Application of Barotropic Tendency Equation to Medium-Range Forecasting

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Abstract

The simple barotropic vorticity equation has been used to compute height tendencies on a few 5-day mean 500 mb charts. The results are approximately as good as similar computations made on synoptic flow patterns despite the fact that the time interval between observed charts is 5 times as great. The order of magnitude of the computed tendencies may be considerably improved provided some account is taken of the effects of transformation of potential into kinetic energy, surface friction and non-adiabatic heating. A first approximation to these factors may be obtained by setting them equal to their climatological or normal values.

A simple method for integrating the barotropic equation is derived upon assuming that the anomalies of the large-scale flow patterns may be approximated by a relatively small number of Rankine vortices.

1. Introduction

In the preparation of weather forecasts for time-intervals longer than one or two days, it is customary to isolate in some way the larger-scale features of the atmospheric circulation. This procedure is common to most of the present schools of long-range forecasting. One way to accomplish this is to prepare time-averaged or mean circulation patterns at selected levels above the earth's surface. This averaging process not only reveals systems having large space dimensions but also those having large time dimensions as well. Such mean patterns have very slow periods of evolution and may be extrapolated for longer periods in advance.

It is beyond the scope of this article to discuss in detail the pros and cons of this particular approach to the problem of extended-range forecasting. The reader must be referred to two sources of the extensive literature on this subject (NAMIAS and CLAPP, 1951; NAMIAS, 1951). The purpose of the present paper is to report on further investigations of the use of physical-dynamical methods in forecasting the evolution of mean patterns. In particular, tests have been made of the simple barotropic vorticity equation which already promises to be reasonably successful when applied to short-range forecasts using synoptic charts (BOLIN and CHARNEY, 1951; STAFF MEMBERS, UNIVERSITY OF STOCKHOLM, 1952). A very great advantage of this method is that it can be carried out in an extremely short time by the use of high-speed electronic computing machines.

In this paper, only the evolution of 5-day mean circulation patterns at the 500 mb level will be discussed.

2. The vorticity equation for time-averaged flow

The vorticity equation for laminar (frictionless) flow and for constant pressure surfaces may be expressed in the following form (ELIASSEN, 1952):

$$\frac{\partial \zeta}{\partial t} = -\mathbf{v} \cdot \nabla \eta - \eta \nabla \cdot \mathbf{v} - w \frac{\partial \eta}{\partial p} \quad (1)$$

Here ζ is the component of relative vorticity normal to the pressure surface $\left(\frac{\partial v}{\partial x_p} - \frac{\partial u}{\partial y_p}\right)$; $\mathbf{v}$, the horizontal wind vector; η, the absolute vorticity $(\zeta + f)$; f the Coriolis force; and w the vertical "wind" normal to the pressure surface $\left(\frac{dp}{dt}\right)$. The symbol ∇ refers to quasi-horizontal gradients as measured on the pressure surfaces. The only approximation made in this equation for laminar flow is to neglect the term in $\frac{\partial w}{dx_p}$ and $\frac{\partial w}{\partial y_p}$ which represents the turning into the vertical of the absolute vorticity vector.

In applying this equation to turbulent or time-averaged flow, a procedure is used exactly similar to that employed in developing the well-known Reynolds stresses. First, each term in (1) is averaged for the desired time interval. The resulting arithmetic mean values may be represented symbolically by placing a "bar" over each term. Now let each of the meteorological quantities—for example the absolute vorticity—be represented by its arithmetic mean value $(\bar{\eta})$ for the period of averaging plus a departure from this mean (η'):

$$\eta = \bar{\eta} + \eta' \quad (2)$$

Equation (2) is now substituted in the averaged expression for (1), together with identical equations for the other quantities, and use is made of the fact that in all cases the sum of the departures from the mean $(\Sigma\eta')$ is zero. This follows from the definition of the arithmetic mean. Then, after some algebraic manipulation the following equation is obtained:

$$\frac{\partial \bar{\zeta}}{\partial t} = -\bar{\mathbf{v}} \cdot \nabla \bar{\eta} - \bar{\eta} \nabla \cdot \bar{\mathbf{v}} - \bar{w} \frac{\partial \bar{\eta}}{\partial p} - \nabla \cdot \overline{\eta' \mathbf{v}'} \quad (3)$$

The bar over each term indicates that it is measured directly on the mean flow pattern. Thus it is found that the vorticity equation for laminar flow may be applied unchanged to mean flow provided that a term is added (the last term in (3)) which represents local convergence of vorticity due to the average vorticity transport by the eddies which have been smoothed out of the mean flow. An additional approximation has been made, however. The term $\overline{w' \frac{\partial \eta'}{\partial p}}$ has been neglected, so that the last term in (3) represents the convergence of vorticity due to horizontal eddies only. It is assumed that in the mid-troposphere, where this equation is applied, the eddy transport of meteorological quantities takes place in quasi-horizontal surfaces. The third and fourth terms in (3) are determined indirectly by such physical processes as non-adiabatic heat sources and sinks, surface friction, flow over mountain barriers, and transformations between potential and kinetic energy.

Since (3) applies to any mean flow it is assumed to apply also to "normal" circulation patterns. These are defined as mean flow patterns for a very long number of years for one particular month or season. To distinguish between "normal" and "mean" quantities the subscript N will be used to replace the bar when referring to normals. The quantity $\frac{\partial \zeta_N}{\partial t}$ is usually quite small compared to the other terms (less than 5 % of $\mathbf{v}_N \cdot \nabla \eta_N$), and so for convenience it will be omitted here. Thus, with some re-arrangement the vorticity equation for normal flow becomes:

$$-\mathbf{v}_N \cdot \nabla \eta_N = \eta_N \nabla \cdot \mathbf{v}_N + w_N \frac{\partial \eta_N}{\partial p} + \\ + \nabla \cdot (\eta' \mathbf{v}')_N \quad (4)$$

It is assumed, as in applying (3) to synoptic charts, that the first two terms may be expressed by the height contour field of a constant pressure surface by making the following geostrophic approximations:

$$\bar{u} = -\frac{g}{f} \frac{\partial \bar{z}}{\partial y_p}; \; \bar{v} = \frac{g}{f} \frac{\partial \bar{z}}{\partial x_p} \quad (5)$$

$$\bar{\zeta} = \frac{g}{f} \nabla^2 \bar{z} = \frac{g}{f} \left(\frac{\partial^2 \bar{z}}{\partial x_p^2} + \frac{\partial^2 \bar{z}}{\partial y^{p\,2}}\right) \quad (6)$$

In order to keep the theoretical development within one section it is necessary to anticipate the results a little. It has been found in applying (3) to longer-interval mean charts that it is not possible to neglect the last three terms (as has been done on synoptic charts). However, to a first approximation these terms may be replaced by their normal equivalents, given by (4). Making this substitution, and replacing the relative vorticity in the first term of (3) by its geostrophic value (6) one obtains a very simple equation for the local height tendency:

$$\frac{g}{f}\nabla^2\frac{\partial \bar{z}}{\partial t} = (-\mathbf{v}\cdot\nabla\bar{\eta}) - (-\mathbf{v}_N\cdot\nabla\eta_N) \quad (7)$$

The last term may be evaluated once and for all from existing normal flow patterns, and so in practice this equation is not much more difficult to apply than the simple "barotropic tendency equation" obtained by dropping this last term.

The principal assumption involved in the derivation of (7) implies that the sum of the last three terms of (3) does not depart far from its normal value, while the vorticity advection (first term on the right) may vary within wide limits. Much argument, both theoretical and practical, may be brought to bear against this hypothesis. For example STARR and his co-workers (LORENZ, 1952) have shown fairly conclusively that the eddy transport of momentum, equivalent to the last term of (3), is not only important for maintaining the normal zonal flow in the Northern Hemisphere but in determining its day-to-day changes as well. Nevertheless evidence will be presented in the present paper that such an hypothesis as that given above must be made if the simple barotropic formula is to be applied at all to longer-interval mean flow patterns, and also if it is to be applied to synoptic charts on a hemisphere scale.

3. Tests of the barotropic tendency equation

Equation (7) with the last term omitted is called the "barotropic tendency equation". As has already been mentioned, it has been applied with fair success to synoptic weather charts, which represent mean flow patterns for time intervals of several hours. The first attempt to apply it to longer-interval mean patterns was made by AUBERT (1951), who obtained some encouraging results in computing tendencies on a single 5-day mean 700 mb chart. The next test was made by BERSON (1952) using 5-day mean 500 mb patterns for the summer season. To a certain extent the research reported on here represents an addition to that of Berson, as I had the benefit of working very closely with him during the latter portion of his study.

In the present study a preliminary test of the barotropic formula was made on three successive 5-day mean 500 mb charts for November 1951 and for the eastern Atlantic and Europe (Area 2 of fig. 8). These were chosen for the same area and dates used in a previous investigation employing synoptic charts (STAFF MEMBERS, UNIVERSITY OF STOCKHOLM, 1952). Also, the same procedure was employed. The results indicate that the average correlation coefficient between computed and observed tendencies (0.6) is about the same despite the fact that the "tendency interval" (interval of 2 or 3 days between successive mean charts) is 4 to 6 times that for the synoptic tests (12 hours). This indicates at once that further tests on mean patterns are justified. However, it was also found that the computed mean tendencies were about 40 % too large, as compared to only 10 % for the synoptic tendencies. The explanation for this discrepancy probably lies in the three last terms of (3) which have been omitted from the calculations. One possible explanation is that the sum of these three terms remains constant (perhaps close to its normal value) while the vorticity advection term is known to decrease rapidly in magnitude with the length of the averaging interval. Considerations of this kind have led to the development of (7).

Using these same cases, tendencies were also estimated from (7) using the normal vorticity advection term as well. While this showed a distinct improvement over the barotropic formula, especially with regard to the magnitude of the tendencies, the test was considered unsatisfactory mainly because of the smallness of the chosen area. It was decided that a further test should be made using a winter situation for the entire Northern Hemisphere.

4. Procedure for evaluating the barotropic equation with normal vorticity advection

In computing mean tendencies for the hemispheric case from the barotropic equation the procedure used is the same as that described by BOLIN and CHARNEY (1951) with only minor variations. Briefly, this consists of analysing the mean contours on a polar-stereographic projection. Horizontal gradients in contour height used in estimating wind speeds and relative vorticities from (5) and (6), as well as in obtaining the mean vorticity advection, are estimated by the method of finite differences using a rectangular grid having a spacing whose absolute magnitude is 620 km at 50° N. In contrast to the study of Bolin and Charney the local vorticity and vorticity advection is obtained for every point in the grid instead of for every other point. The tendencies are then computed for each point by the method of "relaxation". It should be noted in the example which follows that the tendencies so obtained usually have the opposite sign to that of the vorticity advection. This is due to the fact that the latter determines the curvature of the tendency profile.

In making the computations some assumption must be made regarding the tendencies for the gridpoints at the boundaries of the chosen region. In this study it has been assumed that these tendencies are zero.

The verification of the tendencies is also slightly different from that used in the synoptic study. They are compared to an observed "centered tendency" which is defined as the difference in contour height between mean charts one day later than and one day previous to that on which the computations are made.

In computing mean tendencies from (7) it makes no difference theoretically whether the mean and normal vorticity advection terms are first added and then relaxed, or whether they are relaxed separately and the results then added. In practice it would be preferable to adopt the former method since then only one relaxation is necessary. However, in the present study the second procedure was chosen, since it was desirable to investigate separately the importance of the two terms. The results of relaxing the normal vorticity advection yields a fictitious normal tendency which in general has no direct relation to the true normal tendency $\left(\frac{\partial z_N}{\partial t}\right)$. If it may be assumed that (7) represents an improvement of the simple barotropic formula, then it is clear that the error in the latter should be related directly to the relaxed normal advection. For this reason this will be referred to as the "normal correction term".

5. Test of tendency equation for a winter situation and for the Northern Hemisphere

The case chosen was the 5-day mean 500 mb circulation pattern for Feb. 7—11, 1949, shown as the thin solid lines in fig. 1. These lines are drawn for every 300 feet and are labeled in 100's of feet. This mean pattern represents the average of 5 0300 Z synoptic charts obtained from the printed historical map series (U. S. WEATHER BUREAU, 1949). The synoptic situation was one of rapid breakdown of a large blocking high over northern Europe with a return to strong zonal westerlies over Britain and Scandinavia. Equally rapid changes, but in the opposite direction, were taking place over the Pacific and North America, where a very strong belt of westerlies was giving way to an extensive ridge in the Gulf of Alaska and a deep trough over the western United States. A deep low over western Asia was filling slowly. Thus this situation was quite abnormal, especially over the eastern Pacific and North America, and should provide a severe test of (7). The observed 2-day height changes centered on the mean chart are shown as the dashed lines in fig. 4. These are drawn for every 5 units and expressed in 10's of feet per 12 hours.

Before discussing the synoptic situation further it is desirable to consider first the normal chart for February together with the derived normal correction term. The normal contours for February, obtained from the latest available normal chart series (U. S. WEATHER BUREAU, 1952) are shown in fig. 2 as the thin solid lines. The reader is referred to the above mentioned publication for a complete discussion of these normal contours, but it must be mentioned here that they differ in some important respects from previously

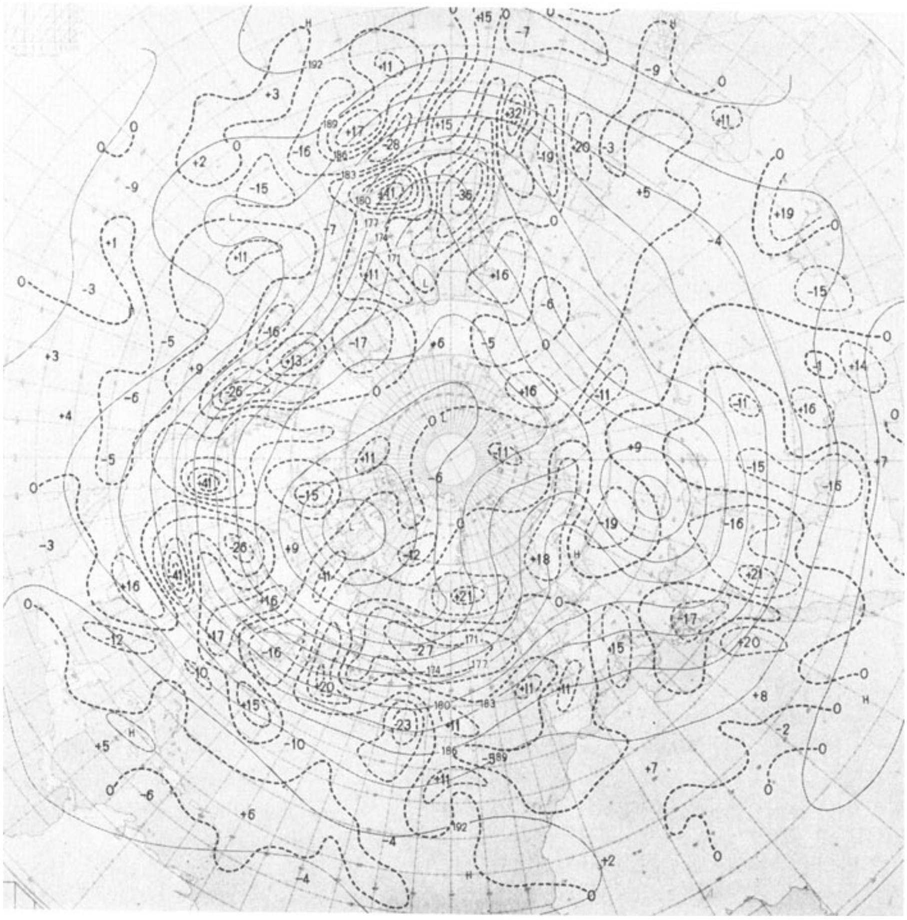

Fig. 1. Five-day mean 500 mb chart for Feb. 7—11, 1949. Contours (thin solid lines) labeled in 100's of feet and drawn for every 300 feet. Vorticity advection (dashed lines) in units which on relaxation give tendencies in 10's of feet per 12 hours, and drawn for every 10 units.

published monthly normals (JOINT METEOROLOGICAL COMMITTEE, 1944; SCHERHAG, 1948) particularly in Canada and northern Asia, where upper air data have increased significantly in recent years. Because of the limitations of the data with which these charts were constructed it is safe to say that these contours are most accurate over North America, the North Atlantic and Europe; of less value over the Pacific; and least reliable in the Asiatic region. The same remarks, with only slight modification, may be said to apply to the individual 5-day mean chart shown in fig. 1.

The normal vorticity advection term is shown as the dashed lines in fig. 2. The values shown on the chart, with isolines drawn for every 10 units, are in such units that when relaxed they yield tendencies in 10's of feet per 12 hours. It is natural to attempt to interpret this field in terms of the physical processes leading to the establishment of the normal patterns. However, the task is rendered extremely difficult for two principal reasons: there is no way to tell from (4) alone which of the three terms on the right hand side is responsible for the normal vorticity advection; and more important, the various physical processes (such as non-adiabatic heating and cooling and flow over mountains) may each contribute to all three of the terms. To illustrate the diversified interpretations that have been placed on the normal

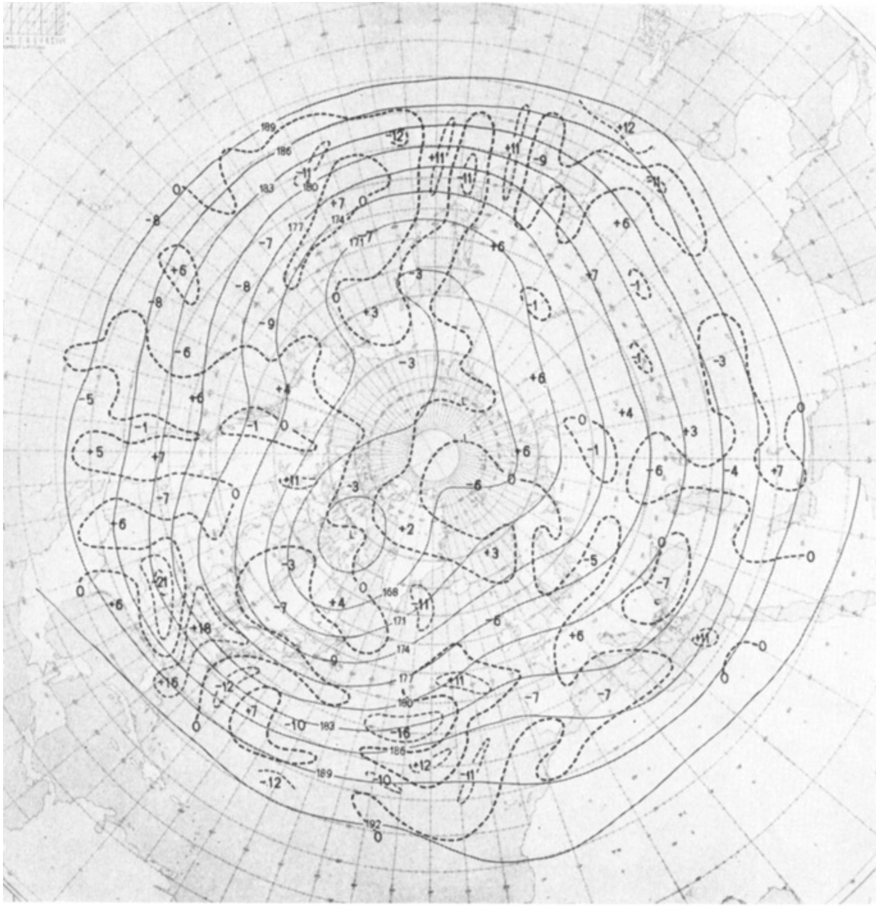

Fig. 2. Normal chart for February. For explanation of units, see fig. 1.

vorticity advection term it may be mentioned that WIPPERMANN (1952) has considered it to be largely the result of surface friction, non-adiabatic heat sources and sinks, or forced flow over mountain barriers. BERSON (1952) has presented evidence indicating that the frictional transfer term is considerably larger than the others for long-term averaged flow. Finally, NAMIAS and CLAPP (1946) have interpreted the normal advection at 700 mbs in terms of fields of divergence which must accompany stationary waves in a baroclinic current.

In spite of these difficulties the question is of great importance since its answer may determine to a large extent the emphasis which should be placed on attempts to approximate a true 3-dimensional atmospheric model for numerical computations. To date such attempts have been confined to introducing some of the aspects of transformation between potential and kinetic energy (PHILLIPS, 1951). For this reason a few remarks will be made regarding the broad features of fig. 2.

There seems little doubt that the meridional bands of positive and negative vorticity advection in western North America and the eastern Pacific are due to forced flow over the mountain ranges. For example, the extensive area of positive advection just to the west of the continental divide in the United States is clearly due to lifting, which produces divergence and upward motion through a thick layer in the lower troposphere. Both of the first two terms to the right of (4) make positive contributions in this case. This opinion is strengthened when it is noted that the maximum geostrophic wind speed normal to the

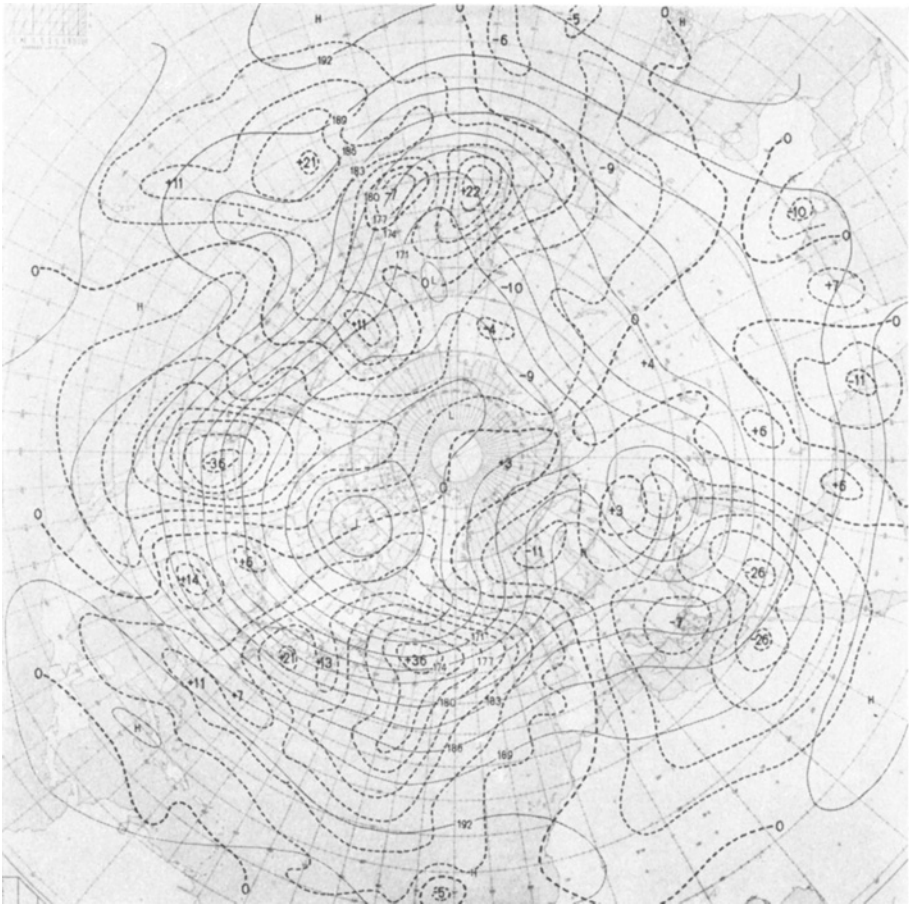

Fig. 3. Five-day mean chart for Feb. 7—11, 1949. Contours (thin solid lines) as in fig. 1. Tendencies from barotropic formula without normal vorticity advection (dashed lines) in 10's of feet per 12 hours, drawn for every 5 units.

mountain crests is located at about 40° N, just to the south of the maximum vorticity advection (+ 7 units in fig. 2).

The prevailingly negative values over the oceans and positive values over the relatively flat continental areas are most likely due to heating and cooling from below. This is supported by the fact that the largest values are located near the principal heat and cold sources. Thus, maximum negative values are located east of Greenland, in the central portions of both oceans, and over the Mediterranean, Black, and Caspian Seas; while maximum positive values are located near the known cold sources in northern Canada and Siberia. However, the exact way in which these large-scale heat sources and sinks contribute to each of the three last terms in (4) is not yet understood.

Many of the smaller-scale features of fig. 2 are no doubt due to local uncertainties in the normal charts, although the reader's attention is called to the geographical similarity of certain features over the two oceans.

Figure 6 shows the normal correction term, or relaxed values of the normal vorticity advection. This illustrates the very extensive smoothing which results from the relaxation procedure. Most of the minor details of fig. 2 have disappeared leaving only the large-scale features which were discussed previously. The magnitude of the normal correction term is quite large. Comparison with fig. 4 shows that it is of the same order as the observed

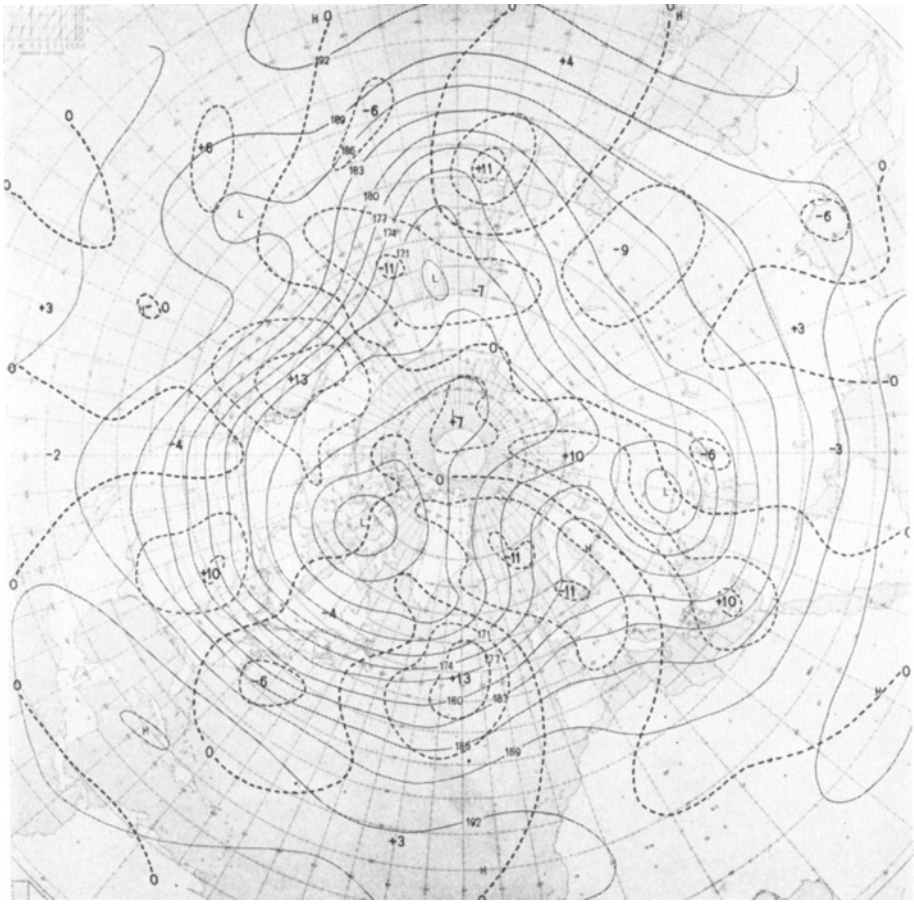

Fig. 4. Five-day mean chart for Feb. 7—11, 1949. Contours (thin solid lines) and observed two-day centered height change (dashed lines). For explanation of units see fig. 3.

tendencies. It is approximately one fourth the size of 12-hour height changes observed on synoptic charts. The reason for the great smoothing and large tendencies is due of course to the extensive fields of predominantly positive or negative vorticity advection over the continents and oceans. Since in accordance with (6) the vorticity advection gives the curvature of the tendency rather than the tendency itself, it is clear that uniform fields of small values of the advective term will produce very large tendencies in the centers of these fields. Such extensive fields also lead to great difficulties in completing the relaxation procedure, as anyone familiar with this process will know. In the present case time limitations prevented complete relaxation of fig. 2, but this cannot affect the sign and general order of magnitude of the tendencies.

The advective fields obtained from the 5-day mean hemisphere chart of Feb. 7—11, 1949 are shown in fig. 1. In its large-scale features and even in some of the minor details the pattern bears much resemblance to the normal fields of fig. 2. The values are mainly negative over ocean areas and positive over land areas. However, the positive values due to lifting over the Rockies are much larger than in fig. 2, due of course to the much greater wind speeds. It is of interest to note that the largest value in this area (+ 41) is located a little to the north of the maximum wind normal to the mountain crests, which is again located near 40° N. This feature is in

Fig. 5. Error in tendencies computed from barotropic formula without normal vorticity advection, defined as computed minus observed tendencies. See fig. 3 for explanation of units.

general agreement with the distribution of geostrophic winds normal to the mountain ranges. These increase rapidly to their peak at 40° N but then appear to level off into a broad maximum with but slowly decreasing speeds until they decrease rapidly in magnitude north of 47° N. While the maximum 5-day mean winds in this area are about twice their normal value the mean vorticity advection is almost six times as large as normal.

It is not surprising from what has been said that the computed barotropic tendencies (fig. 3) obtained by relaxing the field of fig. 1 should bear a marked resemblance in many areas to the normal correction term of fig. 6. However, over the Rocky Mountains the mean tendencies are 7 times as large as their normal counterparts, and the accompanying large field of negative tendencies seems to spread out over a large area on either side of the ranges.

Comparing these computed tendencies with those observed (fig. 4) it will be seen that over Europe and the Atlantic the general pattern is in fair agreement. However, as found in the preliminary test, the values are in a general much too large, in some cases exceeding the observed by several hundred per cent. Over the relatively flat areas in the eastern United States and western Atlantic the agreement is not very good, although the rises in the central United States are estimated fairly well. The worst error in this region is the failure to predict the broad area of falls

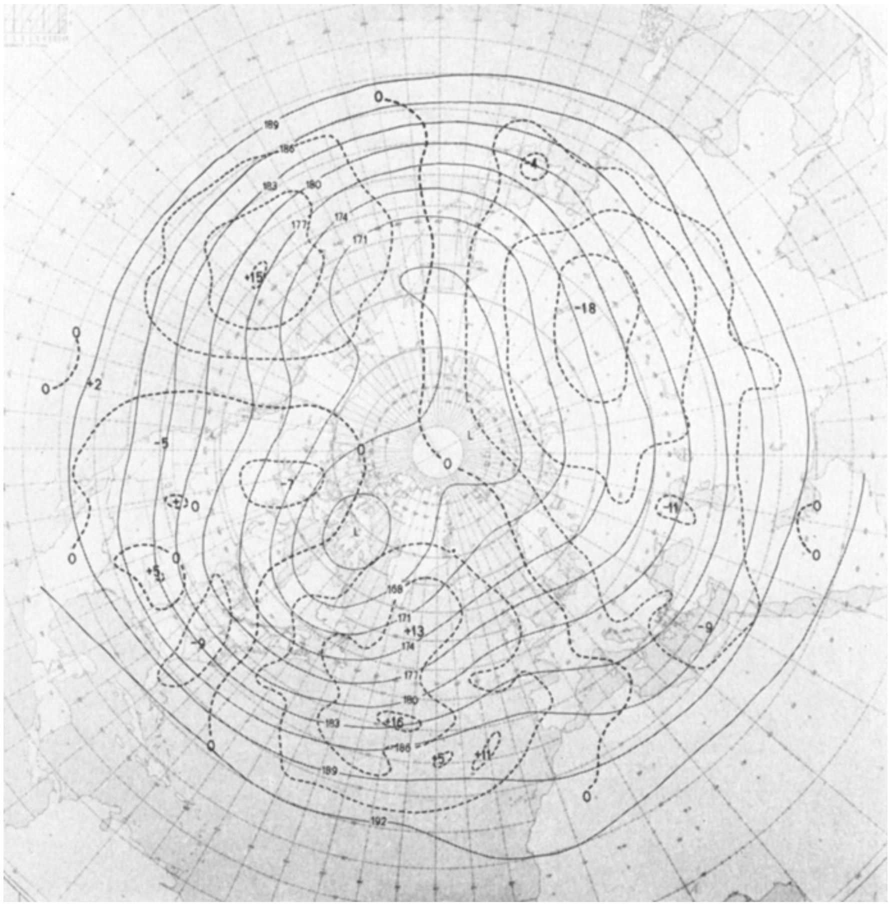

Fig. 6. February normal correction term, defined as relaxation of normal vorticity advection. For explanation of units, see fig. 3.

extending from northeastern Canada to the Antilles, with extremes east of Hatteras and southeast of Hudson Bay. However, near these two areas there are weak relative minima in the field of computed positive tendencies.

In the western United States and extreme eastern Pacific the general pattern of computed tendencies is fair although the negative values are all several times too large. It is interesting to note that an observed fall area is located near the minimum in the computed field. The marked rises which occurred in the Gulf of Alaska were not computed, in spite of the fact that fig. 1 shows a rather broad area of negative vorticity advection in this region. This result is due to the profound influence of the mountain effect, which upon relaxation "spread" to the east and west and produced large errors in adjacent regions.

Over the Pacific the agreement is in general unsatisfactory although the eastward motion of the relatively sharp trough east of northern Asia is qualitatively anticipated. The most conspicuous error in this area is the failure of the "cut-off" low northwest of the Hawaiian Islands to move north-eastward as expected. This may be due in part to poor analysis since during the period in question there were only three upper-air stations over this broad area of the Pacific, and these were to the south of the low center.

Over Asia the agreement is poor, although there is some indication of the slow filling of the deep vortex in western Russia.

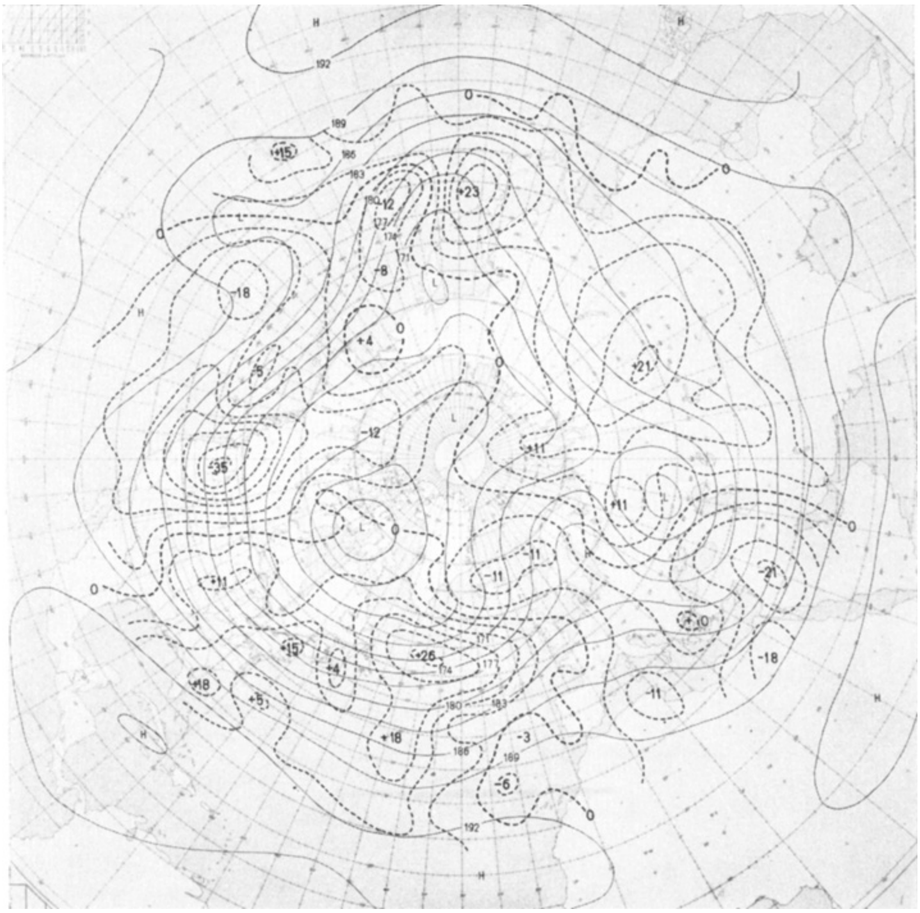

Fig. 7. Tendencies from barotropic formula with "normal correction term" added. For explanation of units, see fig. 3.

As expected from (7) the pattern of the errors in the computed tendencies (fig. 5) is in fair agreement with the normal correction term (fig. 6) over the areas of plentiful data from eastern Europe to central North America. The magnitude of the errors is also in fair agreement although somewhat too large. Over the Rocky Mountains where data is also plentiful, the agreement in pattern is again fair, although the magnitudes are poor. Some agreement in general pattern can be noted in the Pacific area while over Asia, where the data is least reliable, the results are poor.

As a final test of the method, the complete equation (7) was evaluated (fig. 7) by subtracting the field of fig. 6 from that of fig. 3. Figure 7 should be compared to the observed tendencies (fig. 4). The marked improvement over Europe and the Atlantic can be noted, although the order of magnitude of the computed tendencies, while greatly improved, is still somewhat too large. Over North America and the western Atlantic there is again some improvement, especially in the magnitudes, although the negative tendencies in eastern North America are still lacking. However, there is some sharpening of the relative minima close to those observed. The errors in the Rocky mountain region are still quite large, and no marked improvement can be noted over the Pacific and Asia.

These results have been summarized statistically in Table 1. The Northern Hemisphere

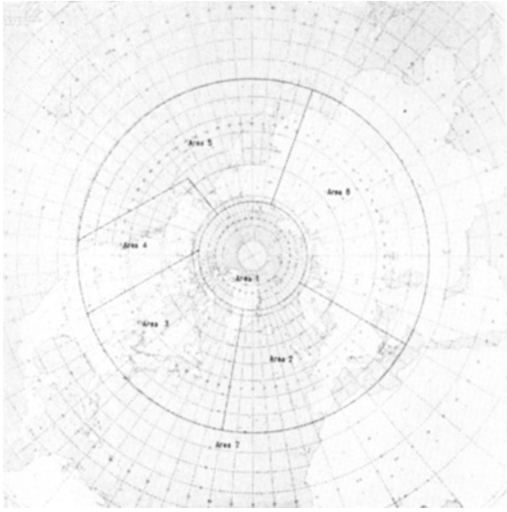

Fig. 8. Areas of Northern Hemisphere used in computing statistics of Table 1.

was divided into 7 areas, shown in fig. 8. Statistics were computed for only the middle latitude regions (Areas 2 to 6). The boundaries between the areas were chosen more or less arbitrarily although the northern and southern boundaries as well as those defining the Asiatic region were chosen on the basis of the distribution of available data. The statistics shown in the table are defined as follows: X_1 in the first and second columns under each area refers to the observed centered tendency, while in the last column it represents the normal correction term; X_2 in the first to third columns is respectively the computed barotropic tendency, the "corrected" tendency from (7) and the error in the barotropic tendency; σ_{x1} and σ_{x2} are the corresponding standard deviations; and $\overline{X}_1$ and $\overline{X}_2$, the arithmetic means. The correlation coefficient is an indication of how well the observed tendencies may be estimated from those computed, while the standard deviations are a measure of the gross error or "bias" of the computed values.

In general the numbers in Table 1 confirm the qualitative conclusions reached in previous paragraphs. In areas 2 and 3 the results agree with what one would expect from (7) while in areas 5 and 6 they do not. This result is certainly in agreement with what one would expect from the distribution of data over the globe. It is interesting to note that although fair correlations are obtained relating errors to normal correction term in areas 2 to 4 there is no corresponding improvement in correlation for the corrected tendencies. However, in areas 2 and 3 there is marked improvement in the order of magnitude of computed tendencies.

The results in eastern North America are rather disappointing in view of the fact that this is a relatively flat region with plentiful data. However, in all probability this is an exceptionally poor case because tests on synoptic charts over the same area (Bolin and Charney, 1951) average somewhat better than those over Europe (Staff Members, University of Stockholm, 1952), while all tests on mean charts made so far indicate about the same degree of skill as corresponding daily computations for the latter area.

6. Short-cut methods of computation

Solution of (7) by the method of relaxation is of course designed for use with high speed electronic computers. Because of difficulties such as the thousands of arithmetical operations (each of which must be carefully checked), and the slowness of the relaxation procedure when dealing with large areas of uniform vorticity advection, the length of time necessary to carry out a hemispheric analysis by hand, is prohibitive except for very limited research studies. It is out of the question for use in practical forecasting. For this reason, and because electronic machines will probably remain generally unavailable for several years, it is desirable to develop short-cut methods for solving (7). Fortunately several such methods have been proposed in recent months. One of these, developed in connection with the present study, will now be briefly outlined.

Let us designate the arithmetic mean value of any meteorological quantity, say absolute vorticity ($\bar{\eta}$) by its normal value (η_N) plus a departure from normal (η°):

$$\bar{\eta} = \eta_N + \eta^\circ \tag{8}$$

Substitution of (8) in the first term on the right of (7) yields four new terms, one of which cancels with the normal advective term in (7). The following equation results:

$$\frac{g}{f}\nabla^2\frac{\partial \bar{z}}{\partial t} = -\mathbf{v}^\circ\cdot\nabla\eta^\circ - \mathbf{v}_N\cdot\nabla\eta^\circ - \mathbf{v}^\circ\cdot\nabla\eta_N \quad (9)$$

The first term on the right of (9) represents the advection of departure from normal (d. n.) vortex elements by the d.n. flow field. The practical solution of this term has been discussed in a previous paper (CLAPP, 1952) where it was assumed that one may represent the d.n. flow pattern as a collection of simple symmetrical Rankine vortices. A glance at the 5-day mean d.n. contour patterns shown in the above paper will suggest that this assumption seems quite reasonable. It was shown that based on this assumption the displacement of one individual vortex center can be obtained in 15 to 20 minutes. This represents the local solution of the first term on the right of (9). The second term represents the advection of the d.n. vortex centers by the normal flow field. This term is solved locally by simply computing the normal wind speed and direction at the center of a given d.n. vortex. Since the normal winds are zonal in character this will give displacements mainly towards the east. However, as pointed out (CLAPP, 1952) these displacements will be about two times as large as those observed as well as those computed from the first term on the right of (9). The reason for this becomes clear on considering the last term in (9) which represents the advection of the normal vorti-

Table 1. Relation between 5-day mean computed and observed tendencies, and between estimated and observed errors in tendency. Units: 10's of feet per 12 hours. Number of cases indicated by n. X_1 = observed tendency in columns 1 and 2 under each area, and "normal correction term" in column 3. X_2 = respectively computed barotropic tendency, "corrected" tendency from (7) and error in barotropic tendency.

Area 2: Eastern Atlantic and Europe.
$n = 51$

	Computed vs. observed tendency	Corrected vs. observed tendency	Error vs. correction term
$r_{x_1x_2}$	0.67	0.73	0.86
σ_{x_2}	13	9	10
σ_{x_1}	6	6	6
$\sigma_{x_2}/\sigma_{x_1}$	2.2	1.5	1.7
$\bar{X}_2$	4	0	4
$\bar{X}_1$	0	0	4

Area 3: Eastern North America and western Atlantic.
$n = 44$

	Computed vs. observed tendency	Corrected vs. observed tendency	Error vs. correction term
$r_{x_1x_2}$	— 0.20	— 0.12	0.84
σ_{x_2}	11	6	12
σ_{x_1}	3	3	6
$\sigma_{x_2}/\sigma_{x_1}$	3.1	1.8	2.0
$\bar{X}_2$	10	8	10
$\bar{X}_1$	1	1	3

Area 4: Mountain region.
$n = 30$

$r_{x_2x_1}$	0.61	0.58	0.68
σ_{x_2}	12	9	10
σ_{x_1}	4	4	3
$\sigma_{x_2}/\sigma_{x_1}$	2.8	2.3	2.9
$\bar{X}_2$	— 13	— 12	— 18
$\bar{X}_1$	5	5	— 1

Area 5: Pacific.
$n = 57$

$r_{x_1x_2}$	0.12	0.14	0.12
σ_{x_2}	8	9	8
σ_{x_1}	5	5	6
$\sigma_{x_2}/\sigma_{x_1}$	1.6	2.0	1.4
$\bar{X}_2$	2	— 2	3
$\bar{X}_1$	— 1	— 1	4

Area 6: Asia.
$n = 67$

$r_{x_1x_2}$	— 0.13	— 0.33	— 0.46
σ_{x_2}	5	7	7
σ_{x_1}	4	4	3
$\sigma_{x_2}/\sigma_{x_1}$	1.3	1.7	2.2
$\bar{X}_2$	— 3	8	— 2
$\bar{X}_1$	— 1	— 1	— 11

city field by the d.n. flow pattern. If the assumption is again made that the d.n. vortex centers are simple Rankine vortices, and if it may be assumed that $\eta_N \approx f$, then a very simple expression can be obtained for the displacement of an individual vortex due to the last term in (9). This expression will contain β, the north-south gradient of f, as well as the dimensions of the system; and will produce a displacement towards the west. It is equivalent to the dynamic term $\left(\frac{-\beta L^2}{4\pi^2}\right)$ in Rossby's well-known wave equation (ROSSBY, 1939), while the second term to the right of (9) represents the advective term (U). It is known that for large-scale systems these two terms nearly balance one another.

In the manner outlined above a forecast may be obtained for the displacement of a single d.n. vortex center in approximately 25 minutes. Since these centers often cover a very large area of the globe, such a forecast if successful would be of material value to the forecaster.

7. Conclusions

The basic assumption in the derivation of (7) and (9) is that the main role of terrestrial heat sources and sinks and the transformations between potential and kinetic energy is to maintain the normal circulation against the dissipative effects of friction. Otherwise the motion and development of the large-scale atmospheric systems is determined largely by the internal redistribution of vorticity within the atmosphere itself. It is clear that if this hypothesis is even partly true then the way is open to forecasting much longer weather changes, such as secular trends. Indeed, upon making similar assumptions STEWART (1943) showed theoretically that large-scale vortex systems having the general character of the atmospheric centers of action could have very long periods of evolution, perhaps several hundred years.

It is not difficult to extend this hypothesis to include variations from normal of the physical processes mentioned at the beginning of the last paragraph provided it can be assumed that these processes are a function of the circulation itself. To give a simple example of this, it is possible to account entirely for the "mountain effect" since this is a function of such factors as the wind velocity and direction relative to mountain crests.

It has been mentioned that the large areas of small but uniform values of the normal vorticity advection, which seem to act as a "background" to the much larger but less regular values of the individual advection, have a very important effect on the computed tendencies when these are carried out on a hemisphere-wide scale. Prior tests of the barotropic formula, especially with respect to synoptic circulation patterns, have been made for relatively small areas. If this formula is to be applied to synoptic charts on a hemispheric scale, an equation similar to (7) must be used.

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