Tendency Computations with a Continuous Two-Parametric Atmospheric Model

By SIGURD J. SMEBYE
University of Stockholm and Det Norske Meteorologisk Institutt

(Manuscript received December 19, 1952)

Abstract

In a case of strong development the tendencies at the 500 and 1000 mb surfaces are computed with a two parametric model. It is possible to take the thermal structure of the atmosphere into account in a crude manner by expressing the vertical temperature distribution as a function of the deviation of the 1,000—500 mb thickness from a standard value. The height of the 500 mb surface and the 1,000—500 mb thickness are used as variables. The tendencies are computed for two maps 12 hrs apart, the intermediate tendencies interpolated for every 2 hrs and the 12 hr change of the 1000 and 500 mb surfaces are then computed. The tendencies computed represent an improvement over those computed with the barotropic model, but the 12 hr changes are still only about $^2/_3$ of the observed changes.

The following tendency computations were started during a stay at "The Institute of Meteorology, University of Stockholm" in spring 1952 and finished in Oslo.

The different atmospheric models have been treated by Dr. ELIASSEN 1952. The model I have used in the following computations is given there. I will therefore only repeat the main assumptions and the equations which form the basis for the computations.

We try to represent the temperature field as a sum of a "normal" vertical temperature distribution and a deviation from this normal.

$$T = T_n + p\frac{g}{R}A_1'(p)(h - h_n) \text{ or spec. vol.}$$

$\alpha = \alpha_n + g\, A_1'(p)\,(h - h_n)$ where $A_1', (p)$ is a function of p only and $h - h_n$ a function of x, y, t, chosen to give the deviation of the 1,000—500 mb thickness from the "normal". The $'$ means derivation with respect to p. The assumption leads to the following vorticity equation.

$$\nabla^2 \bar{Z}_t + A\nabla^2 h_t + \mathbf{k}\times(\nabla\bar{Z} + \quad (1)$$

$$+ A\nabla h)\cdot\nabla\left(f + \frac{g}{f}\nabla^2\bar{Z} + \frac{g}{f}A\nabla^2 h\right) +$$

$$+ 2A'\omega\nabla^2 h -$$

$$\frac{\partial}{\partial p}\cdot\left[\left(\frac{f^2}{g} + \nabla^2\bar{Z} + A\nabla^2 h\right)\omega\right] = 0$$

where $A = A_1 - \bar{A}'$, $\quad Z_t = \dfrac{\partial z}{\partial t}$, $\quad h_t = \dfrac{\partial h}{\partial t}$

We can now in different ways from this equation form two equations which can be used to find Z_t and h_t. We have, however, to assume something about the vertical velocity ω. We put $\omega = W(x\ y\ t)\cdot B(p)$ and assume adiabatic motion. Then

$$\frac{\Theta_t}{\Theta} + \mathbf{v}\cdot\frac{\nabla\Theta}{\Theta} + \omega\frac{\Theta_p}{\Theta} = 0$$

or

$$\frac{\alpha_t}{\alpha} + \mathbf{v} \cdot \frac{\nabla \alpha}{\alpha} + \omega \frac{\Theta_p}{\Theta} = 0, \quad \Theta_p = \frac{\partial \Theta}{\partial p}$$

We ignore the variation of $\frac{\Theta_p}{\Theta}$ with x, y, t, and the deviation of α from the chosen normal. We then get

$$\frac{g\, A_1'\, h_t}{\alpha_n} + \mathbf{v} \cdot \frac{g\, A_1' \nabla h}{\alpha_n} + W \cdot B \frac{\Theta_p}{\Theta} = 0$$

$$h_t + \frac{g}{f} \mathbf{k} \times \nabla Z \cdot \nabla h = h_t +$$

$$+ \frac{g}{f} \mathbf{k} \times \nabla Z_5 \cdot \nabla h = - W \cdot B \frac{\Theta_p}{\Theta} \frac{\alpha_n}{g A_1'}$$

Here the left hand side is a function of x, y, t, only, and we must have

$$B = -\frac{g A_1'}{\alpha_n} \frac{\Theta}{\Theta_p}, \quad W = h_t + \frac{g}{f} \mathbf{k} \times \nabla Z_5 \cdot \nabla h$$

If we now apply the equation (1) to the 1,000 mb level and also take the mean value of equation (1), we get the following two equations.

$$\nabla^2 \bar{Z}_t + \mathbf{k} \times \nabla \bar{Z} \cdot \nabla \left(f + \frac{g}{f} \nabla^2 \bar{Z}\right) +$$

$$+ \overline{A^2}\, \mathbf{k} \times \nabla h \cdot \nabla \frac{g}{f} \nabla^2 h + 2\, \overline{A' \omega}\, \nabla^2 h = 0 \quad (2)$$

$$\nabla^2 \bar{Z}_t + A_0 \nabla^2 h_t + \mathbf{k} \times (\nabla \bar{Z} +$$

$$+ A_0 \nabla h) \cdot \nabla \left(f + \frac{g}{f} \nabla^2 \bar{Z} + \frac{g}{f} A_0 \nabla^2 h\right) -$$

$$- \left(\frac{f^2}{g} + \nabla^2 \bar{Z} + A_0 \nabla^2 h\right) \omega_{p0} = 0 \quad (3)$$

In the following we shall use the equations (2) and (2)—(3). If we now introduce the expression for W in these two last equations, we get the following two equations which form the basis for the computations.

$$\nabla^2 h_t - \left[\frac{B_0'}{A_0}\left(\frac{f^2}{g} + \nabla^2 \bar{Z}\right) + \right.$$

$$\left. + \left(B_0' + 2 \frac{\overline{A' B}}{A_0}\right) \nabla^2 h \right] h_t + \mathbf{k} \times \nabla \bar{Z} \cdot$$

$$\cdot \nabla \frac{g}{f} \nabla^2 h - \left[\frac{B_0'}{A_0}\left(\frac{f^2}{g} + \nabla^2 \bar{Z}\right) + \left(B_0' + \right.\right.$$

$$\left.\left. + 2 \frac{\overline{A' B}}{A_0}\right) \nabla^2 h \right] \frac{g}{f} \mathbf{k} \times \nabla Z_5 \cdot \nabla h +$$

$$+ \mathbf{k} \times \nabla h \cdot \nabla \left[f + \frac{g}{f} (\nabla^2 \bar{Z} + A_0 \nabla^2 h)\right] -$$

$$- \frac{\overline{A^2}}{A_0} \mathbf{k} \times \nabla h \cdot \nabla \frac{g}{f} \nabla^2 h = 0 \quad (4)$$

$$\nabla^2 Z_t + \mathbf{k} \times \nabla \bar{Z} \cdot \nabla \left(f + \frac{g}{f} \nabla^2 \bar{Z}\right) +$$

$$+ \overline{A^2}\, \mathbf{k} \times \nabla h \cdot \nabla \frac{g}{f} \nabla^2 h + 2\, \overline{A' B}\, (h +$$

$$+ \mathbf{k} \times \frac{g}{f} \nabla Z_5 \cdot \nabla h = 0 \quad (5)$$

Here $\bar{Z}$ can be expressed by Z_5, the height of the 500 mb surface. We now can solve the equations if we can find values of A and B. B is, as we have seen, given when A is given. To get a first estimate of the values to choose for A, I used the ascents for about a month from an arbitrary station, Berlin, and calculated $\frac{T - T_n}{h - h_n}$. T_n and h_n were the mean values for all ascents. I could then find a mean value of A_1'. The values of A_1' thus found started from near zero at the ground and increased gradually to a maximum at about 300 mb. From there it dropped to zero between 225 and 250 mb. Higher up the values were negative. From A_1' we may get A. With some small adjustments these values of A were used in the computations. Since

$$\nabla Z = \nabla Z_5 + (A_1\ (500) - A_1)\ \nabla h$$

we must have A_1 (500) — A_1 (1,000) = —1. The function $A(p)$ must also give a reasonable variation of wind with height. The functions $A(p)$ and $B(p)$ are shown in the figures 4 and

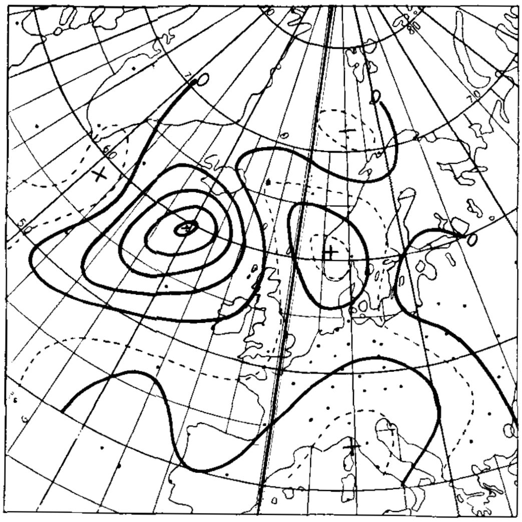

Fig. 1. Observed change in 500 mb contour height 03-15 G. M. T. 4-12-1951. Lines are drawn for every 80 m with some intermediate 40 m lines stippled in.

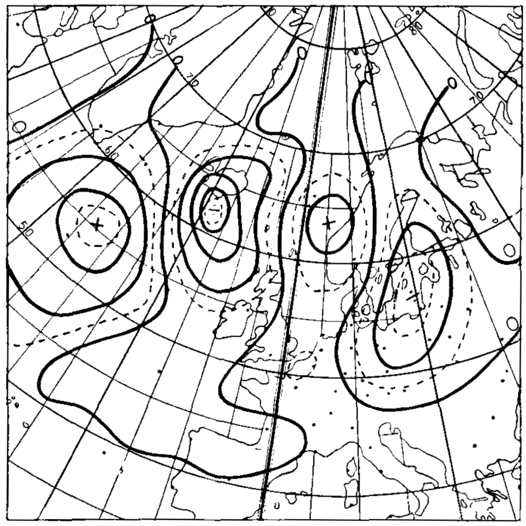

Fig. 2. Observed change in surface pressure 03-15 G. M. T. Lines are drawn for every 10 mb with some intermediate 5 mb lines stippled in.

5 in Dr. Eliassens paper. The constants which enter into the equations are.

$$A_0 = -0.8, \ \overline{A^2} = 0.6, \ \overline{A'B} = -4.4 \cdot 10^{-2}, \ B_0' = -6.7 \cdot 10^{-2}$$

The unit for B is mb/decameter. A is a pure number.

I have used the same area and gridsize in my computations as those that were used in the barotropic tendency-computations in Stockholm. The distance between two points in the grid is about 300 km. From the map we must read Z_5, the height of the 500 mb surface and h, the thickness 1,000—500 mb. Then the following terms, or combinations of terms must be computed for every point in the grid

$$\mathbf{k} \times \nabla h \cdot \nabla \left(f + \frac{g}{f} \nabla^2 Z_5\right), \ \mathbf{k} \times \nabla Z_5 \cdot \nabla \left(f + \frac{g}{f} \nabla^2 Z_5\right), \ \mathbf{k} \times \nabla h \cdot \nabla \frac{g}{f} \nabla^2 h,$$

$$\mathbf{k} \times \nabla Z_5 \cdot \nabla h, \ \mathbf{k} \times \nabla Z_5 \cdot \nabla \frac{g}{f} \nabla^2 h$$

We solve first the equation for h_t. It has the form $\nabla^2 h_t - a h_t + b = 0$ where a is positive almost everywhere, and there are no difficulties in finding h_t by relaxation. The values we find for h_t may now be used in equation (5) which is also solved by relaxation.

For a numerical test of the model I picked a situation with fairly strong development. The day chosen was Dec. 4, 1951. In the figures 1 and 2 we see the observed 12 hrs changes at 500 mb and at the surface from 03 to 15 G. M. T. The 500 mb and 1,000 mb tendencies were computed at the beginning and end of the period.

A major difficulty is to verify the results. The computed tendencies are instantaneous tendencies, and they can only be compared with a difference between two maps 12 hrs apart. The best procedure would of course be to make a series of short time steps. The amount of work involved in such a scheeme is, however, very large.

If we simply take the mean of the two tendencies, we can not expect to get a good result in a case like the one I have treated with rapid movement of strong tendency centres. We would get a picture with two maxima of falls and an area with smaller falls between them. To get a better verification I have tried to interpolate the intermediate tendencies for every 2 hrs. 05-07-09-11-13 G. M. T. The isallobaric centres were assumed to move with constant speed and to change intensity linearly. In the same way the position of the 0 lines were

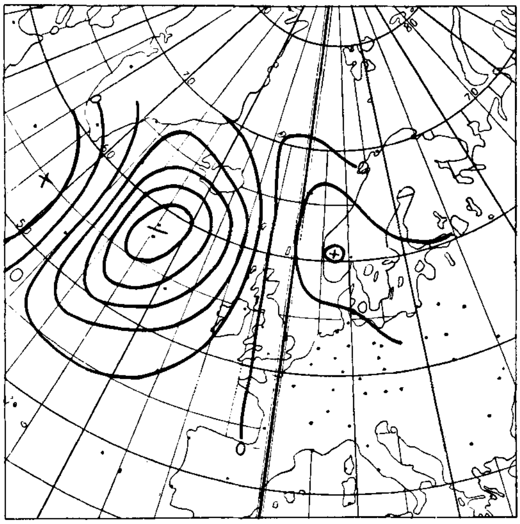

Fig. 3. Interpolated 12 hrs change of 500 mb contour height 03-15 G. M. T. Lines are drawn for every 40 m.

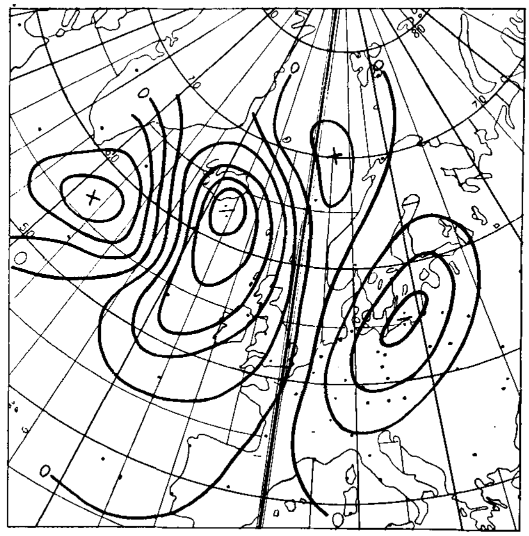

Fig. 4. Interpolated change of 1,000 mb contour height 03-15 G. M. T. Lines are drawn for every 40 m.

found by letting each point on them move parallell to the centres with constant speed. The rest of the intermediate tendencies were then interpolated on the maps. The change from 03—15 is then found by letting the first tendency work 1 hr, the next 5 2 hrs and the last 1 hr. The 12-hour changes obtained in this way are shown in the figures 3 and 4. In these two figures the lines are drawn for every 40 m.

These two maps may now be compared with the two observed maps in fig. 1 and 2. We see that the position of the 0 lines and the centres of rise an fall are in fairly good agreement with the observed positions. Both rises and falls are, however, too small. Except for the falls over Balicum on the 1,000 mb map, they seem to be about 2/3 of the observed values. The two 500 mb tendencies computed with the barotropic model are both much smaller than the two computed with the two — parametric model. 12 hrs changes computed from them seems then be much too small.

It is possible that the two-parametric model might give better agreement with another choise of A. In the equation for h_t the terms containing

$$\mathbf{k} \times \nabla h \cdot \nabla \left(f + \frac{g}{f} \nabla^2 Z_5 \right),$$

$$\mathbf{k} \times \nabla Z_5 \cdot \nabla \frac{g}{f} \nabla^2 h, \quad \mathbf{k} \times \nabla Z_5 \cdot \nabla h$$

are usually the most important. The constants in these terms depend on A and B. It is the values near the ground which come in as a consequence of the way I have chosen to apply the equation. That is perhaps not a very good choise, as the values near the ground may be difficult to estimate.

In the equation for Z_t, it is, apart from the barotropic term, usually the terms with the constants $\overline{A^2}$ and $\overline{A'B}$ which are most important. B is proportional to A' as we have seen. The windshear along the vertical is determined by A'. Since A is zero at the mean level, big values of A' will also give big values of $\overline{A^2}$. Another possibility is to choose B without letting the adiabatic equation be fullfilled for every value of p.

It may be of some interest to study the vertical motion which results from the adiabatic equation with the assumptions made earlier. We have $\omega = W(x\, y\, t) \cdot B(p) = (h_t + \frac{g}{f} \mathbf{k} \times \nabla Z_5 \cdot \nabla h) \cdot B(p)$. Here all quantities are known and we can calculate ω. The vertical motion at the beginning and the end of the period is shown in the figures 5 and 6. The surface fronts are shown as dashed lines. The

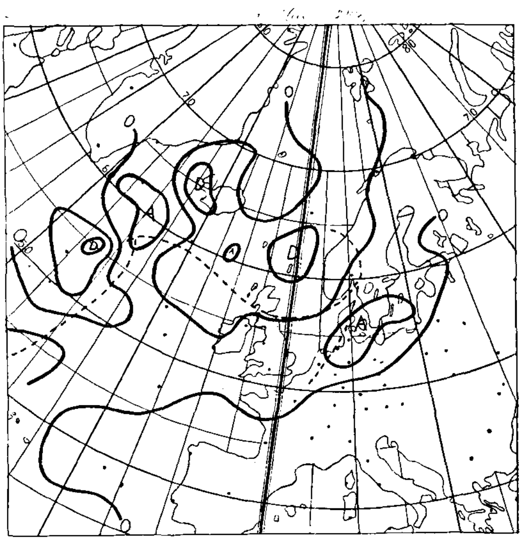

Fig. 5. Computed vertical motion 03 G. M. T. Units are given in the text. A stands for ascending motion, D for descending motion below 250 mb. Surface fronts are stippled in.

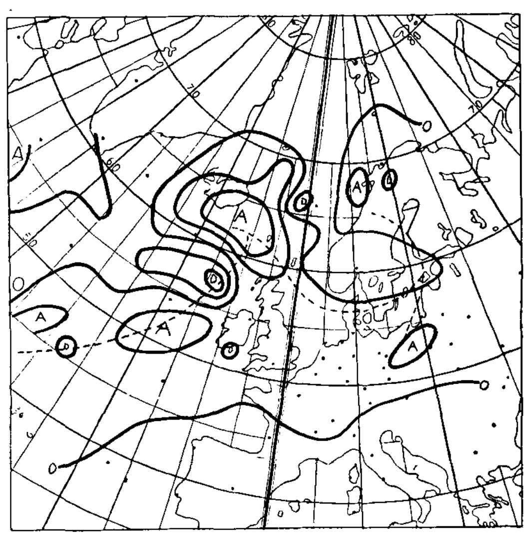

Fig. 6. Computed vertical motion 15 G. M. T.

model gives the same direction of the vertical motion up to 250 mb and above this level it is in the opposite direction. The figures will therefore give the horizontal distribution of the vertical velocity at all levels. The unit used on the map will, however, change from level to level and is given in the table below.

800 mb	1 cm/sec
700	1,7
500	6
300	2,8
250	0
225	—3
100	—0,1
75	0

Let us look at the conditions below 250 mb. In the fig. 5 we see an area of ascending air above the tip of the deepening cyclone. Ahead of it and behind it there are areas of descending motion. There is also an area of fairly strong descending motion behind an old depression off Norway. Above and ahead of the warm sector near Denmark we see an area of ascending air. A considerable pressure fall occurred over Balticum. In the fig. 6 we find the same general picture. In the troposphere the ascending motion over the deepening cyclone is considerably stronger, and the descending motion behind it is also quite strong. We have still the area of descending motion ahead of the deepening cyclone.

REFERENCE

ELIASSEN, A., 1952: Simplified Models of the Atmosphere, Designed for the purpose of Numerical Weather Prediction. *Tellus*. 4, 145—156.