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Cloud radiative forcing ratio

An analytical model

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ABSTRACT

The ratio of the cloud radiative forcing at the surface to the cloud radiative forcing at the top of the atmosphere has been recently deduced from a set of satellite and surface measurements, and calculated using detailed radiative transfer models. We use an adding method of radiative transfer to derive an analytical expression for the cloud radiative forcing ratio. The advantage of an analytical expression is its simplicity. The analytical expression provides insight onto how the cloud radiative forcing ratio depends on surface albedo, cloud microphysics, and atmospheric composition.

1. Introduction

The absorption of solar radiation by the atmosphere, oceans, and land surfaces is a source of energy for atmospheric and oceanic flow. The total amount of solar radiation absorbed by the earth-atmosphere system can be deduced from satellite observations (Harrison et al., 1990). The fraction of the total energy absorbed individually by the atmosphere, land surfaces, and oceans is less certain. The major source of uncertainty is due to clouds (Kiehl, 1994; Wielicki et al., 1995). Clouds reflect a large fraction of solar radiation back to the space which is not available for absorption by the atmosphere below the clouds, and by the surface. Clouds also absorb a portion of solar radiation themselves. The total effect of clouds can be an increase or a decrease of solar radiation absorbed by an atmospheric column. The ratio of the cloud radiative forcing at the surface to the cloud radiative forcing at the top of the atmosphere, denoted R, has been recently deduced from a set of satellite and surface measurements and calculated using detailed radiative transfer models (Li and Moreau, 1996).

Our investigation is based on a simplified model that allows an analytical solution depending on the radiative properties of the surface, the cloud layer, and the atmospheric layers above and below the cloud. The conclusions are similar to those of Li and Moreau (1996), derived from detailed radiative transfer calculations.

We use an adding method and a plane parallel model to derive an analytical expression for the cloud radiative forcing ratio, R, as a function of various atmospheric parameters. The adding method (Van de Hulst, 1980; Goody and Yung, 1989) is a suitable formulation of energy conservation for a plane parallel atmosphere. If the optical properties of individual atmospheric layers are known, energy conservation can be used to calculate the optical properties of the combined layers. For radiative fluxes the adding method is equivalent to solving the radiative transfer equation.

2. Cloud radiative forcing

In the plane parallel atmosphere model, the atmosphere is homogeneous in its horizontal

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extent and physical properties are allowed to change only in the vertical direction. To treat the problem of partial cloudiness, the radiative fluxes for clear and cloudy sky conditions are calculated separately and the radiative fluxes for partial cloudiness are obtained as an average of the clear and cloudy sky cases.

Consider a three layer plane parallel atmosphere with clouds confined to the middle layer (Fig. 1). Let the transmittance and the reflectance of the layer above the cloud be T_a and R_a, and the transmittance and reflectance of the layer below the cloud be T_b and R_b. Let the transmittance, reflectance, and absorptance of the cloud layer be denoted by T_c, R_c and A_c. Let us further assume that the transmittance, reflectance, and absorptance functions of these layers are known. The vertical position of the cloud layer within the atmosphere (for given atmospheric composition) determines T_a and T_b. Each of the T_a, T_b, T_c, R_a, R_b, R_c and A_c is a function of the solar zenith angle and includes direct and diffuse components.

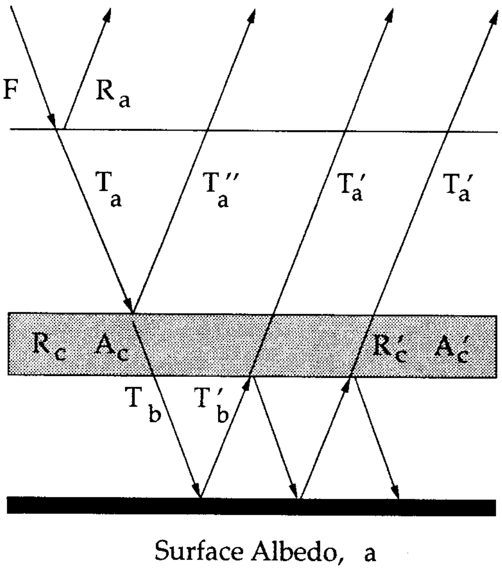

Fig. 1. The used model of the plane parallel cloudy atmosphere. *T*, *R*, and *A* stand for the transmittance, reflectance, and absorptance of individual layers. The subscripts $_a$, $_b$, and $_c$ refer to the atmospheric layer above, below, and containing the clouds. The primed symbols refer to the radiation reflected by the ground and the double primed symbols refer to the radiation reflected by clouds. The surface albedo is *a*.

The surface albedo is denoted by *a*, and *F* is the incident solar flux. All of the above defined quantities are functions of wavelength. The corresponding broadband quantities can be obtained by appropriate averages using the spectrum of incoming solar flux as a weighting function.

The broad band transmittance and reflectance differ from each other due to the different composition of the appropriate atmospheric layers, and to the different spectral distribution of the solar radiation entering the layer. Therefore, the transmittance through the same layer will be different for each multiple path of radiation through the layer. In the first approximation, the first path through the layer can be described by the T_a, T_b, T_c, R_a, R_b and R_c as specified above. The upper layer transmittance for radiation reflected by the cloud layer is denoted by T_a''. The transmittance and reflectance for radiation that is reflected at least once by the surface are denoted by primed symbols T_a', T_b', T'_c, R'_a, R_b' and R_c'.

With the above notation the downward and the upward solar fluxes at the surface and the top of the atmosphere, with the presence of a cloud layer, can be written as:

$$F^{\text{cld}}_{-\text{S}} = \frac{F T_a T_b T_c}{1 - a R_c' T_b'^2}, \tag{1}$$

$$F^{\text{cld}}_{+\text{S}} = \frac{a F T_a T_b T_c}{1 - a R_c' T_b'^2}, \tag{2}$$

$$F^{\text{cld}}_{-\text{T}} = F, \tag{3}$$

$$F^{\text{cld}}_{+\text{T}} = F R_a + F R_c T_a T_a'' + \frac{a F T_a'' T_b T_c T_a' T_b' T_c''}{1 - a R_c' T_b'^2}, \tag{4}$$

where the minus and plus symbols stand for the downward and upward directions and the $_\text{S}$ and $_\text{T}$ subscript denote the surface and the top of the atmosphere, respectively. In the derivation of these expressions, multiple reflections between individual layers are neglected with the exception of the multiple reflections between the surface and cloud layer.

Similarly, for the clear sky case, *t* and *r* denote the clear sky transmittance and reflectance for the incoming solar radiation. The upward transmittance of radiation reflected by the surface is denoted by t'. The multiple reflections by the surface and the clear atmosphere are neglected. The surface and the top of the atmosphere fluxes under the clear sky conditions are now given by

$$F^{\rm clr}_{-S} = Ft\,, \quad (5)$$

$$F^{\rm clr}_{+S} = aFt\,, \quad (6)$$

$$F^{\rm clr}_{-T} = F\,, \quad (7)$$

$$F^{\rm clr}_{+T} = Fr + aFtt' \quad (8)$$

To evaluate the broad band clear sky atmospheric transmittance a model described by Brine and Iqbal (1983) that includes Rayleigh scattering, water vapor, ozone, and carbon dioxide absorption is used with an atmospheric profile close to the midlatitude summer atmosphere. The solar spectrum is divided into 14 spectral bands (Winter and Chylek, 1997) and the transmittance is calculated within each band separately. The total broad band transmittance is obtained by averaging spectral band transmittance using the incoming solar flux within each band as a weighting function. For the downwards clear atmosphere transmittance at zero and 60° solar zenith angle we obtain $t=0.78$ and 0.72 respectively. For the upward transmittance $t'=0.90$ and 0.86 is obtained, assuming that the water vapor absorption bands are saturated after the first pass.

The cloud radiative forcing is defined as $\mathrm{CRF} = F^{\rm all} - F^{\rm clr}$ where F denotes the net flux ($F = F_- - F_+$). The average all sky radiative fluxes are approximated, in the plane parallel atmosphere, by an average of the cloudy and clear sky fluxes: $F^{\rm all} = NF^{\rm cld} + (1-N)F^{\rm clr}$ where N is fractional cloud cover. The surface cloud radiative forcing, $\mathrm{CRF_s}$, is obtained using eqs. (1) to (4):

$$\mathrm{CRF_S} = \mathrm{FN}(1-\mathrm{a})\left(\frac{\mathrm{T_a T_b T_c}}{1-\mathrm{a}R'_\mathrm{c}\mathrm{T}'^2_\mathrm{b}} - \mathrm{t}\right). \quad (9)$$

Similarly, the top of the atmosphere cloud radiative forcing, $\mathrm{CRF_T}$, is:

$$\mathrm{CRF_T} = FN\left(r - R_\mathrm{a} + att' - R_\mathrm{c} T_\mathrm{a} T''_\mathrm{a} - \frac{aT_\mathrm{a} T_\mathrm{b} T_\mathrm{c} T'_\mathrm{a} T'_\mathrm{b} T'_\mathrm{c}}{1 - aR'_\mathrm{c} T'^2_\mathrm{b}}\right). \quad (10)$$

3. Cloud radiative forcing ratio

The cloud radiative forcing ratio defined by (Cess et al., 1995; Ramanathan et al., 1995; Li et al., 1996):

$$R = \frac{\mathrm{CRF_S}}{\mathrm{CRF_T}}, \quad (11)$$

can now be written, using eqs. (9) and (10), as

$$R = \frac{(1-a)[T_\mathrm{a} T_\mathrm{b} T_\mathrm{c} - t(1 - aR'_\mathrm{c} T'^2_\mathrm{b})]}{(1 - aR_\mathrm{c} T'^2_\mathrm{b})(att' + r - R_\mathrm{a} - R_\mathrm{c} T_\mathrm{a} T''_\mathrm{a}) - aT_\mathrm{a} T_\mathrm{b} T_\mathrm{c} T'_\mathrm{a} T'_\mathrm{b} T'_\mathrm{c}}. \quad (12a)$$

The ratio, R, characterizes the change in the amount of solar radiation absorbed by a clear sky and cloudy atmospheric column. $R>1$ (or $R<1$) means that the cloudy atmospheric column absorbs more (or less) solar radiation than the clear sky column.

From the form of eq. (12) it is apparent that the cloud radiative forcing ratio R is independent of the amount of cloudiness, N. This is a direct consequence of how partial cloudiness is treated in the plane parallel atmosphere model. Numerical results obtained from detailed radiative transfer models (Li and Moreau, 1996) are in agreement with this conclusion.

The dependence of R on the solar zenith angle, cloud microphysics, cloud top and base heights, surface albedo and other parameters is implicit within the various symbols used in eqs. (9–10). Comparison with a four-stream radiative transfer calculation suggests that the accuracy of results given by eq. (12a) is about 10%.

For small values of surface albedo the cloud radiative forcing ratio can be approximated by

$$R = \left[\frac{T_\mathrm{b}}{T''_\mathrm{a}}\left(1 + \frac{A_\mathrm{c}}{R_\mathrm{c}}\right) - \frac{T_\mathrm{a} T_\mathrm{b} - t}{R_\mathrm{c} T_\mathrm{a} T''_\mathrm{a}}\right] \times \left(1 + \frac{r - R_\mathrm{a}}{R_\mathrm{c} T_\mathrm{a} T''_\mathrm{a}} + ad\right) \quad (12b)$$

where ad represents a small correction term with

$$d = \frac{tt'}{R_\mathrm{c} T_\mathrm{a} T'_\mathrm{a}} + \frac{T_\mathrm{b} T'_\mathrm{b}}{R_\mathrm{c}}(1 - R_\mathrm{c} - A_\mathrm{c})(1 - R'_\mathrm{c} - A'_\mathrm{c}) - 1 - \frac{T_\mathrm{b} T'^2_\mathrm{b}}{T_\mathrm{a}}(1 - R_\mathrm{c} - A_\mathrm{c}). \quad (13)$$

The terms proportional to $(T_\mathrm{a} T_\mathrm{b} - t)$, $(r - R_\mathrm{a})$, and the ad term in eq. (11) are all small compared to one (for clouds that are not geometrically too thick). Consequently, the cloud radiative forcing ratio can be, in the first approximation, written as

$$R = \frac{T_b}{T_a}\left(1 + \frac{A_c}{R_c}\right). \tag{12c}$$

Eq. (12c) is a crude approximation which reduces the physics of the problem into a simple transparent form. In the first approximation the cloud radiative forcing ratio depends only on the ratio of the broad band atmospheric transmittance of the below the cloud layer to that of the above the cloud layer, and on the ratio of cloud absorptance to cloud reflectance. The first term depends on a location of the cloud in the atmosphere, on the distribution of gases and aerosols in the atmosphere and on the solar zenith angle. The second term depends on cloud microphysics, geometrical parameters of a cloud and on the solar zenith angle.

4. Discussion

4.1. Thick clouds

First consider the case of a thick cloud. Assume that most of the atmospheric mass is within the cloud layer. Consequently the atmospheric layer above the cloud is assumed to be completely transparent to incoming solar radiation. Thus we have $T_a \simeq T'_a \simeq T''_a \simeq 1$. Most of the absorption due to water vapor and carbon dioxide as well as most of the Rayleigh molecular scattering takes place within the cloudy layer. Consequently, the atmospheric layer below the cloud will have a transmittance also close to one, leading to the approximation $T_b \simeq 1$. Finally, the reflectance of the atmospheric layer above the cloud will be small and the approximation $R_a \simeq 0$ is used. Thus, for a thick cloud we obtain

$$R^{\text{thick}} = \left(1 + \frac{A_c}{R_c} - \frac{1-t}{R_c}\right)\left(1 + \frac{r}{R_c} + ad\right). \tag{14}$$

To calculate the range of possible numerical values for cloud radiative forcing ratio, R, we consider the cloud layer reflectance in the range from 0.4 to 0.8 and the cloud layer absorptance in the range from zero to 0.4. The clear atmosphere downward transmittance, t, for the incoming solar radiation is taken to be $t = 0.78$ and the clear atmosphere reflectance $r = 0.06$. Three values of surface albedo ($a = 0.0$, 0.05 and 0.10) are considered.

Numerical results are displayed in Fig. 2. In general, the cloud radiative forcing ratio, R, shows a strong dependence on the cloud layer absorptance and only weak dependence on the cloud layer reflectance. Changes in the value of R due to a nonzero surface albedo are relatively small, as expected from eqs. (12b) and (12c). For the range of cloud layer absorptance between 0.10 and 0.16, R is between 0.8 and 1.0. To obtain values around 1.5 (as deduced from observational data analysis for the southern Pacific by Ramanathan et al., 1995), cloud layer absorptance above 30% is needed. The maximum value of a thick cloud layer absorptance obtained in plane parallel atmosphere models is around 0.20. To increase this value to 0.30 requires some un-accounted for mechanism, like finite inhomo-

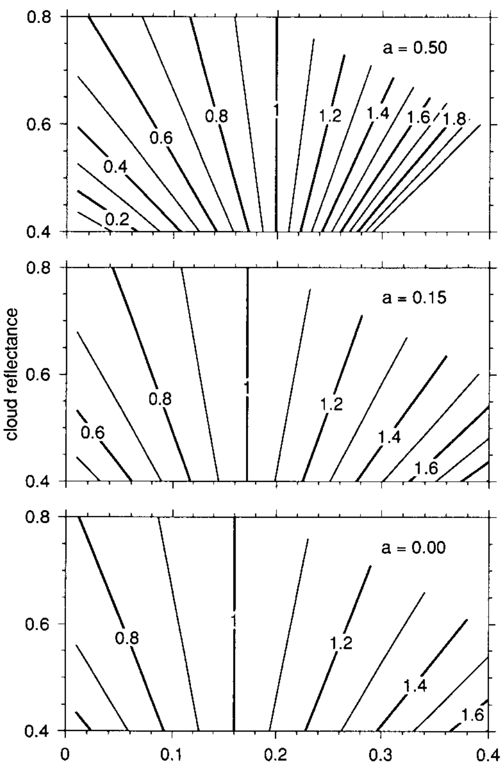

Fig. 2. The cloud radiative forcing ratio, R, for a geometrically thick cloud and zero solar zenith angle, as a function of cloud layer absorptance and reflectance for surface albedos of 0.50 (top panel), 0.15 (middle panel), and zero (bottom panel).

geneous cloud effects, the presence of large drops, black carbon, or water vapor clusters (Chylek and Geldart 1997).

4.2. High clouds

The height of the cloud top determines the transmittance of the atmospheric layer above the cloud. For a thin cloud near the top of the atmosphere $T_a \simeq T''_a \simeq 1$ and $T_b \simeq t$. The reflectance of the atmospheric layer above the cloud can be approximated by $R_a \simeq 0$. The cloud radiative forcing ratio in this case reduces to:

$$R^{\mathrm{Hcld}} = t\left(1 + \frac{A_c}{R_c}\right)\left(1 + \frac{r}{R_c} + ad\right). \tag{15}$$

4.3. Low clouds

For a low cloud, the absorptance of the atmospheric layer below the cloud can be taken as $T_b \simeq 1$. The exact value of T_a and T''_a depends on atmospheric composition. For numerical calculation we consider values of $T_a \simeq T''_a \simeq 0.90$. The reflectance of the atmospheric layer above the cloud is approximated by the reflectance of the clear sky atmosphere $R_a \simeq r$. In this case, the cloud radiative forcing ratio is simplified as:

$$R^{\mathrm{Lcld}} = \frac{1}{T''_a}\left(1 + \frac{A_c}{R_c}\right)(1 + ad). \tag{16}$$

The major change compared to the high cloud case is a factor $1/T_a$ in eq. (16) instead of t in eq. (15). Since the clear atmosphere transmittance for the downward solar radiation is around 0.78 for an overhead sun, the cloud radiative forcing ratio for low clouds will be higher than that for high clouds. The numerical values are shown in Fig. 3 as a function of the above the cloud layer transmittance of T_a.

The low level cloud with the above the cloud layer atmospheric transmittance $T_a = 0.9$ and a cloud layer absorptance between 0.10 and 0.20 leads to values of R between 0.9 and 1.2. On the other hand, if the above the cloud atmospheric transmitance is reduced to $T_a = 0.80$, the R increases to values between 1.5 and 1.7.

If smoke from tropical biosphere burning gets into and above the low level cloud layer, it will increase the cloud layer absorptance and at the same time decrease the transmitance, T_a, of the above the cloud layer. Both of these effects will cause R to increase. Smoke above the cloud layer cannot be easily removed by washout and may stay in the atmosphere for some time. Thus tropical region biosphere burning may contribute significantly to the increase of R for low level clouds.

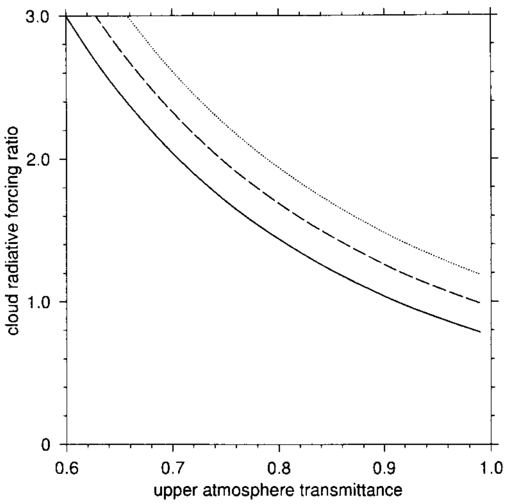

Fig. 3. The cloud radiative forcing ratio, R, for a low cloud over a zero surface albedo as a function of transmittance of the layer above the cloud. The cloud reflectance is $R_c = 0.50$ and the cloud absorptance $A_c = 0.10$ (solid line), 0.20 (dashed line) and 0.30 (dotted line).

5. Summary

By using a three layer plane parallel model atmosphere over a reflecting surface, analytical expressions (12a) to (12c) for the cloud radiative forcing ratio, R, are derived. The ratio R has an especially simple form for the case of small surface albedo (eqs. (12b) and (12c)). The advantage of these analytical expressions is their simplicity. Effects of cloud radiative properties, surface albedo and atmospheric transmission can be separated and their individual effects on the ratio R can be evaluated.

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