

On the electrification of snow

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Abstract

Electrification of snow and related phenomena were investigated by measuring: (1) the charge of snow when pouring it from a vessel into a funnel, (2) the charging of an insulated target when letting snow hit it as well as the charge of the scattered snow, and (3) the charge in air from blowing snow. Specific, basic phenomena were separated. The charge of larger snow particles collected was dependent on the temperature and on the material of the target. The properties of the surfaces seem to be of importance in these phenomena. Enhanced charges of both polarities were measured in air by ion-counters. It seems that these charges are not due to ions, but may be attributed to very small snow particles suspended in air. A complete insight into the whole problem of electrification of snow might be obtained by a systematic consideration of all these specific phenomena.

1. Introduction

The problem of electrification of snow, known for a long time, has recently been investigated anew, taking into consideration different points of view. Some of these investigations (1), (8) are connected with the so called precipitation static, i. e. the electric perturbances arising in telecommunication when an airplane flies in a snow cloud. Another field, in which it is of importance to know the phenomena of electrification of snow, is in connection with the production of electrical charges during thunderstorm conditions. Several investigations in this field have recently been published. (7), (3).

One of the authors (H. NORINDER) has discussed the possibilities of the genesis of a special type of atmospherics within snow showers (6). These ideas led us to undertake the present investigation, but some other more general points of view will also be discussed.

Some difficulties arose at the beginning when using snow as the substance of electrification and therefore some preliminary experiments were carried out by using sugar. Measurements were made to clarify some general phenomena observed when pouring substances; it was as-

sumed that similar properties may also be expected for snow. This assumption appeared not to be fully correct because of a difference in order of magnitude of electric resistivity between sugar and snow. The resistivity of snow is smaller and therefore some difficulties which arose when working with sugar (rest charge), do not appear when using snow. To clarify this subject, the resistivity of snow was also measured. The electrification of snow and related phenomena were investigated by measuring:

- (1) the charge of snow when pouring it from a vessel into a funnel,
- (2) the charging of an insulated target on which snow was blown and the charge of the scattered snow, and
- (3) the charge in air during condition of blowing snow.

2. Arrangement of the experiments

1. *Measurements of the resistivity of snow.* If a charged Wulf electrometer was connected with a metallic rod embedded in the given substance placed in a grounded metallic vessel the fibres of the electrometer dropped together in the case of sugar, but the dropping could be seen and studied. In case of snow, however, the dropping

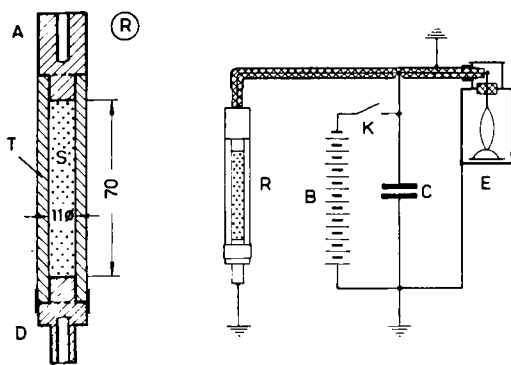


Fig. 1. Device for measuring the resistivity of snow: (R) resistor of the substance under consideration S, T trolitul tube, A and D brass plugs; E electrometer, C high quality condenser, B battery, K key.

was instantaneous. It was even impossible to determine the resistivity of the snow, by means of a bridge by which resistivities up to $10^7 \Omega$ could be measured. Therefore a method must be used by which higher resistances can be measured. The device used is shown in Fig. 1. A tube T made of very good insulating material (trolitul) was closed at both ends with the brass plugs A and D and filled with the substance under investigation. One plug was grounded, the other was connected to the Wulf-electrometer E with the aid of a screened cable of very good quality. A condenser $C = 0.1 \mu F$, also of very good quality, was installed in parallel with the electrometer. The whole system was charged by a battery B. Resistivity of the substance under examination was determined from the fall of the potential.

2. *Pouring snow.* The device used is shown in Fig. 2. Snow was poured from a vessel into the funnel F. A cylindrical collector C was placed on the polystyrene insulators I in the screening vessel Sc. The collector was placed outdoors and connected with the Wulf bifilar electrometer through a high quality screened cable. The electrometer was placed in a closed room at room temperature. The following vessels and funnels were used: (1) metallic, (2) paper coated, and (3) paper coated impregnated first with water which was frozen and after that covered with ice by repeated immersion in water and freezing. Pouring snow was quite difficult. The vessel from which the snow was poured, had to be shaken all the time by hand, and in the funnel the snow had to be scraped from time to time with a rod.

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3. *Blowing snow.* A device for snow blast was built as shown in Fig. 3. Snow was poured into a metallic funnel F connected with a flexible tube T. Air was blown into the tube by an electric driven blower B. Snow did not always pour continuously into the gap between the funnel and the nozzle of the blower, and it had therefore to be poked by a stirrer S from time to time. But on the whole the device was suitable for obtaining a jet of snow which could be blown in the air off on to different targets, for instance on an insulated plate. The plate was charged up to a comparatively high potential even by using small quantities of snow so that once by slight carelessness the Wulf electrometer connected with the plate (Situation I, Fig. 3) was so highly charged that one fibre was attached to the field plate. Shorter sparks were occasionally obtained from the plate. The charging of the plate was measured by a Wulf electrometer E (Situation I, Fig. 3). The charge of the scattered larger snow particles was measured in the collector C (Situation II, Fig. 3). The charge varied depending upon whether the plate was insulated or grounded (Situation III, Fig. 3).

4. *Measurements of the charge in the air.* It was evident that from blowing snow particles of

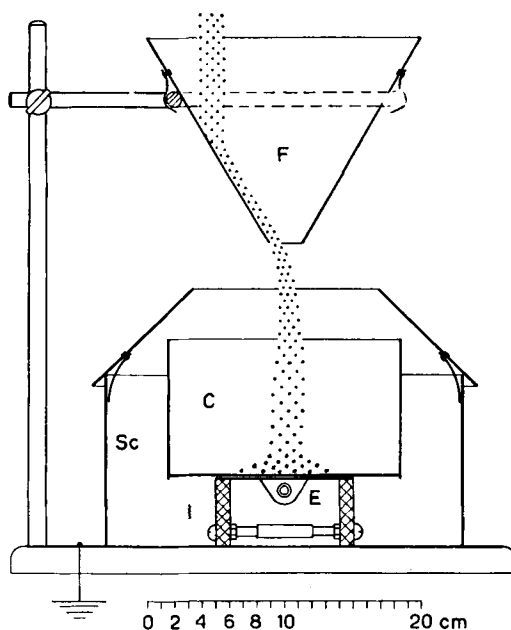


Fig. 2. Device for experiments by pouring snow: F funnel, C collector, I polystyrene insulator, Sc screening vessel, E connector for cable to electrometer.

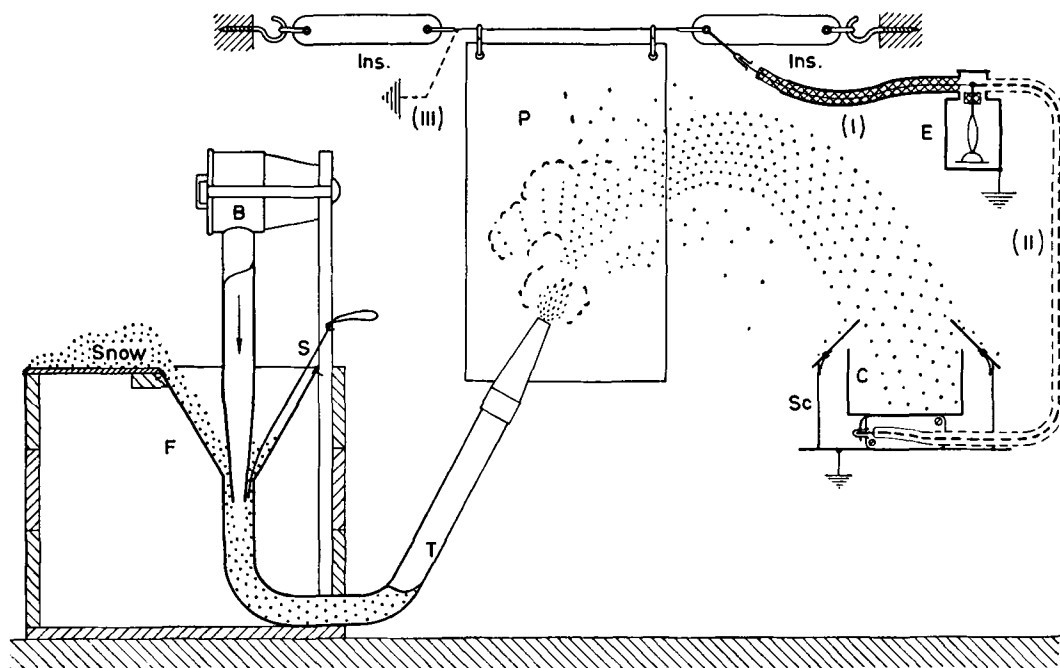


Fig. 3. Arrangement for experiments by blowing snow: *B* air blower, *F* funnel, *S* stirrer, *P* plate of various materials, *Ins* polystyrene insulators, *E* electrometer, *C* collector, *Sc* screening vessel.

smaller size also could be expected to be present in the air and it was impossible to collect these suspended particles in a collector. The charge carried by these small snow particles and possibly also by the air itself, if ionized, may be measured as the net charge, as done by other authors (7), but it is preferable, if charges of both polarities are present, to separate them. It was attempted to make this separation by measuring the charge in air with ion counters by which method it also could be determined if ions were produced by blowing snow. The ion counters used were those used at the Institute for measuring ions in the atmospheric air: one ion counter for small ions equipped with a Weger condenser (10) and one Israël large-ion counter (9). The snow blast was arranged in a gap between the observation cabin, where the counters were placed, and a wagon on one side of it (Fig. 4). In this way the space, where snow was blown, was partially screened from winds, although a slight draught prevented us from obtaining an even "snow atmosphere". Therefore, if more definite conditions were desirable, a better screening would be necessary in order to eliminate the difficulties caused by

the wind. The snow jet was blown on a plate *P* and in some experiments also directly into the air.

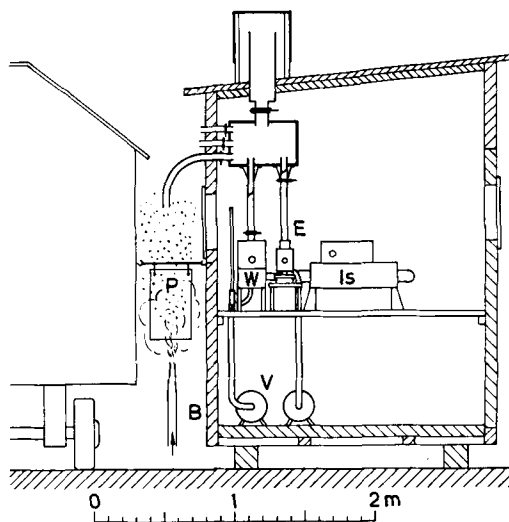


Fig. 4. Arrangement for measuring of charges in the air with ion counters when blowing snow: *B* pipe of snow blast, *P* plate, *W* small-ion counter with Weger condenser, *E* Ebert counter, *Is* Israël large-ion counter, *V* vacuum cleaners.

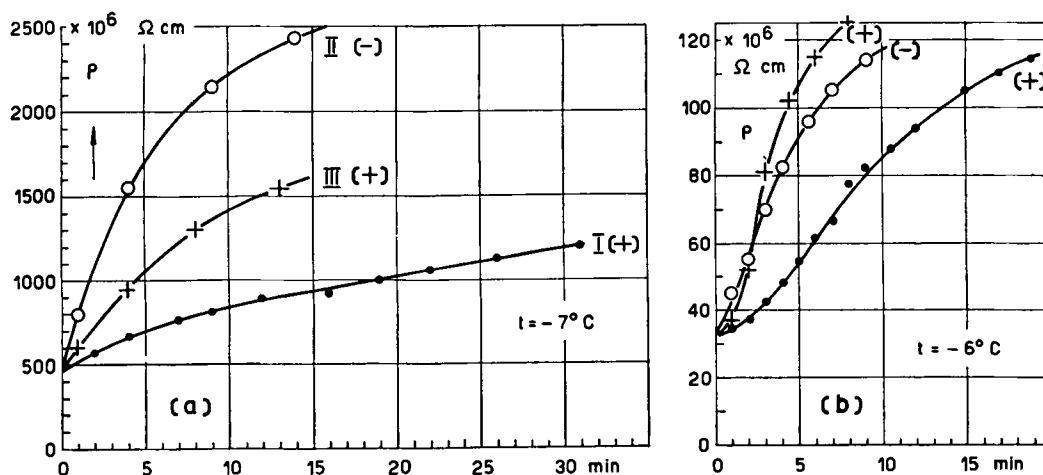


Fig. 5. Increase of the resistivity of snow with time and variation by changing polarity of the system; (a) loosely poured snow, (b) more shaken snow.

3. Some considerations on the electrification of snow

a. *The reason for electrification.* Snow or ice crystals may be electrified in various ways:

(1) The ice may be charged when water is freezing (14), (5).

(2) Completely formed and uncharged ice crystals may be electrified when falling through ionized air, as has been assumed by several authors. A summary and references on this subject may be found for example in a paper by WICHMANN (13).

(3) Completely formed and uncharged snow particles will be charged by touching other bodies. Contact potential, friction and fragmentation of snow crystals may be the cause of electrification in this case.

Only the last effect is of interest in our experiments.

b. *Electrification in our experiments.* There are three entities which may be electrified and the charge of which can be measured when snow is falling on a target:

(1) The snow itself, particularly the larger snow particles which may be collected and measured;

(2) The target may be electrified because, if snow particles are charged, at least some other object must be present which could be electrified with the opposite electricity and this can be the target touched by the snow;

(3) The air; electrification of snow and target takes place in air and therefore the air may

also be electrified. From our measurements it was not possible to determine whether the air was really electrified, i. e. if ions were formed in air by electrification of snow, or the charges measured in air could be attributed to very small snow particles suspended in the air. It seems to us that the latter assumption is more likely.

4. Results and discussion

1. *Resistivity of snow.* As might be expected, the resistivity of snow was highly variant depending upon the variation of its structure, but the order of magnitude (Table 1) was the same as that of ice measured by JOHNSTONE (4). It was attempted to characterize the snow by

Table 1. Resistivity (in $10^6 \Omega \text{ cm}$) of snow.

Date	Temperature $^\circ \text{C}$	Snow density	Loosely packed	Shaken	Packed
1953					
Jan. 29	-2	—	2600	470	70
Feb. 4	-3, -2	0.13	620	300	28
Feb. 3	-7, -4	0.18	62	15	—
		0.22	200	29	67
Feb. 9	-8	0.51	77	44	27
		0.47	100	60	70
		0.09	440	170	60
Feb. 5	-9	0.25	2430	—	400
Feb. 6	-12	0.29	170	90	500
Feb. 7	-16	0.17	900	440	200
		0.30	700	210	190

its density, which also is variable. A definite dependence on the temperature could not be found.

The obtained variations of the resistivity of snow may also be related to some other phenomena (Fig. 5). The resistivity of snow poured into the holder increased with time. If the sign of the electrical charge applied to the system was changed, the resistivity returned to the smaller initial values but thereafter increased again.

Resistivity of sugar was of the order $1-3 \times 10^{12}$ Ω cm. It is evident that in order to obtain more definite information about the electrical resistivity of snow, more careful measurements must be carried out. Electrical resistivity is also a quantity by which electrical properties of the substance are characterized, and, as has been shown, no difficulties arise in measuring it for snow if one uses modern insulation materials. In our case it was only important to obtain a general idea of the problem and the most important conclusion was the following one: although the resistivity of snow is comparatively high, it is considerably smaller than that of sugar, and therefore difficulties caused by the rest charge observed in case of sugar should not be expected to be of importance when working with snow. Thus the difficulties one might expect from working with sugar did not appear because it may be expected that if electrified snow particles fall in a heap, the charges of the particular particles will be completely distributed over the heap and hence the charge would disappear if the system were grounded. In addition we may also conclude that results of investigation of electrification obtained by using substances of very high electrical resistivity (as sugar, perhaps also quartz) cannot be applied to substances with low resistivity, (e. g. snow) without particular considerations.

2. *The electric charge on snow falling into a tunnel.* Under the same conditions snow was negatively charged when falling on a metallic surface or an ice surface and positively when falling on a paper surface (Table 2. Jan. 26, 30, Feb. 9), although under other conditions it was negatively charged when falling on a paper surface (Table 2. Feb. 7). Thus the same snow obtained charges of different sign depending on the character of the surface which was touched. The properties of the other entities determined the charge and also the polarity of

Table 2. Electric charge in 10^{-10} coulombs per 1 g snow when pouring as dependent on various factors.

Funnel			Snow density	Temperature C°	Date Weather
Metal	Ice	Paper			
(-) 0.38	(-) 0.35 (-) 0.29	(+) 0.37	0.5	-4	1953 Jan. 26
(-) 0.7	(-) 0.3	(+) 1.3	0.24	-1	Jan. 30
(-) 0.96 (-) 1.00			0.22 drifted	-7, -4	Feb. 3
(-) 18.4 (-) 8.6 (-) 15.2	(-) 1.0	*)	0.25	-8	Feb. 5 strong wind
(-) 35.4 (-) 38 (-) 39.4	(-) 0.88 (-) 0.95 (-) 0.45 (-) 0.42 (-) 0.93 (-) 1.19 (-) 0.96	*)	0.29	-12	Feb. 6
(-) 21.0 (-) 23.0 (-) 22.0 (-) 19.0	(-) 0.72 (-) 0.55	(-) 1.56 (-) 1.36 (-) 1.35 (-) 1.44 (-) 2.24 (-) 2.40	0.3 old granulated-	-16	Feb. 7 wind
(-) 29.0 (-) 26.6	(-) 1.25 (-) 0.53	(-) 0.20 (-) 0.12	0.17 fresh soft		
(-) 1.35 (-) 1.05 (-) 1.15		*)	0.47 old granulated	-9.6 -8	Feb. 9 windless, snow falling
(-) 1.10 (-) 0.47 (-) 0.94 (-) 8.7 (-) 7.1 (-) 7.6	(-) 0.39 (-) 0.15 (-) 0.26 (-) 1.45 (-) 0.93	*) (-) 1.7 (-) 1.5 Funnel heated up (+) 0.67 (+) 0.58 (+) 0.45 (+) 0.60	0.09 fresh		

*) Indefinite, collector discharged independent on the polarity of the charge at the beginning.

the falling snow. The magnitude of the effect was also dependent on the structure of snow and on the temperature. At lower temperatures the effect was greater.

3. *Charge of an insulated plate when hit by drifting snow.* A similar and more marked effect was established when a jet of blowing snow

Table 3. Charge on insulated plates of various materials when blowing snow.

Plate	Charge in 10^{-10} coulombs per 1 g snow			Date and temperature 1953
	Plate at the beginning			
	(+) charged	(−) charged	un- charged	
Aluminium	(+) 6.2 (+) 8.5 (+) 5.8	(+) 16.8 (+) 12.8 (+) 7.2		Feb. 13, t = —10° C
			(+) 12.2 (+) 13.0	Feb. 14, t = — 5° C
Pasteboard	(−) 40.0 (−) 37.4 (−) 36.1	(−) 21.7 (−) 23.0 (−) 24.4		Feb. 13, t = —10° C
			(−) 22.8 (−) 25.0 (−) 29.6 (−) 25.6	Feb. 14, t = — 7° C
Pasteboard covered with ice	(+) 10.0 (+) 6.3 (+) 6.3 (+) 8.5	(+) 18.6 (+) 13.0 (+) 11.7	(+) 10.2 (+) 11.8 (+) 14.9	Feb. 13, t = —10° C
Wood			(−) 13.6 (−) 12.4 (−) 12.2 (−) 14.6	Feb. 14, t = — 7° C
Bakelite			(−) 11.9 (−) 11.8 (−) 11.2 (−) 16.8	Feb. 14, t = — 6° C
Plexiglas			(+) 9.0 (+) 6.3 (+) 7.1 (+) 6.7	Feb. 14, t = — 6° C

struck different targets: metal and pasteboard covered with ice were positively charged, pasteboard, wood, bakelite were negatively charged, but a better insulator such as plexiglas was charged positively (Tables 3 and 4). The charge obtained on pasteboard was very large. A snow block also obtained a very large charge (Table 4). On the other hand the charge of a metal plate was very dependent on the quality of the surface, a greasy iron plate obtained sometimes positive and sometimes negative charges. Also here the effect was, in general, larger at lower temperatures.

4. *The importance of the surface.* It seems that these experiments conclusively show the importance of the character of the surfaces touched

by the snow particles. This is shown still better by experiments at temperatures close to zero. In such cases a dry pasteboard and a dry wood plate were negatively charged, but the charge became positive if the plates were wet (Table 5). A similar effect of changing polarity was also observed on metallic surfaces although not so marked. It may seem that this effect could not be of importance for electrification of snow under natural conditions, but we do not think so, because the properties of the surface of individual snow flakes, snow crystals, particularly graupel grains may also be different depending on various circumstances of this effect, possibly modified, may therefore also be expected under natural conditions in the air and in clouds. The different charging of various targets when struck by blowing snow is of basic importance because so many varying conditions have been observed to affect snow electrification. If the properties of the surface of the bodies for electrification are of such importance as shown by these experiments, the conception of the role of the surface layers for electrification of bodies by friction as pointed out recently by BÖNNING (2) may help us in trying to explain this phenomenon.

5. *Charge of larger snow particles scattered from the plate.* One fact seems to be contradictory to this conception, viz. that the larger snow particles scattered from the target by blowing snow were in all cases positively charged independent of whether the target was charged positively (metallic surface, ice block) or negatively (pasteboard, wood) (Table 4). It should be added that the larger snow particles were negatively charged only in one case when scattered from a grounded ice block (Table 4, Feb. 20). The charge of the scattered larger snow particles per gram of snow was in general larger than that produced on the target per gram of snow. But as yet we have not considered the possible effect of the air or the smaller snow particles in the air.

6. *The charge in the air.* The presence of charges in air, and simultaneously charges of both polarities was shown by measuring with the small-ion counter (Figs. 6, 7, 8). Sometimes the positive charge was predominant, sometimes the negative. Similar effects have also been observed by several authors during natural snow storms and also in experiments. SCHAEFER (8) has shown that the air-to-ground current during snow

Table 4. Charge of an insulated plate when blowing snow and charge on larger snow particles blown away from the plate and collected in a collector, when the plate was insulated and when grounded.

Plate	Charge of insulated plate	Charge carried by larger snow particles		Date and temperature
		Plate		
		insulated	grounded	
Aluminium		Total charge in 10^{-10} coul. of the same unknown quantity of snow		1953 Feb. 20, t = -2.5° C
		(+) 60	(+) 60	
		(+) 48		
	Pasteboard		(+) 130	(+) 190
		(+) 115		
	In 10^{-10} coul. per 1 g snow			
Snow block	(+) 44	(+) 145	(-) 25	t = -1.5° C
	(+) 55	(+) 185	(-) 36	
Iron very carefully cleaned	in 10^{-10} coul. per 1 g snow			Feb. 24, t = -2° C
	(+) 1.1	(+) 3.8	(+) 1.15	
	(+) 1.0	(+) 3.0	(+) 4.7	
	(+) 2.3			
Pasteboard	(-) 5.2	(+) 10.5	(+) 27	t = -1.5° C
	(-) 4.4	(+) 8.2	(+) 33.5	
Wood	(-) 10.2			t = -1° C
	(-) 12.2			
Wood	(-) 4.7	(+) 85	(+) 83	March 4, t = -1.5° C
		(+) 106	(+) 75	
Snow block		(+) 72		t = $+1^{\circ}$ C
	(+) 0.82			
	(+) 0.35			
	(+) 5.5	(+) 3.2	(+) 1.6	
	(+) 7.0	(+) 1.9	(+) 1.8	March 5, t = -1.5° C coarse-grained snow, snow block very dense

Table 5. Charging of an insulate plate from blowing snow. Date March 10, 1953. $t = 0^{\circ}\text{C}$, strong NW wind.

Plate		Charge in 10^{-10} coulombs per 1 g snow
Pasteboard	Dry	(-) 5.1 (-) 3.2
	moistened by water	(+) 1.5 (+) 2.1
Wood	dry	(-) 3.5 (-) 3.8
	moistened by water	(+) 1.6 (+) 2.1

storms usually was positive, but it may also be negative in some cases. STÄGER (11), (12) has established sometimes a positive, sometimes a negative charge in a cloud of fine snow in a whirl. By measuring the space charge during snow storms PEARCE and CURRIE (7) have established that the space charge density becomes strongly positive, when the surface snow starts to drift, but the rate of charging of the collector either decreased or changed its sign temporarily when very strong gusts occurred which indicates that the proportion of negatively charged particles entering the collector increased in such cases.

We have not succeeded in clarifying the con-

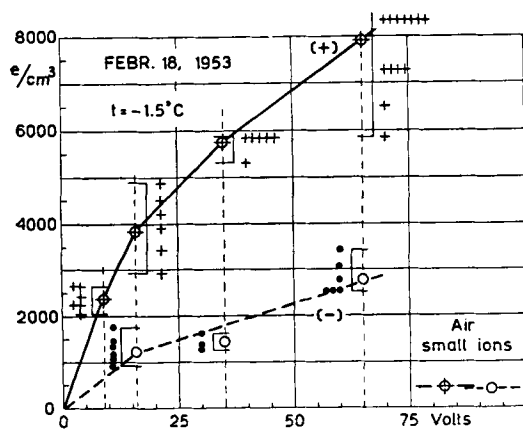


Fig. 6.

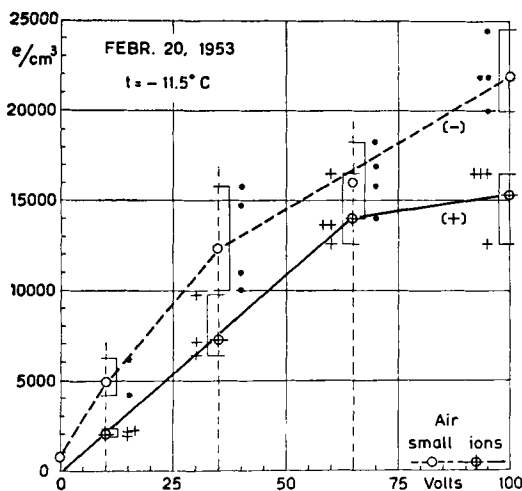


Fig. 7.

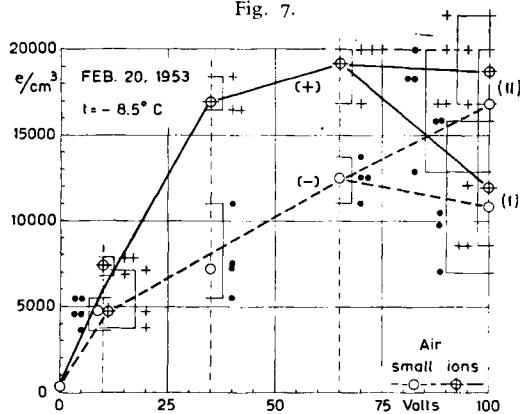


Fig. 8.

Figs 6, 7, 8. Number of elementary charges e per cm^3 in air when blowing snow measured with Weger condenser at different potentials (in Volts) of the control electrode. Beside the average values points for individual measurements are also shown.

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Table 6. Number of elementary charges e per cm^3 measured with Israël large-ion counter adjusted to a limit mobility $k_* = 13.6 \times 10^{-4} \text{ cm sec}^{-1}$: Volt cm^{-1} . Feb. 20, 1953, $t = -5^\circ \text{C}$.

	I series		II series	
	(+)	(-)	(+)	(-)
In the air	760	1030		
When blowing snow	2380	3700	3020	3020
	4430	3320	3170	2550
	3900	3020	2770	1580
	4400	4150	2660	2660
	5100	3500	2890	2880
	4430	2890	3910	2660
	5540		3700	3500
	3700		3700	3170
	3900			
	4430			
	4430			
Average	4070	3380	3170	2620

ditions under which the positive or the negative charge is predominant in the air. The maximum value of the positive space charge or the net charge estimated by us in the air may be of the order $0.5 \times 10^{-5} \text{ e. s. u. per cm}^3$; PEARCE and CURRIE (7) have measured values up to $10^{-5} \text{ e. s. u. per cm}^3$ occurring with moderate drift during snow storms. If only the net charge were measured, the charge opposite to the predominant one would be overshadowed. Therefore measuring of separated charges as done by us has a certain advantage. Charges of both polarities were also measured in air with the large-ion counter (Table 6).

No definite conclusions would be drawn whether real ions were measured in air by ion counters or not. It seemed to us as if smaller snow particles with larger charges (up to several 100 000 elementary charges per particle) carried the charge on to the collector electrode when measuring with the Weger condenser. Larger particles with properties of large atmospheric ions might be measured by the large-ion counter.

The importance of the role of the charges or possibly the smaller snow particles produced in the air by electrifying snow is also shown by the following phenomenon observed by us: The measured charge of the collector (when snow was falling into the funnel) and of the plate (when blowing snow) was larger when the collector or the plate had been discharged

during the measurement (collector or plate were charged at the beginning with a charge opposite to the charge collected), than when collector or the plate became more strongly charged, (i. e. when they had had at the beginning a charge of the same polarity as that of the collected charge).

The great importance of our knowledge of the properties of the charges produced in the air by electrification of snow may also be seen from another point of view, viz. the problem of separation of charges obtained by electrification of snow. This problem is very important for various thunderstorm theories, but it seems that it cannot be clarified in a simple way.

In addition it must be noted here that we have not observed the effect of electrification by erosion of snow as shown by PEARCE and CURRIE (7) when an air stream strikes a snow

block. Perhaps our jet was too weak and the sensitivity of our electrometer may also have been too low, but our intention was only to get an idea of the more pronounced effects. By using more effective devices certainly many interesting details could be found.

A more detailed account of the investigation presented here will appear in *Arkiv för Geofysik* published by Kungl. Svenska Vetenskapsakademien, Stockholm.

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