A Remark on the Energy Transport from a Warm River Surface into Cold Air

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Abstract

Temperature measurements in very cold air above an open river surface indicate that in spite of very high temperature lapse rate (temperature of river, in the 2 cm, 10 cm, and 12 m levels are + 0.5, — 14.0, — 15.3, and — 18.0° C respectively), the eddy conductivity is of the same order of magnitude as in extremely stable conditions (e. g. above a snow surface during nocturnal cooling).

The vertical transfer of energy in the atmosphere is due to vertical variations in the mean kinetic energy (vertical velocity shear caused by friction or other effects) or due to vertical statical instability (thermal convection). The transport through a horizontal surface of unit area can be expressed as $-c_p A_z \frac{\partial \Theta}{\partial z}$ where c_p is the heat capacity of air at constant pressure, A_z the eddy conductivity and $\frac{\partial \Theta}{\partial z}$ the lapse rate of potential temperature. The eddy conductivity is a function of the lapse rate. In stable conditions the dynamical turbulence is damped whereas in unstable conditions there is a tendency for convection cells to form. In unstable conditions the thermal convection is generally supposed to be the dominating factor. When in an air layer the density increases with height auto-convection is expected to set in, i.e. the colder upper air should break through to the bottom of the layer and the warm air rise to the top of the layer. This condition is reached when the lapse rate amounts to 3.4°/100 m.

Some measurements, recently made at the river Indalsälven near Östersund in central Sweden, seem to indicate, that considerable thermal convection does not start as soon as the lapse rate 3.4° in 100 m is reached.

Fig. 1 shows the location of measurements and the estimated prevailing wind. The surface of the river is not frozen due to the

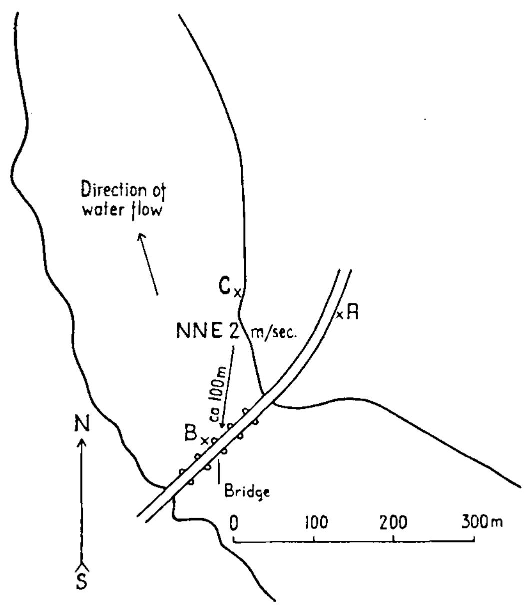

Fig. 1. Observational site.

regulation of the water flow system for the water power industry. The water from the lake Storsjön which is covered by a sheet of ice almost half a meter thick has a temperature of about $+0.5°$ C when leaving the lake. The flow in the river is so rapid that no ice cover is formed until more than 2 km from the lake even at air temperatures of $-20°$ C.

The measurements on 12 February 1953 were made at the point A on land 100 m on the windward side of the river and at the point B on the bridge over the river.

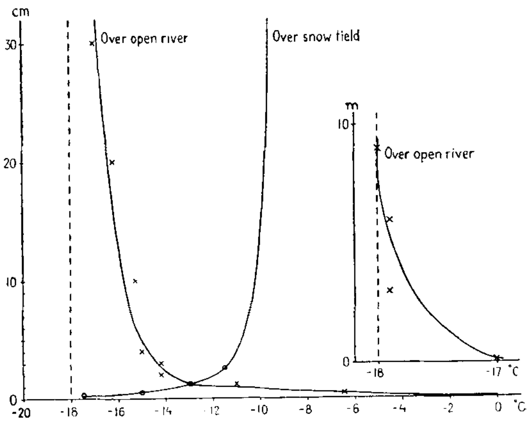

Fig. 2. Temperature height curves in the two cases refered to in the text.

Fig. 2 shows the observed temperature profile. (In fig. 2 is also shown the temperature curve obtained at an extremely stable situation, see NYBERG (1939). The values are given also in Table 1 which in addition shows some values obtained on 11 February at a point C over the river 2 m from the shore. See fig. 1. The instrument used here was a direct indicating resistance thermometer with three thermistors (Western Electric 14 A) coupled to a standard ohmmeter circuit. The thermistors were mounted equally spaced on a 6 m bamboostick. The measurements on the 6 m high bridge (point B in Fig. 1) were made by directing the bamboostick down to the water surface and touching the water surface with the top thermistor. The bamboostick was then successively raised and the temperature variation followed. The height above the water surface was measured with an estimated absolute accuracy of 5 mm. The accuracy of the height difference was about 2 mm.

From these measurements approximate values of the vertical eddy conductivity may be derived under certain assumptions.

Consider a horisontal surface of unit area in the level z. The vertical energy transport through the surface due to eddy motion is in unit time

$$-c_p \cdot A_z \frac{\partial \Theta}{\partial z}$$

Let this surface move with the horizontal wind u_z in the x-direction during the time t_1.

The total vertical eddy transport of energy through the area is then

$$-\int_0^{t_1} c_p A_z \frac{\partial \Theta}{\partial z} dt$$

Assuming at first that the horizontal wind u_z is independent of z this energy transport is equal to the increase in energy in a vertical air column above the surface from the level z to the top of the atmosphere. We did estimate the wind in the level 6 m being about 2 m/sec but no measurements took place. However, it is clear from similar cases that the wind must have increased with height roughly at a logaritmic rate. (The wind profile must be fairly complicated in the lowest centimeter layer as the speed of the water flow is as high as about half a meter per second.) The column in which the energy has been stored above the level z_1, is then not vertical but in a twodimensional picture forms an area between the curves $u_z t_1$ and $u_z t_1 + \frac{u_z}{u_{z_1}}$

The temperature increase from A ($t=0$) to B ($t=t_1$) in the level z is $\triangle T_z$. Then we have

$$-\int_0^{t_1} c_p A_z \frac{\partial \Theta}{\partial z} dt = \int_{z_1}^{\infty} c_p \varrho \triangle T_z \cdot \frac{u_z}{u_{z_1}} dz$$

The data available are not sufficient for a direct computation of A_z from this equation. We therefore make the following simplifications. At first we put ϱ equal to the constant value $1.2 \cdot 10^{-3}$ g/cm³. We further assume that A_z is independent of time, that $\frac{\partial \Theta}{\partial z}$ is equal to zero at $t=0$ and that $\frac{\partial \Theta}{\partial z}$ has a measured value at $x=x_1$.

Table I

Height above water surface	Profile			ΔT $T_B - T_A$	$\int_{z_1}^{\infty} \Delta T_z \cdot u_z \cdot dz$	$\frac{\partial \Theta}{\partial z}$	A_z
	A	B	C				
1,200 cm	— 17.9	— 18.0		(— 0.1)			
900	— 18.1	— 18.0		(0.1)			
600	— 17.8	— 17.9	— 16.5	0.05	20	0.02°/m	1.2
400				0.25	80	0.10°/m	1.0
300		— 17.6	— 16.6	0.4	150	0.18°/m	1.0
200					245	0.24°/m	1.2
50					415	1.04°/m	0.34
30		— 17.0		1.0	445	4.5°/m	0.18
20		— 16.3		1.7	470	8.5°/m	0.12
10		— 15.3		2.7	505	16 °/m	0.072
4		— 15.0		3.0	525	0.4°/cm	0.048
3		— 14.2	— 16.0	3.8	530	0.65°/cm	0.030
2.5		— 14.7		3.3	532	1.4°/cm	0.018
2		— 14.2		3.8	533	2.0°/cm	0.013
1.5		— 13.5		4.5	534	2.6°/cm	0.011
1		— 11.0		7.0	536	5.5°/cm	0.006
0.5		— 6.5	— 14.0	11.5	539	11.0°/cm	0.0035
0		± 0.0		18.0	541		

$x = x_1 \cong 100$ m at $z = 6$ m corresponds to $t = t_1$. We now assume that $\frac{\partial \Theta}{\partial z}$ and T at the time t_1 in each level has the value measured at $x = x_1$ in that level. We then get the following estimates of A_z at different levels. $t_1 = 50$ sec. See Table I. From the measurements at C we can see that $\frac{\partial \Theta}{\partial z}$ in the lowest levels rapidly approaches the values at $x = x_1$ but in higher levels this is not the case. The values of A_z in the lowest layers must be considered more reliable than the values in the uppermost levels.

According to Exner $A_z = \dfrac{C}{3.4 \cdot 10^{-4} + \dfrac{\partial \Theta}{\partial z}}$.

This is an empirical law valid in certain cases. However, if the absolute value of the lapse rate is greater than $3.4 \cdot 10^{-4}$ it is obviously not valid.

Sverdrup, see LETTAU (1938), gives the expression

$$A_z = \frac{A_z^i}{(1 + \beta Ri)}$$

where A_z^i is the eddy conductivity at indifferent stratification, β is a constant and Ri is the Richardson number

$$Ri = \frac{\frac{g}{\varrho} \cdot \frac{\partial \Theta}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2}$$

Ri is negative over the warm water surface and the absolute value is so large that A_z gets unlimited. Thus even Sverdrup's formula does not hold.

Jeffreys, see BRUNT (1944), p. 244, has given another criterion on the greatest possible gradient. If the actual lapse rate is large compared with the dry adiabatic lapse rate, the condition is:

$$-\frac{\partial T}{\partial z} < \frac{27 \pi^4 A^2 T}{4 g h^4 \varrho^2}$$

Here g is the acceleration of gravity and h is the thickness of the layer.

The formula is valid in a homogeneous free layer with $\frac{\partial T}{\partial z}$ and A constant throughout the layer. The formula is thus not directly applicable when there is a great variation with height of the two quantities $\frac{\partial T}{\partial z}$ and A. However mean values of $\frac{\partial T}{\partial z}$ and A in the layer considered have been used here.

The formula is valid for a layer with free upper and lower boundaries. In our case the lower boundary is the water surface and the constant $\frac{27\pi^4}{4}$ should be replaced by the constant 1,100.

Applying the formula for three different thicknesses of the bottom layer we obtain:

Table II

h cm	A_{mean} cgs	$\frac{\partial T}{\partial z}_{\text{mean}}$ °/cm	$\frac{1{,}100 A^2 T}{g h^4 \varrho^2}$ °/cm
2	0.01	7	1,000
20	0.07	0.8	5
50	0.08	0.34	1.2

From Table II we see that the actual lapse rate never reaches the critical value at which convection starts, according to Jeffreys' formula. The discrepancy between the theoretical and actual gradients may be due the above described limitations of Jeffreys' formula.

Studies of cold air outbreaks over warm water by G. I. Taylor and others, see BRUNT, p. 228 (1944), have given values of A_z amounting to 50 – 100 cgs in the lowest 300 m deep layers whereas values of A_z amounting to 1 – 3 cgs have been obtained in a similar layer when warm air flows over cold water. That is to say A is about 50 times larger in slightly unstable conditions than in slightly stable conditions.

We have obtained about the same low values of A in extremely cold air over a very warm water as NYBERG (1939) found at extremely stable conditions. Nyberg found in his group 6 and 7, which had the same winds as in this case, $A_{10} = 29 \cdot 10^{-4}$ and $37 \cdot 10^{-4}$ cgs. Here A_{10} is equal to $60 \cdot 10^{-4}$ cgs. The value of the molecular heat transport coefficient corresponding to A is $\frac{\lambda}{c_p} = 2 \cdot 10^{-4}$ cgs and this value of A_z is reached at about the level 1 mm over the snow field and at a level less than half a mm over the water.

Our suggestion is that the air over the warm water is not in a very unstable state and that A_z is a function of time, or of x, that is to say that convection cells do not start on too small a scale. They have to have a considerable space dimension. An observation of a Swedish glider pilot Mr Söderholm indicates that a length of a warm bay of about half a km is needed to get warm bubbles that can be used by gliders.

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