On the Budget of Angular Momentum in the Zone between Equator and 30° N

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Abstract

A complete budget of angular momentum in the atmosphere is possible only by considering both the eddy transfer of angular momentum and the transport due to "mean meridional circulations". An attempt has been made in the present paper to achieve such a complete budget for the zone 20—30°N using the data available. The mean mass transport in the tropical circulation cell accumulates high relative momentum (W wind) in the upper atmosphere of this belt, whereas the eddy transport distributes it northwards and downwards from this subtropical jet.

1. Introduction

Several attempts have recently been made to explain the maintenance of the zonal atmospheric circulation as a result of horizontal and vertical flux of relative angular momentum primarily due to large-scale eddies (MINTZ, 1951; PRIESTLEY, 1951; STARR, WHITE, 1952; WIDGER, 1949). Whereas these attempts seem to give rather satisfactory results at higher latitudes, they have encountered considerable difficulties in the tropical and subtropical belts. As shown by meridional cross sections (MINTZ, 1951; PETTERSSEN, 1950), the strongest westerlies aloft during winter are, on the average, located around latitude 25°—30° N. According to Y. MINTZ (1951) and W. K. WIDGER (1949), this latitude is a zone where the northward angular-momentum flux due to large-scale eddies results in a momentum flux divergence. These eddies therefore tend to destroy the mean hemispheric "jet stream". Other mechanisms of momentum transfer are thus required to compensate for this divergence. In order to avoid this dilemma, Widger postulates the existence of an upward eddy flux of momentum in the subtropical belt.

As pointed out by PRIESTLEY (1951) and (PALMÉN 1951), a complete momentum budget necessitates the inclusion of a mean meridional circulation. Such a circulation, if existing, is instrumental in transforming absolute angular momentum ("ω-angular momentum") into relative angular momentum ("u-angular momentum") of the subtropical jet stream. The further poleward transport of u-momentum from the belt of strong zonal motion along the poleward margin of the tropical circulation cell can then primarily be regarded as some kind of eddy flux, in agreement with principles developed by V. P. STARR (1948), J. BJERKNES (1948), and others.

The aim of the present paper is to investigate the effect of various mechanisms of momentum transfer operating in the lower latitudes. Since no new empirical data are presented here, the paper should be regarded primarily as an attempt to discuss the principles of a complete momentum budget by considering empirical data already in existence. Further, since the best data available are for the month of January, we shall limit our discussion to this month. The general principles, however, can presumably be extended to other months of the year.

Our budget will consider the balance between the following quantities in the belt 0—30° N:

a. The horizontal and vertical flux of angular momentum due to the mass transport resulting from a mean meridional circulation.

b. The horizontal and vertical eddy flux of angular momentum.

c. The angular momentum transfer from earth to atmosphere due to the surface stress of the wind.

2. Mass transport in the tropical circulation cell

Although the mean meridional circulation plays an essential role in all classical theories of the general circulation, very few attempts have been made to compute the mass transport in this mean circulation from real wind data. The discrepancy between the results achieved by C. H. B. Priestley (1951) and by V. P. Starr and R. M. White (1952) concerning the net meridional air flow in the free atmosphere indicates that the network of upper wind stations probably is not good enough for that purpose. By use of surface wind data, however, reasonably good values of the mass transport in the surface layer of the tropical and subtropical belts of both hemispheres can be computed, as H. Riehl and T. C. Yeh (1950) have shown. The greatest difficulty, however, arises when one wants to estimate the change of this horizontal mass transport along the vertical.

In order to obtain a minimum estimate of the masses involved in the meridional circulation, Riehl and Yeh assumed that the mean equatorward wind component decreases linearly with height and vanishes at 1 km. They, however, pointed out that the meridional circulation, computed under that assumption, obviously gives too small values.

Several investigations indicate that the meridional wind component does not essentially decrease from the surface up to about 950 or 900 mb. A study by Riehl and collaborators (1951) showed this to be the case in the trade-wind zone of the eastern Pacific. Also, an investigation by K. M. Nagler, not yet published, gives similar indications. In order to estimate the mass transport involved, we shall therefore use the values of the surface flow during January, given by Riehl and Yeh in the above-mentioned paper. But concerning the vertical extension, we shall adopt two tentative assumptions:

Model a: The surface meridional wind as computed by Riehl and Yeh is constant up to 900 mb and decreases then linearly to zero at 700 mb.

Model b: The surface meridional wind decreases linearly with height and reaches the value zero at 800 mb.

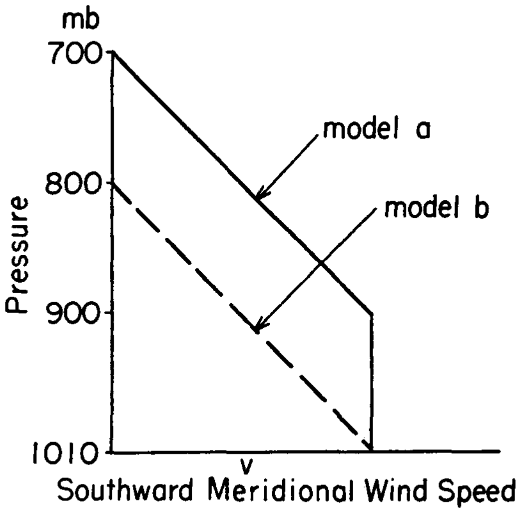

Fig. 1. Model (a) and (b) for the meridional wind component between 1010 and 700 mb.

The mass transport $[m]_\varphi$ across a given latitude φ between the isobaric surfaces p_1 and p_2 is

$$[m]_\varphi = \frac{2\pi a \cos\varphi}{g} \int_{p_2}^{p_1} v\, dp. \qquad (1)$$

Here g is the acceleration of gravity, a the earth's radius, and v the mean meridional wind component at an arbitrary isobaric surface at latitude φ. If $\bar{v}$ denotes a "mean value" of the meridional component between the isobaric surfaces p_1 and p_2 the mass transport between these isobaric surfaces can be expressed by

$$[m]_\varphi = \frac{2\pi a \cos\varphi}{g} \bar{v}(p_1 - p_2). \qquad (2)$$

Fig. 1 gives the profiles of the meridional wind for our two schematic models (a) and (b). The quantity $\bar{v}$ $(p_1 - p_2)$ is determined by the area between the wind profiles and the coordinate axes. If we assume that the mean surface pressure is 1010 mb, it is easy to see that in model (a) the equatorward mass flux is two times that corresponding to model (b). Since no schematic assumption can exactly describe the distribution of meridional wind with height in the total belt Equator — 30° N, the above assumptions were selected in order to give a simple comparison.

Table 1 gives values of the southward mass transport at different latitudes computed on the basis of model (a), if the surface wind v_0 is assumed to be equal to the values for January computed by RIEHL and YEH (1950).

Table 1: Southward mass transport $[m]_\varphi$ (unit 10^6 tons/sec.) in the layer between 1010 and 700 mb in the belt 30—0° N computed on the basis of model (a).

Latitude	30°	25°	20°	15°	10°	5°	0°
$[m]_\varphi$	58	84	126	183	190	134	67

The vertical mass flux through the upper boundary can be determined by the difference in horizontal flux across the parallels of two latitudes, if one assumes that there is no change in mass distribution. The vertical mass transports through the 700-mb surface are presented in Table 2:

Table 2: Vertical mass transport $[m]_\varphi$ (unit 10^6 tons/sec.) through the isobaric surface 700 mb in the belt 30—0° N computed on the basis of model (a).

Latitude	30—25°	25—20°	20—15°	15—10°	10—5°	5—0°
$[m]_\varphi$	— 26	— 42	— 57	— 7	56	67

The mass fluxes according to model (b) can be obtained from Tables 1 and 2 by dividing the corresponding values by two. In this case, of course, the horizontal mass transport will be for the layer between the surface and 800 mb, and the vertical mass transport will be that across the 800-mb surface.

3. Flux of angular momentum due to meridional circulation

If we multiply the quantities in Tables 1 and 2 by the total angular momentum per unit mass

$$\Omega = \omega a^2 \cos^2\varphi + u a \cos\varphi \qquad (3)$$

we obtain the horizontal and vertical transport of angular momentum across the horizontal and vertical boundaries in question. The angular momentum transport can be expressed as the sum of two terms, one giving the transport of ω-angular momentum, the other that of u-angular momentum. In view of the observed zonal wind speeds, the second term is small compared to the first.

Since we assume a steady state, the southward mass transport in the lower layers must be compensated by an equal northward mass transport above the level of reversal of the meridional circulation (700 mb in model [a], and 800 mb in model [b]). Consequently, if we neglect the small effect of the variation in the distance from the earth's axis, the intensity of flow of ω-angular momentum must be the same, though its direction is opposite, in the two layers. The net transport of ω-angular momentum across each latitude circle is therefore approximately zero when summed from the earth's surface to the upper limit of the atmosphere; any net poleward transport of angular momentum across an arbitrary latitude is then determined solely by the flow of u-angular momentum.

The angular momentum budget due to the meridional circulation computed from model (a) is presented in Figure 2. The arrows showing the horizontal and vertical fluxes of angular momentum point in the direction of the mean air movement. The erect figures give the flux of ω-momentum and the slanting figures the flux of u-momentum. In computing the latter values the distribution of zonal wind according to MINTZ (1951) was used. Since Mintz' computation was not extended to the upper limit of the atmosphere and to the

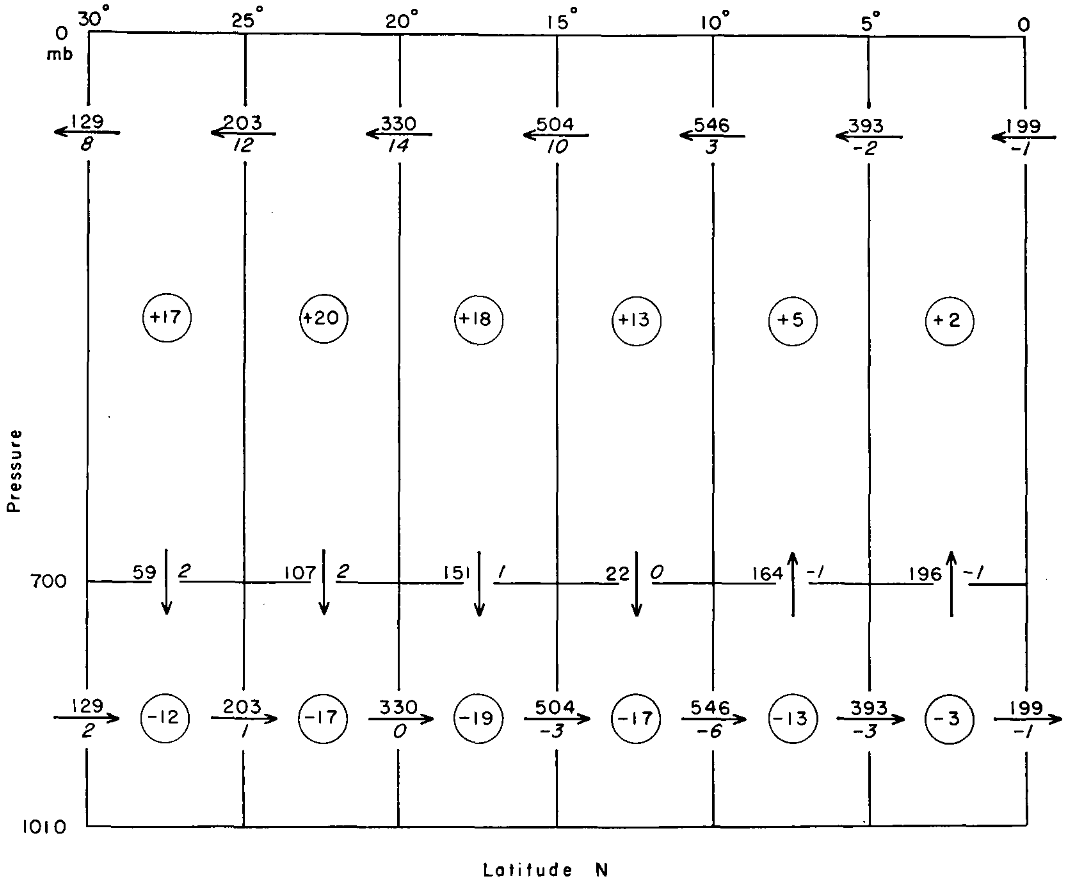

Fig. 2. Flux of ω-angular momentum (erect figures) and u-angular momentum (slanting figures) due to the meridional circulation computed from model (a) in Fig. 1. Encircled figures give the surplus or deficit of angular momentum in each block. Unit: 10^{25} g cm^2 sec^{-2}.

Equator, the missing data were extrapolated. The possible errors made in the extrapolation are of relatively minor importance. If u is negative (east wind) the positive transport of momentum is naturally in a direction opposite to the arrows. All values in Figure 2 are given in units of 10^{25} g cm^2 sec^{-2}. The encircled figures in the different blocks represent the difference between inflow and outflow of angular momentum; thus, they represent the generation or loss of angular momentum in each block as a result of the meridional circulation.

The following conclusions can be drawn from Figure 2:

1) The total deficit of angular momentum in the whole lower layer, represented by the sum of the encircled negative numbers is 81×10^{25} g cm^2 sec^{-2}, while the total surplus in the upper layer is 75×10^{25} g cm^2 sec^{-2}. The difference, 6×10^{25} g cm^2 sec^{-2}, represents the net flux of angular momentum due to meridional drift of air at latitude 30° N. The same result can be obtained by subtracting the slanting figures in the upper and lower layers at latitude 30° N.

2) The angular momentum transferred from the earth to the atmosphere in the belt 30°—0° N, due to the wind stress in the easterlies, is, according to PRIESTLEY (1951) and WIDGER (1949), much greater than the net northward flux of momentum of 6×10^{25} g

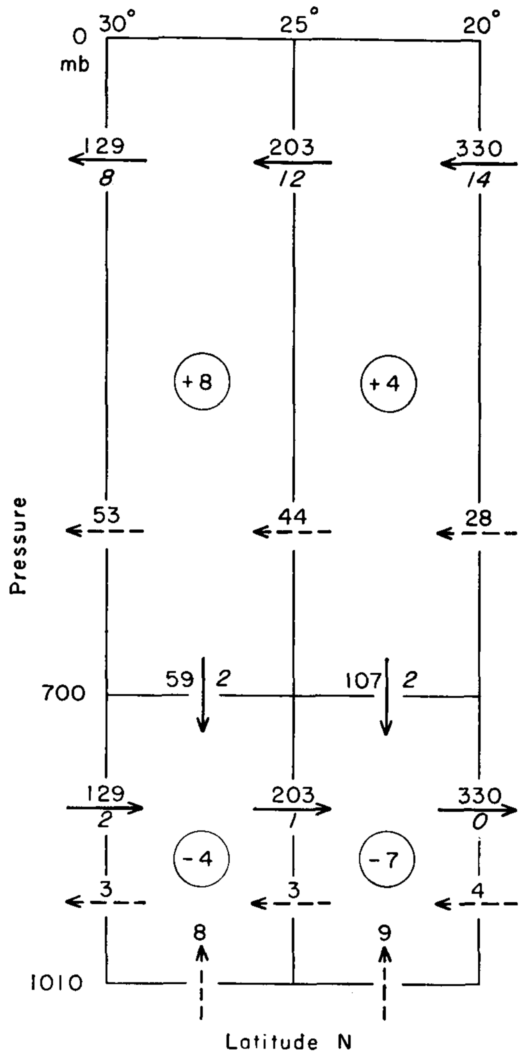

Fig. 3. Budget of angular momentum in the belt 30—20°N computed by use of model (a) and considering the horizontal eddy flux of angular momentum according to Mintz and the transfer from earth according to Widger. Encircled figures give the surplus or deficit of angular momentum if the vertical eddy flux is not considered. Unit: 10^{25}g cm² sec—².

cm² sec⁻². Thus, one can conclude that the meridional circulation computed from our model (a) does not give any satisfactory angular momentum budget. The same can naturally be said about every circulation with weaker mass transport, e.g., our model (b). This negative result is in good agreement with conclusions recently made by STARR and WHITE (1952).

4. Total budget of angular momentum

In order to obtain a satisfactory momentum budget we have to add to the data presented in Figure 2 the horizontal and vertical eddy flux of angular momentum, and also the transfer of angular momentum due to the zonal stress of the surface wind. For the horizontal eddy transfer the data recently published by MINTZ (1951) can be used. These data, which were computed on the assumption that the geostrophic wind approximates the real wind in the free atmosphere, are restricted to the north of latitude 20° N. For the transfer of angular momentum from earth to atmosphere, values computed by WIDGER (1949) are utilized. These values are based on a constant ratio between the geostrophic wind, as determined from the surface isobars, and the actual wind. As shown by C. L. JORDAN (1952), this assumption is quite satisfactory southwards to around latitude 20° N.

In view of the above, we shall confine our complete momentum budget to the zone 30—20° N. Since the data for the geostrophic horizontal transfer of angular momentum computed by Mintz do not extend above 100 mb, the remaining transport above this level has been extrapolated. By this extrapolation we assumed that the flux of angular momentum linearily approaches zero at the 0-mb surface.

In Figure 3 the total budget, with the exception of the vertical eddy flux, is represented on the basis of model (a). The horizontal eddy fluxes and the transports from earth to atmosphere are here represented by broken arrows. In the lower layer between latitude 30 and 20° N the deficit of angular momentum amounts to 11×10^{25} g cm² sec⁻². If our budget is exact, this should correspond to the excess of angular momentum in the upper layer and represent the vertical eddy flux of momentum downwards across the 700-mb surface. Figure 3 shows that the excess in the upper layer is 12×10^{25} g cm² sec⁻². Thus, in spite of the approximate nature of all data used, a very close balance is obtained.

Of great interest is the fact that throughout the belt considered, the vertical eddy flux of momentum is directed downwards across the 700-mb surface, i.e., in the direction from higher to lower u-momentum. This result should be compared with the streamlines for angular momentum flux given by WIDGER (1949). It might be pointed out that our value of the vertical eddy flux of angular momentum is computed from continuity considerations and that it therefore includes everything from molecular viscosity to the effect of large-scale atmospheric disturbances. If A denotes the area of the latitude belt 30—20° N the vertical transfer of angular momentum $[M]_z$ can be written

$$[M]_z = \overline{\varrho u' w'}\, A\, a \cos\varphi, \qquad (4)$$

or if we use a coefficient of eddy viscosity μ,

$$[M]_z = -\overline{\mu \frac{\partial u}{\partial z}}\, A\, a \cos\varphi. \qquad (5)$$

In Equation (4) u', w' denote the fluctuations in zonal and vertical velocity. Using the mean value of 11.5×10^{25} g cm^2 sec^{-2} for the downward eddy transfer of angular momentum through the 700-mb surface we get for the coefficient μ the value 120 g cm^{-1} sec^{-1} if $\frac{\partial u}{\partial z} = 4 \times 10^{-3}$ sec^{-1}

This value is considerably smaller than the value computed by RIEHL and collaborators (1951) for the trade wind belt of the Eastern Pacific Ocean. By order of magnitude it is, however, in agreement with the latter and also with values generally accepted for the free atmosphere above the frictional layer.

There are indications of a positive correlation between u' and w' in the large-scale eddies represented by cyclones, anticyclones, long waves and so on. Such a correlation would, according to Equation (4), result in upward momentum transfer. It is, however, easy to see that such an organized motion could hardly transport upwards more than a relatively small part of the angular momentum transferred from the earth, which according to Widger, amounts to 17×10^{25} g cm^2 sec^{-2}.

From this, it follows that there must be a lower limit of the meridional circulation determined by the possible vertical eddy flux of momentum, if we assume that all other quantities are approximately right. This is illustrated by Figure 4 which is computed on the basis of model (b). The horizontal eddy fluxes are nearly the same as in Figure 3 in all

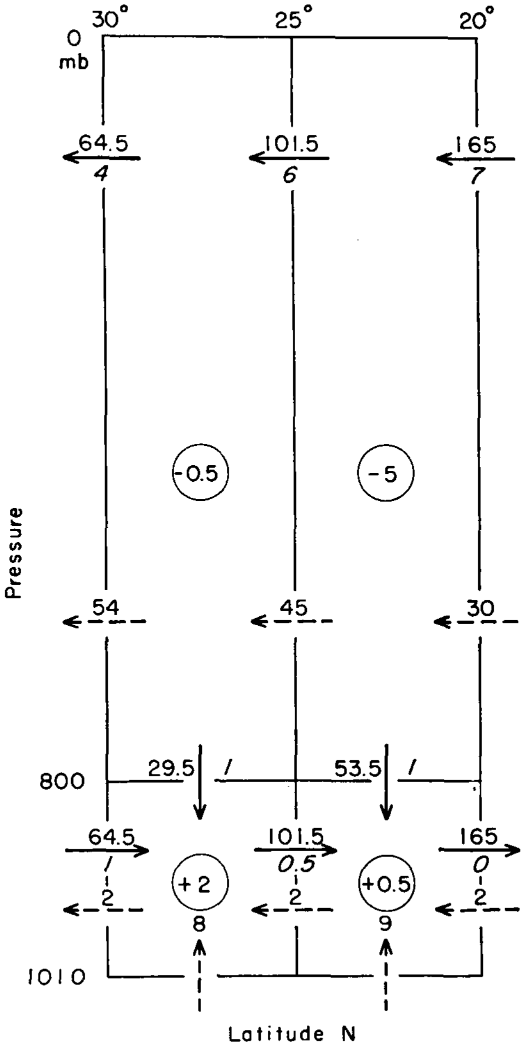

Fig. 4. Budget of angular momentum in the belt 30—20°N computed from model (b) and considering the horizontal eddy fluxes and the fluxes from earth according to Mintz and Widger. Meaning of figures the same as in Fig. 3.

blocks, now separated by the 800-mb surface, but the fluxes due to the meridional mass circulation are reduced to one half of the values in Figure 3. This gives a deficit of 5.5×10^{25} g cm^2 sec^{-2} in the upper layer and a surplus of 2.5×10^{25} g cm^2 sec^{-2} in the lower layer, requiring an upward eddy transfer of momentum across the 800-mb surface. This seems to indicate that the meridional circulation necessary for a complete balance probably should be stronger than according to model (b), but perhaps a little weaker than according to model (a). A decision concerning the strength of the meridional circulation is possible only if exact values of both the horizontal and vertical eddy transport of momentum, including the transfer earth →atmosphere, could be computed.

Figure 3 reveals another interesting fact. Whereas the flux of angular momentum due to the assumed meridional circulation results in convergence of momentum in the upper layer and divergence of momentum in the lower layer, the horizontal eddy transport, according to Widger and Mintz, has the opposite effect. The net gain or loss of angular momentum in the upper layer, especially in the uppermost part of the troposphere, therefore depends on whether the former or latter transport is preponderant.

It should be remembered that Figure 3 represents the mean conditions for January assuming a complete balance between the different components of angular momentum flux. Such a balance does not necessarily prevail at all times. It is conceivable that all quantities involved undergo changes with time. At period when the meridional circulation is etronger than usual, the horizontal and vertical addy fluxes are not vigorous enough to drain all the momentum surplus in the upper troposphere. Continuous accumulation of angular momentum then means strengthening of the upper subtropical jet. This in turn will increase the eddy flux of momentum and act to restore the balance. The opposite is true at periods with subnormal meridional circulation. Thus, although the quantities involved in our mean budget are in balance, instantaneous situations may show pronounced deviations from this balance. This departure from and return to balanced conditions could be responsible for the fluctuations in the intensity of the subtropical jet stream.

The total momentum flux from the tropics to higher latitudes across the parallel circle 30° N is 62×10^{25} g cm^2 sec^{-2}, according to Figure 3. This value appears quite reasonable considering that Widger computed a contribution of 17×10^{25} g cm^2 sec^{-2} from the earth in the belt 30—20°N and that the area between 20° N and the Equator is likely to yield more than twice this amount. PRIESTLEY (1951) recently estimated the surface torque to be at least 10^{34} g cm^2 sec^{-1} per annum in both hemispheres between the Equator and 30°. He pointed out, however, that the real torque probably is considerably greater. Presumably this is the case, since Priestley's estimate corresponds to only about 50 per cent of the flux given in Figure 3.

The total flux of angular momentum from earth to atmosphere in the belt Equator—30° N can also be estimated from Figure 2. The sum of the negative encircled figures in all blocks in the layer between 1010 and 700 mb is -81×10^{25} g cm^2 sec^{-2}. If we reduce this value by the downward eddy flux of momentum, the remaining value must be balanced by the momentum transfer from the earth and the eddy influx across 30°N and the Equator. In the belt 30—20°, we estimated the vertical downward eddy flux at 11.5×10^{25} g cm^2 sec^{-2}. Since the vertical wind shear decreases equatorwards from about latitude 30—25°N, we can assume that the contribution of the vertical eddy flux in the total belt 20°—Equator hardly could be larger than the contribution between 30 and 20° N. If we thus estimate the total downward eddy flux through the 700-mb surface to be about 20×10^{25} g cm^2 sec^{-2} and assume that the horizontal eddy flux is small in the layer in question (compare Fig. 3), the transfer of angular momentum from the earth to the atmosphere due to the surface wind stress in the total belt would amount to 61×10^{25} g cm^2 sec^{-2}, which is in agreement with the total flux of angular momentum across latitude 30° N which, as shown above, amounts to 62×10^{25} g cm^2 sec^{-2}. If all these rather crude estimates are approximately valid, no great net exchange of angular momentum between both hemispheres is necessary, at least in January.

It could also be of interest to compare the net northward flux of angular momentum due to the assumed meridional circulation with the eddy flux. From Figure 3, we get the following values:

The contribution from the circulation is very small compared with the eddy flux at latitude 30° N, but increases both absolutely and relatively equatorwards. This result seems to be in qualitative agreement with results recently achieved by STARR and WHITE (1952).

Table 3: Total northward flux of angular momentum due to meridional circulation and large-scale eddies in January (unit: 10^{25} g cm^2 sec^{-2}).

Latitude	30°	25°	20°
Circulation transport	6	11	14
Eddy transport	56	47	32
Ratio in per cent	11	23	44

Conclusions: In spite of our doubts concerning the different empirical values used here, we feel that our discussion of the maintenance of the zonal motion in the atmosphere could be of interest. The data presented above show, in our opinion, that a total angular momentum budget for the atmosphere can be derived following the principles outlined here. Until more complete observational data are available the different terms in the momentum budget naturally have to be regarded as extremely approximate.

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REFERENCES

BJERKNES, J., 1948: Réunion d'Oslo 1948, Programme et Résumé des Mémoires, pp. 13—14.

JORDAN, C. L., 1952: A Comparison Between Observed and Geostrophic Winds in Low Latitudes. (To be published.)

MINTZ, Y., 1951: The Geostrophic Poleward Flux of Angular Momentum in the Month of January, 1949. *Tellus*, **3**, pp. 195—200.

PALMÉN, E., 1951: The Role of Atmospheric Disturbances in the General Circulation. *Quart. J. Roy. Met. Soc.*, **77**, pp. 337—354.

PETTERSSEN, S., 1950: Some Aspects of the General Circulation of the Atmosphere. *Quart J. Roy. Met. Soc.*, Cent. Proc., pp. 120—155.

PRIESTLEY, C. H. B., 1951: Physical Interactions Between Tropical and Temperate Latitudes. *Quart. J. Roy. Met. Soc.*, **77**, pp. 200—214.

RIEHL, H., and YEH, T. C., 1950: The Intensity of the Net Meridional Circulation. *Quart. J. Roy. Met. Soc.*, **76**, pp. 182—188.

RIEHL, H., et al., 1951: The North-east Trade of the Pacific Ocean. *Quart. J. Roy. Met. Soc.*, **77**, pp. 598—627.

STARR, V. P., 1948: An Essay on the General Circulation of the Earth's Atmosphere. *J. of Met.*, **5**, pp. 39—43.

STARR, V. P., and WHITE, R. M., 1952: Meridional Flux of Angular Momentum in the Tropics. *Tellus*, **4**, pp. 118—125.

WIDGER, W. K., 1949: A Study of the Flow of Angular Momentum in the Atmosphere. *J. of Met.*, **5**, pp 291—299.