Tellus (1999), 51A, 314–325
Printed in UK – all rights reserved

TELLUS
ISSN 0280–6495

On thermobaric production of potential vorticity in the ocean

By DAVID N. STRAUB*, *Atmospheric and Ocean Sciences, McGill University, 805 Rue Sherbrooke O, Montreal, QC H3A 2K6, Canada*

(Manuscript received 21 November 1997; in final form 20 July 1998)

ABSTRACT

Because of the two-constituent nature of seawater, there is no conservative potential vorticity for the ocean, even when viscous and diffusive effects are ignored. Specifically, nonconservation arises in association with the thermobaric effect, which is related to nonlinearities in the equation of state. While this is well-known, it is generally supposed that this forcing is far too small to be of importance for most problems of interest. A careful scale analysis is presented to suggest that thermobaric forcing is larger than is generally supposed, particularly at the high wavenumber end of the spectrum. The spatial structure of the thermobaric potential vorticity source is dependent on an arbitrary choice, such as the choice of a reference pressure in the definition of potential vorticity. To address the effect of thermobaricity on the dynamics, which must be independent of arbitrarily chosen parameters, it therefore seems desirable to consider how balanced dynamical systems are affected. In this spirit, an analog to quasigeostrophy that is applicable to thermobaric fluids is derived and its conservation properties are discussed. It is argued that thermobaricity can lead to O(1) changes in the vorticity distribution at the high wavenumber end of the spectrum. Additionally, integral constraints that can influence basin scale circulation are affected. For example, isopycnal mixing affects the (QG) energetics and potential enstrophy is lost as a conserved quantity. The size of the enstrophy source (or sink) and the wave number at which thermobaric effects produce an O(1) effect both depend on the size of a nondimensional parameter that is introduced. This parameter, although typically small, can be large with respect to the Rossby number appropriate to the mid-latitude eddy field. That is, it is suggested that thermobaric effects may be larger than finite Rossby number effects on the dynamics of the mid-latitude eddy field, particularly below the thermocline.

1. Introduction

Conservation of potential vorticity plays a central role in theories describing oceanic circulation. The most general form is due to Ertel (1942),

$$\Pi \equiv \frac{\omega_{\text{abs}} \cdot \nabla\lambda}{\rho}, \tag{1}$$

where ω_{abs} is the absolute vorticity, ρ is density and λ is a scalar function. Following a fluid parcel, Π evolves according to

$$\frac{\mathrm{d}\Pi}{\Pi t} = \mathscr{V} + \mathscr{N} + \mathscr{T}, \tag{2}$$

where $\mathscr{V}$, $\mathscr{N}$ and $\mathscr{T}$ are associated with viscous effects, nonconservation of λ, and thermobaricity, respectively (e.g., Pedlosky, 1987):

$$\mathscr{V} = \frac{\nabla\lambda}{\rho} \cdot \left(\nabla \times \frac{F}{\rho}\right)$$

$$\mathscr{N} = \frac{\omega_{\text{abs}}}{\rho} \cdot \nabla\Psi \tag{3}$$

$$\mathscr{T} = \frac{1}{\rho^3} (\nabla\lambda \cdot (\nabla\rho \times \nabla p)).$$

* Also at Centre de Recherche en Calcul Appliqué (CERCA), 5160 Boulevard Décarie, bureau 400, Montréal, QC H3X 2H9, Canada.

Here, F is the viscous force and Ψ is defined by

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = \Psi . \quad (4)$$

For example, if λ is taken to be potential density then Ψ is associated with the absorbtion of solar radiation and with molecular diffusion of heat and salt*.

Far away from the air-sea interface, direct solar heating can be ignored and $\mathscr{V}$ and $\mathscr{N}$ can be considered small, except where the flow develops small scale structure. Of course, scales on which molecular processes can affect the potential vorticity are impossible to resolve for most problems of interest to oceanographers and the net effects of $\mathscr{N}$ and $\mathscr{V}$ on the resolved flow are generally parametrized, with each leading to "forcing" terms (linked to air-sea fluxes of momentum, heat, and fresh water), as well as to "dissipative" terms (relating to eddy viscosities and diffusivities). The thermobaric term, $\mathscr{T}$, is generally dismissed as unimportant. The purpose of this paper is to clarify the conditions under which this dismissal is justified and to discuss some of the possible ramifications for geostrophic dynamic when these conditions are not met.

In general, there is no reason to expect flow to remain barotropic and therefore $\mathscr{T}$ vanishes identically only if $\nabla\lambda$ remains perpendicular to the baroclinic vector, $\nabla\rho \times \nabla p$. This implies that λ is expressible as a function of p and ρ:

$$\lambda = a_0(p, \rho) . \quad (5)$$

(We will further assume that λ varies monotonically with ρ for fixed p.) Alternatively, $\mathscr{T}$ vanishes provided that the equation of state can be written as a function of p and λ:

$$p = a_1(p, \lambda) . \quad (6)$$

Generally λ is taken to be potential density, σ, and in fact it is straight-forward to show that any λ satisfying (6) can be written as a function of potential density. To see this consider an adiabatic, isentropic displacement of a fluid parcel to a point on the pressure surface, $p = p_{\mathrm{ref}}$. By assumption λ is conserved during the displacement, so that at $p = p_{\mathrm{ref}}$, $\rho = a_1(p_{\mathrm{ref}}, \lambda)$. Then, since ρ and σ are identical at p_{ref},

$$\sigma = a_2(\lambda) , \quad (7)$$

which can be inverted to give λ as a function of σ. Furthermore, since the choice of the reference pressure is arbitrary, conservation of Π requires λ to be a function of potential density irrespective of the choice of p_{ref}. This, in turn, implies that σ referenced to one pressure can be written as a function of σ referenced to another. For a thermobaric fluid such as seawater, this is not possible and a conservative Ertel's potential vorticity does not exist.

A fluid is said to be thermobaric if the density of two parcels is the same at one pressure, but differs at another. More formally (Müller, 1995), it is required that $b \neq 0$, where the thermobaric coefficient, b, is defined for a two-constituent fluid by

$$b \equiv \frac{1}{2}\left(\frac{1}{\gamma_1}\frac{\partial\gamma_1}{\partial p} - \frac{1}{\gamma_2}\frac{\partial\gamma_2}{\partial p}\right) . \quad (8)$$

Here,

$$\gamma_1 \equiv \frac{-1}{\rho}\frac{\partial}{\partial\eta}\rho(p, \eta, s) , \quad (9)$$

and

$$\gamma_2 \equiv \frac{1}{\rho}\frac{\partial}{\partial s}\rho(p, \eta, s) , \quad (10)$$

where s gives the concentration of the constituent (e.g., s is salinity for the ocean) and η is a conserved thermodynamic variable such as specific entrophy or potential temperature. Thus, γ_2 is the haline contraction coefficient and γ_1 is analogous to the thermal expansion coefficient. The thermobaric coefficient is related to pressure dependence in the slope, m, of density curves on an η, s diagram. That is, taking ρ to be a function of p, η and s and defining

$$m \equiv \left(\frac{\partial\rho/\partial\eta}{\partial\rho/\partial s}\right) , \quad (11)$$

then

$$\frac{\partial m}{\partial p} = 2mb . \quad (12)$$

That a conservative potential vorticity does not exist for seawater is well known. It is generally

* Effects such as evaporation and radiative cooling act at the sea surface and thus influence potential vorticity only indirectly by influencing boundary conditions on the diffusion of heat and salt.

believed, however, that the thermobaric production of potential vorticity, $\mathscr{T}$, is far too small to be of significance in most problems of interest. Here it is argued that, while generally small, $\mathscr{T}$ can be non-negligible. Additionally, arguments are given to suggest possible consequences to ocean dynamics. This is done in the context of quasigeostrophy. Other dynamical consequences of thermobaricity have been discussed elsewhere in the literature. Gill (1973) and Killworth (1977), for example, discuss effects on static stability and Antarctic Bottom Water formation. Müller and Willebrand (1986, hereafter, MW86) discuss thermobaric effects in the context of the abyssal branch of the thermohaline circulation. They go on to use linear wave analysis to analize an "η–s mode" that exists in the presence of a background vertical velocity. McDougal (1987a,b) discusses the role of thermobaric effects in water mass conversion.

McDougal (1987a) emphasizes the fact that the slope of a locally referenced potential density surface can differ significantly from that of a potential density surface referenced to some arbitrary pressure. He goes on to define "neutral surfaces" and neutral surface potential vorticity, NSPV (see also You and McDougall (1990)). The idea relates to the common assumption that heat and salt diffuse readily in the plane tangent to locally referenced potential density surfaces and that diffusion is much weaker in the orthogonal, dianeutral direction. They argue that choosing λ to correspond to a neutral surface lable then minimizes the potential vorticity source associated with $\mathscr{N}$. Since there is no clear reason to expect that using NSPV will lessen the magnitude of $\mathscr{T}$, and since $\mathscr{T}$ is necessarily nonzero except when λ is chosen to be a function of potential density, this will be assumed to be the case here. That is, we take Π to be defined using potential density as the scalar, λ.

The present paper is organized as follows: In Section 2, an expression relating $\mathscr{T}$ to a pressure integral of mb is derived and used to estimate the magnitude of thermobaric production term for flow in which the horizontal velocity obeys geostrophic scaling. The spatial structure of the source term turns out to be strongly dependent on the arbitrary choice of a reference pressure needed to define potential density. Because of this, it is difficult to ascertain how this source might effect the dynamics. In Section 3, a thermobaric analog to quasigeostrophy is developed and its conservation properties are discussed. This gives us a reasonably tractable dynamical framework from which we can begin to understand how thermobaricity might affect the geostrophic flow. It is argued that there is an O(1) modification of the vorticity field at the high wavenumber end of the spectrum. Also, it is found that energy is conserved only when mixing (including isopycnal mixing) is ignored and that potential enstrophy is lost as a conserved quantity in the inviscid, nondiffusive limit. In Section 4, a more intuitive discussion is presented to demonstrate how thermobarcity can influence the small scale structure of the vorticity field and it is pointed out that the large scales may also be influenced, given that thermobaricity alters various integral constraints on the flow.

2. Scaling of the thermobaric production term

We begin by deriving an expression that helps to clarify the relationship between $\mathscr{T}$ and the thermobaric coefficient, b. The result is then used to scale $\mathscr{T}$. As discussed above, one can only hope to find a conservative potential vorticity if λ is a function of σ and this will be assumed to be the case here. Thus, using (3),

$$\mathscr{T}=\frac{1}{\rho^3}(\nabla p\cdot(\nabla\sigma\times\nabla\rho)), \tag{13}$$

where, without loss of generality, dependence on the arbitrary function $d\lambda/d\sigma$ has been dropped. Next, note that since $\rho=\rho(p,\eta,s)$ and $\sigma=\sigma(\eta,s)$, $\nabla\sigma\times\nabla\rho$ can be written as

$$\nabla\sigma\times\nabla\rho=\left(\frac{\partial\sigma}{\partial\eta}\frac{\partial\rho}{\partial s}-\frac{\partial\sigma}{\partial s}\frac{\partial\rho}{\partial\eta}\right)\nabla\eta\times\nabla s+\boldsymbol{A}, \tag{14}$$

where $\boldsymbol{A}$ is perpendicular to ∇p. We would like to relate the terms in parentheses to b. Integrating (12) from p to p_{ref} (while holding η and s constant) yields:

$$\int_p^{p_{\text{ref}}}(2mb)\,dp'=\frac{\partial\sigma/\partial\eta}{\partial\sigma/\partial s}-\frac{\partial\rho/\partial\eta}{\partial\rho/\partial s}. \tag{15}$$

Rearranging terms and substituting into (14) gives

$$\nabla\sigma\times\nabla\rho=\frac{\partial\sigma}{\partial s}\frac{\partial\rho}{\partial s}(\nabla\eta\times\nabla s)\int_p^{p_{\text{ref}}}(2mb)\,dp'+\boldsymbol{A}. \tag{16}$$

Substituting back into (13) then gives

$$\mathscr{T} = \frac{1}{\rho^3}\frac{\partial\sigma}{\partial s}\frac{\partial\rho}{\partial s}(\nabla p \cdot (\nabla\eta \times \nabla s)) \int_p^{p_{\mathrm{ref}}} (2mb)\,\mathrm{d}p'. \tag{17}$$

Because the $\mathscr{T}$ vanishes when $b = 0$, we refer to this as thermobaric forcing of potential vorticity.

For problems that are localised in the vertical, $\mathscr{T}$ can be made small simply by choosing an appropriate reference pressure. Our interest here is in problems for which the entire water column must be considered. In such cases, it is not obvious that a judicious choice of p_{ref} will suffice; hence, we would like to scale (17) to estimate the size of $\mathscr{T}$. To begin, note that there are 2 horizontal and 1 vertical scales in the triple scalar product. Since all 3 vectors are very nearly vertically aligned, it is not immediately obvious which gradient should be scaled according to its vertical derivative; however, a closer inspection reveals that the dominant term takes ∇P to be scaled as $P_z \approx -\rho g$. This yields

$$\nabla p \cdot (\nabla\eta \times \nabla s) \sim \rho g J(\eta, s), \tag{18}$$

where J is the Jacobian operator with respect to x and y. We can also use the hydrostatic approximation to scale the integral in (17):

$$\int_p^{p_{\mathrm{ref}}} (2mb)\,\mathrm{d}p' \sim (p - p_{\mathrm{ref}})2mb$$
$$\sim \frac{\rho g D}{2} mb. \tag{19}$$

Here, D is the ocean depth and σ is taken to be referenced to a depth $D/2$, so that on average a parcel is a vertical distance of $D/4$ from the reference depth. MW86 estimate $\rho g b$ to be approximately $(15\,\mathrm{km})^{-1}$. Taking D to be 4 km then gives that

$$\int_p^{p_{\mathrm{ref}}} (2mb)\,\mathrm{d}p' \sim \frac{2m}{15}. \tag{20}$$

Substituting this, together with (18) and the definition of m back into (17) gives that

$$\mathscr{T} \sim \frac{2g}{15\rho^2}\frac{\partial\sigma}{\partial s}\frac{\partial\rho}{\partial\eta} J(\eta, s). \tag{21}$$

It remains to scale the Jacobian in (21). To simplify, we take the pressure dependence in ρ to be equivalent to a z dependence, so that ρ may be thought of as a function of z, η and s. Then,

$$\frac{\partial\rho}{\partial\eta} J(\eta, s) \approx J(\rho, s). \tag{22}$$

Care must be taken in scaling $J(\rho, s)$ to account for possible alignment between the density and salinity fields. That is, only that part of the horizontal gradient of s which is perpendicular to the horizontal density gradient contributes to the Jacobian. Thus,

$$J(\rho, s) \sim \frac{\Delta s}{L}|\nabla_{\mathrm{h}}\rho|, \tag{23}$$

where Δs is the range of salinity (over a lengthscale, L on a geopotential surface) for which density is constant. The horizontal density gradient can be scaled using the thermal wind relation:

$$|\nabla_{\mathrm{h}}\rho| \sim \frac{\rho f U}{gH}, \tag{24}$$

where f is the Coriolis parameter, U is the horizontal velocity scale and H is the vertical scale of the flow.

Substituting (22–24) back into (21) gives

$$\mathscr{T} \sim \frac{2}{15\rho}\frac{fU}{H}\frac{\partial\sigma}{\partial s}\frac{\Delta s}{L}. \tag{25}$$

To get a feel for when this might be significant, we estimate the fractional change in Π that results from $\mathscr{T}$ in a time, T_0:

$$\frac{\Delta\Pi}{\Pi} \approx \mathscr{T} T_0 \Pi^{-1}$$
$$\sim \left(\frac{2T_0}{15\rho}\frac{fU}{H}\frac{\partial\sigma}{\partial s}\frac{\Delta s}{L}\right)\left(\frac{fN^2}{g}\right)^{-1}, \tag{26}$$

where Π has been approximated by fN^2/g and N is the buoyancy frequency*. It is convenient to group terms in (26) that are essentially constant over the ocean together:

$$\frac{\Delta\Pi}{\Pi} \approx \left(\frac{2g}{15\rho}\frac{\partial\sigma}{\partial s}\right)\left(\frac{U\Delta s}{N^2 LH}\right) T_0$$
$$\approx (10^{-3}\,\mathrm{psu}^{-1}\,\mathrm{ms}^{-2})\left(\frac{U\Delta s}{N^2 LH}\right) T_0, \tag{27}$$

where we have taken $\partial\sigma/\partial s \approx 8 \times 10^{-4}\,\mathrm{psu}^{-1}\rho$.

* N^2 is approximately proportional to σ_z near the reference pressure to which σ is defined.

We now use (27) to estimate the importance of thermobaric forcing in various situations. Specifically, we consider the importance of $\mathscr{T}$ to the potential vorticity balance in the midlatitude eddy field and find $\mathscr{T}$ to be potentially important below the main thermocline. Because the magnitude of $\mathscr{T}$ scales in proportion to vorticity, it seems less likely that $\mathscr{T}$ will be important for basin scale flows (except, of course, in so much as the basin scale circulation is influenced by the mesoscale eddy field).

To consider the effect on the eddy field, we first need to estimate the relevant Δs. To do this using climatological data, we assume that temporal variance is caused primarily by eddies. Specifically, we take the standard deviation reported by Lozier et al. (1995) for the North Atlantic to approximate the (instantaneous) range in salinity over an eddy length scale*. Typically, this yields values of 0.03 psu in the thermocline and 0.01 psu for the deep ocean.

Other parameters are chosen as follows. For the upper ocean, a typical value of N^2 is $10^{-4}\,\mathrm{s}^{-2}$ and for the abyssal ocean $N^2 \sim 10^{-6}\,\mathrm{s}^{-2}$. The relevant T_0 in both cases is the eddy turnover time, $T_0 \sim L/U$. Also, to get a meaningful comparison, we note that the change in Π associated with advection of relative vorticity is $\mathrm{O}(R_0)\Pi$, where R_0 is the Rossby number. Since advection is an $\mathrm{O}(1)$ process at scales comparable to or smaller than the Rhines scale, $(U/\beta)^{1/2}$ (where β gives the meridional gradient of f), $\mathscr{T}$ must be considered part of the leading order balance (for the eddies) whenever $\Delta\Pi/\Pi$ approaches R_0 in magnitude. It is thus necessary to estimate the Rossby number. For the upper ocean, we take $0.1\,\mathrm{ms}^{-1}$ as a velocity scale, 50 km as a length scale and $7\times10^{-5}\,\mathrm{s}^{-1}$ as a midlatitude value for f. This leads to an R_0 of 0.03. For the deep ocean, both U and L are smaller, so that the Rossby number is probably comparable. Finally, we somewhat arbitrarily take 1000 m as reasonable value† for H to get that

$$\left.\frac{\Delta\Pi}{R_0\Pi}\right|_{\text{upper ocean eddies}} \sim 10^{-2}, \tag{28}$$

$$\left.\frac{\Delta\Pi}{R_0\Pi}\right|_{\text{deep eddies}} \sim 3\times10^{-1}. \tag{29}$$

We thus conclude that it is reasonable to dismiss thermobaric effects as unimportant to eddy field of the upper ocean; however, the same may not be true in the abyssal ocean.

It does not appear that $\mathscr{T}$ provides a significant potential vorticity source for gyre scale flows. Although $\Delta\Pi$ is larger for basin scale flows (since the relevant Δs is larger), the appropriate comparison is between $\Delta\Pi$ and $R_\mathrm{p}\Pi$, where R_p is the planetary number ($R_\mathrm{p}\equiv\beta L/f$). Given that R_p typically approaches unity for basin scales, one concludes that thermobaric forcing is relatively weak. Note, however that this does not necessarily imply that the basin scale flow is unaffected by thermobaricity. For example, frictional processes are also negligible at basin scales, but they affect integral properties which, in turn, constrain the large scale flow (see, for example, Pedlosky, 1996, ch. 2, and the references therein).

It is also of interest to compare the size of $\mathscr{T}$ directly to parametrizations of $\mathscr{N}$ typically used in models. For example, one might be interested in how $\mathscr{T}$ compares with that part of $\mathscr{N}$ associated with mixing. Assuming a constant eddy diffusivity for vertical diffusion of buoyancy leads to

$$\mathscr{N}|_{\text{vertical}} \sim \kappa_\mathrm{v}\frac{fN^2}{gH^2}. \tag{30}$$

Thus,

$$\frac{\mathscr{T}}{\mathscr{N}|_{\text{vertical}}} \sim \left(\frac{2g}{15\rho}\frac{\partial\sigma}{\partial s}\right)\left(\frac{UH\Delta s}{\kappa_\mathrm{v}LN^2}\right)$$
$$\sim(10^{-3}\,\mathrm{psu}^{-1}\,\mathrm{ms}^{-2})\left(\frac{H\Delta s}{T_0\kappa_\mathrm{v}N^2}\right), \tag{31}$$

where as before T_0 is the advective timescale, L/U. $\mathscr{N}|_{\text{vertical}}$ is thought to be a leading-order forcing of the large scale abyssal circulation and it thus makes sense to make the comparison using parameters appropriate to the abyssal circulation. For

* They report standard deviation of potential temperature. To convert to standard deviation of salinity, we use the rule of thumb that 0.015 psu corresponds to about 0.1°C. Also, they report values on surfaces of constant potential density, whereas the relevant Δs here is along a density curve on a geopotential surface. It is assumed that this difference does not lead to large errors in our estimates.

† Recall that H is the vertical scale of the motion and need not correspond to D, the ocean depth.

example, taking $N^2 \sim 10^{-6}\,\mathrm{s}^{-2}$, $H \sim 1000$ m, $\kappa_v \sim 10^{-5}\,\mathrm{m^2\,s^{-1}}$ and $T_0 \sim 10^{10}$ s (or about 300 years) leads to

$$\frac{\mathscr{T}}{\mathscr{N}|_{\text{abyssal}}} \sim 10\ \mathrm{psu}^{-1}\,\Delta s < 1\,. \tag{32}$$

Here, Δs is the range of values over an entire basin (for constant density on a geopotential) which is larger than the value of 0.01 reported for the variance. Again we conclude that therobaric forcing is probably small, but not obviously negligible.

We recognize that these scaling arguments should be treated with caution. Nevertheless, it seems fair to say that $\mathscr{T}$ is not so small as to be obviously negligible. Given this uncertainty, we submit that the problem merits further consideration. Below, we address the issue of how thermobaricity might affect the mesoscale eddy field (which seems plausible below the main thermocline). To do this, we derive a consistent set of balanced dynamics that are analogous to QG, but that allow for thermobaricity.

3. A quasigeostrophic system for a thermobaric ocean

In this section, we derive an analog to quasigeostrophy (QG) for a thermobaric fluid and discuss its conservation properties. In doing this, we do not mean to suggest that the additional dynamics introduced are necessarily of greater importance than other effects that are absent under the standard QG approximation. Obviously, one could ignore thermobaricity altogether and conclude that the assumptions leading to QG dynamics are violated in many problems of interest. For example, midlatitude gyres extend over sufficiently large lengthscales so that the Coriolis parameter varies by a near order one amount and perturbations to the stratification associated with the motion are often comparable to the rest state density gradient. Our motivation here is simply to construct a dynamical framework from which to consider how thermobaricity might affect geostrophic dynamics. The main conclusions for this section are outlined below.

(1) The dynamics can no longer be expressed as a single equation for the stream-function, ψ. Instead, we obtain a system of 2 equations and 2 unknowns, ψ and N^2, where the latter depends on composition of the water.

(2) The thermobaric forcing is scale dependent, being important mainly at high wavenumber end of the spectra.

(3) Energy is conserved only in the absence of mixing (including isopycnal mixing).

(4) Potential enstrophy is lost as a conserved scalar.

3.1. Basics

For a non-thermobaric fluid, Berisford and Marshall (1993) derive a potential vorticity conservation law for QG flow directly from Ertel's theorem. The potential vorticity (PV) that they derive differs from the usual QGPV by only a small amount, provided that the assumptions inherent to QG are respected. One might be tempted, therefore, to make the analogous set of assumptions on (17) to derive a QG equivalent to $\mathscr{T}$. This, however, involves choosing a specific reference pressure and the resulting $\mathscr{T}$ depends strongly on this choice, whereas the dynamics, of course, cannot. Our goal is to derive a consistent set of balanced dynamics that reduce to the standard quasi-geostrophic dynamics when thermobaricity is negligible. We thus proceed by analogy with the standard derivation of QG, which can be thought of as an expansion of the dynamics in a small parameter*, ε, about a rest state.

To begin, we first note that thermobaricity is related to an explicit dependence of the speed of sound, c, on the composition of the water. That is, assuming that $c(p, \eta, s)$ can be written as $\tilde{c}(p, \rho, s)$, then the thermobaric coefficient, b, is zero only if (c.f., MW86),

$$\frac{\partial \tilde{c}}{\partial s} = 0\,. \tag{33}$$

(For completeness, the equivalence between (33)

* The parameter, ε, is the largest of several nondimensional numbers, all of which must be small for the QG expansion to be valid. These include, $R_0 \equiv U/fL$, $R_\mathrm{p} \equiv \beta L/f$, $R_\omega \equiv \omega/f$ and $R_\rho \equiv R_0(f^2L^2/N^2H^2)$. The third of these filters gravity waves (ω^{-1} gives the time scale of the motion) and the fourth is equivalent to the requirement that perturbations to the rest state density profile remain small. Additional parameters relating to the amplitude of topography and frictional coefficients must also be small.

and the condition $b=0$ is clarified in *Appendix A*.) Horizontal and temporal variability can then arise in the base state buoyancy frequency* since this depends on c:

$$N^2 \equiv \frac{-g}{\rho_0}\frac{\mathrm{d}\bar{\rho}(z)}{\mathrm{d}z} - \frac{g^2}{c^2}. \tag{34}$$

Here, $\bar{\rho}(z)$ gives the base state stratification. This can be further simplified by noting that, since p and ρ are both functions of z to leading order, c can be though of as a function of z and s,

$$c \approx c_0(z) + c_1(z, s), \tag{35}$$

where s represents departures from a possible background salinity stratification, $s_0(z)$. Substitution into (34) and use of the fact that $c_0 \gg c_1$ yields:

$$N^2(z, s) \approx N_0^2(z) + N_1^2(z, s), \tag{36}$$

where

$$N_0^2 \equiv \frac{-g}{\rho_0}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}z} - \frac{g^2}{c_0^2}, \qquad N_1^2 \equiv \frac{2g^2}{c_0^2}\frac{c_1}{c_0}. \tag{37}$$

The ratio, α of N_1^2 to N_0^2 gives a measure of the importance of thermobaricity. When α_0 (a typical value of α) is comparable to ε, the leading order QG dynamics are unaffected and traditional QG is recovered. Typically α is small; however, values large compared to the (eddy) Rossby number are plausible, particularly near water mass boundaries in the abyssal ocean. Here, we take the point of view that α is large compared to ε and that an accuracy of $O(\varepsilon)$ is sought. That is, formally, we consider α to be $O(1)$. Nonetheless, in discussing the results it will be borne in mind that α is typically somewhat less than this.

Whether or not thermobaric effects are considered, the QG vorticity equation can be written as

$$\frac{\mathrm{D}}{\mathrm{D}t}(\nabla^2\psi + \beta y) + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\mathrm{D}\psi_z}{\mathrm{D}t}\right) = 0. \tag{38}$$

For the thermobaric case considered here, a predictive equation for N^2 (and hence for s) is needed to close the system. This can be found by using the fact that salinity is conserved in the absence of mixing†,

$$\frac{\mathrm{D}s}{\mathrm{D}t} = 0, \qquad \text{implying} \qquad \frac{\mathrm{D}}{\mathrm{D}t}N^2 = 0. \tag{39}$$

One can then commute the $\mathrm{D}/\mathrm{D}t$ operator in (38) with f_0^2/N^2, to get that

$$\frac{\mathrm{D}}{\mathrm{D}t}(\nabla^2\psi + \beta y) + \frac{\partial}{\partial z}\left(\frac{\mathrm{D}}{\mathrm{D}t}\frac{f_0^2}{N^2}\psi_z\right) = 0, \tag{40}$$

which together with (39) forms a closed system.

3.2. Conservation properties

3.2.1. Potential vorticity. To write out (40) as a potential vorticity equation, we first note that the ∂_z and $\mathrm{D}/\mathrm{D}t$ operators do not commute. Instead,

$$\frac{\partial}{\partial z}\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\mathrm{D}}{\mathrm{D}t}\frac{\partial}{\partial z} + J(\psi_z, \ ). \tag{41}$$

Using this, together with the fact that $J(\psi_z, \psi_z)$ vanishes identically gives

$$\frac{\mathrm{D}}{\mathrm{D}t}Q = \frac{-f_0^2}{2}J(\psi_z^2, N^{-2}), \tag{42}$$

where Q is defined by

$$Q \equiv \nabla^2\psi + \beta y + \frac{\partial}{\partial z}\frac{f_0^2}{N^2}\frac{\partial\psi}{\partial z}. \tag{43}$$

A few comments seem appropriate. First note that while Q is not conserved in a Lagrangian sense, it is conserved in an area integrated sense for many domains of interest. For example, there is no net potential vorticity forcing in a closed basin or on a doublely periodic domain. (The same is not true for Q_0, the standard QGPV, defined below.) Another comment is that the magnitude of the source term is inversely related to the horizontal scale of the salinity field. This is important since one generally expects small scales in the salinity field to develop. For example, if thermobaric effects are ignored (for the moment), then salinity evolves as would a passive scalar

* We emphasize that N^2 is defined in QG context with respect to the rest state density field (which cannot vary in the horizontal). We do not mean to suggest that horizontal variance of N^2 in the real ocean (and calculated using ρ_z in place of $\bar{\rho}_z$) is dominated by the salinity dependence in the speed of sound. The additional variance is, however, necessarily associated with motion, which is assumed weak in the QG context.

† Since we take the point of view that $O(1)$ horizontal variations of salinity are allowed, s_z/s_x scales like L/H. Then, since w scales like $O(\varepsilon)HU/L$, it follows that ws_z/us_x is $O(\varepsilon)$. Thus, vertical advection of salinity can be neglected in (39).

and, given a reasonably complex flow, one expects a cascade of salinity variance to small scales. As this happens, (42) suggests that the thermobaric production of Q "switches on". Once this occurs, salinity is no longer passive and the thermobaric source term becomes large. Thus, assuming thermobaric forcing to be small leads us to the conclusion that this is not the case. It should be emphasized, however, that for small α_0, the implied forcing is at high wavenumbers. For example, the thermobaric term is comparable to $J(\psi, Q)$ when the horizontal lengthscale, l, of the salinity field is smaller than the length scale, L, of the eddies by a factor of α_0.

A third point is that the definition of Q is altered with respect to the standard definition in that the explicit salinity dependence in N is retained. One might also be interested in the temporal evolution of the standard QG potential vorticity, Q_0 (defined by replacing N with N_0 in 43). After a bit of algebra, one gets that

$$\frac{\mathrm{D}}{\mathrm{D}t} Q_0 = -f_0 \frac{\partial}{\partial z}(w\alpha), \tag{44}$$

where

$$w = \frac{-f_0}{N^2} \frac{\mathrm{D}\psi_z}{\mathrm{D}t}. \tag{45}$$

To try to quantify the net effect of this forcing, we take the time integral of $(w\alpha)_z$ to scale like α_z multiplied by a typical vertical displacement (i.e., by εH). This then gives that

$$\Delta Q_0 \sim \varepsilon f_0 \frac{\alpha_0 H}{h} \sim Q_0 \frac{\alpha_0 H}{h}, \tag{46}$$

where h gives the dominant vertical scale of the salinity field and ΔQ_0 is the net change in potential vorticity. Thus, for small α_0 we expect O(1) changes in Q_0 only when h becomes small compared to the vertical scale of the flow. Put another way, it appears that thermobaricity is likely to have an O(1) effect on the Q_0 field at the high wavenumber end of the spectrum. This is similar to what was found above for Q, except that here the implied forcing is at high vertical wavenumber, whereas there, the implied forcing was at high horizontal wavenumber. A physical interpretation of how thermobaricity can lead to potential vorticity forcing at small scales is offered in Subsection 4.1.

3.2.2. Energy. To give us confidence that eqs. (39–40) form a consistent dynamical system, we look for a conserved energy-like quantity. It is straightforward to derive an energy equation by multiplying (40) by ψ and integration in the horizontal. One finds that

$$\frac{\partial}{\partial t} \int \left(\psi_z^2 + \psi_y^2 + \frac{f_0^2}{N^2} \psi_z^2 \right) \mathrm{d}V = 0. \tag{47}$$

Two points should be emphasized. Firstly, in deriving (47), it is necessary to use (39). In other words, changes in salinity due to mixing affect the energetics. As pointed out above, one expects small scales to develop in the salinity field. At some point, therefore, the neglect of mixing becomes unjustified and a conserved energy-like quantity can no longer be found. It is interesting that even mixing that does not directly affect the density field (and hence leaves the potential and geostrophic kinetic energies unchanged) can lead to a non-conservation of QG energy. A physical interpretation of this counter-intuitive result is given in Subsection 4.2.

A second point is that the definition of energy here differs from the standard QG energy in that N includes the explicit salinity dependence. The standard QG energy is not conserved. An equation describing its evolution can be found by noting that $N^2 = (1+\alpha)N_0^2$ and rewriting (47) as

$$\frac{\partial}{\partial t} \int \left(\psi_z^2 + \psi_y^2 + \frac{f_0^2}{N_0^2} \psi_z^2 \right) \mathrm{d}V = \frac{\partial}{\partial t} \int \alpha \frac{f_0^2}{N^2} \psi_z^2 \, \mathrm{d}V. \tag{48}$$

The associated loss (or gain) of standard QG energy will depend on whether a correlation develops between $\partial_t \alpha$ and ψ_z^2 or between $\partial_t \psi_z^2$ and α. Presumably, such correlations, if they were to develop at all, would occur more readily in the high wave number end of the spectrum (i.e., in the same part of the spectrum where thermobaric forcing becomes important).

3.2.3. Potential enstrophy. Not surprisingly, potential enstrophy is lost as a conservative scalar. For completeness and since there are implications for the inverse cascade of quasigeostrphic turbulence, we consider the potential enstrophy equation below. Before proceeding, however, it is convenient to rewrite the source term as the divergence of a flux. To do this, recall that in general

$$J(A, B) = -\nabla \cdot (A\hat{k} \times \nabla B). \tag{49}$$

Using this, (42) may be rewritten as*

$$\frac{\mathrm{D}}{\mathrm{D}t} Q = \nabla \cdot \tilde{\boldsymbol{v}} \tag{50}$$

where

$$\tilde{\boldsymbol{v}} \equiv \frac{-f_0^2}{2N^2} (\hat{k} \times \nabla \psi_z^2). \tag{51}$$

Multiplying (50) by Q and integrating over the domain, one gets that

$$\frac{\partial}{\partial t} Z = \int Q(\nabla \cdot \tilde{\boldsymbol{v}})\, \mathrm{d}V \tag{52}$$

where $\mathrm{Z} \equiv \int \mathrm{Q}^2\, \mathrm{d}V$.

Obviously, if α_0 is truly $\mathrm{O}(1)$, then there is no reason to expect Z to be conserved, even approximately. But we would like to know how a modestly sized α affects the potential enstrophy budget. Taking α_0 to be a small parameter, we can rewrite $\tilde{\boldsymbol{v}}$ as

$$\tilde{\boldsymbol{v}} \approx \boldsymbol{v}_0 + \boldsymbol{v}_1, \tag{53}$$

where $\boldsymbol{v}_0$ is given by (51) with N_0 replacing N and $\boldsymbol{v}_1 \equiv -\alpha \boldsymbol{v}_0$. Then, using the fact that $\nabla \cdot \boldsymbol{v}_0 = 0$, the enstrophy equation becomes

$$\begin{aligned} \frac{\partial}{\partial t} Z &= \int Q \tilde{\boldsymbol{v}}_0 \cdot \nabla \alpha\, \mathrm{d}V \\ &= \int \tilde{\boldsymbol{v}}_0 \cdot \nabla \alpha Q\, \mathrm{d}V - \int \alpha \tilde{\boldsymbol{v}}_0 \cdot \nabla Q\, \mathrm{d}V \\ &= -\int \frac{f_0^2}{2N_0^2} J(\psi_z^2, \alpha Q)\, \mathrm{d}V - \int \alpha \tilde{\boldsymbol{v}}_0 \cdot \nabla Q\, \mathrm{d}V \\ &= -\int \alpha \tilde{\boldsymbol{v}}_0 \cdot \nabla Q\, \mathrm{d}V, \end{aligned} \tag{54}$$

where the Jacobian term has vanished upon integration.

* The thermobaric forcing of Ertel's potential vorticity can also be written as the gradient of a vector field. This allows for (2) to be rewritten as $\partial(\rho\Pi)/\partial t + \nabla \cdot \chi = 0$, where $\rho\Pi\boldsymbol{u} + \omega_{\mathrm{abs}}\Psi + F/\rho \times \nabla\sigma + \sigma(\nabla P \times \nabla\rho^{-1})$. This is discussed in more detail elsewhere (Haynes and McIntyre, 1987; Marshall and Nurser, 1992).

The important point above is that, except for an α dependence in ∇Q, derivatives of α do not appear in the enstrophy source term. Hence when α_0 is small, we expect the net enstrophy source to be small. Using (51) and (54), it is straightforward to obtain a quantitative estimate of the source term:

$$\frac{1}{V} \frac{\partial}{\partial t} Z \sim \frac{\alpha_0 f_0^2 \psi^2 Q}{N_0^2 H^2 L^2}, \tag{55}$$

where V is the volume of the domain. Finally, taking $N_0^2 H^2 / f_0^2 \sim L^2$ and $\psi/L^2 \sim Q$ gives that

$$\frac{\partial}{\partial t} Z \sim \alpha_0 Q Z. \tag{56}$$

Since Q scales like εf_0, this implies that Z can change by an $\mathrm{O}(1)$ amount in a time $(\varepsilon \alpha_0 f_0)^{-1}$. For example, an ε of 0.1 and an α_0 of 0.2 gives a time scale of $50 f_0^{-1}$, or a bout a week at midlatidutes. We emphasize that this represents an upper estimate of the enstrophy source. That is, the scaling used to derive (56) assumes correlations between $\boldsymbol{v}_0 \cdot \nabla Q$ and α to be $\mathrm{O}(1)$. If these two fields are weakly correlated, then the net enstrophy source will be smaller.

As in the previous subsection, we are also interested in how standard QG is modified for the case of small α. That is, we would like to estimate the size of the source term for the standard QG potential enstrophy, Z_0. The Z_0 equation is found simply by multiplying (44) by Q_0 and integrating:

$$\frac{1}{V} \frac{\partial}{\partial t} Z_0 = \int Q_0 \frac{\partial}{\partial z} (f_0 \alpha w)\, \mathrm{d}V. \tag{57}$$

It is not obvious here, however, whether the net source is $\mathrm{O}(\alpha_0)$ (as suggested by 56 for Z) or whether the z derivatives of α in (57) might lead to a larger enstrophy forcing.

4. Discussion

The main point of this paper has been that thermobaric forcing of potential vorticity, although weak for many applications, is considerably larger than is generally supposed and may not always be negligible. The importance of thermobaricity is measured by a the nondimensional parameter, α_0, which can be usually be considered small with respect to unity. Below, we discuss possible consequences of a modest sized α_0 to two regimes of flow. Subsection 4.1 considers O(1) effects that can occur at length scales that are $O(\alpha_0)$ with respect to the energy containing eddies. Subsection 4.2 considers weaker effects that can influence the larger scales.

4.1. *Small-scale effects*

It is suggested that thermobaric forcing can have an O(1) affect on the potential vorticity field at the high wavenumber end of the spectrum, even if α_0 is relatively small. Our reasoning is that, for any reasonably complicated flow, an initially smooth salinity field should develop both small horizontal and vertical scales. Once horizontal scales in the salinity field become $O(\alpha_0)$ with respect to the scale of the energy containing eddies, the source term for Q to "switches on." Similarly, small vertical scales cause the source term for Q_0 to become large.

To get a better physical understanding of this, we consider the following thought experiment. Imagine a small "blob" of cold, low salinity water in the presence of a larger scale background eddy field. The potential density surface containing the low salinity blob is advected up and down by larger scale eddies. As it is raised, it decompresses slightly more than does the surrounding water and hence forms a negative density anomaly. Conversely, as it is lowered, the low salinity blob compresses slightly more. The net effect is analogous to that of a small oscillating "density source." If the timescale of the vertical advection is long compared to the inertial time scale, a geostrophic adjustment occurs. The net effect is that a positive "density source" (e.g., the lifting of a low salinity blob or the lowering of a high salinity blob) produces an anticlonic vortex. Similarly, a negative density source produces a cyclonic vortex.

Were the geostrophic velocity field induced in this manner to remain coupled to the salinity perturbation which produced it, there would be little net effect. That is, the sign of the forcing would simply reverse when the anomalous salinity water was brought back to its original position and the induced vortex would disappear. However, the evolution of the salinity perturbation and induced flow are described by different equations, so that there is no reason to expect two fields to remain coupled. For example, gradients in the background potential vorticity field (i.e., the part not associated with the thermobaric forcing) could provide a surrogate β-effect, allowing for the induced vortex to move away from the anomalous salinity blob (in a manner similar to the way that Rossby waves propagate away from the region where they are produced). Furthermore, the forward cascade of potential enstrophy in geostrophic turbulence suggests that such gradients in potential vorticity should be relatively strong. Assuming this decoupling to occur on a timescale short or comparable to that of the large scale flow, then the induced vortex is not eradicated when the low salinity blob is lowered back towards its original vertical position. In this way, thermobaric forcing can influence the vorticity field at small scales. This, in turn, may influence the internal gravity wave spectrum since interactions between these waves are affected by vorticies (Lelong and Riley, 1991; Bartello, 1995).

To conclude the thought experiment, we note that with each cycle of lifting and lowering, the salinity perturbation can be expected to get increasingly strung out until mixing eventually occurs. Once the anomaly is mixed, the thermobaric effect becomes irrelevant and the mixed water behaves as would a single constituent fluid. Interestingly, the small scale velocity field induced by thermobaric forcing aids in the mixing process (i.e., in the cascade of salinity variance to increasingly smaller scales). In this sense, thermobaric forcing helps to shut itself off.

We should also point out that α_z is likely to be smaller than the estimate of α/h which was used to derive (46). That is, although it is correct to base α_0 on the range of salinity over the entire domain in the QG expansion, the standard deviation at a point is typically small compared to this range. Physically, this implies that the cascade of s to scales at which mixing occurs must be fast compared to the "random walk" time required for a parcel to traverse the basin. Then, salinity is not conserved and although two parcels separated only by a small vertical distance, h, may have originated from opposite sides of the basin, this is not reflected in their salinities. A more correct scaling of α_z (i.e., that takes account of this salinity mixing) associated with the small salinity blobs would probably be to take α to scale according to its standard deviation. Hence, the ratio H/h must grow beyond that suggested by (46) before an

O(1) potential vorticity source is implied. Additionally this implies that this forcing mechanism is likely to be important only in specific areas, such as fronts between water masses, where the variance in s remains relatively high.

4.2. Large-scale effects

To the extent that α_0 can be considered small, one does not expect thermobaricity to have a significant direct affect on flow at large scales (i.e., ranging from the energy containing eddy scales to basin scales). Nevertheless, thermobaricity may affect the large scale flow indirectly by altering various integral constraints on the flow. For example, in the simpler context of wind driven barotropic gyres, model energetics and basin vorticity budgets have been linked to "inertial runaway" — i.e., to the fact that solutions are sensitive to poorly known frictional parameters and fail to converge in the limit that these parameters become small (e.g., Pedlosky (1996) and references therein). It seems reasonable to suspect that similar constraints could also play an important role in the stratified problem. Hence, to the extent that thermobaricity alters these constraints, it is likely to affect the large scale flow. As mentioned in Section 3, whether or not thermobaricity plays a role in the basin (potential) vorticity budget depends on whether one is considering Q or Q_0. When Q_0 is considered, there is a net source, whereas when Q is considered, the thermobaric forcing integrates to zero (for many domains of interest).

With regard to the energetics, there are 3 ways that thermobaricity may play a role, all of which appear to be at least potentially $O(\alpha_0)$. Firstly, the standard QG energy is simply not conserved in the thermobaric system (see eq. (48)). Next, the lack of a conserved potential enstrophy may affect the reverse cascade of geostrophic turbulence. This in turn, could conceivably allow for a small amount of energy to cascade forward to dissipative scales or could retard the rate at which energy cascades upscale. This is important since energy dissipation is facilitated. That is, without a forward cascade, one expects energy dissipation to be dominated by bottom friction (Holland, 1978) whereas any energy cascaded forward can be readily dissipated by scale dependent dissipation operators, even when the relevant coefficients are small. A third way that the energetics is affected by thermobaricity is in association with mixing, whether or not the density field is altered in the mixing process. That is, isopycnal mixing affects the energetics in a thermobaric QG fluid. The physical mechanism here is that the ratio of changes in internal energy to changes in potential energy (as a parcel moves vertically through the water column) depends on the speed of sound and hence on the composition of the water. In the QG context, this implies that isopycnal mixing alters the energy of the base state. That is, if one imagines mixing η and s in such a way that the density field is not affected, then neither the gravitational potential energy nor the geostrophic kinetic energy is affected. But, if one then rearranges the parcels in such a way that ρ is a monotonically decreasing function of z, there is no guarantee that the original profile, $\bar{\rho}(z)$, is recovered.

5. Acknowledgements

Numerous discussions with Tom Warn greatly benefited this work. My interest in this subject stems from conversations with Trevor McDougall. Susan Lozier and Eddy Carmack provided helpful information with regard to oceanographic data. Finally, helpful comments from 2 anonymous reviewers are also gratefully acknowledged. Financial support was received from the following funding agencies: NSERC, AES (Canada) and FCAR (Québec).

6. Appendix A

In this appendix, we show that the condition, (33), on the speed of sound is equivalent to the condition $b = 0$, where b is defined by (8). A similar equivalence, namely that $b = 0$ implies

$$\frac{\partial}{\partial s}\hat{c}(p, \sigma, s) = 0\,, \tag{A1}$$

was noted in MW86. The motivation for restating this result using ρ instead of σ is that the explicit salinity dependence in $\tilde{c}(p, \rho, s)$ is important to the discussion in Section 3.

Using (11) and (12), we have that

$$2mb = \left(\frac{\partial \rho}{\partial s}\right)^{-1} \frac{\partial^2 \rho}{\partial p\, \partial \eta} - \left(\frac{\partial \rho}{\partial s}\right)^{-2} \left(\frac{\partial \rho}{\partial \eta}\right) \frac{\partial^2 \rho}{\partial p\, \partial s} \tag{A2}$$

or

$$2b = \left(\frac{\partial \rho}{\partial \eta}\right)^{-1} \frac{\partial}{\partial \eta}\left(\frac{\partial \rho}{\partial p}\right) - \left(\frac{\partial \rho}{\partial s}\right)^{-1} \frac{\partial}{\partial s}\left(\frac{\partial \rho}{\partial p}\right). \tag{A3}$$

To relate this to the speed of sound, recall that

$$\left.\frac{\partial \rho}{\partial p}\right|_{\eta,s} = \frac{1}{c^2}. \tag{A4}$$

Substituting for $\partial\rho/\partial p$ in (A3) gives

$$-bc^3 = \left(\frac{\partial \rho}{\partial \eta}\right)^{-1} \frac{\partial c}{\partial \eta} - \left(\frac{\partial \rho}{\partial s}\right)^{-1} \frac{\partial c}{\partial s}. \tag{A5}$$

Above, both c and ρ are considered functions of p, η and s. Assume next that the speed of sound can also be written as a function of p, ρ and s:

$$c(p, \eta, s) = \tilde{c}(p, \rho(p, \eta, s), s). \tag{A6}$$

Then,

$$\frac{\partial c}{\partial \eta} = \frac{\partial \tilde{c}}{\partial \rho} \frac{\partial \rho}{\partial \eta}, \tag{A7}$$

$$\frac{\partial c}{\partial s} = \frac{\partial \tilde{c}}{\partial s} + \frac{\partial \tilde{c}}{\partial \rho} \frac{\partial \rho}{\partial s}. \tag{A8}$$

Substituting back into (A5) leads to

$$\frac{\partial \tilde{c}}{\partial s} = \left(\frac{\partial \rho}{\partial s}\right) bc^3. \tag{A9}$$

Hence, $b = 0$ implies that

$$\frac{\partial \tilde{c}}{\partial s} = 0 \tag{A10}$$

and the proof is completed.

REFERENCES

Bartello, P. 1995. Geostrophic adjustment and inverse cascades in rotating stratified turbulence. *J. of Atmos. Sci.* **52**, 4410–4428.

Berrisford, P. and J. C. Marshall, 1993. Quasigeostrophic potential vorticity in isentropic coordinates. *J. Atmos. Sciences* **50**, 778–782.

Ertel, H. 1942. Ein neuer hydrodynamischer Wirbelsatz. *Meterologische Zeitschrift* **59**, 277–282.

Gill, A. E. 1973. Circulation and bottom water production in the Weddell Sea. *Deep-Sea Res.* **20**, 111–140.

Gill, A. E. 1982. *Atmosphere-ocean dynamics.* Academic Press, 662 + xv pp.

Haynes, P. H., and M. E. McIntyre, 1987. On the evolution of vorticity and potential vorticity in the presence of diabatic heating and frictional or other forces. *J. of Atmos. Sci.* **44**, 828–840.

Holland, W. R. 1978. The role of mesoscale eddies in the general circulation–numerical experiments using a quasigeostrophic model. *J. Phys. Oceanogr.* **8**, 363–392.

Killworth, P. K. 1977. Mixing on the Weddell Sea continental slope. *Deep-Sea Res.* **24**, 427–448.

Lelong, M. Pascale and James J. Riley, 1991. Internal wave-vortical mode interactions in strongly stratified flows. *J. Fluid Mech.* **232**, 1–19.

Lozier, M. S., W. B. Owens and R. G. Currey, 1995. The climatology of the North Atlantic. *Prog. in Oceanogr.* **36**, 1–44.

Marshall, John C. and A. J. George Nurser, 1992. Fluid dynamics of thermocline ventilation. *J. Phys. Oceanogr.* **22**, 583–595.

McDougall, T. J. 1987a. Neutral surfaces. *J. Phys. Oceanogr* **17**, 1950–1964.

McDougal, T. J. 1987b. Thermobaricity, cabbelling and water mass conversion. *J. Geophys. Res.* **92**, 5448–5464.

Müller, P. 1995. Ertel's potential vorticity theorem in physical oceanography. *Reviews of Geosphys.* **33**, 1, 67–97.

Müler, P. and J. Willebrand, 1986. Compressibility effects in the thermohaline circulation: a manifestation of the temperature-salinity mode. *Deep-Sea Res.* **33**, 559–571.

Pedlosky, J. 1987. *Geophysical fluid dynamcis.* Springer-Verlag, 710 + xiv pp.

Pedlosky, J. 1996. *Ocean circulation theory.* Springer-Verlag, 443 + xi pp.

You, Yuzhu and Trevor J. McDougall, 1990. Neutral surfaces and potential vorticity in the world's oceans. *J. Geophys. Res.* **95**, Ch. 8. 13,235–13,261.