

Identification of Sunspots

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Abstract

According to the magneto-hydrodynamic theory of sunspots there should be a correlation between the spots on one hemisphere during a certain cycle and spots on the opposite hemisphere during the following cycle. This has been statistically confirmed by Galvenius and Wold, and by Whittle. It is pointed out that *individual* sunspot groups may be reproduced at the opposite hemisphere during the following cycle. An analysis of the sunspots between -4° and $+4^\circ$ seems to support this conclusion, and a number of sunspots are "identified" as being produced by the same original disturbance in the solar core.

1. Magneto-hydrodynamic theory of sunspots

In the magneto-hydrodynamic theory of sunspots a rather complicated mechanism must be postulated by which the sunspot producing whirl-rings should originate in the solar core¹. The relatively small latitude displacement of sunspots shows that the plane of the rings must be parallel to the magnetic field. The difference in magnetic polarity between the northern and the southern hemispheres indicates that the spots occurring during a certain cycle at the northern hemisphere cannot have originated at the same time as the spots of the southern hemisphere during the same cycle. This leads to the conclusion that the magnetic lines of force must pass *two*

activity regions in the solar core, one to the north and one to the south of the equatorial plane (cf. Fig. 1). The production of a disturbance e.g. in the northern region gives rise to magneto-hydrodynamic waves which about 40 years later appear on the surface of the northern hemisphere and produce sunspots. The same disturbance, however, also proceeds southwards and passes the southern activity region, where it becomes amplified, and finally reaches the surface of the southern hemisphere where it produces sunspots. These spots appear about 11 years after the spots due to the same original disturbance have appeared at the northern hemisphere.

2. Statistical confirmation

Although a mechanism of about this kind seems to be an inevitable consequence of the magneto-hydrodynamic theory, it is obvious that such a complicated mechanism could not be accepted without very strong empirical confirmation. The first direct support of the theory was obtained by an analysis of the latitude distribution of sunspots on the northern hemisphere, compared with the corre-

¹ Alfven (1948). Summarized in *Cosmical Electrodynamics*, Oxford 1950. Other theories of sunspots have recently been proposed by Walén and Schwartzschild. Walén's earlier papers (Ark. f. mat., astr. o. fysik 30 A, No 15; 31 B No 3 1944) are reviewed in *Cosmical Electrodynamics*. The later development of his ideas, with which the present author is not in agreement, are published in Ark. f. mat., astr. o. fysik 33 A No 18, 1946, and On the vibratory rotation of the sun, Lindstahls, Stockholm 1949. Schwarzschild's contribution was presented at the Solar physics conference in Florence 1952.

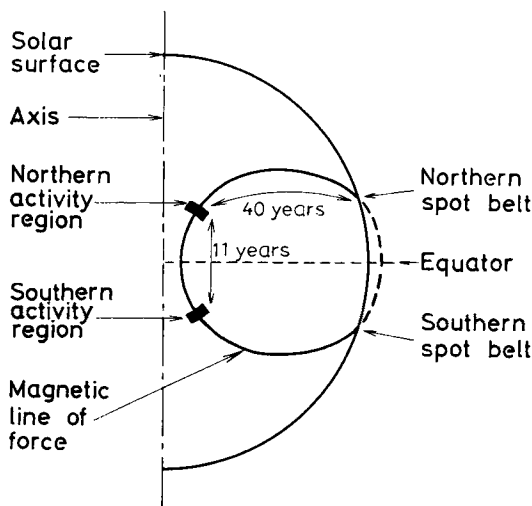


Fig. 1. Magnetic field within the sun. Meridional projection.

sponding distribution of the southern hemisphere 11 years later (ALFVÉN, 1948). According to the theory these patterns should be similar because the sunspots of one hemisphere have the same origin as sunspots of the other hemisphere 11 years later. A statistical investigation by GALVENIUS and WOLD (1948) showed that the similarity of the patterns was statistically significant.

Next confirmation was obtained by WHITTLE (1954) who investigated the correlation between the time series of spots in the latitude belts 16° – 20° at the northern and at the southern hemisphere. He was able to demonstrate in a very convincing way that there exists a correlation between the two hemispheres of the kind required by the theory.

3. Possible methods for spot correlation

The statistical correlation between the two hemispheres which consequently has been demonstrated in two independent ways, gives a strong support to the general lines of the magneto-hydrodynamic theory. There could no longer be any doubt that the spot activity on one hemisphere is coupled with the activity on the other hemisphere during next cycle. So far it has only been demonstrated that there is a statistical connection and this may be due to some factor which in a general way regulates the activity. It is also possible, however, that

there is a much more direct connection between the spots of the opposite hemispheres. The most extreme case would be that *each individual spot on one hemisphere is directly connected with a certain spot on the other hemisphere one cycle later*. The purpose of the present paper is to investigate whether such a connection exists. The work was started as soon as WHITTLE had got his positive results and inspired by this.

Suppose that a disturbance in e.g. the northern activity region of the solar core produces a whirl ring. This moves out along the magnetic lines of force to the surface and appears as a spot A_N in the northern hemisphere. The same disturbance also produces a wave which moves through the core, passes the southern activity region, where it may be in part amplified and in part reflected, and finally reaches the surface and produces a spot A_S on the southern hemisphere one cycle later. The problem is: how could we identify the spots A_N and A_S as being caused by the same original disturbance. There are two different possibilities:

1. *Identification by position.* The two spots should occur at the two intersections with the solar surface of the same magnetic line of force with a time difference corresponding to the time of travel through the core (which is of the order 11 years). If we could find that point of the southern hemisphere which is interconnected by a line of force through the interior with a certain point of the northern hemisphere we could study whether spots at these two points preferably occur with a certain time interval (of the order 11 years).
2. *Identification by structure.* The two spots which originate from the same disturbance in the solar core may have some similarity in structure. For example, their size, life time, or proper motion, may be correlated in some way.

4. Regularity of solar magnetic field

The identification by position would be easy if we knew the shape of the magnetic field inside the sun. This is not the case: We could, however, make a reasonable guess about it under the condition that it is rather regular. On the other hand if the solar field is very irregular, we have little hope to succeed. The first question to discuss is whether we should

expect the field in the sun's interior to be regular or not.

At first sight there seems to be an overwhelming probability that the solar field is very irregular. Zeeman effect measurements of the surface field indicate an irregular structure and strong variations from one day to next. (Whether Zeeman effect measurements of weak fields are reliable at all is still an open question (ALFVÉN, 1951). Further the structure of the corona, which probably is governed to a considerable extent by the magnetic field, indicates a variable and irregular structure. These arguments are not decisive, however, because what is important is the regularities of the "general" field in which the magneto-hydrodynamic waves proceed. This could be thought of as the time average of the actual field. It is superimposed by a variable field which the waves carry with them. Hence the irregularities and rapid changes of the actual magnetic field are quite consistent with a regular general magnetic field.

Further, the magnetic lines of force pass at least two layers, which according to current solar models should be turbulent, *viz.* in the core and in the photosphere. This seems to exclude the possibility of a regular field. Again it must be observed that the field of interest is the time average. A turbulence should be described as a series of waves travelling in the general field, but it does not contradict the assumption of a regular general field.

Having thus found that there is no decisive argument *against* a regular field we shall now examine the arguments in favour of it. According to Ferraro's theorem all parts of a certain line of force must in average rotate with equal angular velocity. As the sun has a non-uniform rotation it is likely that this brings with it that the rotational axis and the magnetic axis must coincide (although this has not been rigidly proved). If the solar magnetic field is of Cowling's decaying type it is quite reasonable that it is very regular because at the decay all irregularities should disappear rapidly. LUNDQUIST has discussed the shape of "magneto-hydrostatic fields" which can exist in a liquid medium without producing a hydrodynamic motion (LUNDQUIST, 1951). Such fields are in general "twisted". If the sun's general field is of this type it is quite reasonable that it should be regular but "twisted", i. e.

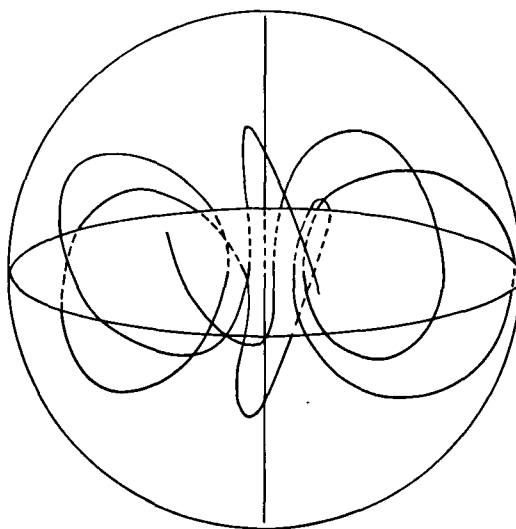


Fig. 2. Twisted magnetic field.

the field has a longitudinal component, so that a magnetic line of force intersects the surface at two points which do not have the same longitude. In fact, there is no reason to suppose that the longitude difference between the northern and the southern intersections with the solar surface should be zero. It is important to observe that Fig. 1 shows only the *projection* of a line of force on the meridional plane. The real shape is obtained if we rotate this plane around the axis when we proceed along the line of force. The spiral structure of a line of force is shown in Fig. 2.

Cowling has shown that a general magnetic field cannot be sustained if it is regular and symmetrical. The models by ELSASSER (1946, 1947), BULLARD (1949), and others are rather complicated and irregular. A regular field which does not decay systematically (counted over very long periods) is compatible with a model proposed some time ago, which assumes a decaying field, at intervals reinforced by a "convulsion" (ALFVÉN, 1950).

The irregularities and disturbances of the general field according to the mentioned models need not necessarily affect the lines of force which intersect the solar surface. They can very well be confined to the lines of force which altogether are hidden in the sun's interior.

Hence theoretical considerations rather sup-

port the view that the general field may be regular and symmetrical but possibly twisted. The conclusions are not decisive, however.

5. Empirical arguments for a regular field

There are quite a few empirical arguments for a regular field. During a certain cycle bipolar sunspots have a certain polarity at the northern hemisphere and the opposite polarity at the southern hemisphere. The line of division is the equator. Only very seldom does a spot of the "northern" polarity proceed as much as one degree to the south of the equator, and vice versa (WALDMEIER). Even independent of the magneto-hydrodynamic theory of sunspots this indicates that there is a division line in magnetic respect very close to the rotational equator. With this theory as a background we must suppose that waves with a certain polarity reach the northern hemisphere exactly down to the equator, and that waves of opposite polarity reach the southern hemisphere. Hence the magnetic equator must almost coincide with the rotational equator.

A second argument is obtained from the sunspot progression curve. According to the theory this curve should be given purely by the geometry of the field (Cosm. El. p. 118). The fact that the progression curve is very well reproduced during all the sunspot cycles, which have been studied, speaks in favour of a regular magnetic field.

Our third—and most important—argument is derived from the correlation between the spottedness pattern of the two hemispheres. An excess of sunspots at a certain latitude of one hemisphere is connected with a corresponding excess at the same latitude of the opposite hemisphere during next cycle. Judging from Fig. 5.16 in *Cosmical Electrodynamics* there is an agreement between the corresponding latitudes within a few degrees. This can only be interpreted in such a way that magnetic lines of force intersecting one hemisphere at a certain latitude, appear at the other hemisphere at the same latitude.

The accuracy can also be judged from the fact that if the spottedness is plotted as a function of the latitude with latitude intervals of 2° , the correlation is poor, but if the curves are smoothed with the weight 1:3:5:3:1 a good correlation is obtained. This indicates that the

accuracy by which a certain latitude is connected with the corresponding latitude of the other hemisphere is a few degrees.

A further confirmation of the regularity of the solar field is given by WHITTLE's analysis. The fact that the time series of 16° — 20° north is correlated with the series of 16° — 20° south shows that lines of force must connect these belts in a regular way.

The first ZEEMAN effect measurements of the general magnetic field indicated an angle of 5° between the magnetic and rotational axes. It is now generally agreed that the accuracy was insufficient to establish this, and there is no later confirmation. ABETTI has made an interesting study of the shape of the magnetic field near the poles as found from the structure of the corona (ABETTI, 1952). The difference between the magnetic pole and the heliographic pole which he finds refers to the instant structure of the field and not to the time average, which is what we discuss.

6. Identification by position

Our survey has shown that it is likely that magnetic lines of force of the general field intersect the two hemispheres at the same latitudes. Hence one of the necessary conditions for a sunspot identification by position is given. On the other hand we do not know the difference in longitude between the two "ends" of a line of force, so we must introduce this quantity as an unknown parameter. It should be kept in mind that the period of the solar rotation is not very well known. Further the rotation is non-uniform. Hence it is difficult to keep account of the longitudes.

We must now ascertain how an attempt to identify sunspots should be made in order to have some chance of success.

The regularity of the magnetic field is probably best near the equator. This is indicated by the fact that the equator strictly separates spots of different polarities (WALDMEIER). The pattern analysis discussed above (ALFVÉN, 1948) which refers to latitudes in the range 10° — 30° does not give a higher accuracy than a few degrees for the correspondence in latitude.

The non-uniform rotation introduces a difficulty into the analysis. Suppose that a disturbance travels along a line of force which intersects the solar surface at latitude φ and

another disturbance travels along a line of force through $\varphi + \Delta\varphi$. Suppose further that at a certain time the two disturbances have the same longitude. A certain time t later a longitude difference $\Delta\lambda$ will be produced by the non-uniform rotation. Adopting for the rotational velocity ω NEWTON's formula

$$\omega = \omega_0 - \omega' \sin^2 \varphi$$

we find

$$\begin{aligned} \Delta\lambda &= \omega' t [\sin^2 (\varphi + \Delta\varphi) - \sin^2 \varphi] \\ &\approx \omega' t \sin 2\varphi \cdot \Delta\varphi \end{aligned}$$

Putting $t = 4,000$ days and $\omega' = 2.8$ degrees per day we find

$$\Delta\lambda = 195 \sin 2\varphi \cdot \Delta\varphi$$

where $\Delta\varphi$ as well as $\Delta\lambda$ is expressed in degrees. It is unlikely that we can find or even define the latitude of a sunspot group within less than a couple of degrees. Putting $\Delta\varphi = 2^\circ$ we find that $\Delta\lambda \geq 90^\circ$ unless $\varphi < 7^\circ$. This means that for latitudes higher than 7° the uncertainty in longitude becomes more than 90° after one solar cycle. Consequently the possibility of identifying spots by their longitude becomes small unless we confine ourselves to very low latitudes.

There is also a third argument for choosing low latitudes, *viz.* that the number of sunspots is relatively small near the equator. This should make an eventual identification more unambiguous.

7. Construction of spot diagrams for the equatorial belt

Hence we confine the analysis to a small belt near the equator. All spot groups which in Greenwich "General Catalogue of Sunspots" or "Ledgers of groups of sunspots" are listed as having a mean latitude $0 < \varphi < 4^\circ$ or $0 > \varphi > -4^\circ$ are selected. For each cycle and hemisphere a diagram is plotted with the time "First seen" as abscissa and "Mean position of group, longitude" as ordinate. If the "Mean Area, corrected for foreshortening, whole spot" is less than 50 the spot group is represented by a dot. Larger spots are marked by the symbols shown in Fig. 3. The longitude is iterated in order to show the pattern even near 360° . A recurrent spot appears in the diagram as two dots close together.

Tellus V (1953), 4

8. Comparison between the patterns

According to the magneto-hydrodynamic theory of sunspots there is a possibility that the patterns of opposite hemispheres during the two consecutive cycles should be similar. In order to investigate whether this is the case we should compare e.g. the diagram 15 S (for the southern hemisphere during the 15th cycle) with 14 N and 16 N. Further we should compare 15 N with 14 S and 16 S. When comparing the diagrams we do not know *a priori* how the diagrams should be placed in longitude because the solar magnetic field may be twisted around the axis as discussed in § 4. Further it is difficult to keep account of the longitudes especially with regard to the non-uniform rotation. Hence the longitude displacement of two diagrams is arbitrary.

The exact time difference between a spot on one hemisphere and the opposite one is also unknown *a priori*, but it should be of the same order as the sunspot period, say in general between 10 and 12 years.

For the comparison we take the two diagrams (which have been plotted on transparent paper) and place one on the top of the other in such a way that one of the biggest spots of one diagram coincides with one of the biggest spots of the other diagram. If in this position most of the spots coincide the diagrams are judged to be similar.

9. The sequence 13 S—14 N—15 S—16 N—17 S

A comparison between the patterns is most unarbitrary when there are only a few spot-groups, and these form a distinct pattern. This makes it reasonable to start with the diagrams 15 S and 16 N. The biggest spot groups during 15 S occur in 1919 and at the end of 1922. In 16 N there is no big spot group later than 1932. Let us see whether it is possible to identify the spot group of 1932 as being caused by the same disturbance as the spot group of 1922. The difference in time between the two spot groups is about $9\frac{1}{2}$ years, the difference in longitude about 50° . If we place the two diagrams 15 S and 16 N on top of each other we find that the medium size spot group in 1921 falls close to the spot group of 1931. A series of spot groups in 1919 and 1920 could very well be of the same origin as a similar series in 1929 and 1930. So far we have only considered large

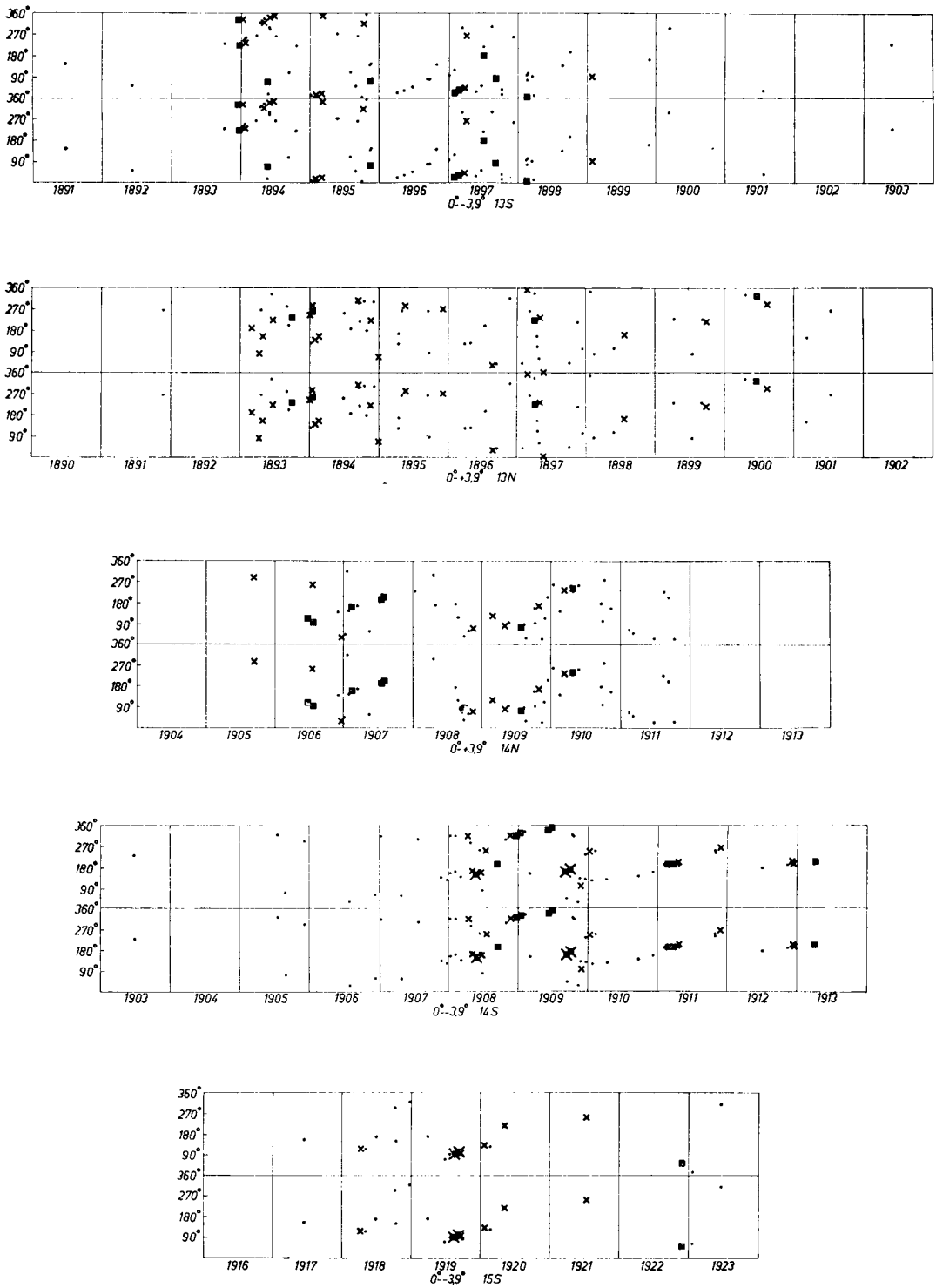
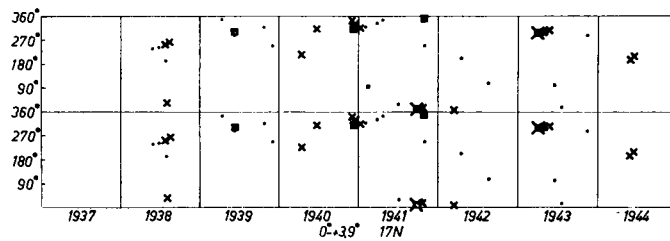
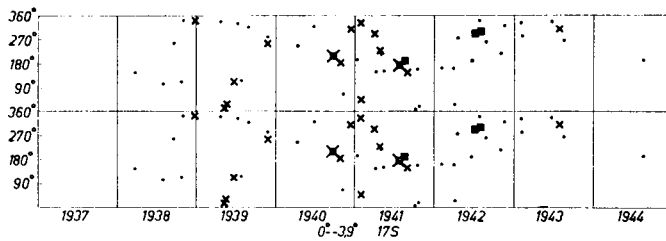
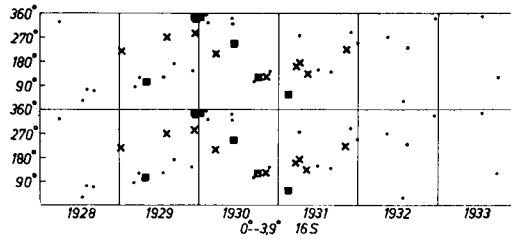
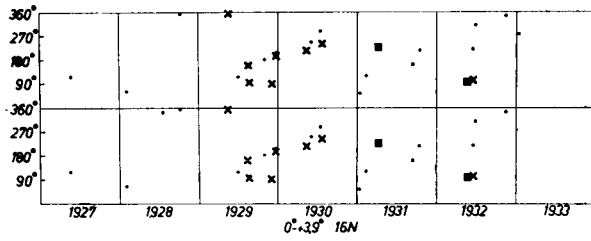
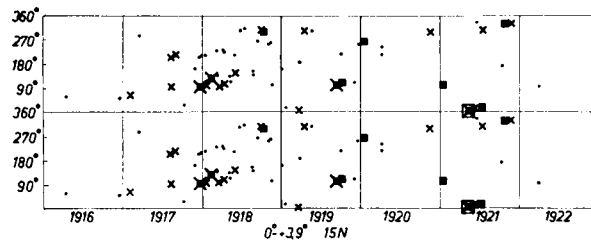


Fig. 3. Spot groups with $[\varphi] < 4^{\circ}$ during the cycles 13—17 in the northern (N) and southern (S) hemisphere.



● Area 50. × Area 50—150. ■ Area 150—500. & Area 500—

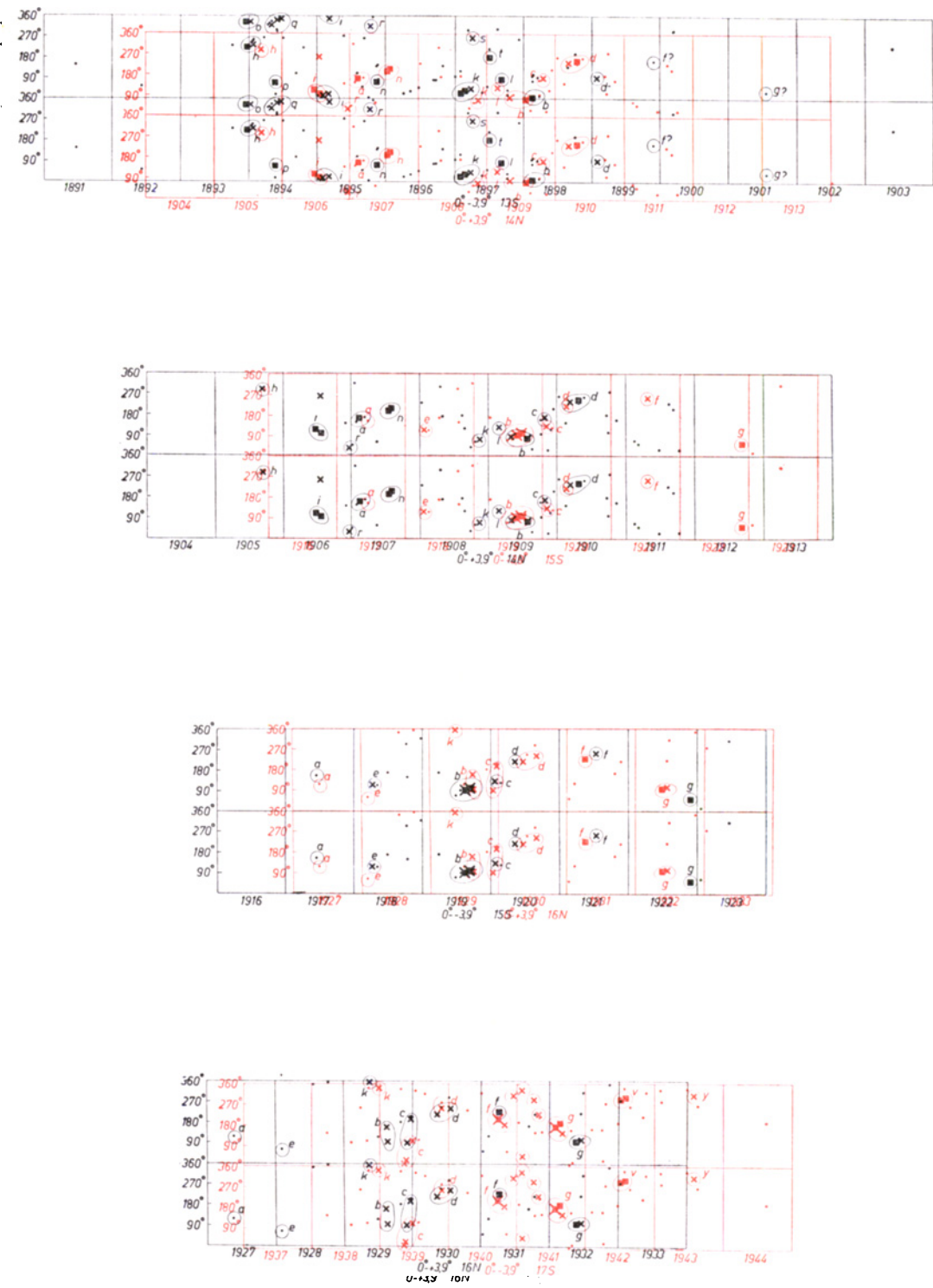
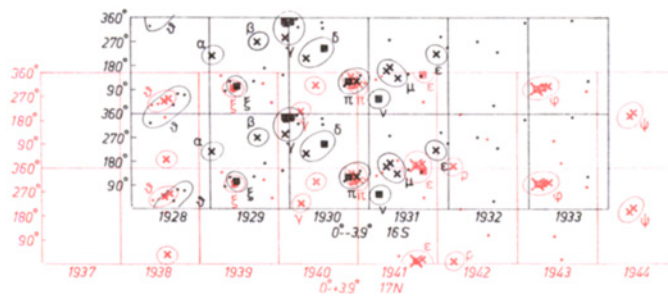
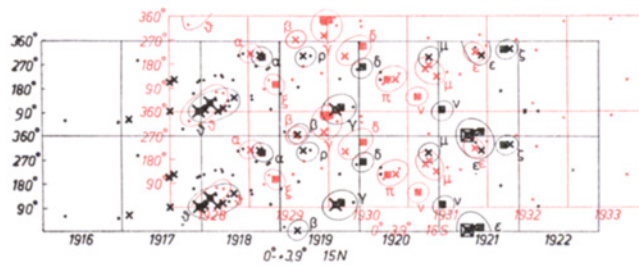
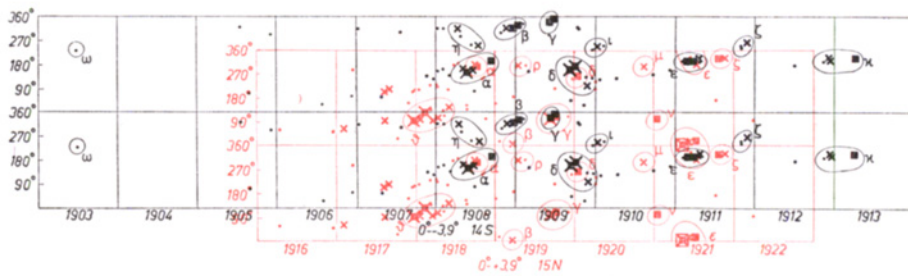
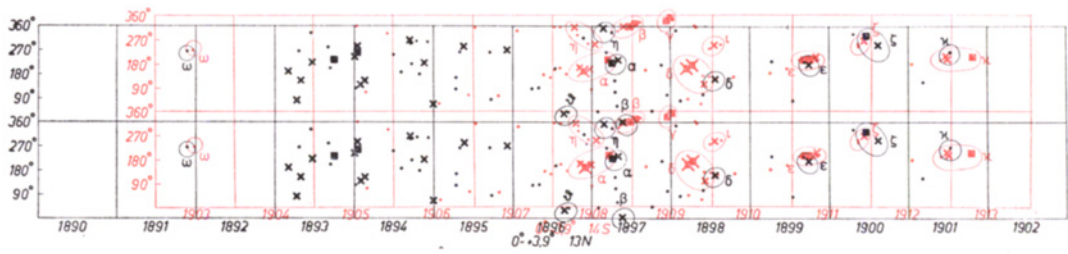


Fig. 4. Comparison between the diagrams of fig. 3. The northern hemisphere is black, the southern hemisphere
Tellus V (1953), 4



red. The diagrams are placed in the positions indicated by the values of A and T given in Table 2.
 Tellus V (1953), 4

or medium size spot groups. There is no indication of a general correspondence between the small spot groups, and these should not in general be used in our analysis. However, in 1917 there is a small spot group which is remarkable because it is the only spot group in that year and it introduces the new cycle. This spot group corresponds in a notable way to the first spot group in the cycle 16 which occurs in 1927. In the same way the first medium size spot group in 15 S, which occurs in the beginning of 1918 corresponds to a small spot group in the beginning of 1928.

The result of the comparison is shown in Fig. 4 where cycle 15 S is marked by black and 16 N by red. The diagrams are placed in such a relative position that the best agreement between the patterns is obtained. The spot groups which are identified are marked by a letter. The identifications a, c, f, and g seem to be unambiguous. In all these cases one group in 15 S corresponds to one group in 16 N. The groups 15 S e and 16 N g are recurrent and as they appear during two consecutive solar rotations they are marked by two dots in the diagram.

The identifications b, c, and d are perhaps more doubtful. It is obvious that there is a general spot activity in 1919 and 1920 at the longitude 100° — 250° and that there is a similar activity during 1929 and 1930 at corresponding longitudes. The big spot group in 1919 was accompanied by some small spots. It is tempting to identify the two medium size spots in the middle of 1929 as being due to the same disturbance. This means that we identify what we could call a "cluster" of spot groups with another "cluster" or single group. Such clusters are b, c, and d in 16 N, identified with the cluster b and the spots c and d in 15 S. The group marked k in the middle of 1929 might belong to the cluster b, but it is more likely that it should be considered as a group with no correspondence. Of course we could not expect *all* spots to be reproduced.

The identifications are listed in Table 1. The difference in time ΔT between a spot in 15 S and its reproduction in 16 N is calculated. We shall call ΔT the *reproduction time*. Also the longitude difference and latitude difference are given. For clusters the weighted means are used, where the weight of the groups in a cluster is put equal to the spotted surface. In

the figures the displacement in longitude and in time is made equal to the average of all the identified cluster (inclusive single groups).

The similarity in pattern between 15 S and 16 N seems to be too striking to be due to chance. We know from GALVENIUS' and WOLD's and WHITTLE's papers that there is a statistical correlation between the two hemispheres. Combined with these facts our results give more support to the general lines of the magneto-hydrodynamic theory. Of course it would be highly desirable to calculate the statistical significance of the similarity between the patterns.

The positive result of this comparison makes it desirable to compare all other pairs of diagrams.

We next proceed to 16 N and 17 S. There are two big clusters in 16 N, *viz.* f and g. In 17 S there are three big clusters, one in 1940, one in 1941, and one in 1942. The relative time of appearance of these clusters agrees rather well with the sequence d, f, and g in 16 N, but if the diagram is placed in such a way that the clusters of 1940 and 1941 coincide with d and f there is a disagreement in longitude of 125° between g and the cluster of 1942. Further there are no other clusters coinciding. Hence we must reject this identification and suppose that the cluster of 1942 (called "v") has no correspondence, in the same way as the group k in 16 N has no correspondence in 15 S.

In next attempt we assume that the clusters of 1940 and 1941 correspond to f and g. If we place the diagrams in this way (Fig. 4) we find in 17 S spots corresponding to k and d in 16 N. Further there are two spots which correspond to the cluster c. As there seems to be no way to obtain a better correspondence between the patterns of 16 N and 17 S we accept these identifications. In 16 N there are five clusters (k, c, d, f, and g) which are reproduced. There are three clusters a, e, and b which are not reproduced, but two of them consist of only one small spot each and were discussed earlier because of their peculiar situation in the beginning of the cycle. In 17 S there is one large spot (v) which is new. (The last spot in 16 N may be a correspondence.) Further there are now some medium size spots (y and five spots between f and g).

The agreement between 16 N and 17 S is not so striking as between 15 S and 16 N. The

identification of 16 N d, f, and g with 17 S f, g, and v could not be altogether ruled out but is definitely less convincing than the identification we have accepted.

As the spot material for the next cycle is not yet available we proceed backwards in time and compare 14 N and 15 S. The big cluster 15 S b could correspond either to one of the big spots in 1906 and 1907 or to one of the big spots in 1909 and 1910. According to the first alternative we could identify b, c, and d with the three big spots in 1906 and 1907. No other identifications are obtained in this way, and the reproduction time becomes as large as 13 years. The second alternative is shown in Fig. 4. The clusters 1909 and 1910 are identified with b, c, and d. The small spot a is considered as a reminiscence of one of the big cluster of 1907. No identification of e, f, and g is obtained. Hence this alternative is only slightly better than the first one. It leaves us, however, with a more reasonable value (10 years) of the reproduction time.

As is evident from the comparison 15 S—16 N the clusters b, c and d form one big region of sunspots and the division of it into clusters is somewhat arbitrary. The comparison 14 N—15 S shows that the corresponding region of sunspots is found in 1909—1910, but the subdivision into clusters is somewhat arbitrary. An identification of 15 S b, c, and d with 14 N m, b, and c is not ruled out although less probable than the identification which we have accepted.

Comparing 13 S and 14 N we find in both cases two maxima of spot activity (1894—1895 and 1897 in 13 S and 1906—1907 and 1909—1910 in 14 N) separated by about one year (1896 and 1908), of low activity. During the two maxima of 13 S the spots have a rather even distribution in longitude so that the pattern is not very distinct. The identification shown in Fig. 4 is no doubt the best one but it is not absolutely convincing because it leaves us with quite a few unreproduced clusters.

The treatment of the sunspot material has not yet been extended more backwards in time. We have thus treated the whole sequence 13 S—14 N—15 S—16 N—17 S. There is no cluster which can be followed the whole way through the sequence. Our identifications depend to a considerable extent on the spot region which in 13 S is represented by k, l,

and b with an extension to d. This is reproduced in 14 N by k, l, b, c, and d. In 15 S and 16 N the same region consists of b, c, and d, whereas it fades out in 17 S with c and d as the only representatives.

A new region of spots appears in 15 S with f and g, which are reproduced in 16 N. By being the only spots in the later parts of each cycle they are very valuable as identity marks. There are no signs at all of f and g in 14 N. Going to 13 S, however, we find in 1901 one single small spot which ends the cycle. This has the same position as a predecessor of g should have, so a genetical connection is possible. (The fact that no corresponding spot is observed in 14 N does not contradict this because there may have been one on the sun's backside.) There is also a possible correspondence to f in 13 S. The spots f and g develop further in 17 S where a number of new spots are created between them. Two new big groups v and y appear in the late part of 17 S.

In 13 S there is an extended spot region from o and h to n and r. This is also prominent in 14 N but it dies away in 15 S. The only remnant is the small spot a which even survives to 16 N.

Consequently in the sequence 13 S—14 N—15 S—16 N—17 S there is a tendency for spot regions to form in the late part of a cycle, proceed to the middle of the cycle where they develop their greatest intensity and finally die at the beginning of the cycle.

10. The sequence 13 N—14 S—15 N—16 S—17 N

We shall now make the same analysis for the sequence 13 N—14 S—15 N—16 S—17 N.

In 13 N there is a number of spots in 1893 and 1894. After a lull in 1895 and 1896 there comes a long train of clusters α , δ , ϵ , ζ , all at about the same longitude. Our earlier results indicate that the first spot of the cycle are in general not very well reproduced, so we should primarily try to find a correspondence to α , δ , ϵ , and ζ . A similar long train is found in 14 S. This leads to the identification shown by fig. 4. A cluster 14 S $\approx$ seems to correspond to the last small spot in 13 N. Further, at about the same time as α appears there is a number of spots also at other longitudes. An identification of the spots η and β seems possible. The small spots in 1905, 1906 and

Table 1. Comparison between the diagrams

CYCLE 13 N										CYCLE 14 S						ΔT	$\Delta \lambda$	$\Delta \varphi$
No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks	No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks					
2351	1891 Nov. 20	3.5	265	33	6	ω	5037	1903 May 21	2.1	229	12	1	ω	11V + 182d	— 36°	— 1° 4		
4415	1896 Aug. 21	2.2	28	130	7		No corresp.	6406	1908 Apr. 18	2.9	308	59					12	
4536	1897 Febr. 27	2.2	352	146	12		Mean	6482	July 17	1.9	249	134					9	
							6427	June 19	2.2	267	193							
4560	Apr. 4	3.3	224	364	13	η	6441	May 7	2.8	156	71	5	η	11V + 112d	— 85°	0.0		
4573	May 1	3.6	232	148	12		» 29	6464	June 23	2.9	154	108					12	
							June 23	6525	Sept. 10	2.5	183	215					13	
Mean	Apr. 12	3.4	226	512			Mean	June 23	2.9	155	956							
4576	May 20	1.6	1	71	8	α	6571	Nov. 21	1.8	312	53	4	α	11V + 72d	— 71°	— 0.5		
							6589	Dec. 25	2.7	320	336	6						
							6602	1909 Jan. 14	2.9	326	192	13						
							Mean	Dec. 29	2.7	321	581							
							6676	June 4	2.7	339	280	7						
							6686	» 25	1.2	347	167	12						
4759	July 22	1.9	162	81	5	β	Mean	June 12	2.1	343	447		β	11V + 223d	— 40°	+ 1.1		
							6723	Sept. 5	2.3	156	667	10						
							6734	Oct. 1	0.9	166	554	10						
							6778	Nov. 27	3.4	97	61	8						
							Mean	Sept. 20	1.7	158	1282							
							6805	1910 Jan. 10	3.9	245	134	12						
4879	Sept. 26	3.1	215	58	6	γ	6928	1911 Febr. 26	1.5	195	193	10	γ	No corresp.		— 0.2		
							6937	Mar. 28	1.7	192	152	10						
							6942	Apr. 21	1.8	202	139	13						
							Mean	Mar. 22	1.6	196	484							
							6972	Nov. 21	2.0	263	140	12						
							7005	1912 Dec. 12	1.8	202	136	11						
4931	June 15	3.6	326	183	10	δ	7006	» 12	1.7	196	115	7	δ	11V + 117d	— 19°	— 1.5		
							7015	1913 Apr. 5	1.9	205	229	3						
							Mean	Febr. 4	1.8	202	480							
							6972	Nov. 21	2.0	263	140	12						
							7005	1912 Dec. 12	1.8	202	136	11						
							7006	» 12	1.7	196	115	7						
4938	Aug. 11	0.6	291	62	3	ϵ	7015	1913 Apr. 5	1.9	205	229	3	ϵ	11V + 148d	— 54°	— 1.8		
							Mean	Febr. 4	1.8	202	480							
							6972	Nov. 21	2.0	263	140	12						
							7005	1912 Dec. 12	1.8	202	136	11						
							7006	» 12	1.7	196	115	7						
							7015	1913 Apr. 5	1.9	205	229	3						
4957	July 8	2.1	266	4	1	ζ	Mean	Febr. 4	1.8	202	480		ζ	11V + 211d	— 64°	— 0.3		
							7015	1913 Apr. 5	1.9	205	229	3						
						κ	Mean	Febr. 4	1.8	202	480		κ	11V + 148d	— 47°	— 0.6		

CYCLE 13 S							CYCLE 14 N							$\Delta\lambda$	$\Delta\varphi$
No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks	No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks		
3305 1893	Dec. 13	0.7	330	268	13										
3352 1894	Jan. 9	2.0	335	135	12										
Mean	Dec. 22	1.1	332	403											
3318 1893	Dec. 21	3.5	221	331	10										
3367 1894	Jan. 24	3.1	233	184	5										
Mean	Jan. 2	3.4	225	515											
3528	May 18	3.9	68	262	12										
3498	Apr. 29	3.8	318	85	13										
3546	May 31	0.6	342	62	6										
3583	June 21	1.1	348	57	10										
Mean	May 23	2.1	334	204											
3888 1895	Jan. 31	3.0	14	139	4										
3924	Mar. 1	3.7	23	67	2										
3929	» 4	3.0	—13	91	2										
Mean	Febr. 16	3.2	8	297											
4192	Oct. 6	2.1	314	137	6										
4221	Nov. 14	2.7	72	239	13										
4517 1897	Jan. 28	2.3	23	246	12										
4532	Febr. 23	0.7	34	288	13										
4549	Mar. 22	2.1	41	76	11										
Mean	Febr. 16	1.5	30	610											
4556	Apr. 1	3.2	262	122	13										
4590	June 28	3.6	179	196	13										
4620	Sept. 3	3.7	82	255	7										
4695 1898	Febr. 17	0.6	3	153	10										
4837 1899	Jan. 30	2.0	91	68	7										
4893	Nov. 25	1.5	163	11	3										
4958 1901	July 22	1.6	32	9	3										
Mean							Mean							+ 0.5	

CYCLE 14 N							CYCLE 15 S							ΔT	$\Delta \lambda$	$\Delta \varphi$
No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks	No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks			
6117	1907 Febr. 17	3.6	163	265	13	a	8117	1917 June 14	3.2	155	30	7	a	10 ^y + 117 ^d	— 8°	— 0° 4
6211	July 2	3.5	194	379	11	n No corresp.										
6230	Aug. 5	3.3	204	161	4											
Mean	July 12	3.4	197	540												
6567	1908 Nov. 13	3.3	75	73	12											
6632	1909 Febr. 26	3.3	125	145	12	k	8489	1918 Apr. 8	1.0	119	113	12	e	No corresp.		
6665	Apr. 30	1.9	83	72	5	l	9007	1919 Aug. 14	2.8	94	764	12				
6700	July 24	2.8	76	152	4		9030	Sept. 8	1.8	105	427	13				
Mean	June 26	2.5	78	224		b	Mean	Aug. 23	2.4	98	1191		b	10 ^y + 58 ^d	+ 20°	— 0.1
6753	Oct. 25	1.6	169	148	13	c	9112	1920 Jan. 24	2.9	133	60	9	c	10 ^y + 91 ^d	— 36°	+ 1.3
6825	1910 Mar. 8	2.5	236	58	7											
6842	Apr. 29	1.9	242	169	12											
Mean	Apr. 16	2.1	240	227		d	9173	May 7	3.8	220	56	6	d	10 ^y + 21 ^d	— 20°	+ 1.7
							9377	1921 Aug. 11	0.5	253	75	8	f	No corresp.		
							9482	1922 Nov. 28	3.0	50	235	13	g	No corresp.		
													Mean	10 ^y + 72 ^d	— 11°	+ 0° 6

CYCLE 14 S							CYCLE 15 N							ΔT	$\Delta \lambda$	$\Delta \varphi$
No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks	No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks			
6406	1908 Apr. 18	2.9	308	59	12	η	8374	1917 Dec. 21	2.6	92	527	13				
							8406	1918 Jan. 17	2.4	97	87	7				
							8433	Febr. 11	3.6	125	730	13				
							8464	Mar. 15	3.0	96	73	9				
							8492	Apr. 12	3.1	101	81	9				
							8552	May 31	3.4	143	77	10				
6427	May 7	2.8	156	71	5		Mean	Febr. 2	3.1	111	1575		No corresp.			
6441	» 29	3.0	144	562	10											
6464	June 23	2.9	154	108	12		8703	Oct. 8	1.2	296	358	6				
6525	Sept. 10	2.5	183	215	13		8727	» 28	0.6	304	91	13				
Mean	June 23	2.9	155	956		α	Mean	Oct. 12	1.1	298	449					$-1^{\circ}.8$
6482	July 17	1.9	249	134	9	λ										
6571	Nov. 21	1.8	312	53	4											
6589	Dec. 25	2.7	320	336	6											
6602	Jan. 14	2.9	326	192	13											
Mean	Dec. 29	2.7	321	581		β	8859	1919 Mar. 17	2.6	4	62	4				-0.1
6676	June 4	2.7	339	280	7		8881	Apr. 13	2.3	304	95	10				
6686	» 25	1.2	347	167	12		9031	Sept. 8	1.6	102	526	13				
Mean	June 12	2.1	343	447		γ	9047	Oct. 5	2.2	106	359	13				-0.3
6723	Sept. 5	2.3	156	667	10		Mean	Sept. 19	1.8	104	885					
6734	Oct. 1	0.9	166	556	10											
6778	Nov. 27	3.4	97	61	8											
Mean	Sept. 20	1.6	146	1284		δ	9102	1920 Jan. 11	2.0	261	219	13				$+0.4$
6805	1910 Jan. 10	3.9	245	134	12		9262	Nov. 12	1.6	297	114	4				
6928	1911 Febr. 26	1.5	195	193	10		9286	Jan. 12	3.2	102	164	13				
6937	Mar. 28	1.7	192	152	10		9334	May 8	0.5	3	1324	14				
6942	Apr. 21	1.8	202	139	13		9345	June 4	0.8	15	138	12				
Mean	Mar. 22	1.6	196	484		ϵ	9360	» 30	3.7	21	264	13				-0.5
6972	Nov. 21	2.0	263	140	12		9369	July 8	2.6	308	56	8				
7005	1912 Dec. 12	1.8	202	136	11		Mean	May 19	1.1	16	1782					
7006	» 12	1.7	196	115	7		9397	Oct. 22	3.9	327	387	12				
7015	1913 Apr. 5	1.9	205	229	3		9400	Nov. 17	3.8	332	104	13				$+1.9$
Mean	Febr. 4	1.8	202	480		κ	Mean	Oct. 28	3.9	328	491					
													Mean	$10^{\circ} + 74^{\circ}$	$+111^{\circ}$	$-0^{\circ}.1$

CYCLE 15 S						CYCLE 16 N						ΔT	$\Delta \lambda$	$\Delta \varphi$					
No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks	No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks						
8117 1917	June 14	3.2	155	30	7	a	Only spot this year	10299 1927	May 18	3.1	117	24	7	a	Only spot this year	10 ^v - 27 ^d	38°	- 0°.1	
8489 1918	Apr. 8	1.0	119	113	12	e	Biggest spot this year	10514 1928	Febr. 3	1.6	59	13	2	e	Only spot this half-year	10 ^v - 72 ^d	59°	+ 0.8	
8519	May 5	0.6	116	49	10			10928 1929	May 6	3.3	349	71	12	k					No corresp.
Mean	Apr. 16	0.8	118	162				11018	Aug. 10	3.6	154	113	10	8/9					
								11027	" 15	2.7	84	60							
9007 1919	Aug. 14	2.8	94	764	12	b	Mean	Aug. 12	3.3	130	173	b		10 ^v - 11 ^d	32°	+ 0.9			
9030	Sept. 8	1.8	105	427	13			11125	Dec. 11	3.2	82						56	4	
Mean	Aug. 23	2.4	98	1191				11144	" 22	3.9	188						142	10	
								Mean	Dec. 19	3.7	158						198	c	
9112 1920	Jan. 24	2.9	133	60	9	c	11235 1930	May 11	2.2	208	85	7	d	10 ^v + 37 ^d	2°	- 1.7			
9173	May 7	3.8	220	56	6	11274	July 24	2.0	239	67	11								
						Mean	June 13	2.1	222	152	f	10 ^v - 126 ^d	30°	+ 2.7					
9377 1921	Aug. 11	0.5	253	75	8	f	11381 1931	Apr. 7	3.2	223					179	3	g	10 ^v - 184 ^d	47°
9482 1922	Nov. 28	3.0	50	235	13	g	11491 1932	May 20	3.6	95	314	13	Mean	10 ^v - 60 ^d	3°	+ 0°.4			
							11497	June 16	0.4	101	142	12							
Mean							Mean	May 28	2.6	97	456	Mean							

CYCLE 15 N							CYCLE 16 S						
No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks	No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks
8374	1917 Dec. 21	2.6	92	527	13								
8406	1918 Jan. 17	2.4	97	87	7		10560	1928 Mar. 25	2.3	—34	8	6	
8433	Febr. 11	3.6	125	730	13		10666	July 12	3.8	32	30	6	
8404	Mar. 15	3.0	96	73	9		10693	Aug. 2	3.8	74	20	5	
8492	Apr. 12	3.1	101	81	9		10729	Sept. 3	1.0	71	32	5	
8552	May 31	3.4	143	77	10		Mean	July 26	2.7	49	90		ϑ
Mean	Febr. 2	3.1	111	1575		ϑ							$10^{\circ} + 174^d$ — 62° — 0.4
8703	Oct. 8	1.2	296	358	6								$10^{\circ} + 87^d$ — 81° 0.0
8727	» 28	0.6	304	91	13								$10^{\circ} + 140^d$ — 94° — 0.2
Mean	Oct. 12	1.1	297	449		α	10837	1929 Jan. 7	1.1	216	60	4	α
8859	Mar. 17	2.6	4	62	4		10922	Apr. 29	1.5	104	295	11	ξ
8881	Apr. 13	2.3	304	95	10	β	11010	Aug. 4	2.4	270	66	4	β
						ϱ	11122	Dec. 10	3.5	347	379	13	
9031	Sept. 8	1.6	102	526	13		11127	» 12	2.6	338	365	11	
9047	Oct. 5	2.2	106	359	13		11130	» 15	3.9	287	106	12	
Mean	Sept. 19	1.8	104	885		γ	11159	1930 Jan. 8	2.3	347	350	11	
9102	Jan. 11	2.0	261	219	13		Mean	Dec. 20	2.9	340	1200		γ
						δ	11207	Mar. 16	1.2	211	52	8	
							11255	June 6	0.3	246	178	6	
							Mean	May 18	0.5	238	230		δ
							11307	Sept. 26	3.0	118	161	12	
							11320	Nov. 2	2.9	123	52	2	
							Mean	Oct. 5	3.0	119	213		π
							11370	1931 Mar. 17	3.6	163	135	12	
							11378	Apr. 1	3.3	172	132	13	
							11392	May 6	1.0	133	56	7	
9262	Nov. 12	1.6	297	114	4		Mean	Apr. 1	3.0	161	323		μ
9286	1921 Jan. 12	3.2	102	164	13		11358	Febr. 15	3.8	57	268	12	ν
9334	May 8	0.5	3	1324	14								
9345	June 4	0.8	15	138	12								
9360	» 30	3.7	21	264	13								
9369	July 8	2.6	—52	56	8								
Mean	May 19	1.1	5	1782		ϵ	11444	Nov. 3	0.6	223	62	10	ϵ

CYCLE 16 N							CYCLE 17 S							ΔT	$\Delta \lambda$	$\Delta \varphi$
No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks	No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks			
110928	1929 May 6	3.3	349	71	12	No corresp.	13124	1938 Dec. 24	2.6	341	144	13	k	10°—133 ^d	— 8°	— 0° 7
11018	Aug. 10	3.6	154	113	10		13268	1939 May 7	2.8	11	86	11				
11027	» 15	2.7	84	60	8/9		13275	» 12	1.1	19	68	6				
Mean	Aug. 12	3.3	130	173			13331	June 23	3.4	119	90	11				
11125	Dec. 11	3.2	82	56	4		Mean	May 26	2.5	50	244		c	10°—207 ^d	— 108°	
11144	» 22	3.9	188	142	10	13479	Nov. 24	2.0	253	71	12	d	10°—201 ^d	+ 31°	— 0.1	
Mean	Dec. 19	3.7	158	198		13711	1940 Sept. 21	3.6	207	409	13					
11235	1930 May 11	2.2	208	85	7	13729	Oct. 24	2.6	181	92	9					
11274	July 24	1.9	239	67	11	Mean	Sept. 27	3.4	202	501		f	10°—192 ^d	— 21°		+ 0.2
Mean	June 13	2.1	222	152		13771	Dec. 12	3.9	307	52	4		No corresp.			
11381	1931 Apr. 7	3.2	223	179	3	13799	1941 Jan. 28	1.4	333	95	9		No corresp.			
						13801	Jan. 29	1.8	44	56	4		No corresp.			
						13839	Apr. 1	1.4	291	62	2		No corresp.			
						13849	Apr. 26	0.6	227	135	13		No corresp.			
11491	May 20	3.6	95	314	13		13904	July 21	3.7	173	498	13				
11497	June 16	0.4	101	142	12		13919	Aug. 16	3.6	186	298	13				
Mean	May 28	2.6	97	456			13927	Aug. 29	2.2	147	61	2				
							Mean	Aug. 1	3.6	176	857		g	10°—300 ^d	+ 79°	+ 1.0
							14074	1942 July 4	3.1	292	330	10				
							14080	» 28	2.0	299	161	13				
							Mean	July 12	2.7	294	491		v	No corresp.		
							14162	1943 July 24	1.8	306	77	6	y	No corresp.		
							Mean						Mean	10°—207 ^d	+ 5°	— 0° 2

CYCLE 16 S							CYCLE 17 N							ΔT	$\Delta \lambda$	$\Delta \varphi$							
No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks	No.	First seen	φ lat.	λ long.	A Area	Dur. Days	Remarks										
10560	1928 Mar. 25	2.4	—34	8	6	θ	No corresp.	12927	1938 July 20	3.7	249	128	13	θ	No corresp.	$10^y + 5^d$ $+ 206^\circ$	$+ 0^\circ 5$						
10666	July 12	3.8	32	30	6			12966	Aug. 15	2.6	262	99	12										
10693	Aug. 2	3.8	74	20	5			Mean	July 31	3.2	255	227											
10729	Sept. 3	1.0	71	32	5			12952	Aug. 5	3.9	33	124	13										
Mean	July 26	2.7	49	90				13317	1939 June 9	3.4	298	162	11										
10837	1929 Jan. 7	1.1	216	60	4	α	No corresp.	13586	1940 Apr. 19	0.8	225	77	3	γ		$+ 194^\circ$	$+ 1.9$						
10922	Apr. 29	1.5	104	295	11																		
11010	Aug. 4	2.4	270	66	4																		
11122	Dec. 10	3.5	347	379	13																		
11127	» 12	2.6	338	365	11																		
11130	» 12	3.9	287	106	12	ξ	No corresp.	13792	1941 Jan. 9	1.6	315	100	4	γ		$+ 245^\circ$	$- 2.1$						
11159	1930 Jan. 8	2.3	347	350	11																		
Mean	Dec. 20	2.9	340	1200																			
11207	Mar. 16	1.2	211	52	8																		
11255	June 6	0.3	246	178	6																		
Mean	May 18	0.5	238	230		δ	No corresp.	13636	June 24	2.5	308	76	12	No corresp.									
11307	Sept. 26	3.0	118	161	12													13758	Dec. 3	1.2	342	89	11
11320	Nov. 2	2.9	123	52	2													13767	» 9	1.0	306	195	3
Mean	Oct. 5	3.0	119	213														13772	» 13	2.2	324	147	3
11358	1931 Febr. 15	3.8	57	268	12													13792	1941 Jan. 9	1.6	315	100	4
11370	Mar. 17	3.6	163	135	12	π	No corresp.	Mean	Dec. 15	1.5	319	531		π									
11378	Apr. 1	3.3	172	132	13																		
11392	May 6	1.0	133	56	7																		
Mean	Oct. 5	3.0	119	213																			
11444	Nov. 3	0.6	223	62	10																		
Mean	Apr. 1	3.0	161	323		μ	No corresp.	13948	Sept. 26	1.4	9	541	13	ε	No corresp.	$10^y - 26^d$ $+ 140^\circ$	$+ 2.4$						
11452	Apr. 27	1.0	300	171	13													13959	Oct. 23	0.6	14	64	10
11457	May 24	3.8	305	50	11													13962	» 31	3.8	—14	235	7
Mean	Apr. 10	2.2	298	698														Mean	Oct. 8	2.0	0.3	840	
11496	1944 May 29	0.7	197	132	5													14023	1942 Mar. 13	2.5	5	83	9
14198	June 16	1.7	206	71	13	φ	No corresp.	Mean	Apr. 10	2.2	298	698	5	φ	No corresp.								
Mean	June 4	1.0	200	203														14148	1943 Mar. 31	2.4	297	477	13
14152	Apr. 27	1.0	300	171	13													14157	May 24	3.8	305	50	11
14157	May 24	3.8	305	50	11													Mean	Apr. 10	2.2	298	698	
14196	1944 May 29	0.7	197	132	5													14198	June 16	1.7	206	71	13
Mean	June 4	1.0	200	203		ψ	No corresp.	Mean	June 4	1.0	200	203		ψ	No corresp.	$10^y + 42^d$ $+ 197^\circ$	$+ 0.2$						
14198	June 16	1.7	206	71	13																		
Mean	June 4	1.0	200	203																			
14152	Apr. 27	1.0	300	171	13																		
14157	May 24	3.8	305	50	11																		

1907 seem not to correspond to the activity in 13 N during 1893–94 but the first small spots ω of each cycle agrees very well. Upon the whole the similarity 13 N–14 S is satisfactory.

Proceeding to 15 N we recognize easily the long train $\alpha, \delta, \epsilon, \zeta$. The cluster γ which was born in 14 S is very well reproduced. An identification of 14 S β , which is also a new-born spot, with a spot in 15 N is indicated. The clusters ϵ and ζ have grown but there is no correspondence of κ . New-born clusters are μ and ν . Further there is a strong activity in the beginning of the cycle, especially a cluster 15 N ϑ containing two very big and four medium groups. There is no correspondence to this in 14 S but the cluster 13 N ϑ coincides with it.

The comparison 15 N–16 S is more difficult. The long train has been dissolved. The biggest cluster in 16 S occurs at Christmas 1929. This could obviously not be identified with the biggest cluster ϵ in 15 N because this would give a reproduction time of only 8.5 years. An identification with the second biggest cluster γ seems to be acceptable (fig. 4). The clusters $\alpha, \beta, \gamma, \delta, \mu$ and possibly ν are reproduced in a reasonable way. Of the big cluster ϵ only one medium group remains, and ζ is not reproduced. The big cluster ϑ may be reproduced by four small spots which start the cycle. A new cluster is ξ .

In 17 N the prevailing feature is again a long train of spots, even longer than in 13 N and 14 S. The best identification seems to be to associate this with ϑ – ξ – π – ϵ (fig. 4). This means that α, β, δ , and ν have died and two new clusters φ and ψ have been born.

In the “Greek sequence” 13 N–14 S–15 N–16 S–17 N, which we have now examined, some clusters have a remarkably long duration. In fact, ϑ, δ , and ϵ have persisted during the whole time (with an interruption for ϑ in 14 S). If 17 N is placed on top of 13 N these three clusters are easily brought to coincide. If all the five diagrams 13 N–14 S–15 N–16 S–17 N are placed on top of each other these three clusters are in about the same place in all of them.

II. Correlation between the two sequences

We have found a good correlation within the “Latin sequence” 13 S–14 N–15 S–16

N–17 S and also within the “Greek sequence” 13 N–14 S–15 N–16 S–17 N. It is of interest to see whether these series are interconnected. A comparison of the type $(n, N) - (n+1, N)$ or $(n, S) - (n+1, S)$ or of the type $(nN) - (nS)$ shows in many cases some degree of similarity.

12. Check of identifications

First check: comparison of non-related patterns

It is of decisive importance to investigate whether the pattern similarities which we have found are significant or not. It is quite possible that two patterns are judged to be similar even if there is no real correlation between them. The first way to check this is to compare diagrams which we do not expect theoretically to be similar. A difficulty is that if there exists a correlation between the two sequences, all diagrams without exception ought to show some correlation. In order to compare diagrams with as small theoretical correlation as possible we should choose them from different sequences, and with as long a time interval as possible. Hence we select the four pairs 13 N–16 N, 14 N–17 N, 13 S–16 S, and 14 S–17 S as material for our check comparison. Proceeding in the same way as earlier we place the biggest spot of one diagram on the top of one of the biggest spots of the other diagram.

The comparison shows that 13 S and 16 S gives a not too bad similarity, 13 S t, l, b , and d forming a pattern similar to 16 S $\gamma, \delta, \pi, \epsilon$. In the three other cases no similarity at all could be traced.

The negative result in three cases indicates that it is probable that our earlier identifications are significant but the fact that in one case a similarity could be found which mainly must be accidental, shows that our method is not quite without danger. Hence other checks are highly desirable.

13. Second check: Calculation of the “twist” of the magnetic field

Our second check is of physical character and concerned with the calculation of the twist of the magnetic field (see § 4).

Suppose that the magnetic field is twisted in such a way that a magnetic line of force which

intersects the southern hemisphere at the longitude λ_S arrives to the northern hemisphere at the longitude

$$\lambda_N = \lambda_S + \Theta \dots \dots \dots (1)$$

where Θ is a constant which we shall call the *twist*. Suppose further that a disturbance travels along a certain line of force and reaches the southern hemisphere during the cycle n at the longitude λ_S^n . The same disturbance reaches the northern hemisphere during the cycle $n+1$ after a time interval $T_{S,n}^{N,n+1}$ (the reproduction time). If the latitude at which the spots occur rotates with the angular velocity ω_d in comparison to the latitude to which the heliographic longitude determination refers, the longitude at the northern hemisphere is

$$\lambda_N^{n+1} = \lambda_S^n + \Theta + \omega_d T_{S,n}^{N,n+1} \dots \dots (2)$$

Consider another disturbance which reaches the northern hemisphere at longitude λ_N^n during cycle n and the southern hemisphere at longitude λ_S^{n+1} during next cycle.

In a similar way we obtain

$$\lambda_S^{n+1} = \lambda_N^n - \Theta + \omega_d T_{N,n}^{S,n+1} \dots \dots (3)$$

where $T_{N,n}^{S,n+1}$ is the reproduction time of the disturbance. We put

$$A_{N,n}^{S,n+1} = \lambda_S^{n+1} - \lambda_N^n \dots \dots \dots (4)$$

and

$$A_{S,n}^{N,n+1} = \lambda_N^{n+1} - \lambda_S^n \dots \dots \dots (5)$$

The quantities A are the longitude differences which are found empirically at the comparison between the diagrams. Subtracting (3) from (2) we obtain

$$\Theta = \frac{1}{2} (A_{S,n}^{N,n+1} - A_{N,n}^{S,n+1}) - \frac{1}{2} \omega_d \Delta T \quad (6)$$

with $\Delta T = T_{S,n}^{N,n+1} - T_{N,n}^{S,n+1}$

We do not know ω_d with certainty. We do not even know whether it is constant. In fact, some of the results of the present analysis indicate that this is not the case. The differential rotation is usually computed from the systematic longitude drifts of sunspots which for our latitude belt is 5° per solar rotation, corresponding to $\frac{1}{2} \omega_d = 0.09$ degrees/day.

From the point of view of the magneto-hydrodynamic theory of sunspots it is con-

ceivable that systematic drifts may be caused also by other effects than a differential rotation. However, all these problems will be reserved for a later analysis. In view of the uncertainty we shall here calculate Θ in two ways, *viz.* by neglecting the second term in (6) and by including this term. The latter value will be referred to as Θ_{corr} .

Table 2 shows the values of Θ calculated with and without the second term. The constancy of Θ is remarkable. In fact, without correction it varies only within the relatively narrow limits 46° and 96° . The excessive agreement between the corrected values is of course only a numerical coincidence. As mean we obtain

$$\Theta = 66^\circ \pm m \ 180^\circ$$

where m is an integer.

The constancy of Θ gives a strong support for our identifications. In fact, if we accept all identifications except one, we could calculate the relative longitude position A for this one from (6). The position would agree with what we have found within an error of less than about 45° . This shows that in no case a serious error can have been made when fixing the relative longitude position of the diagrams.

14. Third check: Correlation between reproduction time and the length of the solar cycle

Having thus shown that the identifications must be correct in longitude, it remains to investigate whether there is any possibility to check the values of the reproduction time. It could be suspected that there should be a positive correlation between this and the length of the solar cycle. In fact, if sunspots are reproduced after a certain time T we should expect that this quantity regulates the length of the solar cycle. It is obviously not the only factor of importance, because as we have seen not all spots are reproduced. There is a general tendency that spots in the beginning of a cycle are not very well reproduced and that new spots are born at the end of the cycle.

Table 2 shows a comparison between the average of $T_{N,n}^{S,n+1}$ and $T_{S,n}^{N,n+1}$ and the length of the solar cycle (= time difference between two sunspot maxima). The correlation is obviously positive. This is satisfactory, but on the other hand the correlation is not good

Table 2. Results of the comparison.

Cycle	$A_{N,n}^{S,n+1}$	$A_{S,n}^{N,n+1}$	θ	$\theta_{\text{corr.}}$	$T_{N,n}^{S,n+1}$	$T_{S,n}^{N,n+1}$	ΔT	T_{mean}	Diff. sunspot max.
13—14	— 47°	+ 84°	66°	64°	11 ^v 148 ^d	11 ^v 169 ^d	21 ^d	11 ^v 159 ^d	12.9
14—15	— 11°	+ 111°	62°	62°	10 ^v 72 ^d	10 ^v 74 ^d	2 ^d	10 ^v 73 ^d	10.6
15—16	— 95°	— 3°	46°	63°	10 ^v 133 ^d	9 ^v 305 ^d	— 193 ^d	10 ^v 36 ^d	10.8
16—17	+ 5°	+ 197°	96°	74°	9 ^v 158 ^d	10 ^v 42 ^d	249 ^d	9 ^v 283 ^d	9.0
		Mean	68°	66°					

enough to give an accurate check of our values of the reproduction time.

15. Identification by structure

In § 3 it was pointed out that it may be possible to identify two spots as produced by the same original disturbance by discovering some peculiar property which is common to both. If for example a sunspot whirl ring makes a certain angle with the magnetic field, this may cause correlated proper motions of one spot of the northern hemisphere and the corresponding spot of the southern hemisphere.

This could preferably be checked by comparing the proper motions of spots which we have identified by their position. Simple spots like f and g are especially suited for this. A preliminary analysis indicates that there may be a correlation between the proper motions, but the problem must be studied much more carefully before any definite conclusions can be reached.

16. Spots at higher latitudes

The whole analysis has so far been confined to the latitude belt 4° S—4° N. An extension to higher latitudes is very desirable. It seems relatively easy to proceed up to 6° or may be even 8° by the same method, but when we approach the maximum belt at 10°—15° the number of spots gets so large, that it is impossible to distinguish distinct patterns. It will be necessary to develop other methods of identification before a real attack towards higher latitudes could be launched. If the identification by structure succeeds, the problem will be easier.

It is of interest to note that the survival of spot groups during several cycles which was found in §§ 9 and 10 is in good agreement with earlier results concerning the latitude pattern of spottedness (cf. Cosmical Electro-

dynamics p. 138). The latitude distribution for cycle 14 is very similar to that of cycle 15 and 16, and there is also a definite resemblance for cycle 12. These results refer to higher latitudes (10°—30°) and indicate that the persistency of spots may be a general phenomenon.

17. Theoretical conclusions

The reproduction of sunspots which has been found here is well explicable according to the magneto-hydrodynamic theory of sunspots. A wave which reaches e.g. the southern activity region becomes in part amplified, and in part reflected. The reflected wave goes to the northern activity region where the same phenomenon occurs and a reflected wave is sent out to the southern region. This can be repeated several times.

In the theory it was assumed that when a whirl ring passes an activity region it usually gives rise to a series of reflected whirl rings. None, or one, or many of these may survive the destruction by damping and other causes, and finally give rise to new spots. The activity regions were thought to consist of the upper parts of the convective core and to be rather thick, perhaps a few tenths of the size of the core.

This idea of the activity regions is well reconcilable with the present results except that the activity regions cannot be very thick. A sunspot group may be reproduced next cycle by a cluster of groups, a single group, or not at all. If it is reproduced by a cluster of groups the spread in time of the spots of the cluster is a measure of the thickness of the activity region, where they are produced. Suppose that a wave pulse with the velocity V passes a region with thickness D in which it may be partially reflected anywhere. The difference in time t between pulses reflected at the beginning and at the end of the region is

$$t = 2D/V$$

In our case t represents the maximum spread in time within a cluster. Judging from the diagrams it is reasonable to put $t = 10^6$ sec. (≈ 4 months). The velocity V is likely to be of the order 60 cm/sec (Cosmical Electrodynamics p. 100). This gives $3 \cdot 10^7$ cm which is only 0.004 of the solar radius.

Consequently the activity region is so thin that it more resembles a surface of discontinuity, which should have the property of reflecting and amplifying waves passing it. The existence of such a surface is hardly reconcilable with current solar models. GAMOW (1945) and REIZ (1948, 1949) have discussed a model in which the energy is released in a spherical shell. It is conceivable that magneto-

hydrodynamic waves passing such an energy producing shell should be reflected and amplified in the way our results require. The possibility to construct a solar model of this type should be seriously discussed. When we learn to correlate more closely the appearance and structure of sunspots with the phenomena in the solar core by which the spots originate, sunspots data will give us valuable information concerning the interior of the sun.

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