For each ascent not rejected the direction in degrees true and velocity in knots at 1, 2 ... etc. km was converted by means of tables to meridional and zonal components.

To find the average components of the perturbation the ascents were then classified in accordance with the only other meteorological variable, namely the pressure, which can give an indication in this respect. An ascent made when the pressure is increasing (decreasing) will, generally speaking, give the vertical distribution of wind in the part of the atmosphere situated between a line of trough (ridge) and the succeeding line of ridge (trough). The facts that the trajectory of the balloon is not vertical, the observation at different heights not simultaneous, and the lines of troughs (ridges) at different levels not in the same perpendicular plane, are here ignored. The average values computed for each of the two groups of ascents given by this form of classification, may be taken as representing the average wind components in the neutral points between a trough and the succeeding ridge, and a ridge and the succeeding trough.

The variation of atmospheric pressure used for the classification of the ascent may, in principle, refer to any level and it would appear that surface pressure values, which are known continuously and with greater accuracy than at any other level, are the most appropriate. However, changes of pressure at the surface are often not representative of the real phase of a perturbation in the atmosphere, due to the influence of erratic, advective changes in the lower layers. Further the writer has shown (1952) that the maximum amplitudes of the average interdiurnal variations of pressure and temperature are greater when the ascents are classified according to the sign of the pressure variation at 8 km and it was expected that this statistical effect would also be operative in the case of wind perturbations. Therefore the variation of pressure at 8 km (to be precise 8 gkm before 1/1/50 and 8 gpkm afterwards) has been used for the classification, although this procedure has resulted in the non-utilisation of some ascents due to lack of pressure data.

The selection of the 8 km level pressure as a basis for the classification introduces also a limitation in the choice of the time interval to which the variation of pressure refers. In effect, radiosonde ascents are made only every twelve hours at Valentia, normally simultaneously with, or in some few cases at very short interval before or after, the radio-wind ascent. It has been considered that to classify the wind data at hour H according to the variation of pressure from $H-12$ hours to

$H+12$ hours, would result in classifying together ascents which in many cases would belong to different halves of the perturbation, due to the short lag with which wind follows the changes of pressure. For this reason it was decided to use the variation of pressure between H and $H-12$ hours.

The values of the pressure at 8 km used for computing the 12-hour variations were obtained by means of Bjerknes' tables and rounded off to whole millibars.

At first only the ascents between 1/10/46 and 31/12/49 were used for this investigation. When they were classified in accordance with the sign of the difference $P_8^{(H)} - P_8^{(H-12)} = \triangle P_8$ between the 8 km pressures at the time of the ascent and 12 hours earlier, it was found that the number of ascents made with increasing pressure was 167 while there were only 85 made with decreasing pressure, i.e. there were approximately twice as many ascents in the group $\triangle P_8 > 0$ as in the group $\triangle P_8 < 0$ (Table 1). This may be due to the wave form of the perturbation having a quick fall and a slow increase of pressure, but it also may be due to a systematic positive error in the values of P_8 during the daytime as a consequence of insufficient ventilation of the temperature element of the radiosonde, and a preponderance of radiowind day ascents (239 including 27 when $\triangle P_8 = 0$) over night ascents (48, including 8 when $\triangle P_8 = 0$).

It was decided then to extend the investigation to cover 4 complete years of observations by considering the ascents made between 1/1/50 and 30/9/50. In this period only day ascents were made, and the proportion of ascents in the groups $\triangle P_8 > 0$ and $\triangle P_8 < 0$ was found to be nearly 3 to 1, i.e. even greater than in the 39 months from 1/10/46 to 31/12/49 considered before.

However, the number of night ascents (21) in the group $\triangle P_8 > 0$ is practically equal to the number of night ascents (19) in the group $\triangle P_8 < 0$ which does not support the hypothesis of a systematic error introducing too large values of P_8 during the daytime.

It is not possible, therefore, to give a satisfactory explanation of the distribution of ascents in the groups $\triangle P_8 > 0$ and $\triangle P_8 < 0$, but as explained in paragraph 7 this distribution is believed not to invalidate the results of the investigation.

6. Table I shows that, in the period of four years to which the investigation refers, there were 405 ascents reaching at least the 16 km level and for which pressure data at 8 km were available for making the classification, and that of these 405 ascents 247 were made when the variation of pressure during the last 12 hours had resulted in an increase at 8 km, 109 when it has resulted in a decrease and 49 when the pressure has not changed. Table II shows the number of selected ascents in each month and season (Winter = December to February, etc.).

The average values in knots of the zonal and meridional component of the wind up to 20 km for the groups $\triangle P_8 > 0$, $\triangle P_8 = 0$ and $\triangle P_8 < 0$ are given in Table III, which refers to the periods 1/10/46 to 31/12/49, and Table IV, which refers to the four-year period 1/10/46 to 30/9/50. From these data Figures 1 and 2, giving the hodographs of the average wind up to 15 km, have been drawn. In these figures the vector wind would be re-

Table I. Number of day and night ascents in the different pressure groups of the classification

	Day ascents			Night ascents			Total		
	1/10/46 to 31/12/49	1/1/50 to 30/9/50	Total	1/10/46 to 31/12/49	1/1/50 to 30/9/50	Total	1/10/46 to 31/12/49	1/1/50 to 30/9/50	Total
$P_8 > 0$	146	80	226	21	0	21	167	80	247
$P_8 = 0$	27	14	41	8	0	8	35	14	49
$P_8 < 0$	66	24	90	19	0	19	85	24	109
Total	239	118	357	48	0	48	287	118	405

Table II. Monthly and seasonal distribution of ascents

Month	J	F	M	A	M	J	J	A	S	O	N	D	Total
Number of ascents	27	23	38	18	54	45	30	37	18	49	39	27	405

Season	Spring	Summer	Autumn	Winter	Total
Number of ascents	110	112	106	77	405

presented, following the meteorological convention, by a straight line drawn from a point of the hodograph to the centre of coordinates. Data up to 20 km have been included in both Tables to illustrate the effect on the average values for the upper four kilometres of a decreasing number of ascents reaching those heights. These data will not be utilized in the discussion. The comparison between the data of Table III and Table IV is also useful in order to estimate the confidence which may be placed in the conclusions derived from them.

The average values of the 12-hour variation of pressure are as follows:

	Period 1/10/46 to 31/12/49	Period 1/10/46 to 30/9/50
$\triangle P_8 > 0$	3.5 mb	3.5 mb
$\triangle P_8 < 0$	3.0 mb	2.9 mb

In Table V and Figure 3 the difference at each height between the respective average wind component of the groups $\triangle P_8 > 0$ and $\triangle P_8 < 0$ for the 39 month period 1/10/46 to 31/12/49 are given. Table VI and Figure 4 refer similarly to the 4-year period 1/10/46 to 30/9/50.

If U and V are the average undisturbed wind components at any level, the average values given in Table III (and Table IV) may be represented by

$$U+u \text{ and } V+v \text{ for } \triangle P_8 > 0$$
$$U \text{ and } V \text{ for } \triangle P_8 = 0$$
$$U-u \text{ and } V-v \text{ for } \triangle P_8 < 0$$

where, as before, u and v stand for the perturbation. Therefore the difference between the average values of the wind components

Table III. Average wind components in knots when the pressure variation at 8 km during the last 12 hours has been positive, zero or negative

Valentia. 1/10/46 to 31/12/49

Height km	$\triangle P_8 > 0$			$\triangle P_8 = 0$			$\triangle P_8 < 0$		
	No. of Ascents	$U+u$	$V+v$	No. of Ascents	$U+u$	$V+v$	No. of Ascents	$U+u$	$V+v$
Surface	167	−1.28	1.80	35	−2.42	1.16	85	−0.89	2.75
1	167	−1.28	4.94	35	−3.75	4.95	85	0.24	5.52
2	167	1.96	4.07	35	−1.29	4.41	85	2.31	4.70
3	167	4.72	2.68	35	0.68	2.62	85	4.34	3.78
4	167	5.70	2.46	35	2.06	1.66	85	5.89	5.65
5	167	6.33	2.19	35	2.51	1.95	85	5.67	6.69
6	167	7.38	1.45	35	4.35	2.75	85	5,27	6.58
7	167	9.10	−0.59	35	4.35	2.67	85	7.22	6.51
8	167	10.16	−1.48	35	3.75	1.54	85	7.97	7.53
9	167	12.10	−5.35	35	7.16	0.51	85	8.72	5.89
10	167	13.42	−7.18	35	10.09	−0.09	85	10.83	4.47
11	167	14.21	−7.17	35	11.10	−0.65	85	13.08	3.59
12	167	14.98	−6.98	35	14.50	−0.30	85	14.84	1.99
13	167	15.17	−5.99	35	15.55	−1.17	85	15.78	0.32
14	167	13.61	−4.82	35	16.54	0.06	85	15.61	−0.16
15	167	13.01	−2.49	35	14.79	−1.64	85	15.47	−0.68
16	167	12.17	−2.83	35	12.33	−2.78	85	14.31	0.43
17	126	9.72	−2.47	24	13.79	−2.82	59	13.29	−2.47
18	90	8.90	−2.34	16	15.75	−2.62	40	11.46	−1.00
19	66	7.66	−2.76	7	10.53	1.87	23	12.28	−2.03
20	35	5.88	−1.42	5	3.34	1.66	9	9.03	−4.30

Table IV. Average wind components in knots where the pressure variation at 8 km during the last 12 hours has been positive, zero or negative

Valentia. 1/10/46 to 30/9/50

Heights km	$\triangle P_8 > 0$			$\triangle P_8 = 0$			$\triangle P_8 < 0$		
	No. of Ascents	$U+u$	$V+v$	No. of Ascents	$U+u$	$V+v$	No. of Ascents	$U+u$	$V+v$
Surface	247	—0.05	1.92	49	—0.97	1.95	109	—0.43	2.53
1	247	0.51	4.21	49	—1.50	4.20	109	1.55	4.81
2	247	3.55	3.79	49	1.68	5.50	109	3.43	3.93
3	247	6.36	2.61	49	3.83	3.82	109	5.74	2.95
4	247	7.71	2.57	49	5.61	2.86	109	7.27	4.24
5	247	8.67	2.06	49	5.86	2.96	109	6.88	5.16
6	247	9.51	1.37	49	7.85	3.14	109	6.98	4.96
7	247	11.26	0.20	49	8.44	3.28	109	8.71	5.29
8	247	12.72	—1.03	49	9.29	2.19	109	9.85	5.81
9	247	14.58	—4.31	49	12.88	1.43	109	11.27	3.79
10	247	15.90	—6.25	49	14.72	1.83	109	13.17	2.35
11	247	16.35	—6.11	49	15.39	1.25	109	14.85	1.50
12	247	16.63	—5.53	49	17.55	1.31	109	16.17	1.00
13	247	16.02	—4.41	49	18.31	0.88	109	16.77	—0.23
14	247	14.25	—3.35	49	17.76	2.09	109	15.58	—0.60
15	247	13.04	—1.66	49	15.30	0.45	109	14.61	—0.86
16	247	12.19	—2.06	49	13.46	—0.09	109	13.31	—0.04
17	195	10.20	—1.06	37	14.49	—0.84	80	12.15	—2.11
18	139	8.69	—1.38	26	13.28	—1.53	57	10.11	—0.73
19	105	7.64	—2.25	11	10.58	1.88	34	9.65	—1.52
20	55	4.19	—0.60	6	12.17	1.95	17	4.46	—2.10

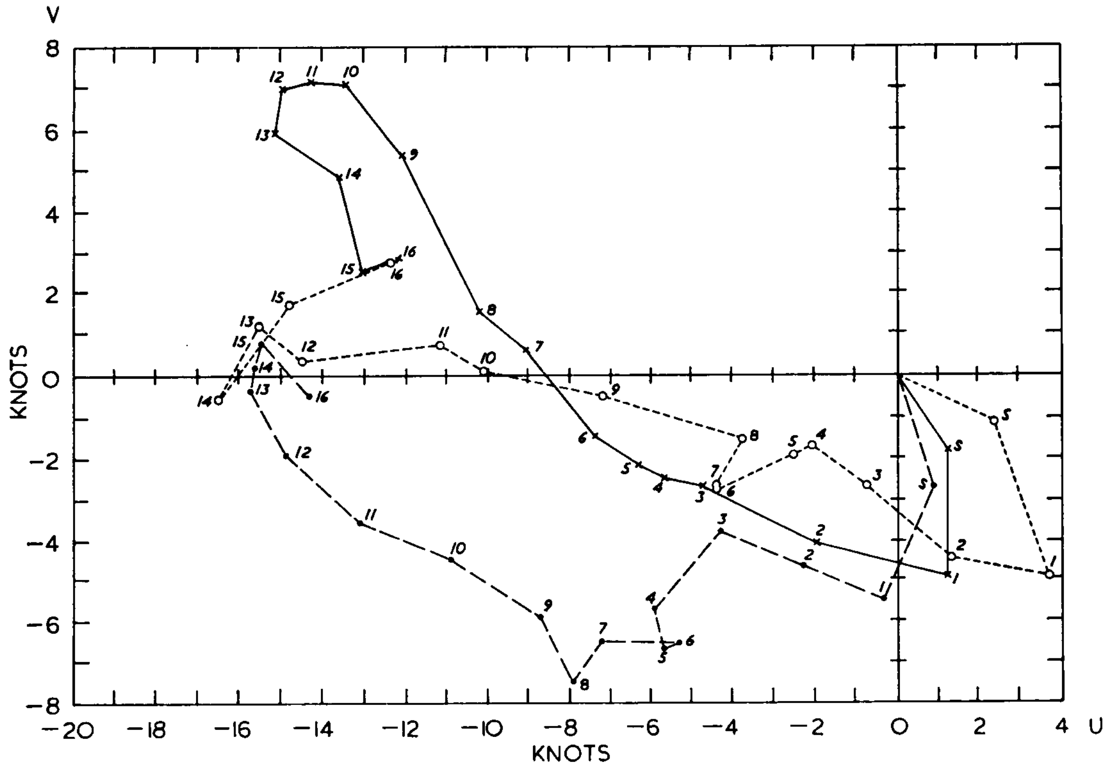

Fig. 1. Hodograph of the average wind when $\triangle P_8 > 0$ (×), $\triangle P_8 = 0$(°) $\triangle P_8 < 0$(·). Valentia Observatory. Period 1/10/46 to 31/12/49. (The vector wind at any height is represented by a straight line drawn from the corresponding point of the hodograph to the origin of coordinates.)

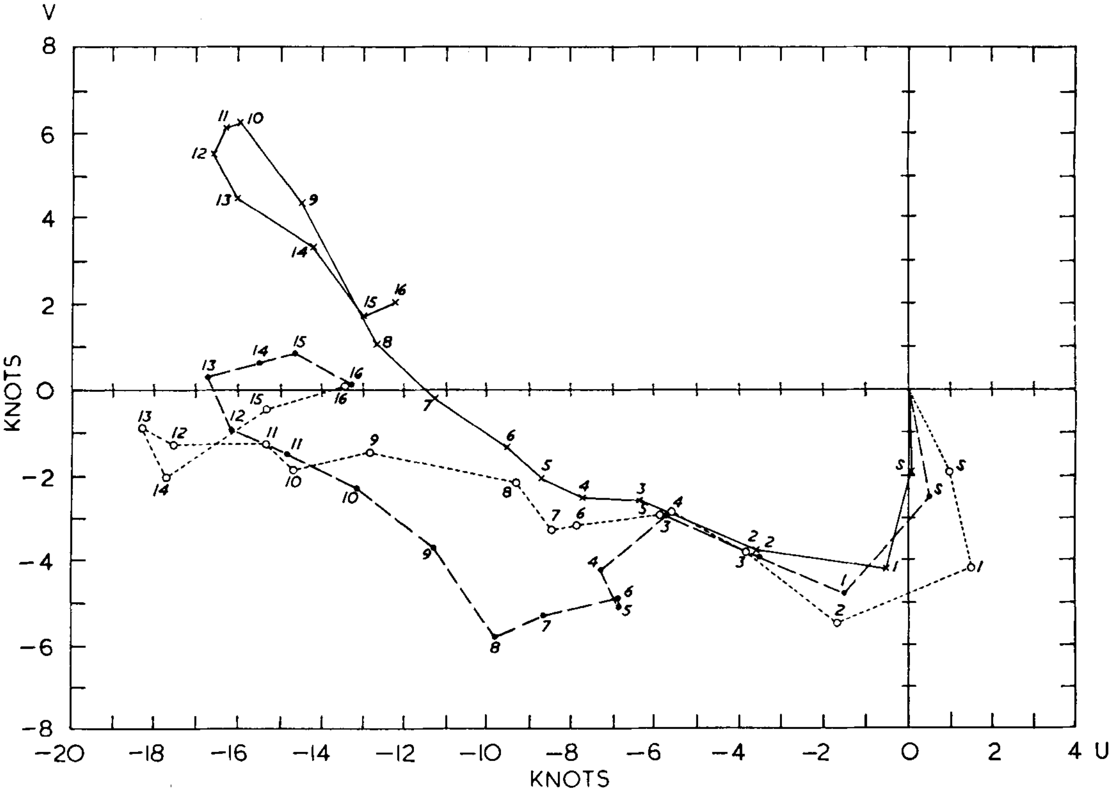

Fig. 2. Hodograph of the average wind when $\triangle P_8 > 0$ (×), $\triangle P_8 = 0$ (°) and $\triangle P_8 < 0$ (·). Valentia Observatory. Period 1/10/46 to 30/9/50. (The vector wind at any height is represented by a straight line drawn from the corresponding point of the hodograph to the origin of co-ordinates.)

for the groups $\triangle P_8 > 0$ and $\triangle P_8 < 0$ gives $2u$ and $2v$ (Tables V and VI and Figures 3 and 4). The relative smallness of the values of U and u is due to the occasional existence of closed circulations, both cyclonic and anticyclonic, at all levels, i.e. the Westerlies at the latitude of Valentia are replaced at times by Easterlies and the algebraic average values of the zonal wind component and of its perturbation are reduced accordingly.

Tables III and IV and Figures 1 and 2 show that the average values of the components for the group $\triangle P_8 = 0$ do not fall as they should between the corresponding values for the groups $\triangle P_8 > 0$ and $\triangle P_8 < 0$. Apart from the large difference between the number of ascents from which the averages have been computed, it is necessary to remember that periods of 39 and 48 months are not long enough to obtain for most meteorological observations average values which may be considered as good approximations to normal values.

For the purpose of this investigation, however, the values of the wind at different levels are not as important as the differences between corresponding wind components at the same level for the groups $\triangle P_8 > 0$ and $\triangle P_8 < 0$ and the variation of these differences with height. Although the corresponding values of these differences given in Tables V and VI show in some cases large variations the general form of the curves in figures 3 and 4 are fairly similar. Only at the lower levels, where the wind circulation is affected by the rugged topography of the Kerry Mountains is the agreement not so good. At other levels the differences between the curves are due, not only to the different number of ascents used for the computation of the average values of the components in the two groups for both periods, but also to the error inherent to the

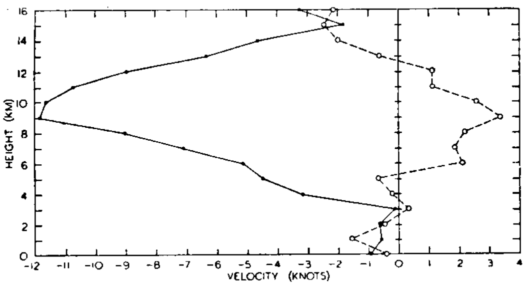

Fig. 3. Variation with height of twice the meridional (·) and zonal (°) components of the vector difference between average winds when $\triangle P_8 > 0$ and $\triangle P_8 < 0$. Valentia Observatory. Period 1/10/46 to 30/12/49.

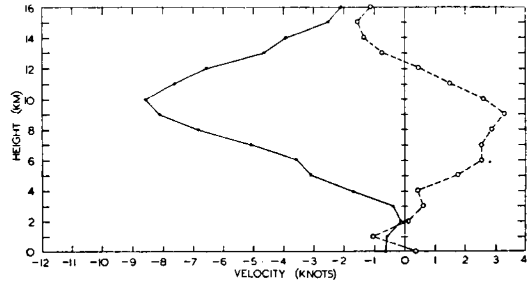

Fig. 4. Variation with height of twice the meridional (·) and zonal (°) components of the vector difference between average winds when $\triangle P_8 > 0$ and $\triangle P_8 < 0$. Valentia Observatory. Period 1/10/46 to 30/9/50.

method of observation. Whatever the claims made regarding the accuracy of the observations by Rawin and radar methods it appears that the average error of individual observations made at Valentia may be in both techniques as large as 5 knots for each component. If this figure is adopted, the average value of the error for the average component would be about 0.4 knots for the $\triangle P_8 > 0$ group and about 0.6 knots for the group $\triangle P_8 < 0$ and, therefore, about 0.7 knots for the differences.

It appears from Figures 3 and 4, and from the above considerations regarding the average errors that

(1) The zonal perturbation, u,
   (a) has opposite directions above and below about 12 km.
   (b) decreases the westerlies, above 12 km and reinforces them below 12 km when the pressure at 8 km in the previous 12 hours has increased. The reverse happens when the pressure has decreased.
   (c) has a zero value at about 4 km and
   (d) has below 4 km the opposite sign to that in the stratum 4—12 km.

(2) The meridional perturbation, v,
   (a) has a maximum value at about 9 km.
   (b) is very small between the surface and 3 km.
   (c) is directed towards the South when the pressure at 8 km has increased during the previous 12 hours. The reverse happens when the pressure has decreased.

7. If the large difference of the number of ascents classified in the groups $\triangle P_8 > 0$ and $\triangle P_8 < 0$ is due to the form of the wave length of the perturbation of pressure at 8 km these results need not be modified. If, as discussed before, this difference were due to a systematic error in the observed pressure at 8 km during daytime, the classification of the ascents into the three groups would no longer be valid; the conclusions, however, would, most likely, be an understatement of the true facts. In effect the averages of the group $\triangle P_8 > 0$ for instance would be based not only on ascents belonging in fact to this group but also on others which more properly would belong to the groups $\triangle P_8 = 0$ and $\triangle P_8 < 0$; therefore the values obtained for the perturbation would be less than the actual values and it would only be necessary to change slightly the heights at which the zonal perturbation is zero.

These observed values of the component of the perturbation of the Westerlies at Valentia show that the exponential solution of the dynamical equations fails in this case, as also in those referring to the existence of nodal and anti-nodal surfaces of the other variables, to account for the data derived from the observations.

The cellular solution, on the other hand, explains the existence of nodal and anti-nodal surfaces of the components of the wind perturbation at heights which are in agreement with the result of the observations. As in some other cases referred to above, the cellular solution, however, fails to give a correct interpretation of the observed circulation.

Taking into account the fact that, neglecting the slant of troughs and ridges, the average values of the components for the group

Table V. Components in knots of the vector difference between the average winds when the pressure variation at 8 km during the last 12 hours has been positive and when it has been negative. (This difference is approximately twice the perturbation components.)

Valentia 1/10/46 to 31/12/49

Height	$2u$	$2v$
Surface	—0.39	—0.95
1	—1.52	—0.58
2	—0.45	—0.63
3	0.38	—0.10
4	—0.19	—3.19
5	—0.66	—4.50
6	2.11	—5.13
7	1.88	—7.10
8	2.19	—9.01
9	3.38	—11.84
10	2.59	—11.65
11	1.13	—10.76
12	1.14	—8.97
13	—0.61	—6.31
14	—2.00	—4.66
15	—2.46	—1.81
16	—2.14	—3.26

Table VI. Components in knots of the vector difference between the average winds when the pressure variation at 8 km during the last 12 hours has been positive and when it has been negative. (This difference is approximately twice the perturbation components.)

Valentia. 1/10/46 to 30/9/50

Height	$2u$	$2v$
Surface	0.38	—0.61
1	—1.04	—0.60
2	0.12	—0.14
3	0.62	—0.34
4	0.44	—1.67
5	1.79	—3.10
6	2.53	—3.59
7	2.55	—5.09
8	2.87	—6.84
9	3.31	—8.10
10	2.63	—8.60
11	1.50	—7.61
12	0.46	—6.53
13	—0.75	—4.63
14	—1.33	—3.95
15	—1.57	—2.52
16	—1.12	—2.10

$\triangle P_8 > 0$ may approximately be equated to the values of these components at the neutral point between a trough and the next ridge, and those for the group $\triangle P_8 < 0$ to the values for the neutral point between a ridge and the next trough, equations (3 a) and (3 b) indicate that the differences of the average components given in Tables V and VI should be:

for the Zonal component, $2u = 0$
for the Meridional component,

$$2\,v = -2\ B\ \cos\,\Theta\,z.$$

It is obtained, however, both from Table V and from Table VI, that it is the zonal component, and not the meridional, that shows nodal surfaces at about 4 and 12 km, while the sign is opposite to that required by the equations. It might be expected in view of this result that the observed meridional component would be zero, as the theory requires the zonal component to be; however the observed meridional component shows a very well defined maximum at about 9 km. Whether this result, together with the sign of the circulation, might be explained by integration of the dynamical equations for a two-layers atmosphere, with sloping tropopause and meridional gradient of temperature, is a problem which awaits the enterprise of a meteorologist who is also an expert mathematician or of a mathematician with an interest in Meteorology.

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