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Implementing convection into Lorenz's global cycle.

Part I. Gridscale averaging of the energy equations

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ABSTRACT

Sub-gridscale processes take place throughout the global atmosphere. Yet they have been neglected in traditional estimates of the global energy cycle on the ground that they can be treated as molecular heat fluxes. This view may cause quantitative underestimates of the efficiency of the global circulation of the atmosphere. In Part I of this two-part study we revisit the classical theory, beginning with the local energy equations. Similar to Lorenz we introduce a barotropic reference pressure p_r and define a generalized field equation for the integrand of available potential energy, without reference to hydrostasy. The emerging energy quantity is new in that it comprises not only the classical correlation between efficiency factor and enthalpy but also an additional potential that depends upon p_r. We then perform mass-averaging over the scale of contemporaneous global models (40–400 km) and come up with averaged field energy equations, valid at the gridscale. Additional global and time-averaging of these removes all divergences and tendencies and yields two equations for the global energy reservoirs. The available potential energy reservoir is fed by gridscale plus sub-gridscale generation. The kinetic energy reservoir is tapped by gridscale plus sub-gridscale dissipation. Exchange between the reservoirs is carried by both gridscale and sub-gridscale conversion terms (C^{grid}, C^{sub}). Generation, conversion and dissipation fluxes are complete, as compared to the approximate quantities in the traditional formulation of the energy cycle. This approach allows to fully exploit Lorenz's original concept. The gridscale equations derived will be the basis for evaluating numerically the classical Lorenz terms plus a couple of new global conversion fluxes, notably C^{sub}, to be presented in Part II of this study.

1. Introduction

This study is concerned with the relative rôle of convection in the global energy cycle. Convection is a sub-gridscale process. Sub-gridscale fluxes can neither be routinely observed (they must be indirectly diagnosed) nor explicitly modelled (they must be parameterized). In the classical paper of Lorenz (1955) the sub-gridscale fluxes have been treated as if they were on the molecular scale. Lorenz (1967) discussed this point explicitly and decided as follows: "In the present treatment, where ... no distinction is made between turbulent and molecular friction, it seems most logical to treat turbulent kinetic energy as neither KE nor a form by itself, but as a portion of the IE, thus effectively grouping the kinetic energy of small-scale motions with the kinetic energy of molecular motions." This viewpoint has been common to all subsequent data evaluations (Oort, 1964; Newell et al., 1970; Arpe et al., 1986; Peixóto and Oort, 1992).

The basic problem with grouping convection and turbulence as molecular processes may be demonstrated by the following consideration. The

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global mean kinetic energy for the hydrostatic atmosphere is (Lorenz, 1955; Lorenz, 1967):

$$KE \equiv \frac{1}{2}\{V_2^2\}. \tag{1.1}$$

V_2 is the horizontal wind vector, the curly brackets denote the global mass average. The time rate of change of KE can be written as:

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = \underbrace{-\{\alpha\omega\}}_{C_L} \underbrace{-\{V_2 \cdot F_2\}}_{D_L}. \tag{1.2}$$

α is specific volume, $\omega \equiv \mathrm{d}p/\mathrm{d}t$, F_2 is the horizontal frictional force, the index L refers to Lorenz. Eq. (1.2) is derived from the horizontal hydrostatic equations of motion. All quantities entering the nonlinear expressions for KE and for the conversion term C_L are considered as gridscale. This approach appeared to be consistent since the turbulent component of KE in the planetary boundary layer (PBL, order of magnitude 1 m^2/s^2, see, e.g., Yamada and Mellor, 1975) is negligible as compared to the mean component (100 m^2/s^2, see, e.g., Peixóto and Oort, 1992). Further, since the time change of KE is almost zero for climate conditions, Lorenz argued that both C_L and D_L should be equal. Friction was not specified so that the dissipation rate D_L was not explicitly calculated but rather determined as residual. Thus estimates of C_L yielded for D_L the present textbook value 2.0 W/m^2 (Peixóto and Oort, 1992).

On the other hand, independent estimates of the global dissipation rate (e.g., Table 5 in Newell et al., 1970, based largely upon PBL measurements, thus including sub-gridscale contributions, mean value of all estimates 6.0 W/m^2) yielded 3× that figure.

It appears that nobody in the past has tried to resolve this striking difference. The difference did not even seem striking. The reason may have been that the various estimates reported by Newell et al. (1970) had not only a large scatter but were not representative either; for example, most authors used exclusively extratropical data. In particular, nobody tried to explain the discrepancy by noting that Lorenz's estimates did not comprise any sub-gridscale contributions while Newell's estimates did. This is astonishing because eq. (1.2) is limited to the 2D-kinetic energy and to hydrostatic conditions so that improvements of the original theory of Lorenz were often discussed (Van Mieghem, 1956; Dutton and Johnson, 1967; Kucharski, 1997).

One further argument in favour of neglecting possible sub-gridscale contributions to the global energy cycle follows from Fig. 1. Plotted in the figure is the integrand $-\overline{\overline{\alpha''\omega''}}$ of the sub-gridscale conversion rate into kinetic energy (referred to as C^{sub} in the following) together with its gridscale equivalent $-\overline{\overline{\alpha}}\,\overline{\overline{\omega}}$ (corresponding conversion rate C^{grid}, identical to C_L); the double bar operator representing the average over the gridscale and the corresponding deviation will be defined below. Fig. 1 demonstrates that the sub-gridscale flux is more than an order of magnitude smaller than the gridscale flux. Thus C^{sub} does not seem able to influence the global energy cycle. The moderate maximum of $-\overline{\overline{\alpha''\omega''}}$ around 500 hPa under disturbed conditions appears to be of little significance. After all, Fig. 1a is an example for relatively extreme extratropical convection which is a rare event and thus not too representative; this makes it even less probable that such a small quantity can have much influence upon the global energy budget. A practical argument against incorporating it was that data accuracy was not sufficient to estimate this quantity with the necessary precision. In summary, researchers seemed to have, up to now, good reasons to neglect $-\overline{\overline{\alpha''\omega''}}$ in the global energy cycle (and other corresponding sub-gridscale quantities as well).

However, the conclusion just drawn from Fig. 1 may be premature. It has been noted by Lorenz (1967), and is implied in Fig. 1, that the integrand $-\overline{\overline{\alpha}}\,\overline{\overline{\omega}}$ shows strong fluctuations both in time and space; consequently, the global value C^{grid} is the residual of fluxes with opposite sign. On the other hand, the integrand $-\overline{\overline{\alpha''\omega''}}$, while being individually much smaller than $-\overline{\overline{\alpha}}\,\overline{\overline{\omega}}$, tends to be of equal sign throughout most of the atmosphere, in all latitudes. Thus its global equivalent C^{sub} has a priori the potential to be similarly significant, despite the errors remaining.

It is the purpose of our study to provide evidence that the sub-gridscale processes may indeed contribute significantly to the global energy cycle. This has become possible by the availability of high accuracy gridscale data which allow to indirectly estimate sub-gridscale fluxes (to be discussed in Haimberger and Hantel, 2000, from here on referred to as Part II) but also since the theoretical

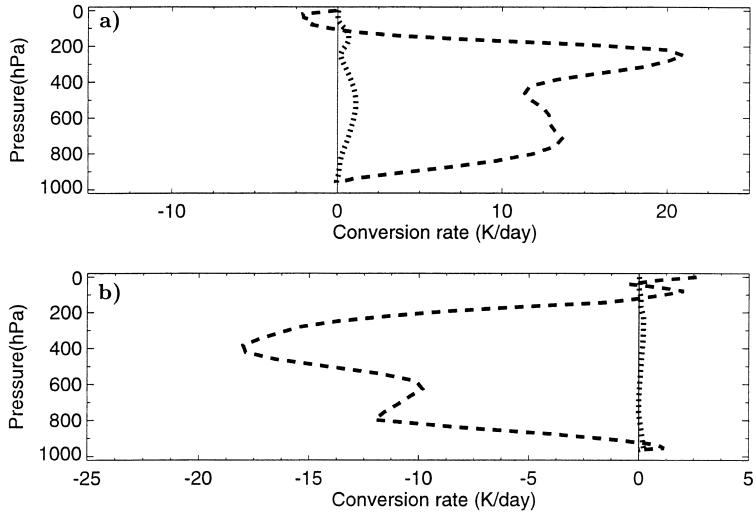

Fig. 1. Vertical profiles of integrands of gridscale conversion rate $-\overline{\overline{\alpha}}\,\overline{\overline{\omega}}$ (long dashed) and of sub-gridscale conversion rate $-\overline{\alpha''\omega''}$ (dotted) (K/day). Profiles valid for atmospheric column over Europe with horizontal area (500 km)^2, 12 h-average, centered over (a) southern Germany (21 July 1996, 00 UTC, situation disturbed, typical for significant convection) and (b) northern Germany (21 July 1996, 12 UTC, undisturbed, typical for PBL convection).

framework of Lorenz's energy cycle may be generalized to include sub-gridscale processes (the present Part I).

In order to have a unified framework we shall derive in Part I the global energy equations, free of 2D- and hydrostatic approximations, in three steps. The first step (Section 2) will be to develop a set of two local energy equations of the kind originally used by Lorenz but with a straightforward generalization. The second step (Section 3) will be to average this set of equations over the space-time interval represented by the grid size of a typical global forecast model like the one run at the European Centre for Medium-Range Weather Forecasts (ECMWF). This averaging will be done not over the volume but over the mass; the concept has been introduced by Hesselberg (1926) and has been applied by Van Mieghem (1973) to the same problem considered here. In the third step (Section 4) we shall arrive at global interaction equations that are but slightly more elaborate than Lorenz's equations and yet contain, without approximation, the sub-gridscale fluxes. This will be the basis for a re-evaluation of the global energy cycle to be executed in Part II.

2. The hydrodynamic energy equations

The classical equations for thermodynamic and mechanical energy of a dry atmosphere read in local form (Batchelor, 1967; Glansdorff and Prigogine, 1971):

$$\frac{\mathrm{d}c_v T}{\mathrm{d}t} = \underbrace{T\frac{\mathrm{d}s}{\mathrm{d}t}}_{\equiv Q} - p\frac{\mathrm{d}\alpha}{\mathrm{d}t}, \tag{2.1}$$

$$\frac{\mathrm{d}(k+\Phi)}{\mathrm{d}t} + \alpha\frac{\partial}{\partial x_j}(pv_j + \pi_{ij}v_i) = p\frac{\mathrm{d}\alpha}{\mathrm{d}t} + \underbrace{\alpha\pi_{ij}\frac{\partial v_i}{\partial x_j}}_{\equiv -\varepsilon}. \tag{2.2}$$

We use standard cartesian tensor notation (p = pressure, T = temperature, s = specific entropy, α = specific volume, k = 3D-kinetic energy, Φ = geopotential); π_{ij} = tensor of molecular momentum flux (opposite in sign to the Stokes' stress tensor, see Herbert, 1975). ε is local dissipation, Q (following Lorenz, 1967) is net heating.

It is often useful to replace the equation for internal energy $c_v T$ by the equation for enthalpy

$c_p T$. This transforms (2.1), (2.2) into:

$$\frac{dc_p T}{dt} = Q + \alpha\omega, \tag{2.3}$$

$$\frac{d(k + \Phi - p\alpha)}{dt} + \alpha \frac{\partial}{\partial x_j}(pv_j + \pi_{ij}v_i) = -\alpha\omega - \varepsilon. \tag{2.4}$$

In the first set of equations $p\, d\alpha/dt$ is the exchange between the energy forms $c_v T$ and $k + \Phi$; in the second set $-\alpha\, dp/dt$ is the exchange between the energy forms $c_p T$ and $k + \Phi - p\alpha$.

Following Lorenz (1967) and Boer (1975) we transform to a third set of local energy equations by introducing the *efficiency factor*:

$$N \equiv 1 - (p_r/p)^\kappa \qquad \text{with}$$

$$\frac{dN}{dt} = -\kappa(1 - N)\left(\frac{1}{p_r}\frac{dp_r}{dt} - \frac{\omega}{p}\right), \tag{2.5}$$

p_r is a reference pressure chosen arbitrary with the first condition that the global mean N vanishes; $\kappa = R/c_p$. By combining eqs. (2.3), (2.5) we obtain with the dry gas equation:

$$\frac{dNc_p T}{dt} = NQ + \alpha\omega - R\Theta\left(\frac{p_r}{p_o}\right)^\kappa \frac{1}{p_r}\frac{dp_r}{dt}. \tag{2.6}$$

Θ is potential temperature, $p_o = 1000$ hPa. As second condition we require that p_r corresponds to a barotropic reference state, like the one defined by Lorenz (1955). Implementing this by putting $p_r = p_r(\Theta)$ and further introducing the new potential function $P(\Theta)$ through the following differential equation:

$$dP(\Theta) = R\Theta\left[\frac{p_r(\Theta)}{p_o}\right]^\kappa \frac{1}{p_r(\Theta)}\, dp_r(\Theta), \tag{2.7}$$

we arrive at the generalized energetic quantity:

$$a \equiv Nc_p T + P. \tag{2.8}$$

The potential P is implicitly time-dependent since $p_r = p_r[\Theta(t)]$. With a, eq. (2.6) reads:

$$\frac{da}{dt} = NQ + \alpha\omega. \tag{2.9}$$

By further introducing the mechanical energy in slightly generalized form:

$$b \equiv k + \Phi - p\alpha, \tag{2.10}$$

eq. (2.4) reads:

$$\frac{db}{dt} + \alpha \frac{\partial}{\partial x_j}(pv_j + \pi_{ij}v_i) = -\alpha\omega - \varepsilon. \tag{2.11}$$

We consider eq. (2.9) the local equation for the integrand of available potential energy (Dutton and Johnson, 1967; Boer, 1975; Boer, 1976); available potential energy is a global quantity. In this formulation $-\alpha\omega$ is the exchange between the thermodynamic energy a and the mechanical energy b.

All 3 sets of energy equations are equivalent. There is, at the present local level, no point in trying to resolve the ambiguity of our three sets of equations. The ambiguity is due to the fact that the energy cannot be objectively separated into parts (Van Mieghem, 1973, or Falk and Ruppel, 1976); also, the exchange between arbitrary energy components cannot uniquely be fixed. Consequently, still different energy forms can, and have been, discussed (Kucharski, 1997).

For the present purpose we prefer, following Lorenz, the third set (2.9), (2.11). One reason is that the global mean of Q in the first and second set is replaced by the global mean of NQ in the third set; it is useful to consider NQ because its global mean can be well interpreted physically and well evaluated numerically. The main advantage of the third set is that we can (after regionally averaging) evaluate the sub-gridscale component of the exchange term $-\alpha\omega$; this would not be possible for the sub-gridscale component of the exchange term $p\, d\alpha/dt$ in the first set.

3. Gridscale and global mass averaging

The gridscale in the sense considered in this study is a space-time interval of a size typical for present global weather and climate forecast models like the model regularly run at ECMWF. The observations as routinely analysed by ECMWF are representative for grid sizes of about $\Delta x \approx 40$–400 km in horizontal direction, $\Delta p \approx 10$–100 hPa in vertical direction and $\Delta t \approx 6$–12 h in time. We shall not try to exactly fix this size but rather to stress that there is considerable meteorological activity on scales below the grid size (Van Mieghem, 1973; Herbert, 1975). These processes, however, cannot be resolved, neither by routine observations nor by deterministic models.

In order to have a formalism for this familiar fact we consider a field of locally defined mass-specific kinetic energy k. Also, we introduce a

finite mass ΔM and a finite time interval Δt over which the following integrals are to be extended. We use these for defining the Reynolds mass-average operator ($t =$ time, $x_i =$ Cartesian space coordinates):

$$\bar{\bar{k}}(t, x_i) \equiv \frac{1}{\Delta M\, \Delta t} \times \int_{\Delta t} \left[\int_{\Delta M} k(t, x_i; \tau, \xi_i)\, \mathrm{d}M(\tau, \xi_i) \right] \mathrm{d}\tau. \tag{3.1}$$

This definition includes the usual running average but is more general (Herbert, 1975). It makes sense only for mass-specific quantities like k or specific volume α. The mass elements of the interval ΔM over which the integration is extended are to be considered functions of the space coordinates ξ_i with fixed time coordinate τ; once the mass integration has been carried out the time integration follows.

The Reynolds mass-average corresponds to, but is different from, the familiar Reynolds volume-average defined for an arbitrary volume-specific quantity like ordinary density ρ or volume-specific kinetic energy $\kappa = \rho k$:

$$\hat{\kappa}(t, x_i) \equiv \frac{1}{\Delta V\, \Delta t} \times \int_{\Delta t} \left[\int_{\Delta V} \kappa(t, x_i; \tau, \xi_i)\, \mathrm{d}V(\tau, \xi_i) \right] \mathrm{d}\tau. \tag{3.2}$$

We imply that the extensive quantities ΔM, ΔV belong to the same fluid parcel. The differentials of the extensive quantities M, V can be transformed into each other through:

$$\mathrm{d}M = \rho\, \mathrm{d}V, \qquad \mathrm{d}V = \alpha\, \mathrm{d}M. \tag{3.3}$$

Thus the relation between the two averaging operators can be expressed by one of the following formulae:

$$\bar{\bar{k}} = \frac{\widehat{\rho k}}{\hat{\rho}}, \qquad \hat{\kappa} = \frac{\overline{\overline{\alpha\kappa}}}{\bar{\bar{\alpha}}}. \tag{3.4}$$

Eqs. (3.4) demonstrates the formal duality between the Reynolds mass-averaging and the Reynolds volume-averaging.

While earlier treatments (Van Mieghem, 1973) occasionally use both definitions (3.1), (3.2) in the same equation we shall restrict the present study to using only the mass-average operator since the other is implied. The actual function (e.g., the mass-specific kinetic energy k or the wind component v_j) within the averaging interval now reads:

$$k = \bar{\bar{k}} + k''. \tag{3.5}$$

The deviation or eddy quantity k'' represents the sub-gridscale component of k. The operator (3.1) is assumed to fulfil the familiar rules:

$$\overline{\overline{(\bar{\bar{k}})}} = \bar{\bar{k}}, \qquad \overline{\overline{k''}} = 0, \qquad \overline{\overline{kv_j}} = \bar{\bar{k}}\bar{\bar{v}}_j + \overline{\overline{k''v_j''}}. \tag{3.6}$$

It is well-known that these rules are only valid under the restrictive assumptions of either the existence of a spectral gap (Vinnichenko, 1970; Van Mieghem, 1973; Leonard, 1974) or the possibility of applying ensemble averaging (Herbert, 1975; Bernhardt, 1979; Cotton and Anthes, 1989). Following most authors we leave this point open here and accept that (3.6) is valid throughout the remainder of this study.

For later convenience we introduce the following nomenclature which should be self-explaining:

$$\frac{\bar{\bar{\mathrm{d}}}}{\mathrm{d}t} \equiv \frac{\partial}{\partial t} + \bar{\bar{v}}_j \frac{\partial}{\partial x_j}. \tag{3.7}$$

With this definition it follows:

$$\overline{\overline{\frac{\mathrm{d}k}{\mathrm{d}t}}} = \frac{\bar{\bar{\mathrm{d}}}\bar{\bar{k}}}{\mathrm{d}t} + \bar{\bar{\alpha}} \frac{\partial}{\partial x_j} \left(\frac{1}{\bar{\bar{\alpha}}} \overline{\overline{k''v_j''}} \right), \tag{3.8}$$

$$\overline{\overline{\alpha \frac{\partial k}{\partial x_j}}} = \bar{\bar{\alpha}} \frac{\partial}{\partial x_j} \left(\frac{1}{\bar{\bar{\alpha}}} \overline{\overline{\alpha k}} \right). \tag{3.9}$$

These rules are implied in earlier treatises (Van Mieghem, 1973; Herbert, 1975) and will be used in the next section.

For the global time- and mass-average of k, denoted by $\{k\}$, we use a definition similar to (3.1). However, the time interval Δt is now to be interpreted as an interval long enough (1 year or more) so that $\{k\}$ can be considered stationary; likewise, the mass interval is to be interpreted as the mass M of the global atmosphere. This yields the global averaging operator:

$$\{k\} \equiv \frac{1}{M\, \Delta t} \int_{\Delta t} \left[\int_{M} k(t, x_i)\, \mathrm{d}M(t, x_i) \right] \mathrm{d}t. \tag{3.10}$$

When applying (3.10) to a total time derivative we obtain after some algebra plus the Gaussian

theorem:

$$\left\{\frac{\mathrm{d}k}{\mathrm{d}t}\right\} = \left\{\frac{\partial k}{\partial t}\right\}. \tag{3.11}$$

Eq. (3.11) can be equivalently expressed as:

$$\left\{\alpha \frac{\partial F_j}{\partial x_j}\right\} = 0. \tag{3.12}$$

Eq. (3.12) is valid for material fluxes F_j since these vanish across the boundaries of the global atmosphere; only these will be used below. Concerning (3.11) we stipulate that global mass averages of state quantities, when time averaged over a climate interval Δt, should vanish: The right-hand side of (3.11) is also zero.

The derivation of these formulae has been independent upon the scale of the averaged quantities involved. However, we shall apply the average operator (3.10) only to gridscale quantities which implies validity of the hydrostatic equation. In these cases by introducing the Earth's surface area A and writing $\mathrm{d}M = g^{-1}\,\mathrm{d}p\,\mathrm{d}A$, eq. (3.10) reads:

$$\{\overline{\overline{k}}\} = \frac{g}{p_s A\,\Delta t}\int_{\Delta t}\int_A\int_0^{p_s} g^{-1}\overline{\overline{k}}(t, x, y, p)\,\mathrm{d}p\,\mathrm{d}A\,\mathrm{d}t. \tag{3.13}$$

The constants p_s, g are global mean surface pressure and gravity, respectively. In the evaluations of Part II we have used only the global averaging in the form (3.13). By multiplying with p_s/g one can transform global averages like $\{k\}$ or $\{\mathrm{d}k/\mathrm{d}t\}$ (which come in units J/kg or W/kg, respectively) into units J/m^2 or W/m^2, respectively, which are more common in general circulation studies.

4. The averaged energy equations

With the definitions just introduced we are in the position to derive equations for the regionally and globally averaged energy components. In contrast to the classical treatments of Lorenz we shall retain the sub-gridscale fluctuations. We start with the regional equations (Subsection 4.1) and proceed to the global equations (Subsection 4.2).

4.1. The regional energy equations

We apply the gridscale mass-averaging operator (3.1) to eq. (2.9), the local equivalent of the equation for available potential energy; along with formula (3.8) we obtain the regional equivalent of the equation for available potential energy:

$$\frac{\overline{\overline{\mathrm{d}}}\,\overline{\overline{a}}}{\mathrm{d}t} + \overline{\overline{\alpha}}\frac{\partial}{\partial x_j}\left(\frac{1}{\overline{\overline{\alpha}}}\overline{\overline{a''v_j''}}\right) = \overline{\overline{NQ}} + \overline{\overline{\alpha\omega}}. \tag{4.1}$$

Similarly the equation for $\overline{\overline{b}}$ is gained from (2.11):

$$\frac{\overline{\overline{\mathrm{d}}}\,\overline{\overline{b}}}{\mathrm{d}t} + \overline{\overline{\alpha}}\frac{\partial}{\partial x_j}\frac{1}{\overline{\overline{\alpha}}}(\overline{\overline{k''v_j''}} + \overline{\overline{\alpha p}}\,\overline{\overline{v_j}} + \overline{\overline{\alpha\pi_{ij}v_i}}) = -\overline{\overline{\alpha\omega}} - \overline{\overline{\varepsilon}}. \tag{4.2}$$

It is the regional equivalent of the local mechanical energy eq. (2.11). In deriving (4.2), we have used eqs. (3.8) and (3.9) and have observed the fact that fluctuations of Φ vanish. Both regional equations (4.1), (4.2) comprise a variety of gridscale averages over nonlinear expressions like the kinetic energy which is part of b. With eq. (3.6) this implies, for example:

$$\underbrace{\frac{1}{2}\overline{\overline{v_i^2}}}_{\overline{\overline{k}}} = \underbrace{\frac{1}{2}\overline{\overline{v}_i}^2}_{k^{\mathrm{grid}}} + \underbrace{\frac{1}{2}\overline{\overline{v_i''^2}}}_{k^{\mathrm{sub}}}. \tag{4.3}$$

k^{grid} (k^{sub}) represents the kinetic energy of gridscale (sub-gridscale) motions. Specifically, k^{sub} is identical to the turbulence kinetic energy discussed in PBL theory. A similar separation applies to the thermodynamic energy quantity $\overline{\overline{a}}$. Likewise, the generation, conversion and dissipation terms consist of two components since they comprise both gridscale and sub-gridscale mechanisms.

It would be possible to proceed from here with developing separate equations for the reservoirs a^{grid}, a^{sub}, b^{grid}, b^{sub}, in analogy to the method followed in PBL theory (Van Mieghem, 1973; Kraus and Businger, 1994). However, we shall be satisfied at this stage with the total budgets.

Up to now we have discussed the properties of $\overline{\overline{a}}$ and $\overline{\overline{b}}$ that are a bit more general than the energy forms orginally considered by Lorenz. On the other hand, some properties of $\overline{\overline{b}}$ are practically identical to Lorenz's. For example, when integrating eq. (4.2) vertically over an entire gridscale atmospheric column the components $\overline{\overline{\Phi}}$ and $\overline{\overline{p\alpha}}$ contained in $\overline{\overline{b}}$ cancel each other with the result:

$$\int_0^{p_s}\overline{\overline{b}}\,\mathrm{d}p = \int_0^{p_s}\overline{\overline{k}}\,\mathrm{d}p. \tag{4.4}$$

The reason is that the vertical integral over gridscale quantities can be performed hydrostatically. This shows that Lorenz's choice to consider k as

the central mechanical energy reservoir was the relevant choice.

4.2. The global energy equations

The various quantities on the left of eqs. (4.1) and (4.2) can be huge in individual grid cells. However, as we turn now to the corresponding global average, we shall not discuss them in detail because each term on the left vanishes in the global climate mean due to eqs. (3.11), (3.12). Thus the globally averaged energy equations read:

$$\begin{aligned} 0 &= \underbrace{\{\overline{\overline{NQ}}\}}_{G} + \underbrace{\{\overline{\overline{\alpha\omega}}\}}_{-C}, \\ 0 &= \underbrace{-\{\overline{\overline{\alpha\omega}}\}}_{C} - \underbrace{\{\bar{\bar{\varepsilon}}\}}_{D}. \end{aligned} \tag{4.5}$$

Only three non-zero flux terms remain: the generation G of available potential energy, the conversion C between available potential and kinetic energy, and the dissipation D of kinetic energy. Following the arguments of Lorenz (1967), D is positive since the integrand is positive definite. Since from the equations (4.5) the 3 conversion terms G, C, D should be equal, it follows that C, and also G, must be positive.

This familiar result (Lorenz, 1967, his Fig. 51) is reproduced in our Fig. 2. The difference of the present theory to the original one of Lorenz (1967) and Peixóto and Oort (1992) is that both our averaged generation and conversion terms contain sub-gridscale correlation components $\overline{\overline{N''Q''}}$ and $-\overline{\overline{\alpha''\omega''}}$. These are physically relevant; we shall demonstrate in Part II that they are also of significant size. Concerning dissipation we will not be able to explicitly evaluate D^{grid} and D^{sub} from data. However, it is obvious a priori that the dissipation happens on the smallest scales so that D^{grid} will practically be negligible while $D \approx D^{\mathrm{sub}}$.

5. Conclusions

This study has been Part I of a companion paper on how to implement convective-scale conversion rates into the classical Lorenz energy cycle. The corresponding gridscale energy equations comprise both directly observable gridscale quantities (like, e.g., the mean kinetic energy) as well as not directly observable quantities generated by sub-gridscale interactions (like, e.g., the vertical eddy heat flux). We have not tried to specify the exact size of the gridscale; it may be of the order used in global models of international weather centers like the ECMWF (i.e., $\Delta x \approx 40$–400 km, $\Delta p \approx 10$–100 hPa, $\Delta t \approx 6$–12 h).

We have considered two elementary energy forms: thermodynamic energy $a \equiv Nc_{\mathrm{p}}T + P$ ($N =$ a generalized efficiency factor, $P =$ an additional thermodynamic potential) and generalized mechanical energy $b \equiv k + \Phi - p\alpha$. While locally a is very different from $Nc_{\mathrm{p}}T$ and b is very different from k, we have argued that, globally, a reproduces $Nc_{\mathrm{p}}T$ and b reproduces k.

The corresponding equations for a and b can be written such as to contain source terms, sink terms, and flux divergence terms. Also, both contain Lorenz's conversion term $-\alpha\omega$ that appears with opposite sign in both.

When mass-averaged over a representative space-time gridcell the interaction term separates

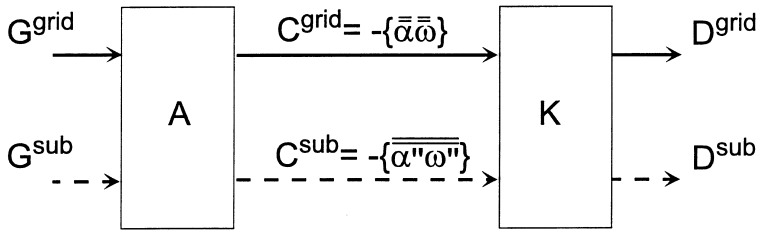

Fig. 2. Completed global energy cycle. Conversion rates with superscript grid comprise the traditional terms. Available potential energy A may be created through generation rates $G^{\mathrm{grid}} = \{\overline{\overline{N}}\,\overline{\overline{Q}}\}$ and $G^{\mathrm{sub}} = \{\overline{\overline{N''Q''}}\}$ and converted into kinetic energy K through conversion rates $C^{\mathrm{grid}} = -\{\bar{\bar{\alpha}}\bar{\bar{\omega}}\}$ and $C^{\mathrm{sub}} = -\{\overline{\overline{\alpha''\omega''}}\}$. K may be destroyed through dissipation rates $D^{\mathrm{grid}} = \{\nu(\partial\bar{\bar{v}}_i/\partial x_j)^2/2\}$ and $D^{\mathrm{sub}} = \{\overline{\nu(\partial v_i''/\partial x_j)^2}/2\}$ for a Navier–Stokes fluid ($\nu =$ kinematic viscosity).

into two components:

$$-\overline{\overline{\alpha\omega}} = -\overline{\overline{\alpha}}\,\overline{\overline{\omega}} - \overline{\overline{\alpha''\omega''}}. \quad (5.1)$$

The global mean of $-\overline{\overline{\alpha}}\,\overline{\overline{\omega}}$ represents Lorenz's classical conversion term C (referred to in this study as C^{grid}), the global mean of $-\overline{\overline{\alpha''\omega''}}$ represents the new sub-gridscale conversion term C^{sub}. This causes a problem for the naive application of Lorenz's original theory: Do we have to include C^{sub} in the global energy cycle or not? In the original APE-theory (Lorenz, 1955) there has been no room for C^{sub}.

Our answer is that we indeed have to include, not only the conversion rate C^{sub}, but the entire sub-gridscale branch of the energy cycle since C^{grid} and C^{sub} are of about equal size, as will be shown in Part II. To include it was however not particularly useful in the times of Lorenz because the accuracy of the available data was not sufficient to obtain reliable estimates of quantities like C^{sub}.

The nontrivial problem of the proper choice of p_{r} has been discussed by various authors (Van Mieghem, 1956; Kucharski, 1997). In the present treatment we did not specify how the reference pressure and the corresponding distribution of N should be gained in a nonhydrostatic atmosphere. We have restricted discussion to relatively general properties, predominantly the fact that $G = \{NQ\} = \{\overline{\overline{NQ}}\}$ is independent upon p_{r} as long as p_{r} is barotropic and chosen such that the global average of N vanishes. Formally, the choice of p_{r} is best when the integrand NQ is of uniform sign throughout all latitudes; the physical implication of a smart choice of the reference pressure is that the unavailable potential energy is maximized.

The new aspect in this study has been that there does not seem to be a formal argument that would allow to drop sub-gridscale interactions from the outset. The sub-gridscale does exchange with the gridscale and it is not evident a priori that, or to what extent, both scale arrays can be cleanly separated. The spectrum of kinetic energy (Vinnichenko, 1970) shows high energy density on large scales and low energy density on small scales; but this gives no clue for the intensity of interaction between the reservoirs.

We may speculate about the analogy between the mechanisms on both scales. In the gridscale domain the location of the generation of potential energy eventually available for conversion into kinetic energy is far separated in horizontal direction (scale: radius of the earth) from the location where it is eventually dumped into turbulent kinetic energy. Similarly, in the sub-gridscale domain the available energy generation is far separated in vertical direction (scale: depth of the atmosphere) from the location where it is transformed into turbulent kinetic energy and dissipated. In other words, the sub-gridscale kinetic energy is not only fed through the corresponding gridscale component but also through flux from the sub-gridscale reservoir of potential energy.

It has long been known from deep convection studies (Arakawa and Schubert, 1974) that there must be significant interaction between convective and large-scale processes. Central quantities in deep convection theory like the cloud work function or convective available potential energy (CAPE) are conceptually similar to the sub-gridscale component of A as defined in this study. These reservoirs can be changed through processes like: surface evaporation, radiative cooling of the free atmosphere and large-scale ascent forced by synoptic disturbances (Emanuel, 1994).

For these reasons, the theory developed here is not new in a conceptual sense. It rather combines various ingredients extensively discussed earlier. The basic ideas of Margules (1903) and of Lorenz (1955); the approximation-free mass-averaging formalism as advanced by Hesselberg (1926) and applied by Van Mieghem (1973) to the local and global energy cycle; the separation of kinetic energy into its mean and turbulent components (gridscale and sub-gridscale in the present context) as extensively used in PBL-theory (Stull, 1988) and deep convection theory (Emanuel, 1994); the significance of barotropy for the interpretation of the reference pressure and the efficiency factor (Dutton and Johnson, 1967); or the analyses of Boer (1975); Boer (1976). Thus the present theory does not try to improve the basic concept of Lorenz but rather to bring it to its completion.

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REFERENCES

Arakawa, A. and Schubert, W. H. 1974. Interaction of a cumulus cloud ensemble with the large-scale environment, Part I. *J. Atmos. Sci.* **31**, 674–701.

Arpe, K., Brankovic, C., Oriol, E. and Speth, P. 1986. Variability in time and space of energetics from a long series of atmospheric data produced by ECMWF. *Beitr. Phys. Atmos.* **59**, 321–355.

Batchelor, G. K. 1967. *An introduction to fluid dynamics.* Cambridge University Press, 615 pp.

Bernhardt, K. 1979. Zur Frage der Gültigkeit der Reynoldsschen Postulate. *Zeitschr. Meteorol.* **30**, 361–368.

Boer, G. J. 1975. Zonal and eddy forms of the available potential energy equations in pressure coordinates. *Tellus* **27**, 433–443.

Boer, G. J. 1976. Reply to J. Egger. *Tellus* **28**, 379–380.

Cotton, W. and Anthes, R. A. 1989. *Storm and cloud dynamics*, vol. 44 of Int. Geophys. Ser. Academic Press, 883 pp.

Dutton, J. A. and Johnson, D. R. 1967. The theory of available potential energy and a variational approach to atmospheric energetics. *Adv. Geophys.* **12**, 333–437.

Emanuel, K. A. 1994. *Atmospheric convection.* Oxford University Press, 580 pp.

Falk, G. and Ruppel, W. 1976. *Energie und Entropie. Eine Einführung in die Thermodynamik.* Springer, 408 pp.

Glansdorff, P. and Prigogine, I. 1971. *Thermodynamic theory of structure, stability and fluctuations.* Wiley, 306 pp.

Haimberger, L. and Hantel, M. 2000. Implementing convection into Lorenz's global cycle, Part II: A new estimate of the conversion rate into kinetic energy. *Tellus* **52A**, 75–92.

Herbert, F. 1975. Irreversible processes in the atmosphere — Part 3 (Phenomenological theory of microturbulent systems). *Beitr. Phys. Atmos.* **48**, 1–29.

Hesselberg, T. 1926. Die Gesetze der ausgeglichenen atmosphärischen Bewegungen. *Beitr. Phys. frei Atmos.* **12**, 141–160.

Kraus, E. B. and Businger, J. A. 1994. *Atmosphere–ocean interaction.* Oxford University Press, 362 pp.

Kucharski, F. 1997. On the concept of exergy and available potential energy. *Quart. J. Roy. Meteor. Soc.* **123**, 2141–2156.

Leonard, A. 1974. Energy cascade in large-eddy simulations of turbulent fluid flows. In: *Turbulent diffusion in environmental pollution* (eds. Frenkiel, F. and Munn, R.), vol. 18A of Adv. Geophys., pp. 237–248. Academic Press.

Lorenz, E. N. 1955. Available potential energy and the maintenance of the general circulation. *Tellus* **7**, 157–167.

Lorenz, E. N. 1967. *The nature and theory of the general circulation of the atmosphere*, vol. 218.TP.115. WMO, 161 pp.

Margules, M. 1903. Über die Energie der Stürme. *Jahrb. k. k. Zent.-Anst. für Meteorol. und Erdmagnet.* **48**, 1–26.

Newell, R. E., Vincent, D. G., Dopplick, T. G., Ferruzza, D. and Kidson, J. W. 1970. The energy balance of the global atmosphere. In: *The global circulation of the atmosphere* (ed. Corby, G. A.), pp. 42–90. London: Roy. Met. Soc.

Oort, A. H. 1964. On estimates of the atmospheric energy cycle. *Mon. Wea. Rev.* **92**, 483–493.

Peixóto, J. P. and Oort, A. H. 1992. *Physics of climate.* American Institute of Physics, 520 pp.

Stull, R. B. 1988. *An introduction to boundary layer meteorology.* Kluwer Academic Publishers, 666 pp.

Van Mieghem, J. 1956. The energy available in the atmosphere for conversion into kinetic energy. *Beitr. Phys. Atmos.* **29**, 129–142.

Van Mieghem, J. 1973. *Atmospheric energetics.* Oxford Monograph on Meteorology. Clarendon Press, 306 pp.

Vinnichenko, N. K. 1970. The kinetic energy spectrum in the free atmosphere — one second to five years. *Tellus* **22**, 128–166.

Yamada, T. and Mellor, G. 1975. A simulation of the Wangara atmospheric boundary layer data. *J. Atmos. Sci.* **32**, 2309–2329.