Note Concerning the Nature of the Large-Scale Eddies in the Atmosphere

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Abstract

The rate of generation of kinetic energy of mean zonal motions by the large-scale horizontal eddies in the atmosphere is computed from three independent sets of hemispheric wind data. The results are larger than certain analogous dissipation estimates would require. Meteorological implications of the material are discussed.

General discussion

The aim of this brief article is to discuss further, with the aid of new additional data, some essential properties of the large-scale horizontal eddies in the atmosphere, which are involved in their relation to the mean zonal motions present. The particular physical principle to be reviewed has been treated previously mainly by my colleague Dr. H. L. KUO (1950, 1951 a, 1953) both theoretically and observationally, although it appears that certain additional remarks and graphic illustrations might be of value, since some residual misunderstandings seem to be prevalent.

The mean zonal motions (i. e., averaged with respect to longitude) of the atmosphere may be pictured as comprising a large vortex about the polar axis in which the angular velocity is a function of elevation, latitude and time. As an example the solid curve in (a) of the figure gives the distribution with latitude of this angular velocity relative to the earth, averaged also with respect to pressure in the vertical from 150 to 900 mb and with respect to time over the six-month period January to June 1950 inclusive (N. H.). Most of what is to be said concerning this profile can be made to apply equally to an instantaneous profile at some given level. One may now ask what the effect of genuine lateral friction alone on such a flow might be. Quite obviously the effect of such viscosity would be to retard the zones of most rapid rotation and to increase the angular velocity of the less rapidly rotating ones, so as to cause the whole to assume a more nearly uniform angular velocity. That is to say, lateral friction would cause a flow of angular momentum northward and also southward away from the zone of most rapid rotation.

One may inquire next as to whether the large-scale eddies in the atmosphere act upon the mean zonal motions in a manner analogous to friction and hence may be regarded as merely giving rise to an eddy viscosity. This is outstandingly not the case, for according to any compilation of data at all adequate for the purpose (and there are a number of such independent ones now available) there exists a strong northward eddy transport of angular momentum to the south of the jet of westerlies (N. H.) and therefore *a fortiori* to the south of the zone of maximum angular velocity, on the average. This is exactly contrary to a frictional effect and would be compatible only with a negative virtual viscosity coefficient, if it were desirable even to use such terminology. The mere existence of this state of affairs is in itself most remarkable and by no means to be lightly dismissed, for regardless of further considerations it is a manifestation of a fundamental process which requires explanation ultimately. The associated angular momentum transport by eddies is given by the dashed line in (a) of Fig. 1.

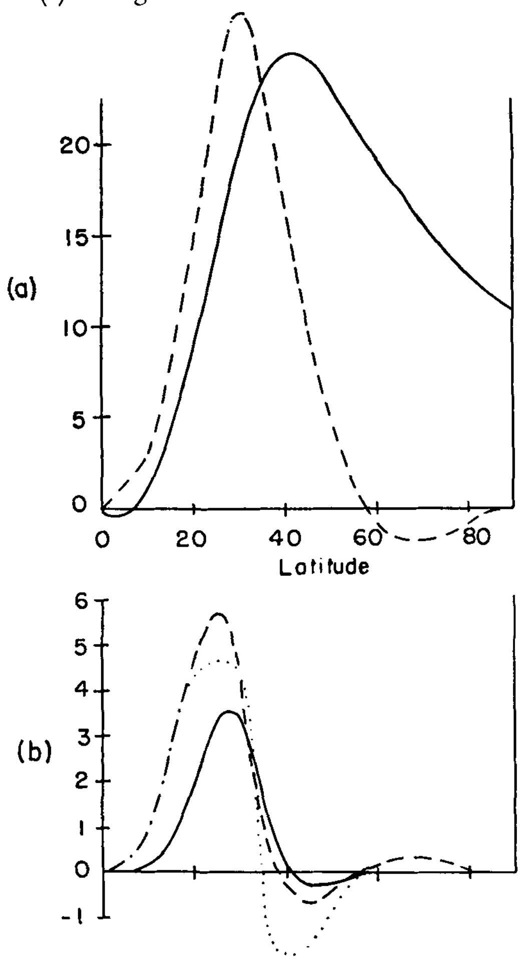

Fig. 1. Relative angular velocity in 10^{-7} sec^{-1} as function of latitude given by full curve in (a). Total angular momentum transport by eddies in 10^{25} c.g.s. units given by dashed curve in (a). Production of zonal kinetic energy using mean wind and total transport (full curve), using 300 mb wind and transport (dashed curve) and using 200 mb wind and transport (dotted curve) shown in (b) in terms of arbitrary units.

Kinetic energy considerations

It is convenient and instructive to study these effects of the eddies upon the *kinetic energy* of the mean zonal motions. Considering the northern hemisphere as acting independently, the net influence on this total kinetic energy is to increase it, if the (positive) action to the south predominates and to decrease it if conditions are the reverse. The rate of production may be written as

$$2\pi r^2 \int\int \varrho [u'v'] \cos^2 \Phi \frac{\vartheta}{\vartheta \Phi}\left(\frac{[u]}{r \cos \Phi}\right) d\Phi dz \quad (1)$$

if small approximations such as the horizontal uniformity of density are allowed in the definition of mean zonal kinetic energy. The notation is that of STARR and WHITE (1951, 1952), u, v, being eastward and northward wind components, ϱ density, r earth's radius, Φ latitude, z elevation, the square brackets denoting averaging along complete latitude circles, and the primes indicating deviations from such averages. This expression may either be set up directly from a consideration of the work done by net forces due to the eddy stresses on elementary rings of air, or it may be obtained by writing the proper form of the balance equation for zonal kinetic energy as done, for example, by KUO (1951 a).

Data and computations

Among the available data for investigating expression (1) the following may be listed.

(1) The first six months of the year 1950 already referred to. These data are based upon (daily) actual wind observations similar to those of Starr and White in the references already given, for Latitudes 13°, 31°, 42°, 55°, and 70° N. The integrand of (1) evaluated from the six-month averages of $[u]$ and $[u'v']$ is given for the 300 and 200-mb levels by the dashed and dotted lines respectively in (b) of the figure. The full line results from using the vertical pressure averages of wind and eddy transport (i.e., corresponding to the curves given in (a) already discussed). Since in (b) the area under each curve measures the production of zonal kinetic energy, it is to be noted that in each case this area is positive by a wide margin.

(2) The second six months of the year 1950. These data are exactly analogous to those for the first six months and were treated in the same way. The integrands (not shown) were again positive without question, i.e., do not represent small differences between large quantities.

(3) One month of data (Jan. 1949) published by MINTZ (1951). These represent geostrophic evaluations obtained by punched card methods. As in the case of the curves (b), here also there is no ambiguity as to the sign of the areas beneath the curves (not shown) in all three instances.

Using the vertically averaged winds and transports the data (1) give 4.2×10^{20} ergs per second for the production over the hemisphere. The data (2) give 4.6×10^{20} units. Data (3) give 10.5×10^{20} units. These magnitudes are to be compared with appropriately defined and properly estimated measures of the dissipation. When the vertical averages are used one deals essentially with the kinetic energy of the mean rotation of the atmosphere, and the dissipation becomes merely the work done against the zonal component of ground stresses in virtue of this mean relative rotation, save for minor other effects. Further discussion of this subject is given by STARR and WHITE (1953). The zonal ground stresses may be obtained in the form of empirical climatological estimates as given by PRIESTLEY (1951), for example. The stresses deduced from the angular momentum balance considerations for the particular period in question as calculated for example by MINTZ (1951) would not give an independent check for the present purpose. Use of Priestley's stresses with the data (1) gives 3.6×10^{20} units for the hemisphere. The data (2) give 1.9×10^{20} units, while data (3) give 4.0×10^{20} units.

If the angular velocity curve in (a) of the figure is assumed to be representative of the entire depth of the atmosphere, we may compute from it the corresponding kinetic energy of mean rotation for the northern hemisphere, which turns out to be about 6.8×10^{26} ergs. Upon appeal to the figure given for the mean rate of production from data (1), it is seen that this amount is normally generated in about 19 days. The corresponding period for data (2) is 12 days and for data (3) it is 17 days. Presumably if the production were suddenly to cease while the dissipation were to maintain its normal rate, the motions of the air here involved would be abolished in about two weeks.

Discussion of results

Relegating certain further technical matters and qualifying remarks to the next section, we may enumerate *certain important inferences* which may be drawn from the general character of the results, if these latter are correct in main outline, as seems most probable. These are:

1. The large disturbances in the atmosphere produce kinetic energy of mean zonal motions, and not the reverse as has been with few exceptions generally supposed in the past.

2. Since the magnitudes of this generation are of the order of the estimated dissipation, and could regenerate the motions involved in about two weeks, it may be stated that any theory for the general circulation of the atmosphere which omits this effect is crucially deficient, lacking an essential correspondence to reality, however interesting it may otherwise be.

3. Through the years there has been current in meteorology an assortment of general circulation theories which do not presuppose this action of the eddies. The singular sterility and lack of promise which have characterized these can perhaps be ascribed largely to the shortcoming under discussion.

4. It is increasingly clear that a valid formal theory of the general circulation showing how the incoming radiational energy supply results in the observed motions (rather than others) will probably be evolved only slowly because of its inherent complexity.

The problem of the mean zonal motions and the large-scale disturbances is one and inseparable, because of the now demonstrated interdependence of these circulations; hence there exists a natural and direct connection to such other modern developments as those in cyclone theory and similar topics, which latter will first have to be not only better understood but also fitted into the more general picture. For the time being it may be necessary to content oneself with the theoretical examination of only portions or phases of the total process. Indeed much of what now may appear as composed of separate and distinct subjects in theoretical meteorology must ultimately form a coordinated and harmonious whole, displaying in true light the proper and natural relationships among these parts.

5. The theoretical efforts of my esteemed friend Dr. H. L. KUO may be mentioned specifically as an example of what has just been said (see KUO, 1950, 1951 b, 1953). These

studies are directed toward the elucidation of the transformation of eddy kinetic energy into mean zonal kinetic energy through the application of the vorticity equation. This procedure may be interpreted also as an effort to apply the general principles, if not the specific techniques, used in what is now called numerical forecasting to the problem of the general circulation. It is to be noted that in this connection, and also from the general results stated in this paper, the seat of the kinetic energy processes is placed in the cyclonic and anticyclonic systems in the atmosphere, which directly or indirectly account for the continued replenishment of the eddy kinetic energy.

6. It may with profit be again contemplated that the positive nature of the production process as measured from data is not contingent upon the attainment of some petty accuracy, but is at once fixed by very gross aspects of the zonal-wind and angular momentum transport profiles. The end result could, in fact, hardly be otherwise if the maximum northward angular momentum transport is found in the vicinity of the mean latitude of the jet stream.

7. Most of the so-to-speak simple theories of the general circulation start from the assumption that the mean zonal motions are maintained by mean meridional circulations. From time to time thinking men have expressed dissatisfaction with such a framework to various degrees, e. g., JEFFREYS (1926), ROSSBY (1947, 1949), although the direct measurement of the mean meridional circulations cannot be carried out with great enough confidence for a direct refutation which would be reliable. Once, however, very extensive independent compilations of data consistently point to the existence of a sufficiently potent alternative, it is quite unscientific to concentrate merely on the older approach. The writer's opinion in this regard does not exclude the possibility that sufficiently extensive observational studies may show that mean meridional circulations perform certain other important functions, or even modify to some extent the eddy-produced zonal flows. In any case it would be highly improper not to give recognition to those indications and suggestions deriving most clearly and unambiguously from purely objective data as already expounded.

Critique and further computations

During the course of the work performed a number of added questions and matters of thoroughness were examined and treated in much greater detail than can be fully entered into for the purpose of this note. Some critical statements and results of corresponding checks in some instances may nevertheless be made for purposes of better orientation.

1. The use of vertical averages for the zonal wind profile is a rather specialized device which introduces valuable simplifications, but general interest still demands that the results be compared with what would be obtained from calculations at individual levels which are then integrated vertically as a final step. Such alternative figures were calculated giving for data (1) 3.9×10^{20}, for data (2) 5.5×10^{20}, and for data (3) 9.8×10^{20} erg sec^{-1}.

2. A second check carried out was to calculate the production using a vertically integrated zonal-wind profile but applying the technique to the instantaneous daily conditions, the long term mean being then formed by averaging the daily figures for the production.

By this procedure an opportunity is also afforded to compute confidence limits defined as twice the standard error of the mean of the daily values, indicating roughly the 95 per cent confidence range. The data (1) were subjected to such a treatment, although at the time this work was done the observations for 70° N latitude had as yet not been tabulated and hence were not included. The figure obtained was $(4.5 \pm 1.0) \times 10^{20}$ erg sec^{-1}.

3. A still further elaboration consists of the use of the analysis by levels, not with the long period averages, but with instantaneous daily data. The long term mean may then again be formed by averaging the daily production figures. The data (1) with the 70° N latitude circle omitted were used also for this purpose. The figure which resulted was 9.8×10^{20} erg sec^{-1}.

4. In the computations it was arbitrarily assumed that $[u]$ and $[u'v']$ are zero at the equator. What the exact values of the momentum transport are at the equator is not known, but the use of more proper negative values of $[u]$ would tend to increase the anticyclonic wind shear immediately to the north where the transport is positive and hence would probably increase the production.

5. Selectivity in the availability of wind observations favoring lower speeds probably leads to an underestimate of the production. It is to be noted that the integrand of expression (1) involves a triple product of quantities in the nature of wind velocity components, hence underestimates of the latter could produce a larger percentual underestimate here than in such quantities as, for example, the kinetic energy itself.

6. It should be most carefully observed that if the effect of the eddy processes on the mean zonal kinetic energy is studied for a narrower belt of latitude, terms representing the meridional transport of this energy across the bounding latitudes should be considered, and the mere adjustment of the limits of integration in (1) does not provide for this requirement. In the case of the whole hemisphere it is doubtful, however, that such a transport across the equator is anything more than a slight correction, which is here neglected (and would necessarily vanish for the entire atmosphere).

7. The surface zonal stresses given by Priestley are according to most proper interpretation values for the oceans. Doubtless the extensive land masses present in the northern hemisphere give rise to larger such stresses as suggested by the angular momentum convergence study of MINTZ (1951), thus helping to bridge the great disparity found here between the eddy production and the dissipation of the energy of mean rotation.

8. The expression (1) is actually nothing more than the spherical polar form of the corresponding term in the familiar theory of OSBORN REYNOLDS (1895) concerning the nature of eddy processes. A review of this material is given also by LAMB (1932) in his textbook on hydrodynamics. Indications of the importance of the process here measured by expression (1) have been obtained also by BLACKADAR (1950) and by VAN MIEGHEM (1953).

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