

Organisation of potential vorticity on the mesoscale during deep moist convection

By CHRIS WEIJENBORG*, PETRA FRIEDERICHS and
ANDREAS HENSE, *Meteorological Institute of the University of Bonn, Bonn, Germany*

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ABSTRACT

Potential vorticity (PV) and its conservation principle elegantly describe large-scale atmospheric dynamics. On the mesoscale, however, PV has received less attention. We describe the characteristics of PV on the convective weather scale (≈ 10 km) as simulated by the non-hydrostatic numerical weather prediction model COSMO-DE. Two weather cases with a different synoptic background are analysed, on 5 June 2011 and 22 June 2011. Composites of PV and other quantities like wind velocity around storm updrafts are calculated to test consistency of PV anomalies associated with storm updrafts.

For the frontal case on 22 June bands of positive and negative PV form approximately along the direction of the wind shear. A possible explanation of these elongated PV bands is the preferential generation of new cells downshear of old cells in an atmosphere with moderate to high vertical wind shear. The PV dipoles are much less consistent in direction in the 5 June case with localised deep convection. For both cases, the wind anomalies around the convective PV anomalies are consistent with the flow around synoptic PV anomalies. The coherent PV anomalies around storm updrafts motivate the use of PV-thinking on smaller scales.

Keywords: dynamic meteorology, potential vorticity, deep moist convection

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1. Introduction

The physical understanding and the forecasting of extreme events in general and those associated with deep convection in particular have been identified as one of the challenges in modern meteorology (Shapiro and Thorpe, 2004). One of the central problems in forecasting deep moist convection and their dynamics is the intense interaction of processes acting on a large range of spatial and temporal scales. The main ingredients necessary for deep moist convection are instability, lift and moisture (Doswell, 1987). Present understanding is often orientated along large-scale indicators like convective available potential energy (CAPE). Parameters such as CAPE combine within a single parameter the information of a thermodynamic profile, but represent only parts of the full dynamics. Conserved quantities like the Ertel potential vorticity (PV) (Schubert et al.,

2004) may offer new insights in the dynamics of convective weather, which have not yet been investigated thoroughly on the convective weather scale (≈ 1 – 10 km). Vorticity dynamics plays an important role in the dynamics of severe weather on the storm scale, for example, for supercells. The combination of dynamical and thermodynamical information and the conservation property of PV might give a relatively simple view on the dynamics without relying on strong assumptions.

Research on PV dynamics focuses mainly on the synoptic to planetary scale, where it is useful in describing, for example, cyclogenesis (e.g. Hoskins et al., 1985; Hoskins, 1997). On the synoptic scale, PV provides powerful conceptual models to describe midlatitude synoptic dynamics (e.g. Hoskins et al., 1985; Haynes and McIntyre, 1987), mainly because of two properties. First, it is materially conserved for frictionless and adiabatic flow, which allows to interpret PV as a dynamical tracer. PV might further be very useful in indicating the effects of diabatic heating on the dynamics (Grams et al., 2011). Secondly, it is tightly related to balanced flow by the invertibility principle.

*Corresponding author.
email: cwborg@uni-bonn.de

Given suitable boundary and balance conditions, a PV field can be inverted to derive standard meteorological quantities such as wind velocity and pressure (Hoskins et al., 1985).

There are some studies of PV dynamics on the mesoscale, mainly focusing on idealised test cases of mesoscale convective vortices (e.g. Raymond and Jiang, 1990; Davis and Morris, 1994; Cram and Montgomery, 2002; Conzemius and Montgomery, 2009), on mesoscale orographic PV anomalies (e.g. Aebischer and Schär, 1998; Schär et al., 2003), and on PV along fronts (e.g. Malardel et al., 1993; Appenzeller and Davies, 1996). On the convective weather scale, there is little research on PV. It is an open question what kind of PV anomalies occur during convection and how they are compared with synoptic PV anomalies, especially for real cases as represented by forecast data. To the best of our knowledge, only Conzemius and Montgomery (2009) and Chagnon and Gray (2009) have analysed PV dynamics on the convective weather scale in the extratropics. They found that horizontal PV anomalies (i.e. anomalies aligned in the horizontal plane) organise as dipoles with a strength of about 10 PVU ($1 \text{ PVU} \equiv 1 \times 10^{-6} \text{ km}^2 \text{ s}^{-1} \text{ kg}^{-1}$) during convection. This is about $O(10)$ larger than typical synoptic PV structures. These PV dipoles are created by the tilting of horizontal vorticity caused by storm-scale updrafts (e.g. Conzemius and Montgomery, 2009), analogously to the creation of a pair of vertical vorticity around an updraft (see Davies-Jones, 1984; Lilly, 1986a). Chagnon and Gray (2009) found those dipoles in an environment with moderate vertical shear of the horizontal flow to be related to heating on the storm scale originating from heating by moist processes inside clouds.

Severe convection is very unsteady; therefore, it is doubtful if the flow around the mesoscale dipoles is balanced. However, one could argue that the flow is at least quasi-balanced. First, Chagnon and Gray (2009) suggested that the PV dipoles created by convection have a longer lifetime than the original updraft which initiated the dipoles. They argued that although the PV dipole is created by the horizontal vorticity component due to the vertical wind shear of the horizontal wind, the vorticity is tilted almost immediately into the vertical. This can possibly lead to balanced dipoles which survive after the original diabatic perturbation has vanished. Secondly, Chagnon and Gray (2009) estimated that the time-scale of adjustment to a balanced flow for the convective weather scale is approximately 0.5 hours, which is shorter than the lifetime of an individual storm cell. Full balance can never be achieved, since latent heating will continuously perturb the flow away from balance. But the ratio of the estimated adjustment time to the life time of a storm cell is indicative for a balanced flow. A third reason to presume quasi-balance is that severe convective weather associated with rotating storms is often

relatively long lasting (e.g. supercells, which can last for several hours). Thus, a PV inversion method of the given PV dipole structure would be useful, as it determines the dynamic and thermodynamic anomalies around the PV anomaly. Unfortunately, PV inversion is problematic or might be even impossible on the convective scale, because the balance condition is not known.

An open question is if and how these PV dipoles organise into larger structures. Chagnon and Gray (2009) discussed a squall line for which the PV dipoles are aligned in positive and negative PV bands. It is generally known that vertical wind shear influences the organisation of mesoscale convective storms. Weisman and Klemp (1982) found that multicell storms are favoured in moderate shear ($10\text{--}20 \text{ ms}^{-1}$ in the lowest 6 km, referred to as bulk shear). Increasing wind shear and directional shear further favours the generation of super cells. New cells in a squall line are favourably generated at the downshear side of a storm system, since here the lifting by the gust front is enhanced due to background vorticity. Rotunno et al. (1988) and Weisman and Rotunno (2004) hypothesised that squall lines are best maintained when there is an equilibrium between cold pool outflow and wind shear. Although the role of cold pool wind shear in determining an optimal state for long-lived squall lines is still a matter of debate, there is general agreement that the wind shear plays an important role in storm organisation (Stensrud et al., 2005). The strength of the dipoles is proportional to the wind shear magnitude and the direction is parallel to the direction of the wind shear (Chagnon and Gray, 2009). Therefore, the wind shear will probably also influence the PV structures on larger scales.

The main goal of the present study is to describe the consistency of PV anomalies during severe convection in a convection-permitting numerical weather prediction model (NWP) using data from operational forecasts. Data from the operational ensemble prediction system (EPS) of the non-hydrostatic NWP model COSMO-DE are used (COSMO-DE-EPS, Baldauf et al., 2011; Peralta et al., 2012). Two cases of severe weather over Germany during June 2011 are selected with a different synoptic background. One case is characterised by local severe convection (5 June 2011) and the other case is convection along a cold front (22 June 2011). We want to investigate a few of the thoughts of Chagnon and Gray (2009) in deeper detail. Firstly, we investigate the coherency of the horizontal PV dipoles by calculating composites of PV and other fields around storm cells. With the use of COSMO-DE-EPS, we have a relatively large sample of convective cells, to calculate robust composites. Secondly, we want to discuss the influence of the synoptic environment on the orientation and other characteristics of the PV dipoles.

The central questions we want to answer in this study are:

- (1) How consistent are the forecasted PV dipoles in strength and direction for non-idealised real weather events investigated in a state-of-the-art convection-permitting NWP model?
- (2) Is there any coherent flow around these PV dipoles; that is, are the wind velocity and θ anomalies around a PV anomaly comparable to quasi-geostrophic (QG) PV anomalies as discussed in Hoskins et al. (1985) and Hoskins (1997)?
- (3) Is there any organisation of PV structures on larger scales?

The article is organised as follows. In Section 2, we shortly review how the PV anomalies are created, and we hypothesise how they will differ in the different synoptic backgrounds of the two cases. Section 3 gives an overview of the model data and the methodology used. In Section 4, we present a synoptic and mesoscale case description of the two weather cases under investigation. Section 5 discusses composites of fields around storm updrafts. Section 6 summarises results and discusses implications and further analysis.

2. Background

The purpose of this section is to review the characteristics of the PV dipoles theoretically described in Chagnon and Gray (2009). We will discuss how the general orientation of the dipoles depends on the characteristics of the synoptic environment like wind shear and storm-relative helicity (SRH).

A schematic depiction of the creation of the dipoles is shown in Fig. 1, which shows a horizontal and vertical cross sections of the dipoles created around a convective cell (depicted in Fig. 1 as a diabatic heating anomaly). Neglecting friction, the PV evolution equation can be written as (e.g. Haynes and McIntyre, 1987),

$$\frac{d\Pi}{dt} = \frac{\zeta_a \cdot \nabla \dot{\theta}}{\rho}, \quad (1)$$

with Π the PV, defined by $\Pi = \rho^{-1} \nabla \theta \cdot \zeta_a$, ρ is the density, ζ_a the three-dimensional absolute vorticity, $\dot{\theta}$ the potential temperature, $\dot{\theta}$ its deviation with time (i.e. diabatic heating), and ∇ denotes the three-dimensional gradient operator. Generally, deep moist convection creates a local heating anomaly with a maximum in the middle troposphere due to condensational heating (Houze, 2004; Xie et al., 2014). This diabatic heating anomaly $\dot{\theta}$ will create PV dipoles in the direction of the absolute vorticity vector ζ_a . We assume that the horizontal vorticity due to the vertical wind shear of the horizontal wind ($\zeta_{ah} = \mathbf{k} \times \mathbf{S}$, with $\mathbf{k}$ as the unit vector in the vertical direction and $\mathbf{S} \equiv \frac{\partial \mathbf{u}_h}{\partial z}$) will dominate the absolute vorticity on the

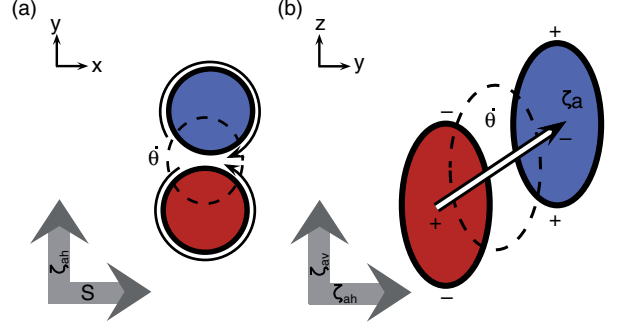


Fig. 1. Schematic overview of PV dipoles generated due to convection, diabatic perturbation indicated by dashed black circle and positive (negative) PV pole indicated by red (blue) circle. The $\pm$ signs indicate regions of high and low static stability, and ζ_{ah} the direction of the (dominant) horizontal vorticity vector. (a) Horizontal cross section, PV dipoles created in the direction of absolute vorticity $\mathbf{k} \times \mathbf{S}$, with $\mathbf{S}$ the vertical wind shear. Storm-relative wind velocities associated with the PV anomalies are indicated by black arrows. (b) Vertical cross section along the direction of $\mathbf{k} \times \mathbf{S}$.

convective weather scale. Therefore, the dipoles are normally created in the horizontal plane (see Fig. 1a). The exact angle with respect to the horizontal plane of the dipole depends on the ratio of vorticity due to wind shear and the planetary vorticity (see Chagnon and Gray, 2009). The vertical component of the background planetary vorticity is positive in the northern hemisphere, because there is always a part related to planetary rotation. Therefore, it is expected that the dipole will be slightly tilted in the vertical (Fig. 1b).

In Fig. 1a and b, it is assumed that the directional wind shear (i.e. changes of the angle of the wind velocity vector with height) is zero. Chagnon and Gray (2009) described the dipoles in unidirectional wind shear (i.e. zero helicity, which is generally not the case). Helicity $H \equiv \zeta \cdot \mathbf{u}$ is defined as the inner product between the three-dimensional wind velocity and relative vorticity vectors. The integrated form is conserved for three-dimensional barotropic fluids. In this study we consistently use the term helicity for the local form, which is sometimes referred to as the helicity density (Lilly, 1986b). The helicity plays a role in the relative persistence of supercells and tornadoes. Droegemeier et al. (1993) found that storms forming in a helical environment are relatively longer lived than storms in an environment without helicity. The reason for this is that the helicity conservation restricts the downward cascade of energy to smaller scales (Betchov, 1961; Lilly, 1986b). Although the energy cascade might be restricted for 2D turbulence, there are indications that there is a joint energy-helicity cascade for 3D turbulence (Chen et al., 2003). It is important to take helicity into account, since we are

interested in the consistency of the PV dipoles in different synoptic backgrounds. SRH, which is defined in terms of storm-relative wind (i.e. the difference between storm motion and total wind velocity), is high for supercells and tornadoes. In case of positive (negative) SRH, the PV dipole will be advected towards the updraft (see e.g. Davies-Jones, 1984; Lilly, 1986b), which would change Fig. 1a in the sense that the positive (negative) PV anomaly is located at the position of the diabatic heating.

Chagnon and Gray (2009) investigated briefly how the PV dipoles can organise themselves into bigger structures. There is a tendency of PV to form bands, either related to orography (e.g. Aebischer and Schär, 1998; Schär et al., 2003), or along fronts (e.g. Appenzeller and Davies, 1996). As already discussed in the introduction, in moderate to high shear flows new cells are most likely to form downshear of old cells due to the lifting by the gust front (Rotunno et al., 1988). These new updrafts downshear of old cells will themselves induce a PV dipole. Moreover, the dipole induced flow might advect the dipoles downshear (Chagnon and Gray, 2009). The combination of these two effects may lead to bands of positive and negative PV along the direction of the wind shear (see Fig. 2). We will be able to investigate these effects, since for the 22 June case the wind shear is much stronger than that for the 5 June case (see next section).

As discussed in the introduction, full balance cannot be expected for the PV dipoles, since there is always unsteady convection. We might still be able to estimate how balanced the flow is around the dipoles by investigating the stationarity of the flow. If composites from different stages of instationary dipoles give on average a significant anomaly pattern, then we have a quasi-stationary picture. Because the flow has a coherent structure we can argue that there is a statistically derived balance. We do know, however, that for steady flow Schär (1993) found that the

wind speed can be given in terms of the PV and the gradient of the potential temperature as

$$\mathbf{u}_{\text{st}} = \frac{1}{\rho\Pi} \nabla\theta \times \nabla B, \quad (2)$$

with $\mathbf{u}_{\text{st}}$ the stationary flow and B the Bernoulli stream function. A stationary flow will therefore be orientated along the surfaces of constant θ and B . The DSI is an index based on this stationary flow (see N  vir, 2004), and is given by

$$DSI = \frac{1}{\rho} \nabla\Pi \cdot \nabla\theta \times \nabla B. \quad (3)$$

Since the DSI is zero for a stationary flow, we can use the DSI to check if the PV dipoles are (quasi-) stationary. The flow cannot be completely stationary, because there is a diabatic perturbation $\dot{\theta}$ in between the two PV poles. But if the dipoles are at least quasi-stationary, generally there should be no DSI perturbation except at the updraft in between the dipoles.

To summarise, we expect that:

- PV dipoles are created around the updraft, orientated in the direction of the horizontal vorticity vector $\zeta_{\text{ah}} = \mathbf{k} \times \mathbf{S}$. The strength of the PV anomalies, and therefore the corresponding flow anomalies, depends on the diabatic heating anomaly and the background wind shear.
- Directional wind shear (i.e. non-zero helicity) advects the dipole pair towards/away from the updraft: in case of a positive (negative) helical environment, the positive (negative) PV is colocated with the updraft.
- Elongated PV bands form in strong shear environments.
- If the dipoles are quasi-balanced, we only expect DSI anomalies associated with the diabatic heating perturbation (i.e. the convective updraft). Moreover, the kinetic energy anomalies should consistently increase for stronger PV anomalies.

3. Data and methodology

3.1. Model and data

We use data of the limited-area non-hydrostatic weather prediction model COSMO-DE (Baldauf et al., 2011) operated by the German Meteorological Service (DWD). The model domain is centred over Germany. It contains 421×461 grid points in the horizontal on an Arakawa C grid with a grid spacing of 0.025° (approximately 2.8 km). The grid is rotated so that the equator lies over Germany, to ensure that grid spacing is approximately equidistant

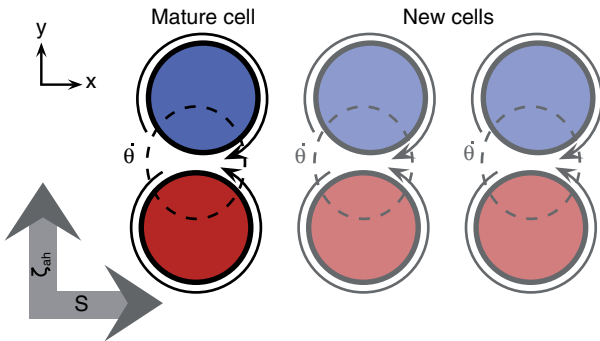


Fig. 2. As in Fig. 1, a diabatic perturbation is indicated by a dashed black circle and a positive (negative) PV pole by a red (blue) circle. New cells (transparent) are preferentially generated downshear, which could, in combination with advection, lead to bands of positive and negative PV.

across the domain. In the vertical, a modified hybrid coordinate is used, which follows the orography at the bottom levels and gradually flattens at higher levels. In total there are 50 vertical levels, with the lowest level at 10 m height and the top level at 21.5 km. With such a high spatial resolution, deep convection can be treated explicitly and no parameterisation is thereof required. Shallow convection is parameterised by a Tiedtke mass flux scheme (Tiedtke, 1989). To initialise convection during the first model hours, latent heat nudging is used to assimilate radar rainfall data (Stephan et al., 2008).

The COSMO-DE ensemble prediction system (COSMO-DE-EPS) aims at improving the prediction of severe weather and quantifying the uncertainty of the forecast. COSMO-DE-EPS consists of 20 ensemble members. These 20 members are driven by four different global models: The global model (GME) of DWD, the Global Forecast System (GFS) of the National Centers for Environmental Prediction, the Integrated Forecast System (IFS) of the European Centre for Medium-range Forecasts and the Global Spectral Model (GSM) of the Meteorological Agency of Japan (Peralta et al., 2012). Moreover, five parameters in parameterisations are altered [see Gebhardt et al. (2008) with further details in Peralta et al. (2012)].

In addition to the EPS forecasts, two single reforecasts are made in analogy to the deterministic COSMO-DE operational forecast starting 00 UTC on 5 and 22 June 2011, respectively. These reforecasts were made to test PV conservation in the COSMO model (not shown). Initial and boundary conditions are provided by the COSMO-DE analysis. A description of the two weather cases is given in the next section. All data presented in this article are from forecasts initialised at 00 UTC. The overview plots in Section 4 are from the reforecasts, all the other plots are based on the COSMO-DE-EPS forecasts. Since the dynamics near the lateral boundaries are largely determined by the boundary forcing, we remove 50 grid points at each lateral boundary. PV, helicity and DSI are calculated on model levels using all three wind components with centred differences for the gradients. The calculation of SRH requires the use of storm-relative wind. An estimation is made of the SRH with help of the perturbation velocities, which estimates the storm velocity with the mean environment wind.

3.2. Methodology

The central aim of this study is to characterise the consistency of PV anomalies associated with deep moist convection. In order to investigate the coherency of the convective PV anomalies and associated flow anomalies, we calculate composites of the PV anomalies associated with storm updrafts. The composites offer an easy and

dynamically unbiased way to check whether the PV anomalies and accompanied flow around the dipoles are coherent or not.

For the composites we define storm cells by local maxima in the vertical mean vertical velocity that exceed a certain threshold. To ensure that we select only deep convective storm cells, we use a threshold of 5 ms^{-1} of the vertical mean vertical velocity between model level 31 ($\approx 3 \text{ km}$) and 20 ($\approx 7.3 \text{ km}$). Since the maximum diabatic heating is expected at around 5 km height during deep convection (Houze, 2004; Xie et al., 2014), it is roughly the vertical height at which we expect the largest anomalies of PV. To prevent that we select vertical maxima too close to each other, the maxima have to be separated by at least three gridpoints ($\approx 8 \text{ km}$).

All relevant fields are extracted in a $50 \times 50 \text{ km}$ (19×19 gridpoints) domain around the updraft. Composites are calculated over all updrafts that are detected in all ensemble members during a specific hour, and provide typical structures of PV, wind velocity, DSI, helicity and static stability. Since we are interested in coherent anomalies, we subtract a typical height profile of the respective variable. The height profile is the average over the 19×19 gridpoints around each updraft. Composites of wind shear are estimated from the 0–6 km wind difference of the horizontal wind velocity (also known as the bulk wind shear). The use of all convective cells that occur in any of the 20 COSMO-DE-EPS members creates a relatively large sample of independent convective cells and should give a good indication of the environment profile associated with the convective cells.

Since we are interested in coherent composite anomalies, it is important to test if they are significant. A standard Student t-test is used to test if the composite anomalies are significantly different from zero (von Storch and Zwiers, 2002). Since a quantile-quantile plot indicates that the data are not normally distributed in the tails, a Wilcoxon signed rank test is performed, too (Wilks, 2011). Both give similar results, as the Student t-test is quite robust against deviations from the normal distribution for large sample sizes (von Storch and Zwiers, 2002). Therefore, only the PV, wind velocity, DSI, helicity anomalies for which the Student t-test can be rejected at the 0.01 significance level are presented in the plots.

4. Case description

4.1. Synoptic situation

The synoptic analysis of DWD for the two selected weather situations is shown in Supplementary Fig. 1. The days preceding 5 June 2011 were characterised by sunny and relatively warm weather associated with a high

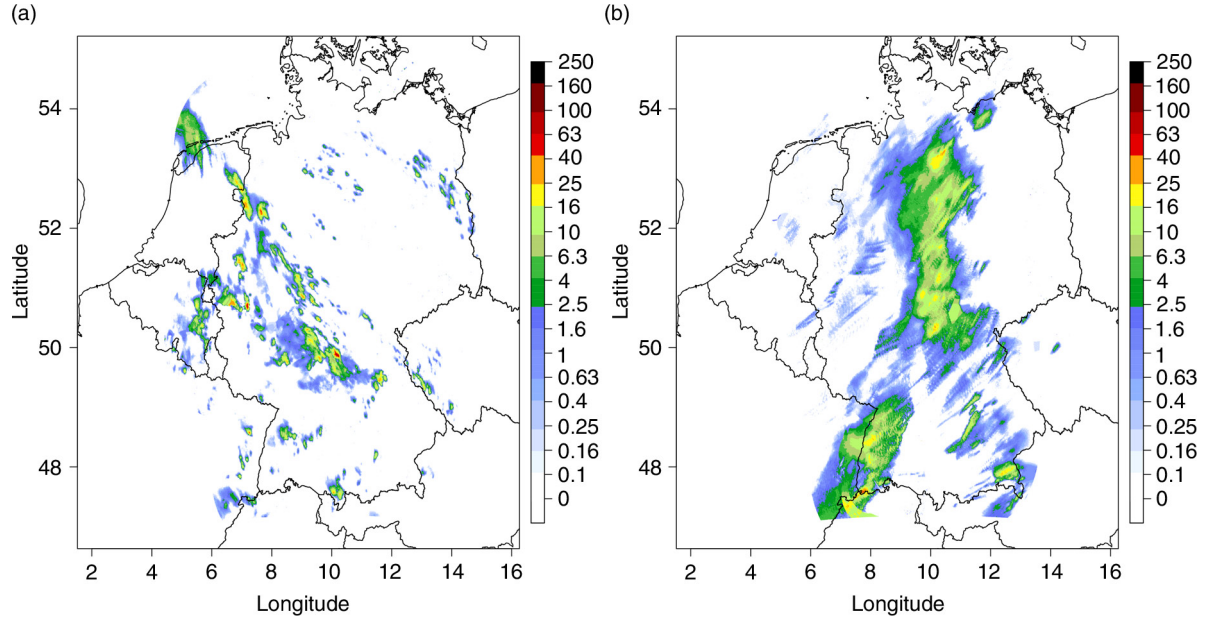


Fig. 3. Rain rates in millimetre per hour, using RADOLAN RW data (Bartels et al., 2004) for (a) 5 June and (b) 22 June 2011 at 14:50 UTC, respectively.

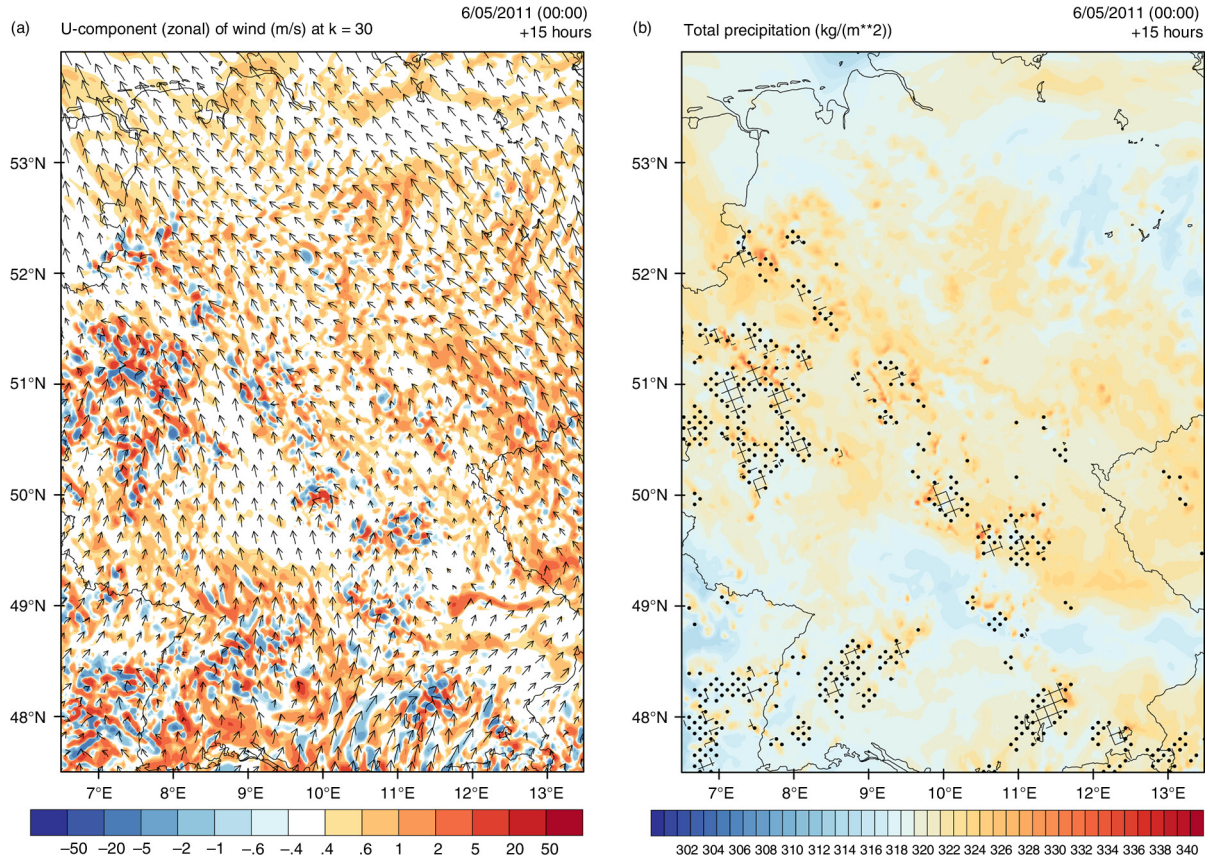


Fig. 4. (a) PV (contours) and wind flow (arrows). (b) Equivalent potential temperature and precipitation (dots indicate hourly precipitation rates above 0.5 mm, dashed patterns above 5.0 mm in the previous hour). Both plots on 5 June 2011 at 15 UTC at model level 30 (about 3 km).

pressure system. East of the high pressure system intensifying south-easterly to southerly winds advected warm and humid air northwards. Dew points of up to 20°C were measured. This humid air, together with the confluent flow of a low pressure system provided the main ingredients for the heavy thunderstorms occurring during 4 June 2011 and the following days. On 4 June, most heavy thunderstorms were located in northern France and southern Germany. The most heavy precipitation in Germany occurred on 5 June, with 87 mm in 24 hours at the weather station Lennestadt-Theten (51.10°N and 8.08°W) and more than 60 mm within a few hours in the neighbourhood of Bonn. Convection was scattered over Germany, although there were some large-scale precipitation areas [Fig. 3a, which shows the radar precipitation rates derived from RADOLAN data (Bartels et al., 2004)].

Responsible for the severe weather on 22 June 2011 was a cold front associated with a low pressure system, which formed around 16/17th June over the western North Atlantic. It moved eastward to the British Isles on 21 June. The cold front associated with the low pressure system crossed Central Europe during the night of 21–22 June. Along the southern part of the cold front, a secondary

low formed. The cold front and the secondary low moved over Germany during 22 June and caused precipitation and intense wind gusts over large parts of Germany. Wind speeds of up to 36 ms^{-1} were measured in southern Germany.

Precipitation was less intense compared to 5 June with 20 mm rain in 24 hours, but extended over a much larger area (Fig. 3b). Besides the heavy wind gusts, large hail stones were observed, too. The main difference to the 5 June case is that the convection was much more organised. Thus, banded structures of PV as discussed in Section 2 are more likely to form on 22 June.

4.2. Mesoscale situation

PV on model level 30 ($\approx 3 \text{ km}$) in the COSMO-DE model is used to get an indication of which PV anomalies are associated with convection. On 5 June PV dipoles of large magnitude (i.e. around 10 PVU on average) are found scattered over the domain (Fig. 4a). As expected, they coincide with regions of diabatic perturbations and precipitation (Fig. 4b). Most convection forms in the middle of the domain, where CAPE is highest with local values of about 1000–1500 J/kg (not shown). There is also a weak

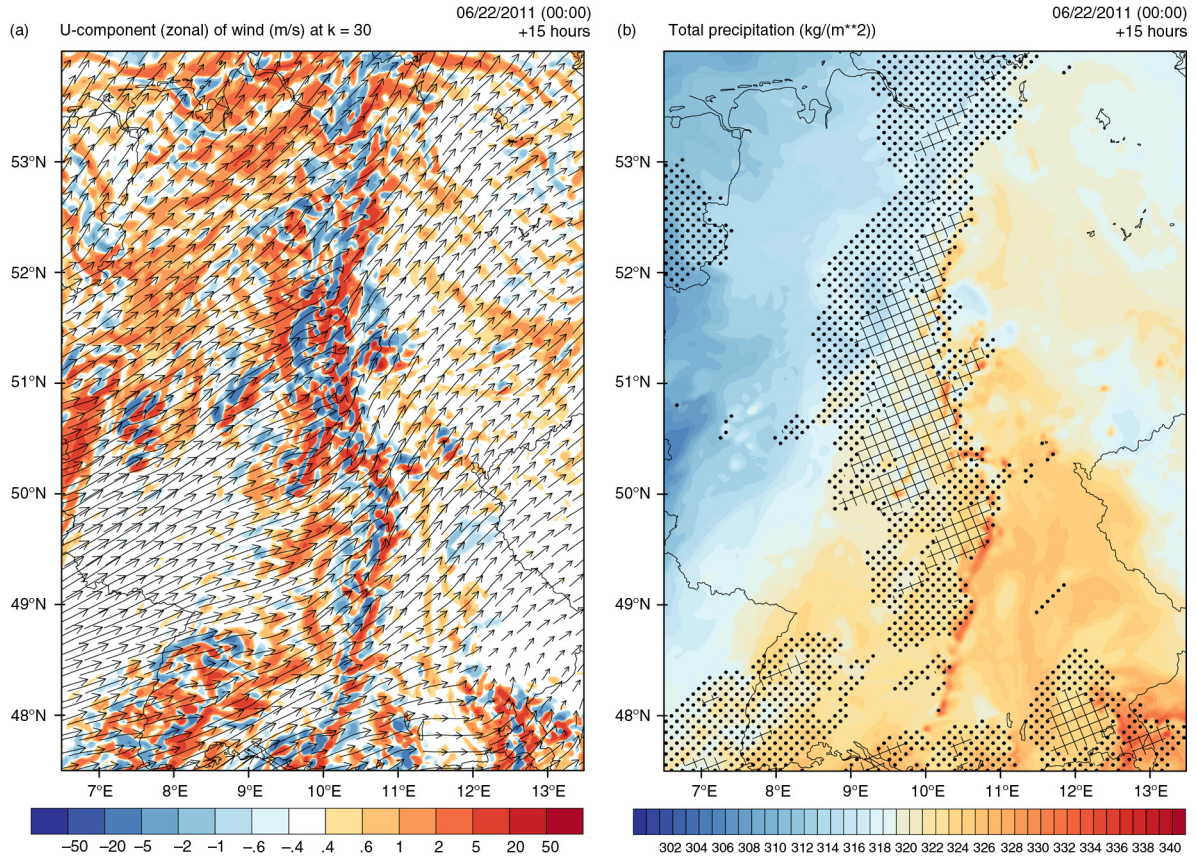


Fig. 5. (a) PV (contours) and wind flow (arrows). (b) Equivalent potential temperature and precipitation (dots indicate precipitation above 0.5 mm, dashed patterns above 5.0 mm in the previous hour). Both plots on 22 June 2011 at 15 UTC at model level 30 (about 3 km).

convergence zone in the middle of the domain which could provide some of the necessary lifting. There are a few larger precipitation areas, but generally convection is scattered over the domain, which is consistent with the measurements during that day (Fig. 3a). The PV dipoles are not very consistent in orientation, but there seems to be a slight preference of dipoles along the direction of the large-scale flow. The bulk wind shear (from 0 to 6 km) over Germany is quite low, about 8 ms^{-1} , which could explain the inconsistent direction of the dipoles. Moreover, the wind direction is not consistent, with large-scale winds from the south-west in southern Germany and from south-east over the rest of the domain. Besides the dipoles, there are also bands of positive PV in the north-east of Germany, which are weak compared to the strength of the dipoles. These bands are not associated with any precipitation. The PV dipoles extend over a large part of the troposphere (Fig. 6a). The PV dipoles maximise in the mid-troposphere, at a height of about 5–6 km.

For 22 June CAPE is weaker compared to 5 June with values of about 500 J/kg , but the large-scale forcing due to the cold front is much larger. On average we observe bulk wind shear of about 20 ms^{-1} with local values of up to $30\text{--}35 \text{ ms}^{-1}$. The large-scale wind shear is orientated in the north-east direction. Along the direction of the large-scale wind, bands of positive and negative PV are visible (Fig. 5a) mainly along the cold front. Higher up in the troposphere at 7 km height these PV bands cover a larger region, and they seem to be advected in the downshear direction (not shown). The cold front is clearly visible as a potential temperature gradient. Precipitation is generated over a much larger area than on 5 June. There are no PV anomalies west of the frontal system. An explanation could be that the precipitation is caused by shallow convection here. No PV anomalies are expected in the middle troposphere, where the diabatic perturbation is zero for shallow convection. The direction of the PV bands are mainly orientated along the large-scale wind direction (i.e. the gradient of PV is perpendicular to the wind direction). As for the 5 June case, the PV dipoles can be seen over the whole troposphere, up to 12 km height (Fig. 6b). Although the storm-scale PV anomalies are much larger, one synoptic PV feature is clearly visible: a strong tropopause fold associated with the cold front system is present for the 22 June case (Fig. 6b). These folds occur especially in regions with strong wind shear and (surface) temperature gradients (Holton, 2004).

5. Storm cell composites

Consistency of flow patterns and anomalies of conserved quantities as PV and H associated with deep convective perturbations is investigated with composites of these fields

around storm updrafts. For both days, the single time step 15 UTC is chosen when there was deep moist convection over a large part of the domain. The composites at this time step are characteristic for the two weather cases discussed; differences with composites at other time steps at the same days are small. These differences are shortly discussed later in this article.

5.1. The 22 June case

We discuss the 22 June weather case first, since for this situation the composites are clearer. The composite is based on 596 storm cells. First of all, a remarkable dipole pattern in PV is visible and its direction is well in the direction of the wind shear (Fig. 7a). The anomalies are significant at the 0.01 significance level. The strength of the PV

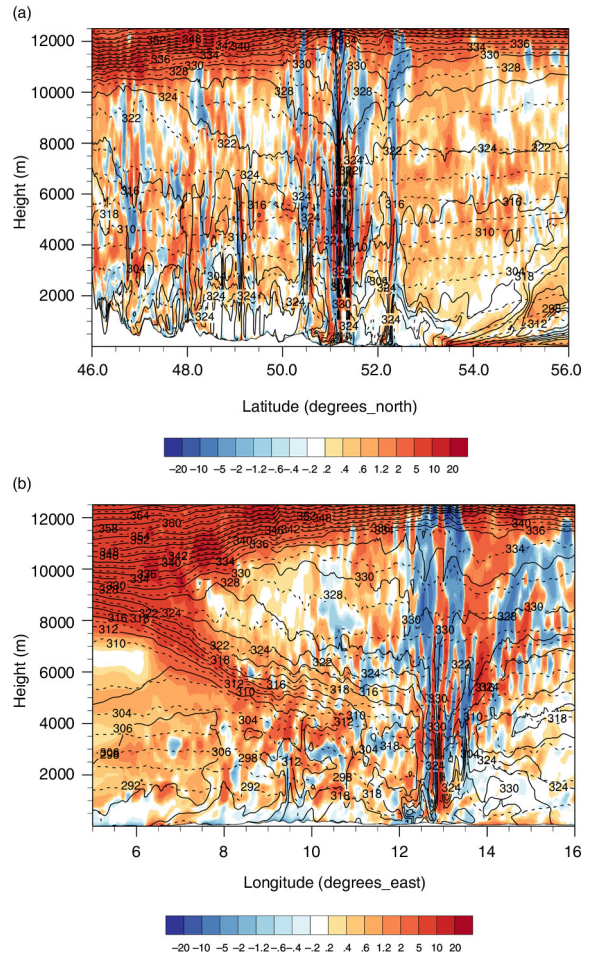


Fig. 6. Figures (a) and (b) show the PV (contours), the equivalent potential temperature θ_e (dashed contour lines) and potential temperature θ (solid contour lines) for a longitudinal cross section at 7° at 5 June 2011 at 15 UTC and a latitudinal cross section at 52° for 22 June 2011 at 18 UTC.

anomalies is more than 10 times larger than the mean environment values of about 0.5 PVU. The positive anomaly (10.9 PVU) is stronger than the negative anomaly (-4.3 PVU) and the positive anomaly seems to be displaced towards the vertical velocity maximum. The dipoles are elongated along the direction of the wind shear (Fig. 7a). The elongated bands mainly appear at the downshear side of the anomalies. At the upshear side the PV anomalies are much smaller. This suggests that advection plays a role here.

Longitudinal and latitudinal cross sections of the composites show that the PV dipoles are visible over a large part of the troposphere with maxima in the middle troposphere just above 5 km (Fig. 7b and c). The updraft is tilted upshear with height. Rotunno et al. (1988) and Weisman and Rotunno (2004) suggested that this can occur in an environment with a relative strong circulation associated with the cold pool compared to the circulation due to low level wind shear. Also clearly visible is a lowered tropopause defined by the 2 PVU surface above the positive anomaly. The 2 PVU surface is approximately 2 km lower above the positive than above the negative PV anomaly.

Chagnon and Gray (2009) found that the PV dipoles are mainly generated as a dipole in the vertical component of the vorticity vector. This is consistent with Hoskins et al. (1985) who stated that balanced flow around a tall QG PV anomaly tends to appear mainly as an absolute vorticity anomaly and not as a static stability anomaly. Our composites seem to confirm this, since there is a very high relation between the 3 and 7.5 km height integrated PV and the vertical component of the vorticity (not shown). This might indicate that the static stability anomalies are small. To check this, the θ difference over the lowest 500 m is used as an estimate for the static stability below the PV anomalies. PV-thinking predicts a decreased (increased) static stability below a positive (negative) PV anomaly. Indeed, the static stability is lower below the positive anomaly and slightly higher below the negative anomaly (Fig. 7d).

The scale effect associated with QG PV inversion states that a small PV anomaly of a given strength has less influence on the flow than a large PV anomaly with the same strength (Hoskins et al., 1985). So, although the PV anomalies are relatively strong compared to the environmental values, the flow anomalies are expected

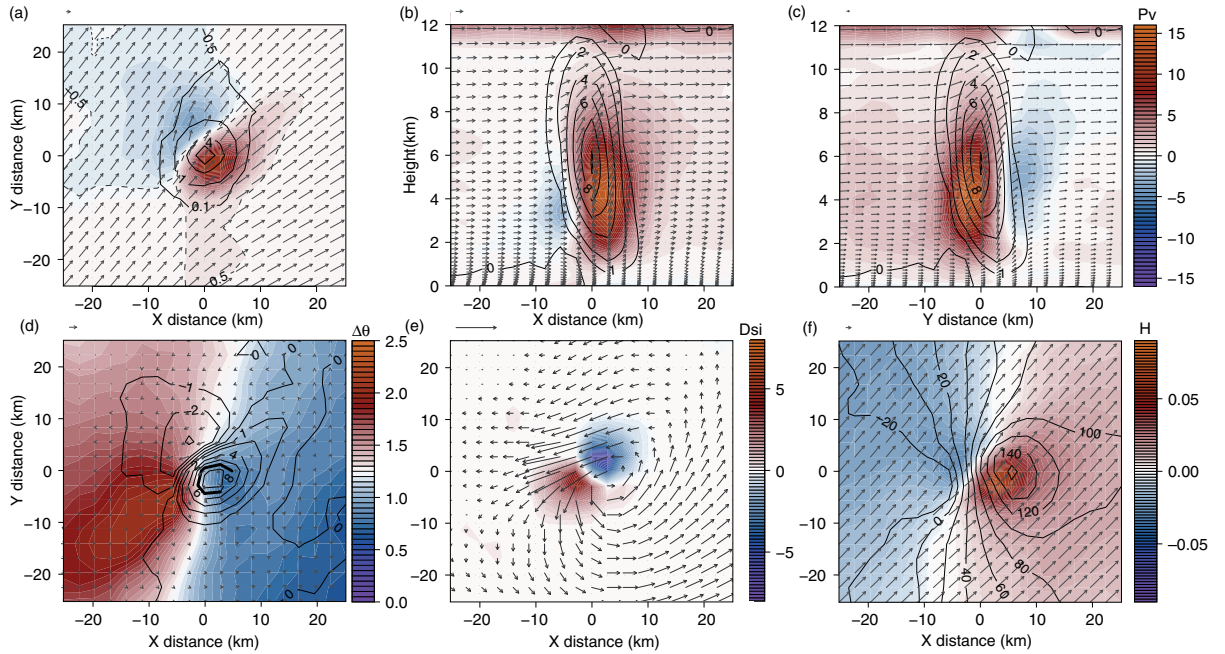


Fig. 7. (a) 3–7.3 km height integrated composites of PV (contours, in PVU), vertical velocity (contour lines, in ms^{-1}) and 0–6 km wind difference (arrows) for 22 June 2011 at 15 UTC. Dashed lines indicate PV contours of -0.5 PVU and 0.5 PVU. (b) Longitudinal cross section at 0 km of (a), with PV (contours, in PVU), vertical wind (black contours, in ms^{-1}) and wind velocity (arrows) for 22 June 2011 at 15 UTC. (c) as (b), but latitudinal cross sections at 0 km. (d) θ difference in the lowest 500 m (contours, in δK), 3–7.3 km height integrated PV (black contours, in PVU) and wind velocity (arrows) for 22 June 2011 at 15 UTC. (e) 3–7.3 km height integrated composites of DSI (contours, in $\text{PVU}^2\text{s}^{-1}$) and perturbation wind (arrows) for 22 June 2011 at 15 UTC. (f) 3–7.3 km height integrated composites of helicity H (contours, in ms^{-2}), 0–3 km SRH (black contour lines) and full wind velocity (arrows) for 22 June 2011 at 15 UTC. Reference arrow at top of each plot of 5 ms^{-1} .

to be quite small compared to the background flow. The largest anomalies of the perturbed flow are of about 5 ms^{-1} (see Fig. 7e). These large values appear between the dipoles, where the anticyclonic flow around the negative and cyclonic flow around the positive flow are superimposed (see Fig. 1). The cyclonic flow around the positive anomaly is dominant, consistent with the much larger magnitude of this PV anomaly.

Other variables show similarly consistent anomalies. For DSI (Fig. 7e), there is a dipole pattern with a magnitude of about $5 \text{ PVU}^2 \text{ s}^{-1}$, but the dipole is 90° turned with respect to the PV dipoles. The DSI anomalies are proportional to the advection of PV^2 (Claußnitzer and Nevir, 2009), where positive (negative) DSI indicates negative (positive) PV^2 advection. The DSI patterns suggest that there is advection of PV^2 downshear of a few $\text{PVU}^2 \text{ s}^{-1}$. The DSI anomalies

are centred around the updraft and confirm that mainly the updraft region is instationary.

Helicity anomalies also exhibit a dipole pattern with positive helicity located at the positive PV anomaly and negative helicity at the negative PV anomaly (Fig. 7f). One source term in the helicity evolution equation is given by $(\mathbf{k} \times \mathbf{u}) \cdot \nabla b$, with b the buoyancy (Lilly, 1986b). The buoyancy b is proportional to the diabatic heating $\dot{\theta}$ and for the 22 June case the large-scale wind is in the direction of the large-scale wind shear $\mathbf{S}$. This explains that the helicity anomalies are co-located with the PV anomalies, since the same term appears in the PV evolution equation. The positive anomaly in Fig. 7a seems to be displaced towards the vertical velocity maximum. Davies-Jones (1984) argued that if the environment has positive SRH, the updraft is advected towards the anticyclonic vorticity pole.

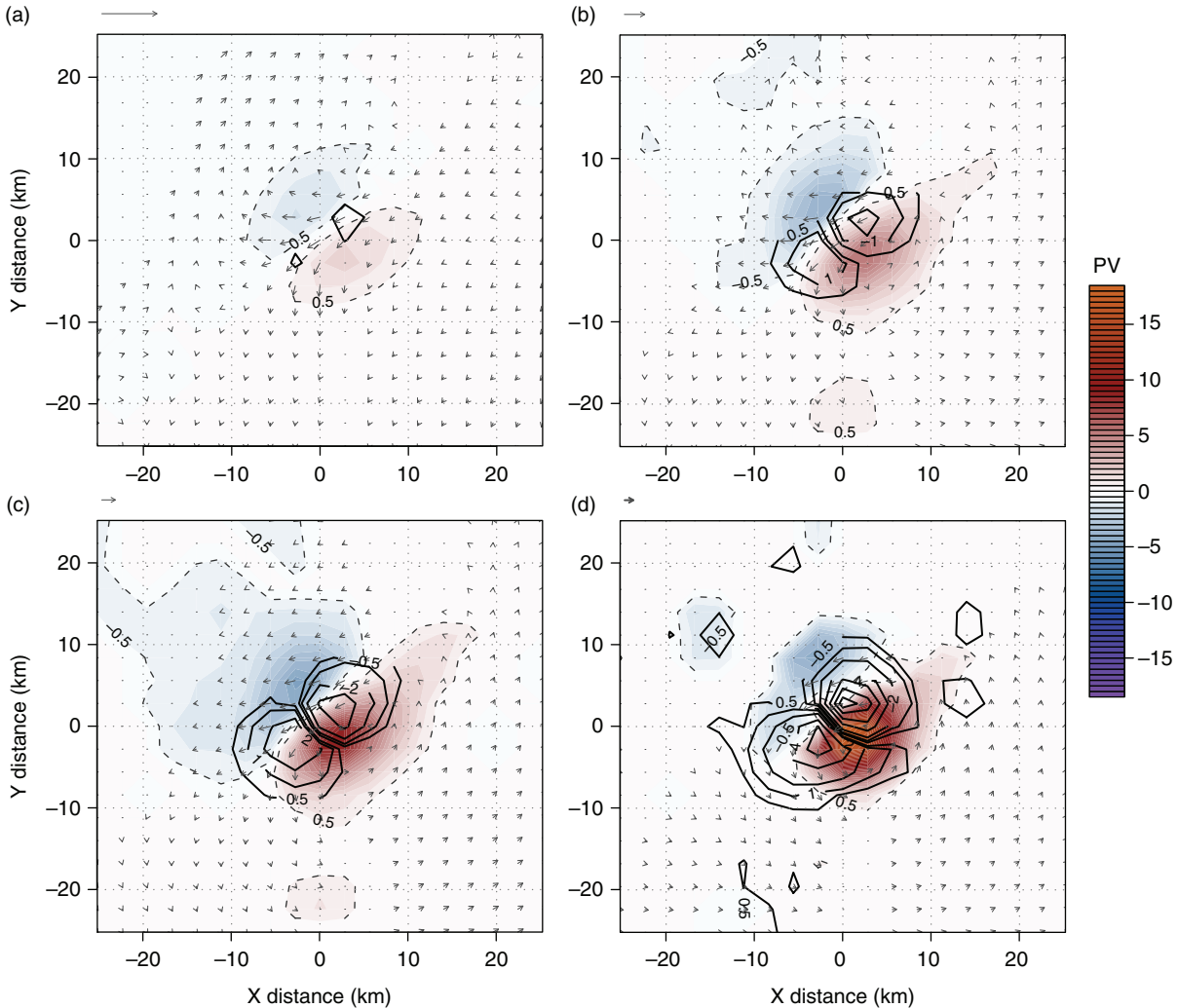


Fig. 8. 3–7.3 km height integrated composites for 22 June 2011 at 15 UTC of PV (contours, in PVU), DSI (black contour lines, in $\text{PVU}^2 \text{ s}^{-1}$) and perturbation wind for height averaged vertical velocity threshold of (a) $1\text{--}3 \text{ ms}^{-1}$, (b) $3\text{--}5 \text{ ms}^{-1}$, (c) $5\text{--}10 \text{ ms}^{-1}$ and (d) above 10 ms^{-1} .

For the environment SRH, we estimate the storm motion by subtracting the 0–6 km mean wind velocity, assuming that the updraft moves with the height integrated velocity. The positive SRH anomaly (Fig. 7f) is consistent with the hypothesis of Davies-Jones (1984) that the positive pole is advected towards the updraft in case of positive environment SRH.

5.2. Different vertical velocity thresholds

In the previous section, the composites were made by selecting updrafts with a vertical velocity of at least 5 ms^{-1} . The strength of the PV anomalies is not only related to the strength of the wind shear, but also depends on the strength of the diabatic anomaly and therefore on the vertical wind velocity. Figure 8 shows the composites for four different thresholds of the vertical wind velocity for 22 June 2011. The PV, DSI, and wind flow anomalies all consistently intensify with increasing thresholds (Fig. 8a–d). When we change the threshold from $1\text{--}3 \text{ ms}^{-1}$ (Fig. 8a) to $5\text{--}10 \text{ ms}^{-1}$ (Fig. 8c) the PV and DSI anomalies increase by a factor of 5. The PV composites with a threshold of 10 ms^{-1} have a PV maximum of more than 18 PVU and a wind

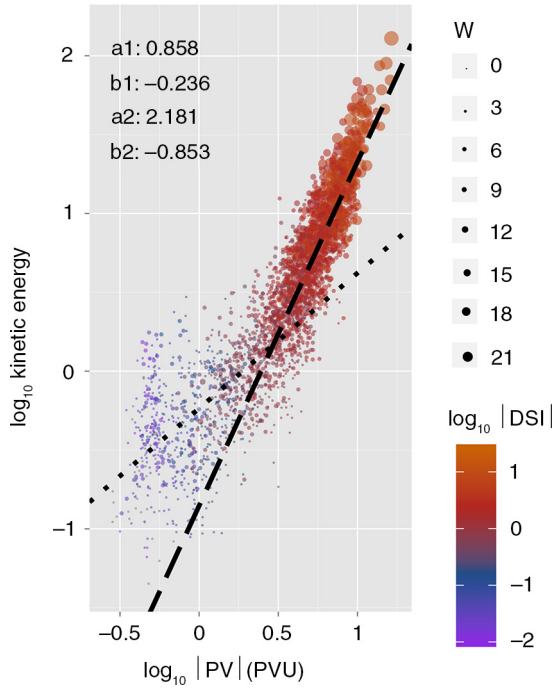


Fig. 9. 3–7.3 km height averaged $\log_{10}(|PV|)$ against height averaged $\log_{10}(U_{kin})$ (in a 7×7 grid point domain around updraft) for 22 June 2011 at 15 UTC. Height averaged $\log_{10}(|DSI|)$ and vertical velocity (ms^{-1}) indicated by dot colour and dot size, respectively. Fit of $\log_{10}(U_{kin}) = a \log_{10}(|PV|) + b$ indicated in plot, for cells with $\log_{10}(|PV|) < 0.5$ (a1, b1, dotted line) and cells for which $\log_{10}(|PV|) > 0.5$ (a2, b2, dashed line).

perturbation maximum of about 8 ms^{-1} (Fig. 8d). There are small qualitative differences between the composites using different thresholds. The dipole asymmetry (i.e. stronger positive pole) is more pronounced for increased threshold.

For both cases there is a correlation between the height averaged absolute value of PV and kinetic energy anomalies (Fig. 9). A linear regression is calculated over all the storm cells of the form $\log_{10}(U_{kin}) = a \log_{10}(|\Pi|) + b$, with U_{kin} the kinetic energy and $|\Pi|$ the PV, both spatially and height averaged (see line in Fig. 9). Since a is approximately equal to 2, the increase of kinetic energy associated with PV anomalies is about quadratical. The explained variance for the fits in Fig. 9 is approximately 85%. This confirms the consistency of the flow anomalies around the PV dipoles. An exception are low magnitude PV storm cells on 22 June, which have a much broader variation in kinetic energy. These have quite low vertical velocity and are therefore probably not always related to storm cells. There is also a consistent increase of $|DSI|$ and vertical velocity with the perturbation PV (Fig. 9).

The longitudinal and latitudinal cross sections also show a higher correlation between vertical velocity and the PV, as shown in Fig. 10 for the 22 June case. For the $1\text{--}3 \text{ ms}^{-1}$ composites, the positive PV anomaly is slightly stronger than the negative PV anomaly, and the updraft is almost centred in the middle. For the strong updraft composites, however, the positive PV anomaly is much stronger and the negative PV anomaly almost inexistent (Fig. 10c–d).

5.3. Composites at other time steps

The composites in the previous sections were made at one specific forecast hour, 15 UTC. The composites of the investigated fields are very similar for other time steps, as seen in Fig. 11 for 11 UTC and 18 UTC. There are no large changes in strength of the PV anomalies or other quantities. As mentioned in Section 2, the updraft can be colocated with either the positive or negative PV anomaly. We measure this with a correlation coefficient r . r in Table 1 between PV and the vertical velocity is calculated in a similar way as in Davies-Jones (1984)

$$r = \frac{\langle \Pi w \rangle}{(\langle \Pi^2 \rangle \langle w^2 \rangle)^{1/2}} \quad (4)$$

where angular brackets denote a height integral (from 3 to 7.5 km, in a 3×3 grid point surrounding of the updraft). There are some qualitative differences, since at 18 UTC the PV dipole is orientated along the longitudinal axes (Fig. 11c). The bands of PV become less clear and change their orientation slightly clockwise when compared to the direction of the wind shear. The change of orientation could be due to a change of the wind direction at the

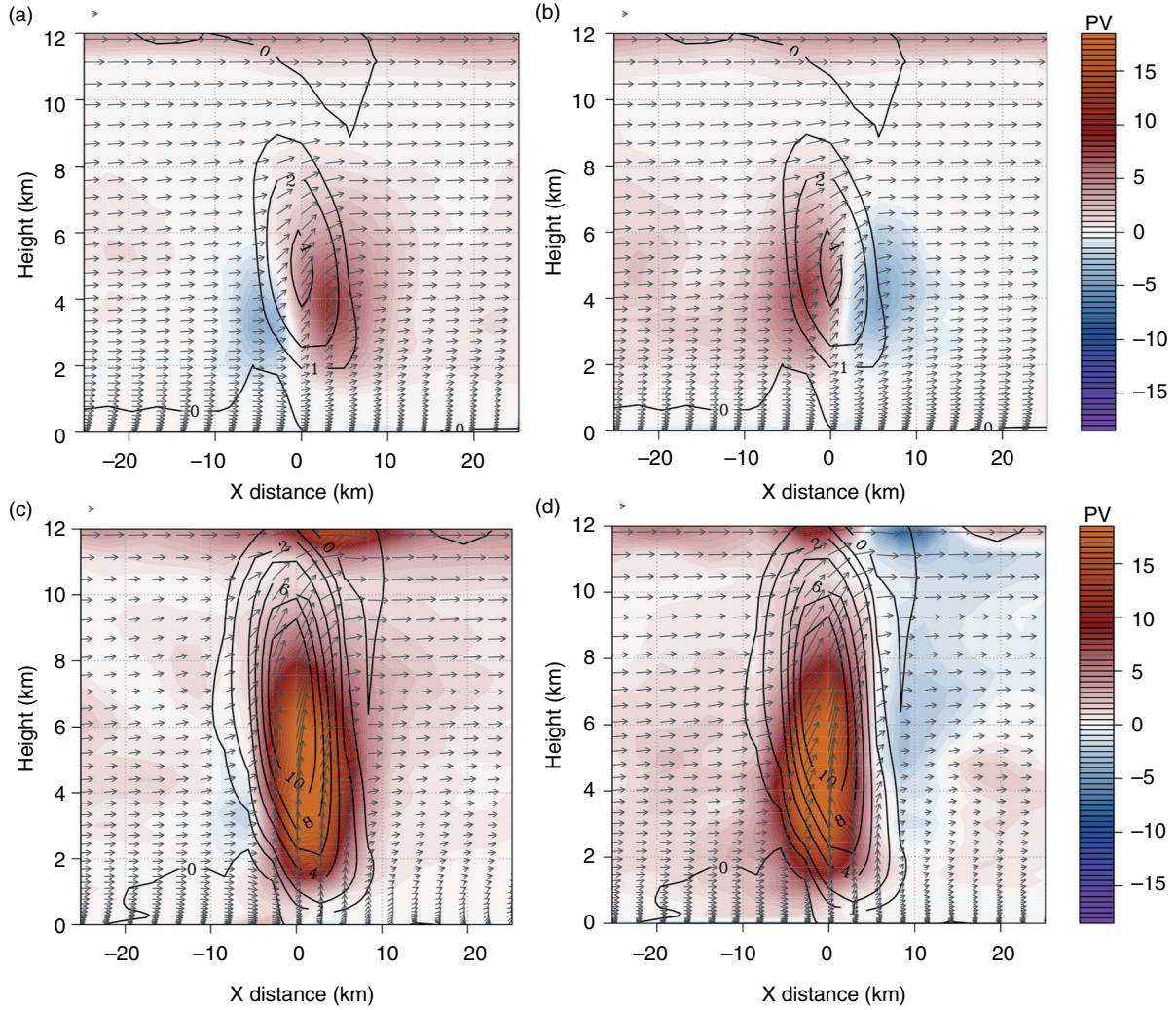


Fig. 10. (a) Longitudinal cross section for composites using a height average vertical velocity threshold of $3\text{--}5\text{ ms}^{-1}$ for 22 June 2011 at 15 UTC (at 0 km of Fig. 8b). PV indicated by contours (in PVU), vertical wind by black contours (in ms^{-1}) and wind velocity by arrows. (b) as (a), but latitudinal cross sections at 0 km. (c–d) as (a–b) but for composites with a vertical velocity threshold above 10 ms^{-1} .

back side of the front (Fig. 5a). However, the wind shear does not change in direction with time (compare Fig. 11a and b). This suggests that other factors play a role in the change of the orientation of the PV dipole. The correlation between the vertical velocity maximum and the positive PV anomaly slightly increases in the afternoon (Fig. 11b and Table 1).

5.4. The 5 June case

The orientation of the dipole in the composites on 5 June 2011 is again consistent with the general wind shear direction (Fig. 12a). For this case, the composites are taken over 1421 convective cells. As expected by the lower wind shear (8 ms^{-1} wind difference between 0 and 6 km

compared to 20 ms^{-1} for 22 June), the magnitude of the PV anomalies is weaker (about 4 PVU for both poles). The direction of the dipole is consistent with the north-westward direction of the wind shear (Fig. 4a). There is no asymmetry; the PV dipole is centred around the updraft and the magnitude of the positive and negative anomalies are equal. Another difference is that the anomalies on 5 June are much more localised around the updraft; no elongated bands of PV are visible. Compared to the 22 June composite, the updraft is more upright (Fig. 12b and c). There is, however, a slight tilt of the PV dipole with height; the negative anomaly is slightly higher than the positive PV anomaly. This is consistent with the schematic picture in Fig. 1b. Static stability anomalies are smaller than for the 22 June case (Fig. 12d), but still consistent with the anomalies

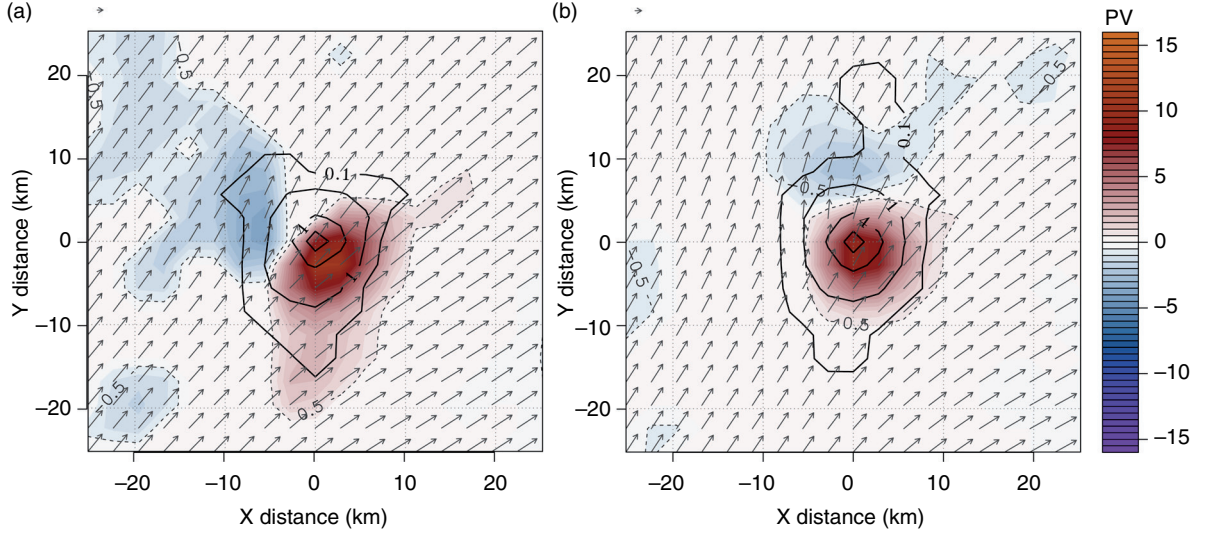


Fig. 11. (a) 3–7.3 km height integrated composites of PV (contours, in PVU), vertical velocity (contour lines, in ms^{-1}) and 0–6 km wind difference (arrows) for 22 June 2011 at 11 UTC. (b) Same as (a) but for 22 June 2011 at 18 UTC. Reference arrow at top of each plot indicates a wind speed of 5 ms^{-1} .

depicted in Fig. 1b. The largest flow anomalies are of a few ms^{-1} on 5 June (Fig. 12e). The DSI dipole is again shifted 90° counterclockwise. Helicity anomalies are much smaller compared to 22 June (Fig. 12f), which is consistent with the more unidirectional background wind shear for this case. This is also confirmed by the much lower values for the 0–3 km SRH.

On 5 June there is a similar increase of PV and DSI anomalies with increasing threshold (Supplementary Fig. 2a–d), though the qualitative differences between the composites using different thresholds are not that high. For this case, the dipole stays centred around the updraft for higher updraft strength, and the strength of the negative and positive anomaly is approximately equal. Again there is a quadratic increase of kinetic energy with increasing strength of the PV dipoles (Supplementary Fig. 3). The main difference between the longitudinal and latitudinal cross section

composites for the different thresholds is that the magnitude of the anomalies of all variables increases (not shown).

At other time steps the composites look similar (Supplementary Fig. 4 and Table 2). The composite at 11 UTC (Supplementary Fig. 4a) seems to be advected downshear. As indicated by the correlation coefficient r in Table 2 the updraft stays centred between the dipoles at all times. At later timesteps, from 18 UTC onwards, the composites for the 5 June case get less consistent (Supplementary Fig. 4b). For these forecast hours, the large-scale wind shear is less consistent, too, which explains the more variable direction of the PV dipoles.

6. Summary and discussion

The main goal of the article is to investigate the consistency of PV dipoles, theoretically described by Chagnon and Gray

Table 1. Storm cell characteristics at 22 June 2011, calculated over all storm updrafts for all ensemble members

Timestep	No. of cells	Max PV (5%/mean/95%)	Min PV (5%/mean/95%)	Correlation W and PV
11	132	3.77/15.53/34.50	−28.11/ −13.11/ −3.43	0.43
12	253	5.98/16.48/30.64	−26.93/ −13.20/ −4.11	0.36
13	399	6.53/16.43/29.58	−23.30/ −13.12/ −4.49	0.45
14	549	7.48/16.63/32.34	−24.48/ −13.76/ −5.20	0.45
15	596	6.95/17.48/35.37	−22.87/ −13.08/ −5.42	0.49
16	613	6.68/17.10/32.84	−25.38/ −12.82/ −4.58	0.50
17	552	6.08/16.82/35.04	−22.54/ −11.97/ −4.04	0.52
18	454	5.66/15.61/31.21	−22.23/ −10.95/ −4.28	0.57
19	390	5.68/15.78/30.57	−21.83/ −11.22/ −4.25	0.54

Maximum and minimum PV (in PVU) are searched in a 3×3 gridpoint surroundings of the vertical velocity maximum. The correlation coefficient between W and PV is calculated with eq. (4), following Davies-Jones (1984).

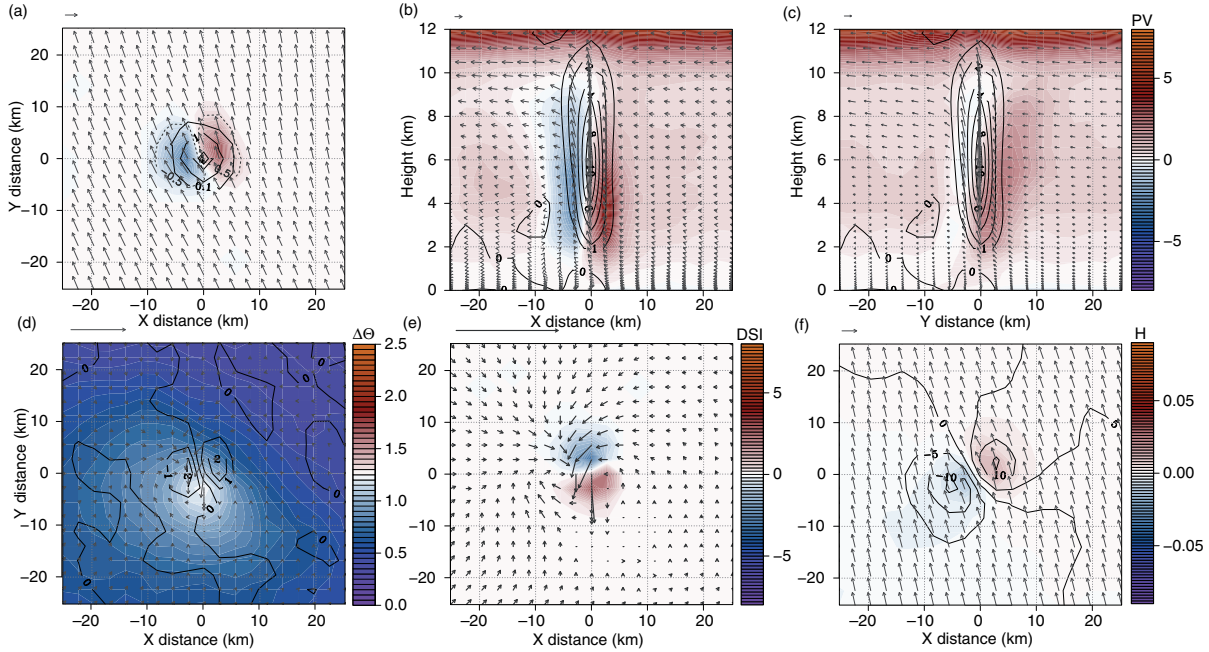


Fig. 12. (a) 3–7.3 km height integrated composites of PV (contours, in PVU), vertical velocity (contour lines, in ms^{-1}) and 0–6 km wind difference (arrows) for 5 June 2011 at 15 UTC. Dashed lines indicate PV contours of -0.5 PVU and 0.5 PVU. (b) Longitudinal cross section at 0 km of (a), with PV (contours, in PVU), vertical wind (black contours, in ms^{-1}) and wind velocity (arrows) for 5 June 2011 at 15 UTC. (c) as (b), but latitudinal cross sections at 0 km. (d) θ difference in the lowest 500 m (contours, in δK), 3–7.3 km height integrated PV (black contours, in PVU) and wind velocity (arrows) for 5 June 2011 at 15 UTC. (e) 3–7.3 km height integrated composites of DSI (contours, in $\text{PVU}^2\text{s}^{-1}$) and perturbation wind (arrows) for 5 June 2011 at 15 UTC. (f) 3–7.3 km height integrated composites of helicity H (contours, in ms^{-2}), 0–3 km SRH (black contour lines) and full wind velocity (arrows) for 5 June 2011 at 15 UTC. Reference arrow at top of each plot of 5 ms^{-1} .

(2009), associated with convective cells in a convection-permitting NWP model. To this end, Ertel PV anomalies on the convective weather scale in a convection-permitting model COSMO-DE have been analysed.

Our results show that PV anomalies can organise themselves in elongated bands of positive and negative PV, with a direction that depends on the large-scale wind shear of the horizontal wind. These bands are particularly visible in the more organised 22 June case. These PV bands are

stronger in magnitude compared to the orographically generated bands described by Aebischer and Schär (1998). A similarity is, however, that advection plays a role in the creation of the PV bands.

Chagnon and Gray (2009) used a linearised model to theoretically describe the generation of PV dipoles. They used quite strong assumptions; for example, they relied on hydrostatic and geostrophic balance. Many of the dipole characteristics are found in the much more complex case

Table 2. Storm cell characteristics at 5 June 2011, calculated over all storm updrafts for all ensemble members. Maximum and minimum PV (in PVU) are searched in a 3×3 gridpoint surroundings of the vertical velocity maximum. The correlation coefficient between W and PV is calculated with Eq. 4, following Davies-Jones (1984)

Timestep	No. of cells	Max PV (5%/mean/95%)	Min PV (5%/mean/95%)	Correlation W and PV
11	389	1.53/5.69/12.33	$-12.58/ -5.72/ -1.53$	0.02
12	807	1.36/5.59/12.25	$-12.53/ -5.91/ -1.52$	0.04
13	1225	1.74/6.75/14.16	$-14.69/ -6.97/ -1.81$	0.06
14	1479	1.94/7.29/15.72	$-14.64/ -7.23/ -2.00$	0.07
15	1421	1.97/7.87/17.29	$-16.00/ -7.76/ -2.11$	0.08
16	1245	1.85/8.40/19.02	$-17.00/ -7.81/ -1.92$	0.12
17	966	1.74/8.31/18.31	$-16.37/ -7.61/ -1.88$	0.13
18	653	1.80/8.24/18.56	$-16.35/ -7.65/ -1.95$	0.12
19	528	2.01/8.11/18.20	$-16.62/ -7.25/ -2.09$	0.13

of the non-hydrostatic COSMO-DE model. Composites around convective cells of our two severe weather cases with a different synoptic background confirm that the strength and direction of the PV dipoles mainly depends on the wind shear. Flow anomalies are also consistent, with cyclonic (anticyclonic) flow around the positive (negative) PV anomaly. As suggested by Chagnon and Gray (2009), the largest flow anomalies are found in between the positive and negative PV anomaly. For the case with a positive background helicity, 22 June, the positive PV anomaly is advected towards the updraft. This is consistent with the hypothesis in Davies-Jones (1984). Besides this, there exists a consistent helicity dipole of equal sign as the PV dipole, which is much more pronounced for the 22 June case.

We deliberately did not address the question: what kind of balance is to be expected for the PV dipoles? The main aim in calculating the composites was to determine the general structure of PV anomalies around storm updrafts. Chagnon and Gray (2009) suggested that the PV dipoles might be balanced to some degree. Our composites confirm this on a statistical basis, since the strength of the PV, DSI, wind velocity and H anomalies consistently increase with increasing updraft strength. There is also a consistent quadratic increase of the kinetic energy with increasing strength of the PV anomalies. One may speculate that this indicates an approximate linear inversion law between the PV and the wind velocity. Further, although potential temperature perturbations are small inside the PV dipoles, there is a decrease of static stability below the positive PV anomaly. The PV anomalies at the mesoscale are very similar to the QG PV anomalies described in Hoskins et al. (1985). DSI anomalies are also consistent and show that the PV dipoles are at least quasi-stationary, since large DSI anomalies exist only close to the updraft. Together with the coherent PV and wind anomalies, this suggests that there might still be some sort of ‘balance’.

As the PV dipoles can only be quasi-stationary, since there is always a diabatic perturbation at the storm updraft, there will be an ongoing adjustment to full balance. A possible mechanism of how balance is restored is by exciting gravity waves and/or sound waves. Gravity waves are generated by convection, but the exact process(es) is still a matter of debate (Fritts and Alexander, 2003). Sound waves are generated in the process of hydrostatic adjustment (Bannon, 1995). Vertical velocity spectra might give an answer if gravity waves play a role for our cases.

We focused on deep moist convection, and it is expected that our results are not valid for shallow convection. Shallow convection is characterised by a different diabatic heating profile (Houze, 2004). This explains why the PV anomalies in the mid-troposphere are mainly generated at the cold front on 22 June, where the precipitation is deep moist convective. For deep moist convection, the

composites give the typical structures associated with a weather situation. It might give the forecaster a more direct way to see if supercells are likely, without using environment characteristics like SRH.

In our study, we averaged all fields independent of the state of the evolution of the storm cell. Our future plan is to investigate the evolution of the balanced structure of PV dipoles further, by tracking individual cells. Clustering on storm environment like wind shear, CAPE, helicity, and/or storm characteristics will provide insights on the differences in storm characteristics in different environments. It is especially interesting to look at the most intense convective cells, as the composites for the two cases discussed in this study suggests that the morphology of the PV dipoles can change with cell intensity.

Our results confirm that ‘PV-thinking’ (Hoskins et al., 1985) is also relevant on convective weather scales. Although there are qualitative differences, the flow anomalies on the convective weather scale are remarkably similar compared to the anomalies on synoptic and subsynoptic scales. PV on the convective weather scale is probably most useful when there is some sort of balance, like the case of a multistorm system along a cold front, or when there is balanced flow along a single vertical updraft, for example, for a supercell. However, this can only be fully exploited if a balance condition between the PV anomalies and the mass and motion fields can be established. Only in that case the interpretation of PV can provide all details of the flow field in and around deep convective cells for impact or warning studies.

7. Acknowledgements

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