

Thermodynamics of a tropical cyclone: generation and dissipation of mechanical energy in a self-driven convection system

By HISASHI OZAWA^{1*} and SHINYA SHIMOKAWA², ¹*Graduate School of Integrated Arts and Sciences, Hiroshima University, Hiroshima 739-8521, Japan;* ²*National Research Institute for Earth Science and Disaster Prevention, Tsukuba 305-0006, Japan*

(Manuscript received 28 February 2014; in final form 18 October 2014)

ABSTRACT

The formation process of circulatory motion of a tropical cyclone is investigated from a thermodynamic viewpoint. The generation rate of mechanical energy by a fluid motion under diabatic heating and cooling, and the dissipation rate of this energy due to irreversible processes are formulated from the first and second laws of thermodynamics. This formulation is applied to a tropical cyclone, and the formation process of the circulatory motion is examined from a balance between the generation and dissipation rates of mechanical energy in the fluid system. We find from this formulation and data analysis that the thermodynamic efficiency of tropical cyclones is about 40% lower than the Carnot maximum efficiency because of the presence of thermal dissipation due to irreversible transport of sensible and latent heat in the atmosphere. We show that a tropical cyclone tends to develop within a few days through a feedback supply of mechanical energy when the sea surface temperature is higher than 300 K, and when the horizontal scale of circulation becomes larger than the vertical height of the troposphere. This result is consistent with the critical radius of 50 km and the corresponding central pressure of about 995 hPa found in statistical properties of typhoons observed in the western North Pacific.

Keywords: thermodynamics, tropical cyclone, Carnot maximum efficiency, thermal dissipation, entropy production

1. Introduction

A tropical cyclone is a large-scale convection system driven by a temperature contrast between the hot tropical sea surface and the cold top of the troposphere and is characterised by a low-pressure centre and inward spiral convergence of strong winds towards the centre. The typical diameter is from 200 to 2000 km, wind speed is from 20 to 85 m s⁻¹ and the typical central pressure generally ranges from 990 to 870 hPa, corresponding to central pressure drops of 20–140 hPa. As a conventional classification, tropical cyclones with maximum wind speeds¹ larger than 33 m s⁻¹ are called typhoons in the western North Pacific, hurricanes in the eastern North Pacific and western North Atlantic and severe tropical cyclones in other regions. A comprehensive

review on this subject can be found in the articles by Bergeron (1954) and Emanuel (2003).

Figure 1 shows a schematic cross-section of a tropical cyclone. There is a large-scale circulatory motion of air in this system, with a convergence of the air mass near the sea surface and a divergence of the air mass at the upper troposphere. Both the sea-level convergence and upper-level divergence are associated with an ascending motion at the centre, and a descending motion at the margin, respectively, thereby closing the circulation. Through this circulation, the air is heated at the hot sea surface and is cooled at the top of the troposphere. With the heating and cooling processes, the air expands near the sea surface with a supply of water vapour, and it contracts in the upper troposphere, with the water vapour condensing to precipitation. Since expansion takes place near the sea surface where the pressure is high, and contraction takes place in the upper troposphere where the pressure is low, part of the heat energy received at the sea surface is converted into mechanical work through the circulation of the atmosphere. The mechanical work thus

*Corresponding author.

email: hozawa@hiroshima-u.ac.jp

¹The maximum wind speed is defined as the maximum speed of wind at an altitude of 10 m, averaged over 10 minutes.

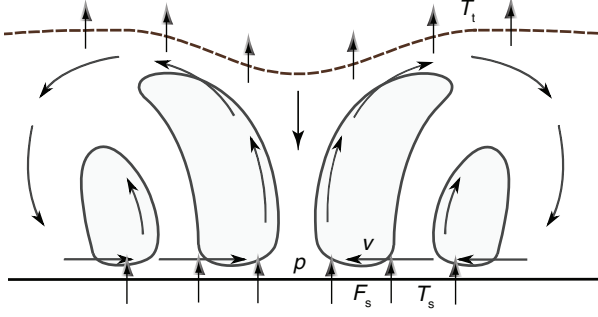


Fig. 1. A schematic cross-section of a tropical cyclone. The air is heated at the sea surface with temperature T_s and is cooled at the top of the troposphere with temperature T_t . The heated air is ascending at the centre, and the cooled air is descending in the margin, thereby forming a circulatory motion.

produced is stored in the background gravitational field as a ‘top-heavy’ density distribution that is gravitationally unstable. (Strictly speaking, this is a *potentially* top-heavy density distribution, whose instability could be realised if the atmosphere were in the isobaric condition.) The maximum amount of mechanical energy that can be extracted from this potentially unstable state was investigated by Lorenz (1955, 1967) and is called available potential energy. Part of this available potential energy is converted into kinetic energy of the fluid motion, thereby sustaining the circulation. In a steady state, the kinetic energy is dissipated into heat (random thermal motion of molecules) by turbulent drag force in the atmosphere with the action of viscosity, and the energy conversion rate is balanced by the energy dissipation rate. The remaining part of the available potential energy is dissipated directly through irreversible transport of sensible and latent heat in the atmosphere. This thermal dissipation process is known to exist in the atmosphere, and its contribution is estimated to be about 70% of the total energy dissipation in the global-scale atmospheric circulation (Pauluis and Held, 2002; Ozawa et al., 2003). Thus, one can expect that the thermal dissipation also plays an important role in the energy conversion process in tropical cyclones. While several studies have been carried out on the thermodynamic properties of tropical cyclones (e.g., Emanuel, 1986, 1987, 1999, 2003; Bister and Emanuel, 1998), the thermal dissipation process seems to have been missing from the previous work. The purpose of this paper is therefore to investigate the thorough energy dissipation processes in a fluid system that exchanges energy with its non-equilibrium surroundings. In this paper we shall start with the basic laws of thermodynamics (the first and second laws), and present a set of equations that describe the generation and dissipation rates of mechanical energy in a fluid system. The derived equations are kept to be in a general form so that one can apply this method to any type of fluid system that interacts with its surroundings. This method is applied to a tropical cyclone, and the formation

process of the circulatory motion is examined from a balance between the generation and dissipation rates of mechanical energy. The relation of thermal and viscous dissipation to entropy production is also discussed.

In what follows, we present a set of equations that express the generation and dissipation rates of mechanical energy in a fluid system from the basic laws of thermodynamics (Section 2). These equations are applied to a tropical cyclone, and the formation process of the circulatory motion is examined in Section 3. Results obtained from this study are examined and compared with the statistical properties of typhoons in the western North Pacific. Implications of the results for general dynamical properties of non-linear non-equilibrium phenomena are discussed in Section 4.

2. Thermodynamic equations

2.1. First and second laws of thermodynamics

Let us first consider a fluid system in which convective motion will be produced and its surrounding system with which the fluid system exchanges heat energy. Entropy of the fluid system and its surrounding system will be changed by the heat exchange process as well as by energy conversion processes in the fluid system. Because of the additive properties of entropy, the entropy change of the whole system can be expressed as a sum of the entropy change in each system:

$$dS_{\text{whole}} = dS_{\text{sys}} + dS_{\text{surr}}, \quad (1)$$

where dS_{whole} , dS_{sys} and dS_{surr} represents the entropy change of the whole system, that of the fluid system and that of the surrounding system, respectively. The second law of thermodynamics states that all spontaneous processes proceed in the direction in which entropy of the whole (isolated) system increases:

$$dS_{\text{whole}} = dS_{\text{sys}} + dS_{\text{surr}} \geq 0, \quad [\text{the 2nd law}] \quad (2)$$

where the equality sign refers to a reversible process with no net increase of entropy, a process that requires infinite time to proceed. All natural processes occur in a way that increases entropy in the whole system. This is a consequence of the second law of thermodynamics. It should be noted that, since the entropy increase of the whole system is associated with irreversible processes, we can call the term dS_{whole} *entropy production* due to irreversible processes that take place in the system. It should be kept in mind, however, that entropy production is identical to the net entropy increase in the whole system, and this increase is a driving force for all spontaneous phenomena in nature. We shall in due course discuss how this condition can account for the emergence of convective motion in a tropical cyclone.

Let us next consider that a small amount of heat energy is supplied from the surrounding system to a fluid parcel

through the system boundary, where the absolute temperature T is assumed in a local equilibrium state. In this case the entropy of the surrounding system will decrease by

$$dS_{\text{surr}} = -\frac{\delta Q}{T}, \quad (3)$$

where δQ is the heat supplied from the surrounding system². The heat supplied to the fluid parcel in the system is changed partly into internal energy of the fluid parcel and partly into mechanical work by volume expansion of the heated fluid. The internal energy of the fluid parcel will also be subject to volume expansion or contraction through the movement of the fluid parcel along its trajectory. Moreover, the internal energy may be transferred to the surrounding cold fluid by heat conduction, thermal radiation, or by latent heat transport. Suppose that all of this internal energy is finally transferred to the coldest region of the system with a minimum reference temperature T_r . This reference temperature can be set to be that at the top of the troposphere (i.e., tropopause) T_1 in the case of a tropical cyclone (Fig. 1). When the fluid expands at the system's surface where the temperature and pressure are high, and it contracts at the cold reference state where the pressure is low, part of the heat energy supplied to the fluid can be converted into mechanical work ($\delta W = \int p dV \geq 0$), even though the total volume change is zero, and the fluid parcel returns to the initial state ($\oint dV = 0$). The heat transferred to the reference state is then less than that gained at the system's surface, because of the first law of thermodynamics:

$$\delta Q' = \delta Q - \delta W, \quad (4)$$

where $\delta Q'$ is the heat transferred to the reference state, and δW is the mechanical work done by the fluid parcel. If we assume a cyclic return of the fluid parcel to the initial state, entropy of the fluid parcel remains unchanged. Entropy increase of the fluid system is then given by the heat transfer to the reference state divided by its temperature:

$$dS_{\text{sys}} = \frac{\delta Q - \delta W}{T_r}. \quad (5)$$

Equation (5) implies that the conversion of heat into mechanical work reduces the entropy of the system, and therefore, an enormous amount of mechanical work conversion is prohibited by the second law [inequality; eq. (2)].

Substituting eqns. (3) and (5) into eq. (2), we can rewrite the second law as:

$$dS_{\text{whole}} = \frac{T - T_r}{TT_r} \delta Q - \frac{\delta W}{T_r} \geq 0. \quad (6)$$

²Here the sign δ denotes a small change of a pass variable whereas d denotes that of a state variable.

The first term on the right-hand-side of the first equality denotes entropy increase by the heat transport from T to T_r , and the second term denotes entropy reduction by the work generation. This inequality suggests that, although the work generation reduces the entropy of the whole system, such a process can proceed provided that a certain amount of heat energy is transferred from a hot to cold temperature, thereby increasing the entropy of the whole system. Inequality [eq. (6)] shows the condition for spontaneous emergence of dynamic motion in a fluid system and is identical to a thermodynamic proposition deduced to account for the emergence of dynamic motions in non-equilibrium systems (Ozawa, 1997).

We can rewrite eq. (6) to express the work generation more explicitly as:

$$\delta W = \left(1 - \frac{T_r}{T}\right) \delta Q - T_r dS_{\text{whole}}. \quad (7)$$

The first term on the right-hand-side represents the maximum work attained through a reversible Carnot cycle, whereas the second term denotes dissipation of the work by entropy increase in the whole system due to irreversible processes in the fluid system. Since in all natural processes $dS_{\text{whole}} > 0$, the actual amount of work generation is always less than the Carnot maximum work. Notice that in the derivation of eq. (7), we did not assume reversibility, but assumed a cyclic return of the fluid parcel to the initial state (i.e., a steady state). In a more general non-steady case, a change of free energy of the fluid system appears in the expression (e.g., Yoshida and Mahajan, 2008), although the framework of the results remains unchanged.

Let us then evaluate the rate of change of mechanical work per unit time in the entire fluid system by diabatic heat exchange. To do so, the small changes in the variables (δQ , δW) are replaced by time derivatives of these variables, and eq. (7) is integrated over the entire volume of the fluid system as

$$\begin{aligned} \dot{W} &= \frac{d}{dt} \int_V w dV = \int_V \frac{\partial w}{\partial t} dV + \int_A w v_n dA \\ &\approx \int_V \left(1 - \frac{T_r}{T}\right) \dot{q} dV - T_r \dot{S}_{\text{whole}}, \end{aligned} \quad (8)$$

where $\dot{W}$ is the net rate of change of mechanical energy (work) in the fluid system, w is the work done by the fluid per unit volume, V is the volume of the fluid system, v_n is the normal component of fluid velocity at the system boundary, A is the surface bounding the system, $\dot{q} = \partial q / \partial t$ is the heating rate per unit volume and $\dot{S}_{\text{whole}}$ is the rate of entropy increase in the whole system. In eq. (8), we have assumed that the fluid velocity normal to the system boundary is negligible ($v_n \approx 0$), and advection of diabatic

heating is negligible compared to the heating rate itself ($\mathbf{v} \cdot \text{grad } q \ll \partial q / \partial t$). The first term on the right-hand-side of eq. (8) represents the maximum rate of generation of mechanical energy by a reversible Carnot cycle. This rate is identical to the generation rate of available potential energy due to diabatic heat exchange (Lorenz, 1955, 1967). The second term denotes the dissipation rate of the available energy through entropy production associated with various irreversible processes in the fluid system.

2.2. Irreversible entropy production and energy dissipation

In a convective fluid system, various kinds of irreversible processes take place, and these processes lead to entropy increase in the whole system, thereby reducing the mechanical energy available for the system, as represented by eq. (8). The irreversible processes relevant to atmospheric convection include transports of sensible and latent heat from hot to cold temperatures, radiation flux from hot to cold materials and mechanical dissipation of kinetic energy into internal energy by the turbulent drag force associated with viscosity. By contrast, heat advection due to fluid motion is, in principle, a reversible process without entropy production if the process proceeds in a quasi-static manner, although a rapid fluid motion often enhances irreversible heat conduction and viscous dissipation around the moving fluid (Ozawa and Shimokawa, 2014). We can take all these irreversible processes into account and express the rate of entropy increase in the whole system as a sum of each contribution [see eq. (A10)]:

$$\dot{S}_{\text{whole}} = \dot{S}_{\text{heat}} + \dot{S}_{\text{rad}} + \dot{S}_{\text{vis}}, \quad (9)$$

where $\dot{S}_{\text{heat}}$, $\dot{S}_{\text{rad}}$ and $\dot{S}_{\text{vis}}$ are the entropy production rates by sensible and latent heat transports, radiation flux and viscous dissipation of kinetic energy, respectively, as:

$$\dot{S}_{\text{heat}} = \int_V \mathbf{F}_{\text{heat}} \cdot \text{grad} \left(\frac{1}{T} \right) dV, \quad (10a)$$

$$\dot{S}_{\text{rad}} = \int_V \mathbf{F}_{\text{rad}} \cdot \text{grad} \left(\frac{1}{T^*} \right) dV, \quad (10b)$$

$$\dot{S}_{\text{vis}} = \int_V \frac{\Phi}{T} dV, \quad (10c)$$

where $\mathbf{F}_{\text{heat}}$ is the diabatic heat flux density due to conduction and latent heat transport³, $\mathbf{F}_{\text{rad}}$ is the radiation flux density, T^* is the effective radiation temperature and Φ is the dissipation rate of kinetic energy into internal energy by viscosity per unit volume of a fluid.

³The reason that entropy production due to the latent heat flux is expressed by Eq. (10a) is explained in Appendix [eq. (A14)].

Substituting eq. (9) into eq. (8), we can rewrite the net rate of change of mechanical energy in the fluid system as:

$$\dot{W} = \dot{W}_C - T_r (\dot{S}_{\text{heat}} + \dot{S}_{\text{rad}} + \dot{S}_{\text{vis}}), \quad (11)$$

where $\dot{W}_C = \oint (1 - T_r/T) \dot{q} dV$ is the Carnot maximum working rate achieved through a reversible process. The first and second terms in the brackets $[T_r (\dot{S}_{\text{heat}} + \dot{S}_{\text{rad}})]$ represent the reduction of the working rate by irreversible fluxes of heat and radiation from hot to cold temperatures, representing thermal (non-mechanical) loss of the available energy that could otherwise be extracted through a reversible process. This thermal loss of available energy can be called *thermal dissipation*. The last term $(T_r \dot{S}_{\text{vis}})$ represents irreversible dissipation of the available energy by the action of viscosity. This term may be called *viscous dissipation*. It should be noted that the viscous dissipation rate $(T_r \dot{S}_{\text{vis}})$ is slightly smaller than the pure mechanical dissipation rate (Φ) when the temperature at the place of dissipation is higher than the reference temperature, since $T_r \dot{S}_{\text{vis}} = (T_r/T) \Phi < \Phi$ when $T > T_r$. The reason for this is that the viscous dissipative heating contributes to additional heating at the place of T , and part of this heat energy can be converted into mechanical work through a Carnot reversible cycle⁴, so that the actual dissipation rate becomes less than Φ by the amount of $(1 - T_r/T) \Phi$, that is, $\Phi - (1 - T_r/T) \Phi = (T_r/T) \Phi$. This apparent reduction in the dissipation rate and the resultant conversion to the available energy was first pointed out for the case of tropical cyclones in an implicit way by Bister and Emanuel (1998), who suggested this conversion by assuming the viscous heating rate to be an additional heat source in the boundary layer. The validity of their arguments was questioned by Makarieva et al. (2010), leading to a debate (Bister et al., 2011). Here we confirm the validity of their arguments directly from the energy balance equation [eq. (11)] deduced from the first and second law of thermodynamics. The general expression for the viscous dissipation rate $[T_r \dot{S}_{\text{vis}} = T_r \int \Phi/T dV]$ describes the direct reduction in the mechanical dissipation rate, and this can be applied to any type of fluid system with arbitrary distributions of Φ and T . The validity of this expression $[T_r \dot{S}_{\text{vis}}]$ and its application to tropical cyclones will be discussed in Section 3.1.

We can consider the *actual* generation rate of mechanical energy, which is actually extracted from the fluid system, as

⁴The additional working is also subject to thermal dissipation, and the actual work generation rate is less than that due to a reversible Carnot cycle. The amount of this reduction has already been taken into account in the thermal dissipation rate $[T_r (\dot{S}_{\text{heat}} + \dot{S}_{\text{rad}})]$ in eq. (11).

the Carnot maximum working rate less the thermal dissipation rate:

$$\dot{W}_{\text{act}} = \dot{W}_C - T_r(\dot{S}_{\text{heat}} + \dot{S}_{\text{rad}}), \quad (12)$$

where $\dot{W}_{\text{act}}$ is the actual generation rate of mechanical energy. Using eq. (12), we can rewrite the net rate of change of mechanical energy in the fluid system [eq. (11)] as

$$\dot{W} = \dot{W}_{\text{act}} - T_r \dot{S}_{\text{vis}}. \quad (13)$$

When the fluid system is in a steady state, there is no net change of mechanical energy in the system: $\dot{W} = 0$. In this case the actual working rate is equal to the viscous dissipation rate:

$$\dot{W}_{\text{act, st}} = T_r \dot{S}_{\text{vis, st}} \quad \text{for a steady state,} \quad (14)$$

where the suffix st denotes the steady state. Equation (14) represents the steady-state energy balance between the actual working rate and the viscous dissipation rate.

For the sake of later convenience, we shall define a relative efficiency factor as the ratio of the actual working rate to the Carnot maximum working rate. Using eqns. (12) and (14), we get

$$k \equiv \frac{\dot{W}_{\text{act}}}{\dot{W}_C} = 1 - \underbrace{\frac{T_r(\dot{S}_{\text{heat}} + \dot{S}_{\text{rad}})}{\dot{W}_C}}_A = \underbrace{\frac{T_r \dot{S}_{\text{vis, st}}}{\dot{W}_C}}_B, \quad (15)$$

where k is the relative efficiency factor ($0 \leq k \leq 1$). The expression (A) shows that the relative efficiency is less than unity when there is thermal dissipation ($\dot{S}_{\text{heat}} + \dot{S}_{\text{rad}} > 0$), and it approaches unity in the thermally reversible limit: $\dot{S}_{\text{heat}} + \dot{S}_{\text{rad}} \rightarrow 0$. The expression (B) shows that the relative efficiency is equal to the ratio of steady-state viscous dissipation rate to the Carnot maximum working rate. Golitsyn (1970) called this relative efficiency factor a *utilisation coefficient* and estimated it to be about 0.1 for the global-scale circulation of the Earth's atmosphere. We shall evaluate this efficiency factor for tropical cyclones using eq. (15B) in Section 3.2.

3. Tropical cyclones

3.1. Energy balance for tropical cyclones

We shall apply the proposed formula of mechanical energy generation to a tropical cyclone. As shown in Fig. 1, the circulating fluid is heated at the hot sea surface and is cooled at the cold top of the troposphere so that the actual working rate can be positive definite. The heating rate at the sea surface per unit surface, F_s , can be expressed by the

empirical bulk formula (e.g., Bister and Emanuel, 1998; Emanuel, 2003) as:

$$F_s = \rho C_h (h_e - h_s) v, \quad (16)$$

where ρ is the air density near the surface, C_h is the heat transfer coefficient, h_e is the specific enthalpy of saturated air at the sea surface, h_s is the specific enthalpy of the near-surface moving air and v is the mean surface velocity. By using eqns. (15) and (16), the actual generation rate of mechanical energy can be expressed as:

$$\begin{aligned} \dot{W}_{\text{act}} &= k \dot{W}_C = k \int_A \left(1 - \frac{T_r}{T_s}\right) F_s dA \\ &\approx \rho A_e C_h k (h_e - h_s) \left(1 - \frac{T_t}{T_s}\right) v, \end{aligned} \quad (17)$$

where T_s is the sea surface temperature, T_t is the temperature at the top of the troposphere (tropopause) and A_e is the effective surface area covered by a tropical cyclone. Here we assume that surface heating is dominant for this system and the surface wind speed is uniform in the effective surface area. The latter assumption may be crude, but is practical for our simple approach, in which the mean energy balance of a tropical cyclone that covers the effective surface area is considered.

The mechanical energy thus generated in the fluid system is stored in the density distribution of the system and is converted into the kinetic energy of the circulatory motion of the atmosphere. The kinetic energy is then dissipated into internal energy by turbulent drag force, mainly in the boundary layer near the sea surface. The turbulent drag force can also be expressed by the empirical bulk formula:

$$f_d = \rho C_d v^2, \quad (18)$$

where f_d is the turbulent drag force per unit surface and C_d is the drag coefficient.

The viscous dissipation rate of the kinetic energy in the fluid system is given by the second term in the energy balance equation [eq. (13)] as:

$$D = T_r \dot{S}_{\text{vis}} = T_r \int_V \frac{\Phi}{T} dV = T_r \int_V \frac{f_d v}{T} dV \approx \rho A_e C_d \frac{T_t}{T_s} v^3, \quad (19)$$

where D is the viscous dissipation rate of the kinetic energy in the tropical cyclone. As we have discussed in Section 2.2., the viscous dissipation rate is smaller than the pure mechanical dissipation rate ($\int \Phi dV$) since the dissipation takes place in the surface boundary layer whose temperature is higher than the reference temperature at the tropopause ($T_s > T_t$). Part of the energy dissipated in the boundary layer can thus be converted into mechanical energy through the energy transport process to the reference temperature. This

is a characteristic feature of a non-equilibrium system – one cannot simply treat the mechanical energy balance when there is an inhomogeneous temperature distribution in the concerned system. In the case of tropical cyclones, the reduction in the dissipation can be about 30% of the pure mechanical dissipation rate, assuming typical values of $T_s = 300$ K and $T_t = 205$ K in eq. (19). This reduction yields an apparent increase in the mean velocity of about 20% according to the balance condition between eq. (17) and eq. (19)⁵. This result agrees with an earlier estimate for the increase of wind speeds by Bister and Emanuel (1998).

It should be noted that we have assumed in the formulation of eq. (19) that the majority of dissipation occurs in the thin boundary layer and neglected other possible dissipation such as that due to falling rain (hydrometeors) or that due to lateral momentum diffusion. While the former dissipation is known to make a certain contribution to total viscous dissipation in the global-scale atmospheric circulation (Pauluis et al., 2000; Lorenz and Rennó, 2002), we can show that this is of a minor contribution in the case of tropical cyclones. Suppose that 80% of the surface heat flux F_s is caused by the latent heat flux. If the water vapour condenses at a mean altitude of H_c , the dissipation rate due to precipitation is approximately $D_p \approx 0.8 g H_c F_s / L$, with g being the acceleration of gravity and L the specific latent heat of vaporisation. Since the total viscous dissipation rate is equal to the actual energy generation rate in the steady state, we get $D \approx k(1 - T_t/T_s)F_s$ from eq. (17). The ratio of D_p to D is then $D_p/D \approx 0.8 g H_c / [kL(1 - T_t/T_s)] \approx 0.08$, using typical values of $H_c \approx 5$ km, $T_t = 205$ K, $T_s = 300$ K and $k = 0.6$ adopted from the observational data shown in Section 3.2. This means that the precipitation-induced dissipation is less than 10% of the total viscous dissipation and can be neglected as the first order approximation. We have also neglected energy dissipation due to momentum diffusion in lateral directions. This contribution may be important in the marginal regions where the mean velocity gradient is very large. However, since we consider the mean dissipation rate in the boundary layer whose horizontal scale is much larger than the vertical convection scale, we have omitted the energy loss through the side margins of a mature tropical cyclone. The marginal energy loss becomes important for an initial growth process of a small tropical cyclone and this effect on the growth process will be discussed from energy balance considerations as follows.

⁵The energy balance condition is given by $C_h k(h_e - h_s)(1 - T_t/T_s) = C_d(T_t/T_s) v^2$. When the energy generation rate (the left-hand-side) remains unchanged, the dissipation rate with the reduction effect (T_t/T_s) should be equal to that without the effect with a corresponding velocity (v_0): $C_d(T_t/T_s) v^2 = C_d v_0^2$. Then, $v = (T_s/T_t)^{1/2} v_0 \approx 1.21 v_0$.

The net rate of change of mechanical energy [eq. (13)] can be expressed by eqns. (17) and (20) as

$$\dot{W} = \underbrace{\rho A_e C_h k(h_e - h_s) \left(1 - \frac{T_t}{T_s}\right) v}_{\dot{W}_{\text{act}}} - \underbrace{\rho A_e C_d \frac{T_t}{T_s} v^3}_D, \quad (20)$$

The first term represents the actual generation rate of mechanical energy ($\dot{W}_{\text{act}}$) and the second term represents the viscous dissipation rate (D). As shown in Fig. 2, $\dot{W}_{\text{act}}$ is proportional to the surface velocity v , whereas D is proportional to v^3 . In the steady state ($\dot{W}_{\text{act}} = D$), there exist two solutions for the velocity v :

$$v = \begin{cases} 0, \\ v_{\text{st}} = \sqrt{(C_h/C_d)k(h_e - h_s)(T_s/T_t - 1)}. \end{cases} \quad (21)$$

The former ($v = 0$) is a static solution with no motion, whereas the latter ($v = v_{\text{st}}$) is a steady moving solution. The static state is possible but meta-stable since, if a small perturbation shifts the state to that with a finite velocity, the generation rate of mechanical energy exceeds the dissipation rate. The net gain of the mechanical energy then accelerates the growth of the velocity, leading to a positive feedback loop. The growth of the velocity can continue until the system reaches the steady state with the steady velocity (v_{st}). This steady state is stable since any further perturbation results in negative feedback. A shift to a larger velocity leads to excess dissipation over generation, whereas a shift to a smaller velocity leads to excess generation over dissipation, thereby letting the system's state go back to the stable steady state (see Fig. 2). Thus, the stable moving state of a tropical cyclone tends to be produced in the tropical region when the sea surface temperature (T_s) is much higher than that at the tropopause (T_t) so that the gradient of $\dot{W}_{\text{act}}$ against v is large enough to

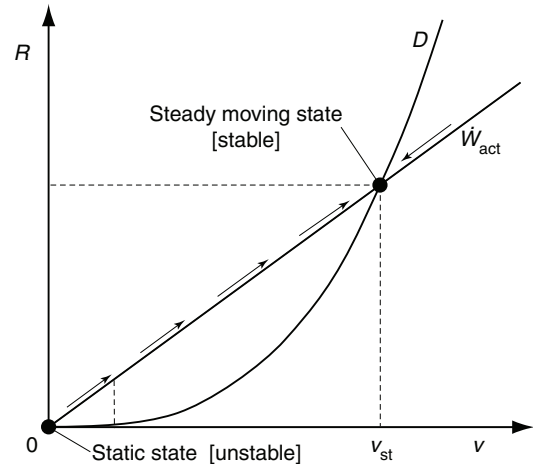


Fig. 2. The actual working rate $\dot{W}_{\text{act}}$ and the viscous dissipation rate D as functions of mean wind velocity v at the surface.

separate the moving state from the static state. The growth process of a tropical cyclone in the transitional period will be discussed in Section 4.1. Here it should be noted that the above argument is valid provided that the dissipation of mechanical energy takes place only in the surface boundary layer and the height of convection reaches the level of tropopause. This assumption may not be valid for an initial developing stage of a small tropical cyclone whose height is lower than the tropopause. In this case, the generation rate of mechanical energy is then less than eq. (17) because the reference temperature cannot reach the tropopause temperature ($T_r > T_t$), and the dissipation rate can be larger than eq. (19) because of additional dissipation at the side margins of the convection cell. It therefore seems that a critical size exists above which the convective motion of a tropical cyclone starts to develop. Presumably, this critical size should be larger than the vertical height of the troposphere. The critical size of tropical cyclones and its statistical significance will be discussed with observational data in Section 3.3.

3.2. Central pressure and relative efficiency

We shall now estimate the central pressure of tropical cyclones. In a steady state, we can expect a mechanical balance between the centrifugal force of the rotating air and the pressure gradient force exerted on the air as:

$$\frac{\Delta p}{l} = \rho \frac{v_{st}^2}{l} = \frac{p_{env}}{RT_s} \frac{v_{st}^2}{l}, \quad (22)$$

where $\Delta p = p_{env} - p$ is the pressure drop at the centre of the tropical cyclone, p is the air pressure at the centre, p_{env} is that in the surrounding environment, l is the distance of the rotating air from the centre and R is the gas constant of air. Substituting the steady velocity of eq. (21) into eq. (22), and eliminating the velocity, we get

$$\Delta p = \frac{p_{env}}{RT_s} \frac{v_{st}^2}{l} = \frac{C_h k (h_e - h_s) p_{env}}{C_d R} \left(\frac{1}{T_t} - \frac{1}{T_s} \right). \quad (23)$$

Equation (23) shows that the central pressure drop increases with increasing the temperature contrast between the surface and the tropopause. It should be noted that eqns. (21) and (23) are similar to those obtained by Emanuel (1986), Emanuel (1987), Bister and Emanuel (1998), Emanuel (1999) and Emanuel (2003), except for the relative efficiency factor k . This difference stems from the fact that they were concerned with the maximum potential intensity (MPI) that a tropical cyclone could attain through a reversible Carnot cycle, whereas here we consider the actual generation process of mechanical energy in a fluid system where thermal dissipation as well as viscous dissipation take place. In addition, the framework of their work is based on the thermal wind

equation that is applicable only to a tropical cyclone. By contrast, the general equation for the mechanical energy generation [eq. (11)] deduced from the basic laws of thermodynamics is in a general form and is applicable to any type of non-equilibrium system that interacts with its surroundings. The general equation can also be applied to the growth process of tropical cyclones, as we shall discuss in Section 4.1.

The maximum potential pressure drop at the centre, Δp_{max} , is obtained by setting $k = 1$ in eq. (23):

$$\Delta p_{max} = \frac{C_h (h_e - h_s) p_{env}}{C_d R} \left(\frac{1}{T_t} - \frac{1}{T_s} \right). \quad (24)$$

and the relative efficiency factor is thereby

$$k = \frac{\Delta p}{\Delta p_{max}}. \quad (25)$$

Using eq. (25) we can estimate the relative efficiency factor, when we know the actual central pressure drop (Δp) by observations and calculate the maximum pressure drop (Δp_{max}) from the temperature field with eq. (24). It should be noted that the direct estimate of the efficiency factor is difficult because of the uncertainty about thermal dissipation due to turbulent mixing in the general expression [eq. (15A)]. As we have discussed in Section 2.2, the thermal dissipation rate can be replaced by the maximum working rate subtracted by the viscous dissipation rate in the steady state, and the alternative expression [eq. (15B)] has been used to derive eq. (25) together with the empirical bulk formulae of eqns. (16) and (18).

Figure 3 shows the relation between the observed pressure drop (ordinate) and the maximum pressure drop (abscissa) estimated with eq. (24) for 633 tropical cyclones (typhoons) observed in the North Pacific Ocean. The observational data are collected from the Regional Specialized Meteorological Center Tokyo–Typhoon Center best track data, for the period 1982–2005 (RSMC Tokyo–Typhoon Center, 2014). The estimations are made with monthly mean temperature and humidity distributions collected from the National Center for Environmental Prediction (NCEP) reanalysis and the National Oceanic and Atmospheric Administration (NOAA) optimum interpolation sea surface temperature (OISST) data sets (Shimokawa et al., in press). In these estimations, the ratio of heat to momentum exchange coefficients is set at $C_h/C_d = 0.9$, being consistent with observational results of $C_h = 1.15 \times 10^{-3}$ and $C_d = 1.3 \times 10^{-3}$ at $v \approx 10 \text{ m s}^{-1}$ by Fairall et al. (2003). Each point in Fig. 3 represents an observed pressure drop and an estimated maximum pressure drop averaged over a developed stage of a tropical cyclone that was observed for a time period longer than 1.5 d (more than 6 data). The developed stage is considered to be the time period when the central pressure is lower than 995 hPa; the reason for this critical

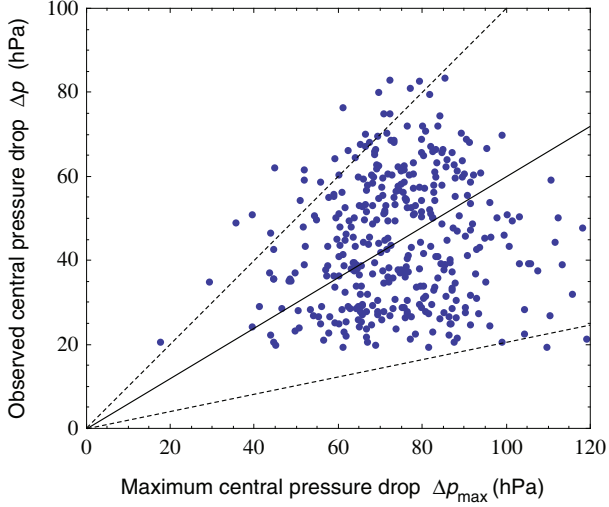


Fig. 3. Relation between the observational central pressure drop Δp and the maximum central pressure drop $\Delta p_{\max}$ estimated from temperature and humidity profiles of the atmosphere at the sea surface. Each point indicates the relationship between Δp and $\Delta p_{\max}$ averaged over a developed stage of a typhoon with an observed central pressure of less than 995 hPa. The gradient corresponds to the relative efficiency factor k in eq. (25). The solid line indicates $k = 0.6$ and the dashed lines indicate $k = 0.2$ and 1, respectively.

pressure will be discussed in the next section. We can see a rather scattered distribution of points in Fig. 3, although they are located roughly in a region between the gradients of $k = 0.2$ and 1 (dashed lines). This large scattering is mainly due to the fact that these tropical cyclones are not in a completely steady state; they show relatively small k values (0.2–0.5) during the early accelerating stages, whereas they show large k values (0.7–1.0) during the later decaying stages. A similar trend was observed in the wind speed analyses by Emanuel (2000). As the entire average of these points, we get a relative efficiency factor of about 0.6 (solid line) as the mean value. This value of $k \approx 0.6$ shows that the actual generation rate of mechanical energy ($\dot{W}_{\text{act}}$) is about 40% smaller than the Carnot maximum working rate ($\dot{W}_C$) because of the presence of thermal dissipation due to irreversible transport of sensible and latent heat in the tropical cyclones. This explains the reason why the observed wind speeds of tropical cyclones cannot reach the maximum potential wind speeds estimated from the Carnot reversible cycle (i.e., $k = 1$), although the reason has been attributed to overestimation of the ratio of the exchange coefficients (C_h/C_d) or insufficient reduction in the surface wind speeds (Emanuel, 2000). The value of $k \approx 0.6$ obtained here is larger than that of 0.1 estimated for the global-scale atmospheric circulation by Golitsyn (1970), suggesting rather higher relative efficiency for tropical cyclones than the global-scale circulation. Although the reason for this

remains unclear, it seems to be related to the feedback growth process of tropical cyclones discussed in the previous section. To investigate this point further, we implement a statistical analysis using the RSMC typhoon data as follows.

3.3. Statistical properties of tropical cyclones

The statistical properties of the intensity of tropical cyclones are investigated using the RSMC Tokyo–Typhoon Center data set, containing central pressures observed at 6-h intervals of 633 typhoons over the period 1982–2005 (23991 data in total). Figure 4a shows the probability density distribution of all typhoons as a function of central pressure over bins of 5-hPa intervals. We can see a general trend that the frequency of occurrence decreases with decreasing central pressure (i.e., increasing intensity) over a pressure range from 900 to 1000 hPa. This trend is consistent with the natural tendency that the probability of occurrence of an event decreases with increasing intensity (free energy) of that event. Above 1000 hPa, the frequency decreases with increasing central pressure (i.e., decreasing intensity) because only strong tropical cyclones with tropical storm intensity (10-minute mean wind speed $v \geq 17.5 \text{ m s}^{-1}$) are compiled in the data set. Apart from this decline towards the high pressure limit, we note a slight decrease in frequency at around 995 hPa; the frequency slightly increases with increasing intensity from 995 to 990 hPa, showing an apparent contradiction to the natural tendency. The same decrease can also be found at 994 hPa in Fig. 4b (inset) in a more precise probability density distribution over bins of 2-hPa intervals for 990–1010 hPa, where the detailed observational data are available. These results suggest that the probability of occurrence of tropical cyclones increases when the central pressure becomes lower than 995 hPa, suggesting the existence of a critical pressure at around $p_c = 995 \text{ hPa}$.

The superimposed line in Fig. 4a shows the best-fit exponential distribution estimated by the least squares method for the pressure range from 900 to 1000 hPa. The exponential distribution can be expected when one considers equilibrium probability density distribution of an event with certain intensity (free energy). Kurgansky (2006), for instance, suggested such exponential probability density distribution as a function of diameters for ‘dust devils’ observed in arid areas on the Earth and Mars. We find a similar general trend for the probability distribution for the intensity of tropical cyclones. However, we can also see that the observed probability distribution deviates from the equilibrium distribution at around the critical pressure $p_c = 995 \text{ hPa}$, suggesting the occurrence of a non-equilibrium growth process of tropical cyclones with the central pressures below this critical pressure. This critical pressure seems to correspond to the critical size, above which tropical cyclones tend to develop with a feedback process, as

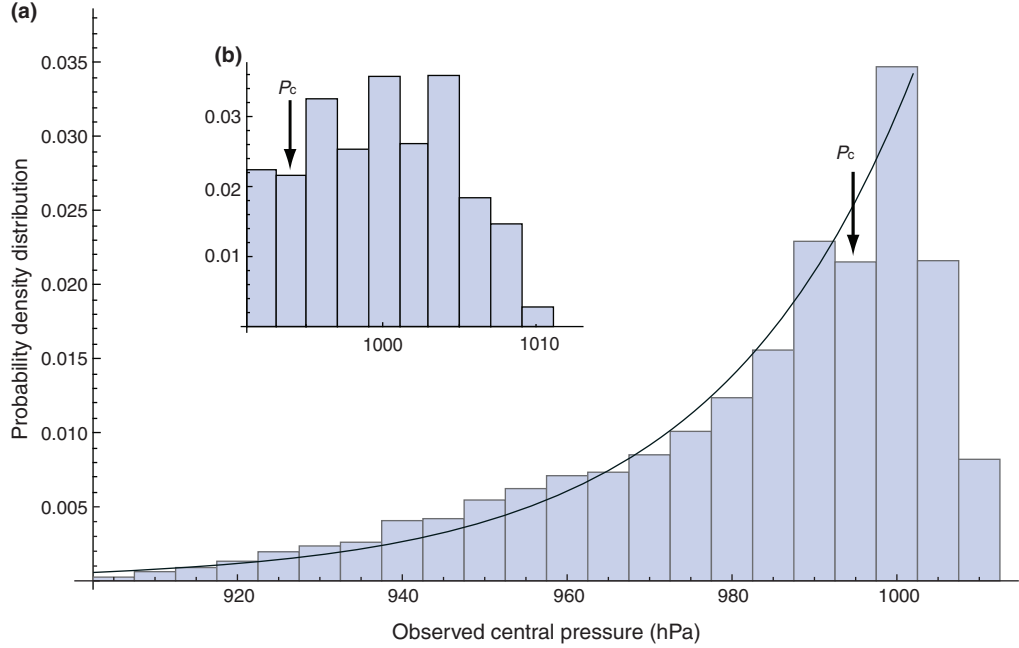


Fig. 4. Probability density distribution of central pressures observed for 633 typhoons over 23 yr from 1982 to 2005. (a) Distribution of central pressures between 905 and 1010 hPa, over bins of 5-hPa intervals; (b) distribution of central pressures between 992 and 1010 hPa, over bins of 2-hPa intervals. The superimposed line shows a best-fit exponential distribution: $P(\Delta p) = C_1 \exp(C_2 \Delta p)$, with $C_1 = 0.0545 \text{ hPa}^{-1}$ and $C_2 = 0.0413 \text{ hPa}^{-1}$, whereby statistical mean pressure depression is $\overline{\Delta p} = 1/C_2 \approx 24 \text{ hPa}$. A slight deviation from the exponential distribution exists at the critical pressure of $p_c = 995 \text{ hPa}$ (arrows).

we have discussed in Section 3.1. To confirm this point, we investigate the relation between the sizes and the central pressures of tropical cyclones using the RSMC Tokyo–Typhoon Center data.

Figure 5 shows the relation between the central pressures and mean radii of two strong wind regions, where the maximum wind speed exceeds 15.4 m s^{-1} and 25.7 m s^{-1} , respectively, observed for 633 typhoons in 6-h intervals over the period 1982–2005 (23991 data in total). We can see, in both cases, the mean radius increases with decreasing central pressure (increasing intensity). A somewhat rapid increase in the radius from 40 to 140 km can be seen with a pressure change from 1000 to 995 hPa for $v = 15.4 \text{ m s}^{-1}$. A similar rise in the radius from 15 to 65 km can be found with a pressure change from 985 to 980 hPa for $v = 25.7 \text{ m s}^{-1}$. These results suggest that tropical cyclones tend to grow when the mean radius of the strong wind regions exceeds a critical radius of about $r_c \approx 50 \text{ km}$. This tendency is consistent with the feedback growth process that can work for tropical cyclones when the horizontal scale exceeds the vertical height of the troposphere ($\approx 15 \text{ km}$)⁶, as pointed out

in Section 3.1. The range of central pressures that correspond to this critical radius is from 980 to 1000 hPa (Fig. 5, shaded region). This pressure range is also consistent with the critical pressure of 995 hPa found in the statistical distribution in Fig. 4, below which tropical cyclones tend to develop. These results suggest the existence of a critical radius and a

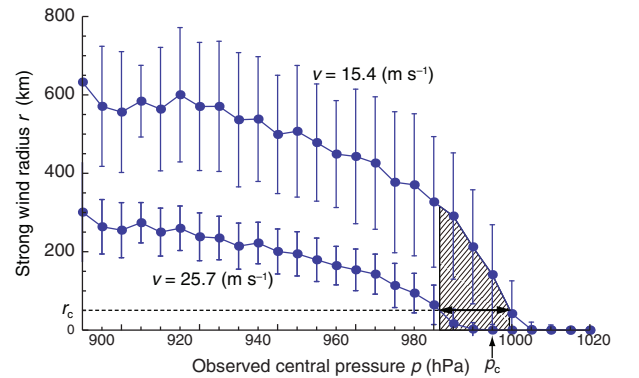


Fig. 5. Relation between observed mean radii of strong wind regions ($v = 15.4 \text{ m s}^{-1}$ and $v = 25.7 \text{ m s}^{-1}$) and central pressures observed for 633 typhoons over 23 yr. Solid curves with dots show the mean values, and the error bars show standard deviation. The shaded region corresponds to a pressure range where the radii exceed the critical size $r_c = 50 \text{ km}$, and where the radii increase rapidly with decreasing central pressure.

⁶The aspect ratio of the radius to the height is $50/15 \approx 3.3$ for the critical-size tropical cyclones. This aspect ratio is consistent with those of 3–5 obtained from numerical simulations of cumulus convection in a conditionally stable atmosphere by Asai and Nakasuji (1982).

corresponding critical central pressure for the genesis of tropical cyclones. Although the genesis process remains unclear, it has been suggested that the amalgamation of individual convective vortices and the resultant formation of a large-scale circulatory motion with an order of 100-km wide is important for tropical cyclogenesis (Fujiwhara, 1923; Bergeron, 1954; Emanuel, 2003). The coalescence of small-scale convective vortices and the genesis of a tropical cyclone with the similar order were also observed in numerical simulations by Nolan et al. (2007). It is therefore worthwhile to investigate this point further when more precise observational data are made available.

4. Discussion and conclusion

4.1. Growth process and surface temperature

We have seen in the preceding sections that a tropical cyclone tends to develop when the size of the circulation exceeds the critical value ($r_c \approx 50$ km) and when the central pressure becomes lower than 995 hPa. The tropical cyclone that meets this threshold can be regarded as a developed one after genesis, and it tends to grow with a feedback supply of mechanical energy as discussed in Section 3.1. We can treat the growth process of a developed tropical cyclone by assuming that the net generation rate of mechanical energy [eq. (20)] is equal to the rate of increase of kinetic energy of the tropical cyclone:

$$\dot{W} = \dot{W}_{\text{act}} - D = \frac{d}{dt} \left[\frac{\rho A_e H v^2}{2} \right] = \rho A_e H v \frac{dv}{dt}, \quad (26)$$

where $H = (1/\rho) \int \rho(z) dz$ is the density scale height of the atmosphere. In eq. (26), we have assumed that the velocity is nearly constant in the density scale height. Substituting eq. (17) and eq. (19) into $\dot{W}_{\text{act}}$ and D in eq. (26), we get

$$H \frac{dv}{dt} = C_h k (h_e - h_s) \left(1 - \frac{T_t}{T_s} \right) - C_d \frac{T_t}{T_s} v^2. \quad (27)$$

We can solve eq. (27) for v as a function of time with an appropriate initial condition ($v = 0$ at $t = 0$)⁷ as

$$v(t) = v_{\text{st}} \tanh \left(\frac{t}{\tau} \right), \quad (28)$$

where v_{st} is the steady-state velocity given by eq. (21), and τ is the characteristic time constant for the velocity growth given by

$$\tau = \frac{H}{\sqrt{C_d C_h k (h_e - h_s) (1 - T_t/T_s) (T_t/T_s)}}. \quad (29)$$

The characteristic time constant can be interpreted as a waiting time that is needed for the velocity to be about 76% of the steady-state velocity under the prescribed environmental conditions⁸. We can see from eq. (29) that the time constant depends mainly on the surface enthalpy difference ($h_e - h_s$) and the temperature difference between the sea surface and the tropopause. It should be noted that a similar equation for the growth of tropical cyclones was obtained from the thermal wind equation and an entropy budget equation in the boundary layer together with an assumption of a critical Richardson number at the outflow region by Emanuel (2012). Here we show that the growth trend of tropical cyclones can also be derived from the simple energy balance equation [eq. (26)] and the mean velocity assumption. As before, we note a difference in the existence of the relative efficiency factor (k) in the time constant [eq. (29)], since we consider the irreversible diffusion processes of internal energy, whereas an isentropic (reversible) process was assumed for the integration of the thermal wind equation by Emanuel (2012). We also note the elongation of the time constant by the temperature factor (T_t/T_s) in eq. (29), which is due to the viscous heating in the boundary layer and the resultant reduction in the total dissipation rate [eq. (19)].

Figure 6a shows the characteristic time constant for the growth of tropical cyclones as a function of the sea surface temperature T_s for different values of the surface relative humidity R_h . Calculations are made with eq. (29) using the following parameter values: $C_d = 1.3 \times 10^{-3}$ and $C_h = 1.15 \times 10^{-3}$ (Fairall et al., 2003), $H = 7500$ m, $T_t = 205$ K and $k = 0.6$ taken from Fig. 3. The enthalpy difference $h_e - h_s$ is calculated as a function of temperature and relative humidity at the sea surface [eq. (A15)]. Each line shows the result for the relative humidity of 0.7, 0.8 and 0.9, respectively. We can see that the characteristic time shortens rapidly with increasing the sea surface temperature; it is longer than 10 d for $T_s \leq 260$ K, whereas it becomes shorter than 5 d for $T_s \geq 280$ K. We can also see that the time constant is shorter for lower relative humidity because of the larger enthalpy difference, resulting in the larger latent heat flux and the faster cyclone growth due to greater mechanical-energy generation. The humidity effect is stronger for the lower sea surface temperatures ($T_s \leq 260$ K), but becomes

⁷For simplicity, the initial velocity is set at $v = 0$. We can also treat the growth process after the genesis by setting $v = v_c$ at $t = 0$, with v_c being the critical velocity at the time of genesis.

⁸Substituting $t = \tau$ in eq. (28), we get $v(\tau) = v_{\text{st}} \sinh(1) = v_{\text{st}} (e^2 - 1)/(e^2 + 1) \approx 0.762 v_{\text{st}}$.

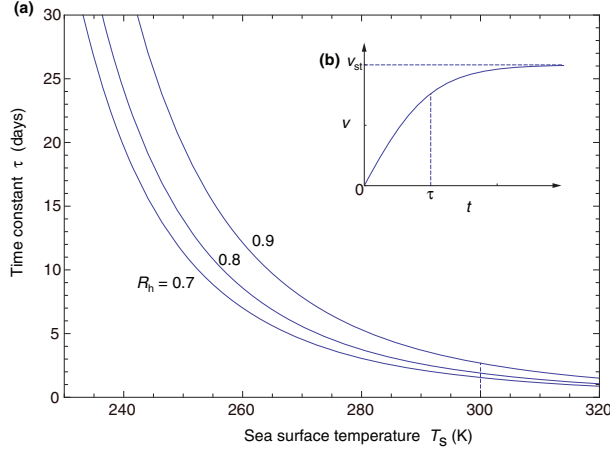


Fig. 6. (a) Characteristic time constant for the growth of tropical cyclones as a function of the sea surface temperature for different values of the relative humidity: $R_h = 0.7$, 0.8 and 0.9 . (b) Time evolution of the mean velocity of a tropical cyclone.

less significant for the higher sea surface temperatures ($T_s \geq 280$ K). When the sea surface temperature exceeds 300 K, the characteristic time constant is about 1–3 d, which is nearly constant and insensitive to the change in relative humidity. This result is consistent with the observational tendency that tropical cyclones are seldom observed over cold oceans whose surface temperature is less than 300 K, whereas they frequently develop within a few days over warm oceans whose surface temperature is larger than this critical value (e.g., Palmén, 1948; Gray, 1968). While the reason for the existence of this temperature threshold has been explained from the depth and extent of a conditionally unstable layer in the moist atmosphere in the static state (e.g., Palmén, 1948; Bergeron, 1954), our result suggests that the dynamical growth process towards the steady state is equally important for the manifestation of the temperature threshold, since the dynamic process and its time-scale are to great extent determined by the sea surface temperature [eq. (29)]. Also shown in Fig. 6b (inset) is the time evolution of the mean velocity of a tropical cyclone under the prescribed environmental conditions. Although very simple assumptions have been made in our calculation, the growth trend shows a certain resemblance to those observed in numerical simulations for the growth of tropical cyclones by Nolan et al. (2007).

4.2. Implications for non-equilibrium phenomena

In this paper, a growth process of a tropical cyclone has been examined from generation and dissipation processes of mechanical energy in the non-equilibrium environment. It is found that a tropical cyclone tends to develop within a few days by a feedback supply of mechanical energy when

the sea surface temperature is higher than 300 K, and when the horizontal scale of circulation exceeds a critical radius of about 50 km. This result is consistent with the observational tendency of tropical cyclogenesis as well as the statistical properties of typhoons observed in the western North Pacific. The emergence of tropical cyclones can therefore be seen to be a consequence of feedback growth of dynamic motion in a fluid system under the large non-equilibrium state. While the existence of such large-scale instability was pointed out through linear stability analyses by Ooyama (1964) and Charney and Eliassen (1964), the actual feedback growth process of a tropical cyclone towards its steady state based on the energy balance equation [eq. (26)] has not been fully examined before.

The tendency to increase the generation and dissipation of available energy has been observed in a variety of non-linear, non-equilibrium phenomena, and has been referred to as a principle of maximum entropy production (Ziegler, 1961; Sawada, 1981; Dewar, 2003; Ozawa et al., 2003; Martyushev and Seleznev, 2006; Ozawa and Shimokawa, 2014). The phenomena include the general circulation of the atmosphere (Paltridge, 1975, 1978; Ozawa and Ohmura, 1997), those of other planets (Lorenz et al., 2001; Fukumura and Ozawa, 2014), oceanic general circulation (Shimokawa and Ozawa, 2002, 2007), boundary layer turbulence (Kleidon et al., 2006), thermal convection, turbulent shear flow (Ozawa et al., 2001) and granular flows (Nohguchi and Ozawa, 2009). Here we present a result that tropical cyclones tend to develop towards steady states with higher rates of dissipation through feedback growth of dynamic motion once the size exceeds a critical value under the non-equilibrium state.

It should be noted that we have so far considered the growth process of tropical cyclones under vertically inhomogeneous distributions of temperature and humidity, that is, vertical non-equilibrium. In this situation, tropical cyclones tend to develop so as to recover the stability by reducing available energy and producing entropy. It seems therefore interesting to see how the trajectories of tropical cyclones are determined under horizontally inhomogeneous distributions of temperature and humidity. Our preliminary analysis suggests that tropical cyclones have a tendency to move along trajectories with higher rates of entropy production (Shimokawa and Ozawa, 2010). The statistical tendency of trajectories of tropical cyclones over inhomogeneous temperature fields is a subject of future studies and will be dealt with in future occasions.

5. Acknowledgements

The authors would like to thank Ralph Lorenz and one anonymous reviewer for valuable comments and suggestions to improve the quality of this paper. This study was supported by the Japan Society for the Promotion of

Science through grants 17540419, 17654090 and 25400465, Hiroshima University project research grants, and by the National Research Institute of Earth Science and Disaster Prevention.

6. Appendix A

A.1. Irreversible entropy production in a fluid system

The time rate of change of entropy of a fluid system is given by the following time derivative:

$$\dot{S}_{\text{sys}} = \frac{d}{dt} \left[\int_V \rho s \, dV \right] = \int_V \frac{\partial(\rho s)}{\partial t} \, dV + \int_A \rho s v_n \, dA, \quad (\text{A1})$$

where ρ is the density of the fluid, s is the entropy per unit mass, v_n is the normal component of surface velocity (positive outward), V is the volume of the system and A is the surface surrounding the system. The first term on the right-hand-side can be expanded and rewritten by using the equation of continuity $[\partial\rho/\partial t = -\text{div}(\rho \mathbf{v})]$

$$\frac{\partial(\rho s)}{\partial t} = \rho \frac{\partial s}{\partial t} + s \frac{\partial \rho}{\partial t} = \rho \frac{\partial s}{\partial t} - \text{div}(\rho s \mathbf{v}) + \rho \mathbf{v} \cdot \text{grad } s.$$

Substituting this in the volume integral of eq. (A1), and transforming the second term by using Gauss's theorem, we get

$$\dot{S}_{\text{sys}} = \int_V \rho \left[\frac{\partial s}{\partial t} + \mathbf{v} \cdot \text{grad } s \right] \, dV. \quad (\text{A2})$$

The expression in the square brackets is the substantial time derivative of entropy per unit mass of a fluid moving about in space (ds/dt). This change rate of entropy can be expressed by using the thermodynamic relation $[ds \equiv \delta Q/T = \{du + p \, d(1/\rho)\}/T]$ as:

$$\frac{ds}{dt} = \frac{1}{T} \left(\frac{du}{dt} + p \frac{d(1/\rho)}{dt} \right), \quad (\text{A3})$$

where u is the internal energy per unit mass, and p is the pressure. Substituting this in eq. (A2), and transforming the substantial time derivative to spatial time derivative ($du/dt = \partial u/\partial t + \mathbf{v} \cdot \text{grad } u$), we get

$$\dot{S}_{\text{sys}} = \int_V \frac{1}{T} \left[\rho \frac{\partial u}{\partial t} + \rho \mathbf{v} \cdot \text{grad } u + p \, \text{div } \mathbf{v} \right] \, dV. \quad (\text{A4})$$

Here the continuity relation $[d(1/\rho)/dt = -(1/\rho^2) \, d\rho/dt = (1/\rho) \, \text{div } \mathbf{v}]$ has been used. The first and second terms in the square brackets can be rewritten using the following relation:

$$\rho \frac{\partial u}{\partial t} + \rho \mathbf{v} \cdot \text{grad } u = \frac{\partial(\rho u)}{\partial t} + \text{div}(\rho u \mathbf{v}).$$

By substituting it in eq. (A4), we get

$$\dot{S}_{\text{sys}} = \int_V \frac{1}{T} \left[\frac{\partial(\rho c T)}{\partial t} + \text{div}(\rho c T \mathbf{v}) + p \, \text{div } \mathbf{v} \right] \, dV. \quad (\text{A5})$$

Here the relation $u = cT$ has been used, where c is the specific heat at constant volume. This equation is valid within the limits of an approximation that the temperature and the velocity are constant in the small volume element dV (Landau and Lifshitz, 1987, Sec. 49).

Entropy of the surrounding system will change by heat flux from the fluid system through the boundary. Following the definition of Clausius (1865), the change rate of entropy in the surrounding system is given by a surface integration of the heat and radiation fluxes divided by the material and radiation temperatures:

$$\dot{S}_{\text{surr}} = \int_A \frac{F_{\text{heat}}}{T} \, dA + \int_A \frac{F_{\text{rad}}}{T^*} \, dA. \quad (\text{A6})$$

where F_{heat} and F_{rad} are the surface heat flux and the radiation flux defined as positive outward (to the surroundings), T^* is the effective radiation temperature, dA is a small surface element and the integration is taken over the whole surface of the system. The effective radiation temperature can be set to a colour temperature defined by the frequency of radiation through Wien's displacement law, or a brightness temperature defined by the flux density of radiation – both temperatures become identical for black body radiation (Landau and Lifshitz, 1980, Sec. 63).

The rate of entropy increase in the whole system (i.e., the entropy production rate) is given by the sum of eqns. (A5) and (A6):

$$\begin{aligned} \dot{S}_{\text{whole}} = & \int_V \frac{1}{T} \left[\frac{\partial(\rho c T)}{\partial t} + \text{div}(\rho c T \mathbf{v}) + p \, \text{div } \mathbf{v} \right] \, dV \\ & + \int_A \frac{F_{\text{heat}}}{T} \, dA + \int_A \frac{F_{\text{rad}}}{T^*} \, dA. \end{aligned} \quad (\text{A7})$$

The first volume integral represents the rate of change of entropy of the fluid system, and the second and third surface integrals represent that of the surrounding system.

The general expression eq. (A7) can be rewritten in a different form. Because of the first law of thermodynamics (the conservation law of energy), the terms in the square brackets on the right-hand-side of eq. (A7) are related to convergences of heat and radiation fluxes, and the rate of heating by viscous dissipation (e.g., Chandrasekhar, 1961, Sec. 7) as

$$\frac{\partial(\rho c T)}{\partial t} + \text{div}(\rho c T \mathbf{v}) + p \, \text{div } \mathbf{v} = -\text{div } \mathbf{F}_{\text{heat}} - \text{div } \mathbf{F}_{\text{rad}} + \Phi, \quad (\text{A8})$$

where $\mathbf{F}_{\text{heat}}$ is the diabatic heat flux density due to conduction and latent heat transport, $\mathbf{F}_{\text{rad}}$ is the radiation flux density and Φ is the rate of dissipation of kinetic energy by viscosity per unit volume of a fluid. Here, the radiation effect has been included in the energy balance equation so as to extend our previous study (Ozawa et al., 2001; Shimokawa and Ozawa, 2001). The surface integral in eq. (A7) can be transformed to a volume integral by using Gauss's theorem:

$$\int_A \frac{X}{T} dA = \int_V \frac{\text{div } \mathbf{X}}{T} dV + \int_V \mathbf{X} \cdot \text{grad} \left(\frac{1}{T} \right) dV. \quad (\text{A9})$$

By substituting eq. (A8) in eq. (A7), and using the relation eq. (A9), we get

$$\begin{aligned} \dot{S}_{\text{whole}} = & \int_V \mathbf{F}_{\text{heat}} \cdot \text{grad} \left(\frac{1}{T} \right) dV + \int_V \mathbf{F}_{\text{rad}} \cdot \text{grad} \left(\frac{1}{T^*} \right) dV \\ & + \int_V \frac{\Phi}{T} dV. \end{aligned} \quad (\text{A10})$$

The first term represents the rate of entropy production due to heat flux, the second term represents the rate of entropy production due to radiation flux and the third term represents the rate of entropy production due to viscous dissipation. In this manipulation, the effective radiation temperature is assumed to be identical to the local material temperature ($T^* \approx T$), that is, local equilibrium. For non-equilibrium radiation, an additional term should be added to the right-hand-side: $-\int \text{div } \mathbf{F}_{\text{rad}} (1/T - 1/T^*) dV$, which represents entropy production due to absorption of radiation with a higher emission temperature (T^*) by an absorber with a lower material temperature (T).

It should be noted that the first term in the right-hand-side of eq. (A10) can include entropy production due to latent heat flux (vapour diffusion) in addition to sensible heat flux. In order to show this, let us suppose a situation in which a small amount of water molecules (n) is evaporated from a place with higher local equilibrium temperature (T_1) to a place with lower local equilibrium temperature (T_2) where the water vapour condenses into water (e.g., clouds). The amount of entropy production due to this water vapour diffusion can be expressed by that of corresponding vapour pressure from $e(T_1)$ to $e(T_2)$ as

$$\Delta S_{\text{diff}} = kn \ln \frac{e(T_1)}{e(T_2)}, \quad (\text{A11})$$

where k is the Boltzmann constant, n is the number of the water molecules, and $e(T_1)$ and $e(T_2)$ are equilibrium water vapour pressure at T_1 and T_2 , respectively (see, e.g., Pauluis and Held, 2002). The equilibrium vapour pressure is related

to the local equilibrium temperature through the Clausius–Clapeyron relation⁹:

$$\ln \frac{e(T_1)}{e(T_2)} = \frac{L}{k} \left(\frac{1}{T_2} - \frac{1}{T_1} \right), \quad (\text{A12})$$

where L is the latent heat of evaporation of a water molecule. Substituting eq. (A12) into eq. (A11), we get

$$\Delta S_{\text{diff}} = Ln \left(\frac{1}{T_2} - \frac{1}{T_1} \right). \quad (\text{A13})$$

We can see from eq. (A13) that entropy production due to diffusion of water molecules from a place with higher vapour pressure [$e(T_1)$] to a place with lower vapour pressure [$e(T_2)$] is expressed by the latent heat flux ($L n$) from the higher corresponding temperature T_1 to the lower corresponding temperature T_2 . We can also treat entropy production due to evaporation of the water molecules under an under-saturated condition as well as that due to condensation under a super-saturated condition. Both entropy productions result in reduction in entropy production due to diffusion of the water molecules, since the local vapour pressure becomes lower than the equilibrium pressure at the evaporation place [$e_1 < e(T_1)$] and it becomes higher than that at the condensation place [$e_2 > e(T_2)$]. The total sum of entropy production (evaporation, condensation and diffusion) is, however, represented by eq. (A13) no matter how the local irreversible processes may differ. Equation (A13) can therefore be seen as a general expression for entropy production due to water vapour transport. Now suppose a distribution of water vapour flux density $\mathbf{j}_v$ in a system with a certain distribution of temperature. The rate of entropy production due to the water vapour flux in the system is then given by

$$\dot{S}_{\text{heat, v}} = \int_V L \mathbf{j}_v \cdot \text{grad} \left(\frac{1}{T} \right) dV, \quad (\text{A14})$$

where $L \mathbf{j}_v$ represents the latent heat flux density in the system.

A.2. Enthalpy difference at the sea surface

The enthalpy difference between the saturated air and the unsaturated air near the sea surface can be expressed as

$$h_e - h_s = L(X_e - X_s) \approx 0.622 L(1 - R_h) \frac{e(T)}{p_a}, \quad (\text{A15})$$

⁹Here, we assume that the molecular volume of liquid water is negligible relative to that of water vapour and that the ideal gas equation is valid for the water vapour (cf. Callen, 1985).

where X_e and X_s are the mixing ratios of the saturated air and the unsaturated air near the sea surface, respectively, R_h is the relative humidity, $e(T)$ is the equilibrium water vapour pressure and p_a is the surface air pressure. In eq. (A15) we have assumed that the water vapour pressure is much smaller than the total air pressure ($e(T) \ll p_a$). We have used eq. (A15) to calculate the enthalpy difference in eq. (29) with an empirical formula for the equilibrium water vapour pressure as a function of temperature given by Hardy (1998).

References

- Asai, T. and Nakasugi, I. 1982. A further study of the preferred mode of cumulus convection in a conditionally unstable atmosphere. *J. Meteorol. Soc. Japan*. **60**, 425–431.
- Bergeron, T. 1954. The problem of tropical hurricanes. *Q. J. Roy. Meteorol. Soc.* **80**, 131–164.
- Bister, M. and Emanuel, K. A. 1998. Dissipative heating and hurricane intensity. *Meteorol. Atmos. Phys.* **65**, 233–240.
- Bister, M., Renno, N., Pauluis, O. and Emanuel, K. 2011. Comment on Makarieva et al. ‘A critique of some modern applications of the Carnot heat engine concept: the dissipative heat engine cannot exist’. *Proc. R. Soc. A*. **467**, 1–6.
- Callen, H. B. 1985. *Thermodynamics and an Introduction to Thermostatistics*. 2nd ed. Wiley, New York.
- Chandrasekhar, S. 1961. *Hydrodynamic and Hydromagnetic Stability*. Oxford University Press, Oxford.
- Charney, J. G. and Eliassen, A. 1964. On the growth of the hurricane depression. *J. Atmos. Sci.* **21**, 68–75.
- Clausius, R. 1865. Ueber verschiedene für die Anwendung bequeme Formen der Hauptgleichungen der mechanischen Wärmetheorie. [On several convenient forms of the fundamental equations of the mechanical theory of heat.] *Ann. Phys. Chem.* **125**, 353–400.
- Dewar, R. 2003. Information theory explanation of the fluctuation theorem, maximum entropy production and self-organized criticality in non-equilibrium stationary states. *J. Phys. A: Math. Gen.* **36**, 631–641.
- Emanuel, K. 2000. A statistical analysis of tropical cyclone intensity. *Mon. Weather Rev.* **128**, 1139–1152.
- Emanuel, K. 2003. Tropical cyclones. *Annu. Rev. Earth Planet. Sci.* **31**, 75–104.
- Emanuel, K. 2012. Self-stratification of tropical cyclone outflow. Part II: implications for storm intensification. *J. Atmos. Sci.* **69**, 988–996.
- Emanuel, K. A. 1986. An air–sea interaction theory for tropical cyclones. Part I: steady-state maintenance. *J. Atmos. Sci.* **43**, 585–604.
- Emanuel, K. A. 1987. The dependence of hurricane intensity on climate. *Nature*. **326**, 483–485.
- Emanuel, K. A. 1999. Thermodynamic control of hurricane intensity. *Nature*. **401**, 665–669.
- Fairall, C. W., Bradley, E. F., Hare, J. E., Grachev, A. A. and Edson, J. B. 2003. Bulk parameterization of air–sea fluxes: updates and verification for the COARE algorithm. *J. Clim.* **16**, 571–591.
- Fujiwhara, S. 1923. On the growth and decay of vortical systems. *Q. J. Roy. Meteorol. Soc.* **49**, 75–104.
- Fukumura, Y. and Ozawa, H. 2014. Entropy production in planetary atmospheres and its applications. In: *Beyond the Second Law: Entropy Production and Non-Equilibrium Systems* (eds. R. C. Dewar, C. H. Lineweaver, R. K. Niven and K. Regenauer-Lieb), Berlin, Springer, pp. 225–239.
- Golitsyn, G. S. 1970. A similarity approach to the general circulation of planetary atmospheres. *Icarus*. **13**, 1–24.
- Gray, W. M. 1968. Global view of the origin of tropical disturbances and storms. *Mon. Weather Rev.* **96**, 669–700.
- Hardy, B. 1998. ITS-90 formulations for vapor pressure, frostpoint temperature, dewpoint temperature, and enhancement factors in the range –100 to +100 C. In: *The Proceedings of the Third International Symposium on Humidity and Moisture*. National Physical Laboratory, London, pp. 1–8.
- Kleidon, A., Fraedrich, K., Kirk, E. and Lunkeit, F. 2006. Maximum entropy production and the strength of boundary layer exchange in an atmospheric general circulation model. *Geophys. Res. Lett.* **33**, L06706, 1–4.
- Kurgansky, M. V. 2006. Steady-state properties and statistical distribution of atmospheric dust devils. *Geophys. Res. Lett.* **33**, L19S06, 1–4.
- Landau, L. D. and Lifshitz, E. M. 1980. *Statistical Physics, Part I*. 3rd ed. Pergamon Press, Oxford. (First published in Russian 1937.)
- Landau, L. D. and Lifshitz, E. M. 1987. *Fluid Mechanics*. 2nd ed. Pergamon Press, Oxford. (First published in Russian 1944.)
- Lorenz, E. N. 1955. Available potential energy and the maintenance of the general circulation. *Tellus*. **7**, 157–167.
- Lorenz, E. N. 1967. *The Nature and Theory of the General Circulation of the Atmosphere*. World Meteorological Organization, Geneva.
- Lorenz, R. D., Lunine, J. I., Withers, P. G. and McKay, C. P. 2001. Titan, Mars and Earth: entropy production by latitudinal heat transport. *Geophys. Res. Lett.* **28**, 415–418.
- Lorenz, R. D. and Rennó, N. O. 2002. Work output of planetary atmospheric engines: dissipation in clouds and rain. *Geophys. Res. Lett.* **29**, 1–4. doi: 10.1029/2001GL013771.
- Makarieva, A. M., Gorshkov, V. G., Li, B.-L. and Nobre, A. D. 2010. A critique of some modern applications of the Carnot heat engine concept: the dissipative heat engine cannot exist. *Proc. R. Soc. A*. **466**, 1893–1902.
- Martyushev, L. M. and Seleznev, V. D. 2006. Maximum entropy production principle in physics, chemistry and biology. *Phys. Rep.* **426**, 1–45.
- Nohguchi, Y. and Ozawa, H. 2009. On the vortex formation at the moving front of lightweight granular particles. *Physica D*. **238**, 20–26.
- Nolan, D. S., Rappin, E. D. and Emanuel, K. A. 2007. Tropical cyclogenesis sensitivity to environmental parameters in radiative-convective equilibrium. *Q. J. Roy. Meteorol. Soc.* **133**, 2085–2017.
- Ooyama, K. 1964. A dynamical model for the study of tropical cyclone development. *Geofis. Intern.* **4**, 187–198.
- Ozawa, H. 1997. Thermodynamics of frost heaving: a thermodynamic proposition for dynamic phenomena. *Phys. Rev. E*. **56**, 2811–2816.

- Ozawa, H. and Ohmura, A. 1997. Thermodynamics of a global-mean state of the atmosphere – a state of maximum entropy increase. *J. Climate*, **10**, 441–445.
- Ozawa, H., Ohmura, A., Lorenz, R. D. and Pujol, T. 2003. The second law of thermodynamics and the global climate system: a review of the maximum entropy production principle. *Rev. Geophys.* **41**, 1018–24.
- Ozawa, H. and Shimokawa, S. 2014. The time evolution of entropy production in nonlinear dynamic systems. In: *Beyond the Second Law: Entropy Production and Non-Equilibrium Systems* (eds. R. C. Dewar, C. H. Lineweaver, R. K. Niven and K. Regenauer-Lieb) Springer, Berlin, pp. 113–128.
- Ozawa, H., Shimokawa, S. and Sakuma, H. 2001. Thermodynamics of fluid turbulence: a unified approach to the maximum transport properties. *Phys. Rev. E*, **64**, 026303–8.
- Palmén, E. 1948. On the formation and structure of tropical hurricanes. *Geophysica*, **3**, 26–38.
- Paltridge, G. W. 1975. Global dynamics and climate – a system of minimum entropy exchange. *Q. J. Roy. Meteorol. Soc.* **101**, 475–484.
- Paltridge, G. W. 1978. The steady-state format of global climate. *Q. J. Roy. Meteorol. Soc.* **104**, 927–945.
- Pauluis, O., Balaji, V. and Held, I. M. 2000. Frictional dissipation in a precipitating atmosphere. *J. Atmos. Sci.* **57**, 989–994.
- Pauluis, O. and Held, I. M. 2002. Entropy budget of an atmosphere in radiative–convective equilibrium. Part II: latent heat transport and moist processes. *J. Atmos. Sci.* **59**, 140–149.
- RSMC Tokyo–Typhoon Center (2014). Best track data. *Japan Meteor. Agency*. Online at: <http://www.jma.go.jp/jma/jma-eng/jma-center/rsmc-hp-pub-eg/besttrack.html>
- Sawada, Y. 1981. A thermodynamic variational principle in nonlinear non-equilibrium phenomena. *Prog. Theor. Phys.* **66**, 68–76.
- Shimokawa, S., Kayahara, T. and Ozawa, H. in press. Maximum potential intensity of typhoon as information for possible natural disaster. *Asian J. Environ. Dis. Manag.* **6**.
- Shimokawa, S. and Ozawa, H. 2001. On the thermodynamics of the oceanic general circulation: entropy increase rates of an open dissipative system and its surroundings. *Tellus A*, **53**, 266–277.
- Shimokawa, S. and Ozawa, H. 2002. On the thermodynamics of the oceanic general circulation: Irreversible transition to a state with higher rate of entropy production. *Q. J. Roy. Meteorol. Soc.* **128**, 2115–2128.
- Shimokawa, S. and Ozawa, H. 2007. Thermodynamics of irreversible transitions in the oceanic general circulation. *Geophys. Res. Lett.* **34**, L12606, 1–5.
- Shimokawa, S. and Ozawa, H. 2010. On analysis of typhoon activities from a thermodynamic viewpoint. In: *Research Activities in Atmospheric and Oceanic Modelling*, CAS/JSC WGNE Report No. 40, World Meteorological Organization, Geneva, Switzerland, pp. 2.07–2.08.
- Yoshida, Z. and Mahajan, S. M. 2008. “Maximum” entropy production in self-organized plasma boundary layer: a thermodynamic discussion about turbulent heat transport. *Phys. Plasmas*, **15**, 032307–6.
- Ziegler, H. 1961. Zwei Extremalprinzipien der irreversiblen Thermodynamik. [Two extremal principles in irreversible thermodynamics.] *Ing. Arch.* **30**, 410–416.