Meridional Flux of Angular Momentum in the Tropics

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Abstract

The results of an attempt to measure the meridional flux of angular momentum across the vicinity of 13° N latitude from actual wind data are presented. It is found that the average flux for one year is directed northward but is much smaller than the corresponding flux at 31° N as obtained from a similar hemispheric study made earlier for that latitude. According to the data the flux at 13° is due mainly to horizontal exchange processes which are most vigorous at an elevation of about 35,000 feet.

1. Introduction

During the course of an extended program of research concerning the angular momentum balance of the northern hemisphere, the writers have reported elsewhere (STARR and WHITE 1951, 1952) the results of flux computations from actual-wind data in the subtropics. In the study described in these two previous papers, henceforth denoted as (A) and (B) respectively, the meridional transports were evaluated from a string of upper-wind stations encircling the earth in the vicinity of 31° N latitude for a period of one year. In the region of the subtropics and at more northerly latitudes the character of the angular momentum balance has also received observational elucidation from the geostrophic-wind studies of WIDGER (1949) MINTZ (1951), WHITE and COOLEY (1952) and LORENZ (1952), and likewise from the actual-wind studies of PRIESTLEY (1949, 1951a, b), STARR (1950) and NYBERG and SCHMACKE (1951).

However, much interest is centered in regard to the character of the transport mechanisms (vertical and meridional) which may be operative in the more tropical regions where geostrophic wind measurements are of doubtful reliability and are more difficult to obtain. For these reasons, and also because of other considerations, recourse must be made to procedures involving the use of actual-wind data in order to throw light upon the problem in this zone. In the present paper an account is given of an attempt to duplicate the studies contained in (A) and (B) for a string of upper-wind stations in the vicinity of 13° N, to the extent that this is possible with existing data. Although an endeavor was made in (B) to show the seasonal variation of the flux, no corresponding attempt was made for 13° N, due to the fact that a somewhat smaller total amount of data was available in the tropics and also for other reasons.

From general physical considerations one is led to certain over-all expectations concerning the total meridional flux across the latitude circle in the vicinity of 13° N. The more important of these are the following. (a) Since the storage capacity of the atmosphere for angular momentum is small when a period of the length of one year is considered. the total

Table 1. List of key stations (numbered) and alternate stations (italics)

Station	Latitude (N.)	Longitude	Altitude (ft)	Type
1. Gao	16° 06′	00° 03′ E	899	pilot balloon
Niamey	13 31	02 26 E	755	»
Zinder	13 48	09 00 E	1604	»
Mopti	14 30	04 12 W	906	»
Bamako	12 38	08 01 W	1086	»
2. El Fasher	13 37	25 20 E	2395	»
Kassala	15 28	36 24 E	1644	»
El Obeid	13 10	30 14 E	1887	»
Kosti	13 10	32 40 E	1253	»
Malakal	09 33	31 39 E	1276	»
Wau	07 42	28 01 E	1440	»
3. Aden Khormaksar	12 50	45 01 E	3	radio wind
Aden Scheikh Othman	12 53	44 58 E	33	pilot balloon
Riyan	14 39	49 23 E	46	»
Kamaran Island	15 20	42 37 E	20	»
4. Trichinopoly	10 49	78 42 E	256	»
Vengurla	15 55	73 40 E	—	»
Anantapur	14 41	77 37 E	1148	»
Madras	13 04	80 15 E	52	»
Mangalore	12 52	74 51 E	72	»
Fort Cochin	09 58	76 14 E	10	»
Trivandrum	08 29	76 57 E	200	»
Nagercoil	08 11	77 26 E	112	»
5. Saigon	10 49	106 39 E	33	»
Mingaladon	16 56	96 07 E	92	»
Tavoy	14 05	98 12 E	112	»
Victoria Point	09 59	98 35 E	122	»
Penang	05 18	100 16 E	13	»
Tourane	16 02	108 11 E	16	»
Pattle	16 33	111 37 E	20	»
Seno	16 40	105 00 E	502	»
Phnom Penh	11 33	104 51 E	33	»
6. Clark Field	15 10	120 34 E	644	radio wind
Laoag	18 11	120 32 E	12	pilot balloon
7. Yap	09 29	138 08 E	0	radio wind
8. Harmon Field	12 31	144 49 E	176	»
9. Kwajalein	08 43	167 44 E	10	»
Wake Island	19 15	166 30 E	0	»
10. Johnson	16 44	169 31 W	20	»
Hilo	19 44	155 04 W	33	»
11. Tehuantepec	16 20	95 14 W	75	pilot balloon
Vera Cruz	19 12	96 08 W	10	»
Ciudad del Carmer	18 39	91 50 W	5	»
12. Managua	12 08	86 12 W	178	radio wind
13. Plato Magdalena	09 48	74 48 W	190	»
Albrook Field	08 58	79 33 W	21	»
Barranquilla	10 55	74 45 W	39	»
14. Waller Field	10 36	61 12 W	0	»
San Juan	18 28	66 07 W	82	»
15. Dakar	14 40	17 26 W	131	»
St. Louis	16 01	16 30 W	10	pilot balloon
Dakar Yoff	14 41	17 25 W	62	»
Sal	16 44	22 57 W	180	»

amount transported into the polar cap north of a given latitude should be equal to the amount removed by surface torques acting on the cap. (b) Since the zone between 13° and 31° N is a region of predominant easterly winds near the surface, the northward flux at 13° N should be smaller (algebraically) than the flux at 31° N. (c) Whether or not the flux across 13° N is actually negative (directed southward) cannot be determined from general considerations except as noted below, since one may not assume complete symmetry of the two hemispheres about the equator in this connection. (d) Since at 31° N angular momentum is transported strongly northward by the very large disturbances in the upper westerlies, one might perhaps expect that this effect would not be suppressed completely in a distance so short as 18° to the south.

The degree to which such anticipations as those listed above are not in conflict with the numerical findings obtained, constitutes one rather general criterion for the plausibility of the results.

2. Observational material

As in the study reported in A) and (B) a string of key stations was used. These 15 stations are listed in table 1. Since reports were often missing from the key stations, alternative stations in the vicinity of the key stations were used in order to increase the amount of observational material. The alternative stations are entered in italics below the key stations in the table. Although it would have been advantageous to use the same period of time as was used for the study at 31° N, various factors made this impractical. The average latitude of the key stations is 13° N.

Wind reports for the levels 2, 6, 10, 14, 20, 25, 30, 35, 40, 45, 50, and 55 thousand feet were taken from the data tabulations of the *Daily Series Synoptic Weather Maps* prepared by the U. S. Weather Bureau, in cooperation with the Army, Navy and Air Force for the period from 1 July 1949 to 30 June 1950 for the hour 0300 GMT for each day. For the 6, 10, 20, 30, 40, and 55 thousand-foot levels supplementary data as reported for the 850, 700, 500, 300, 200, and 100 millibar levels, respectively, were added, although preference was given to constant-height data where available. No attempt was made to correct original data, although in a few cases garbled reports were omitted. The frequency of available observations at the various levels is given in table 2. This table is analogous to table 5 in (B) in that it gives the percentages of the total possible observations, i. e., 365. Each wind observation was resolved into the eastward component u and northward component v, tabulated in m sec^{-1}.

3. Computation of transports

In order to avoid repetition, only a brief statement is made here of the methods used to measure the angular-momentum flux, the reader desiring more details being referred to (A) and (B). The notation used may be summarized as follows:

Table 2. Percentage of total possible observations at each level for each station

Station	Altitude in thousands of feet											
	2	*6*	*10*	*14*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*
Gao	63	65	59	46	15	1	0	0	0	0	0	0
El Fasher	94	92	95	87	60	13	11	11	8	4	1	1
Aden	44	54	53	28	47	22	45	23	38	19	15	23
Trichinopoly ..	99	1	99	0	90	78	73	1	0	0	0	0
Saigon	100	95	95	75	61	11	6	3	0	0	0	0
Clark	84	86	83	75	73	52	44	32	33	23	16	11
Yap	30	25	19	12	10	4	2	1	1	1	0	0
Harmon Field	77	85	83	68	80	63	73	61	68	50	39	28
Kwajalein	92	95	92	86	93	79	82	63	70	49	38	34
Johnson	82	88	86	74	83	66	73	58	64	47	34	28
Tehuantepec ..	78	75	67	47	27	3	0	0	0	0	0	0
Managua	3	11	11	1	9	0	6	1	3	1	1	0
Plato Magdalena	74	84	83	70	83	63	65	50	47	28	9	0
Waller Field ..	93	98	97	86	93	78	88	74	81	65	50	37
Dakar	65	66	62	53	55	47	50	44	43	35	25	20

u eastward component of the wind.

v northward component of the wind.

$[\alpha]$ space average of a quantity α over the length of the complete latitude circle.

α' deviation of a quantity α from its space average $[\alpha]$.

$\bar{\alpha}$ time average of a quantity α. Time and space averaging processes are not necessarily commutative.

$[\alpha]'$ deviation of the space average $[\alpha]$ from the space-time average $\overline{[\alpha]}$.

$\{\alpha\}$ arithmetic mean of all individual observations of α at a given level during entire time period considered.

n number of days for which observations were available at a given level.

N number of individual observations present at a given level during entire time-period considered.

r coefficient of linear correlation for the N pairs of u and v.

In (A) it was shown that the quantity $\overline{[u\ v]}$ which represents the northward transport of linear momentum per unit mass at a given level may be resolved into three components, namely:

(i) Transport of the first species, $\overline{[u]}\ \overline{[v]}$. This component represents the contribution due to a net meridional mass flux $\overline{[v]}$ during the entire period at the level in question. To the extent that a non-zero value of $\overline{[v]}$ at one level is usually compensated by values having an opposite sign at other levels, this component depends upon the existence of so-called mean meridional circulations.

(ii) Transport of the second species, $\overline{[u]'\ [v]'}$. This component represents the contribution from fluctuations in $[v]$ due to its possible correlation with $[u]$ in time.

(iii) Transport of the third species, $\overline{[u'\ v']}$. This component represents the contribution due to a correlation between u and v along the length of the latitude circle. It does not require the presence of net air motions $[v]$ or $\overline{[v]}$.

Since one basic question involved in the subject relates to the role of mean meridional circulations, it is useful to form the quantity $\{u\,v\} - \{u\}\,\{v\}$ which is analogous to the sum of the transports of the second and third species although the mode of averaging used is slightly different as explained in (A).

The results are given in table 3 which is to be compared with table 4 of (B), both being of the same basic form. The table gives the transports of each of the three species in columns 6, 7, and 8 together with various other quantities defined in terms of the symbols already recapitulated here and explained more fully in (A). A number of the quantities carry confidence limits defined as twice the standard error indicating approximately the 95 per cent confidence level. Vertical integrals of several quantities with respect to mass (standard atmosphere pressure) are given at the foot. These figures are to be interpreted as the northward transport of linear momentum per *unit length* of the latitude circle for the layer from 2 to 55 thousand feet. For comparison the corresponding integrals for 31° N as reported in (B) are also given at the foot of table 3.

4. Discussion of numerical results

The appraisal of the numerical results obtained depends upon a simultaneous consideration of various factors which might limit the accuracy of the measurements of the flux and other computations. By and large these factors are similar to those enumerated in (A) and to some extent in (B), although certain points are worthy of especial note in view of the circumstances of the present study, specifically the following:

(a) There exists a rather serious gap in the station network extending from Johnson Island eastward to Central America. There appears to be no source of reports from this region which is known to the writers.

(b) When the twelve values of $\overline{[v]}$ given in column 3 of table 3 are averaged vertically with respect to pressure one obtains the net meridional air flow for the entire layer. This quantity turns out to be $-$ 0.16 m sec^{-1}. Since it is quite improbable that a return flow exists below 2,000 ft., it would be necessary that an average northward flow of rather large magnitude should exist in the remainder of the atmosphere above 55,000 feet, if this figure represents actual conditions. This appears to be improbable. The most likely inference to be drawn is that it represents evidence of a slight lack of randomness of the observations with respect to the troughs and ridges of the streamline patterns, which in turn might be related to the gap in the observational network mentioned in (a) above. In any event the figure is rather excessive, the corresponding value at 31° N being only + 0.04 m sec^{-1} as obtained from data in (B).

(c) The total number of individual wind observations is 27,985 at 13° N as compared with a total of 38,301 at 31° N. This is a reflection of a poorer station network and more missing reports. This and other shortcomings such as the one discussed in (b) might be made less serious by using more than one year of data.

(d) Resort had to be made to the use of relatively more pilot balloon stations than were used at 31°. This fact contributes to a paucity of observations at high levels and may be a factor contributing to a slight lack of randomness discussed in (b).

Table 3. Numerical analysis for entire year. The levels are given in thousands of feet. All velocities are in m sec.$^{-1}$ Internal consistency of figures given is limited by rounding-off approximations

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[u]}$	$\overline{[v]}$	$\overline{[u]\,[v]}$	$\overline{[u\,v]}$	$\overline{[u]}\;\overline{[v]}$	$\overline{[u]'\,[v]'}$	$\overline{[u'\,v']}$	n	$\{u\}$	$\{v\}$	$\{u\,v\}$	$\{u\,v\} - \{u\}\,\{v\}$	r	N
55	+ 0.1	— 0.42 ± 0.56	— 2 ± 6	— 2 ± 11	0	— 2	0 ± 7	242	+ 0.2	— 0.38	— 1	— 2	— 0.02	664
50	+ 3.3	— 0.56 ± 0.53	— 1 ± 5	— 5 ± 8	— 2	+ 1	— 5 ± 6	279	+ 3.6	— 0.65	— 5	— 3	— 0.03	831
45	+ 6.9	+ 0.09 ± 0.60	+ 3 ± 6	0 ± 10	+ 1	+ 2	— 3 ± 6	342	+ 6.3	— 0.03	— 3	— 3	— 0.02	1174
40	+ 6.7	+ 0.24 ± 0.41	+ 4 ± 5	+ 10 ± 8	+ 2	+ 3	+ 6 ± 6	359	+ 6.4	+ 0.17	+ 11	+ 10	+ 0.08	1664
35	+ 5.3	+ 0.37 ± 0.42	+ 5 ± 5	+ 12 ± 8	+ 2	+ 3	+ 7 ± 5	355	+ 5.1	+ 0.41	+ 13	+ 11	+ 0.11	1532
30	+ 2.5	— 0.08 ± 0.28	0 ± 2	+ 6 ± 4	0	0	+ 6 ± 3	365	+ 2.4	— 0.09	+ 6	+ 6	+ 0.09	2251
25	0.0	— 0.35 ± 0.23	— 1 ± 1	+ 4 ± 2	0	— 1	+ 4 ± 2	363	0.0	— 0.32	+ 4	+ 4	+ 0.08	2113
20	— 2.0	+ 0.08 ± 0.18	0 ± 1	+ 3 ± 2	0	0	+ 3 ± 2	365	— 1.9	+ 0.10	+ 3	+ 3	+ 0.10	3200
14	— 2.9	+ 0.22 ± 0.15	0 ± 1	+ 1 ± 1	— 1	+ 1	+ 1 ± 1	365	— 2.9	+ 0.22	+ 1	+ 2	+ 0.08	2950
10	— 3.1	— 0.05 ± 0.12	+ 1 ± 1	+ 3 ± 1	0	0	+ 2 ± 1	365	— 3.1	— 0.04	+ 3	+ 2	+ 0.10	3954
6	— 3.9	— 0.51 ± 0.15	+ 3 ± 1	+ 4 ± 1	+ 2	+ 1	+ 1 ± 1	365	— 4.0	— 0.42	+ 4	+ 2	+ 0.08	3718
2	— 2.4	— 1.24 ± 0.21	+ 7 ± 1	+ 7 ± 1	+ 3	+ 4	+ 1 ± 1	365	— 2.8	— 1.22	+ 7	+ 4	+ 0.12	3932
Integral (10^7 CGS units)				+ 3.5	+ 0.5	+ 0.9	+ 2.2				+ 3.5	+ 3.0	Sum	27,985
Same for 31° N				+ 16.8	— 0.4	+ 0.7	+ 16.5				+ 16.9	+ 17.8		38,301

(e) The range of latitude of the individual key stations is only 8 degrees which is much less than for the network at 31° N.

(f) A large number of alternative stations was used. This in itself should not be necessarily detrimental to the results, except to the extent that a lack of randomness might be introduced in the data.

(g) The confidence limits for the flux $\overline{[u\ v]}$ and the component $\overline{[u'\ v']}$ are smaller at 13° than at 31° N, but the values of the quantities are also smaller so that on the whole less confidence is indicated. On the other hand, the alternative method of averaging given in columns 10—14 of table 3 gives results which compare very well with the first technique.

Generally speaking the data fulfill plausible expectations such as those mentioned in the introduction. The following points may be noted:

(1) The vertical integral of the flux $\overline{[u\ v]}$ may be converted into the total flux of angular momentum by multiplying by the length of the torque arm and the length of the latitude circle. The result is $+ 8.5 \times 10^{25}$ gm cm^2 sec^{-1}, which is considerably smaller than the corresponding value at 31° N, i. e., $+ 31.5 \times 10^{25}$. The difference represents essentially the rate at which angular momentum is acquired from the surface, since probably only small contributions to the fluxes are made by the portions of the atmosphere not considered.

(2) The largest contribution to the angular momentum flow across 13° N is due to the transport of the third species, the value being $+ 5.2 \times 10^{25}$ gm cm^2 sec^{-1}. The next largest is due to the second species, the value being $+ 2.2 \times 10^{25}$. The smallest is due to the first species of transport, i. e., $+ 1.2 \times 10^{25}$. Thus about 14 per cent of the total is brought about by mean meridional circulations, according to the data.

(3) If use is made of parentheses to indicate vertical averaging with respect to standard atmosphere pressure, the quantity $(\overline{[v]})$ has a value of -0.16 m sec^{-1} as already stated. Similarly $(\overline{[u]})$ turns out to be $+ 0.50$ m sec^{-1}. Since one may write that $\overline{[u]} = (\overline{[u]}) + \overline{[u]}'$ with a similar relation involving v in place of u, it follows that

$$(\overline{[u]}\ \overline{[v]}) = (\overline{[u]})\ (\overline{[v]}) + (\overline{[u]}'\ \overline{[v]}')$$

The term on the left is a measure of the transport of the first species integrated vertically. The first term on the right represents a portion of this quantity depending on $(\overline{[v]})$ which as mentioned before may be spurious. Although this "correction" procedure is not recommended as a substitute for better observational material, it is of interest to note that in the present case the correction is of negligible magnitude compared to the last term which contains only departures from the vertical averages.

(4) In the process of calculation, individual monthly averages of $\overline{[u'\ v']}$ were obtained for the twelve levels used. Of these 144 monthly values 98 were positive in sign, whereas at 31° N 138 were positive. There appears to be a tendency for this quantity to be negative at high levels as is also shown in table 3. However, caution must be used in accepting this feature in view of the large confidence limits associated with these data.

(5) The correlation coefficients r for the N pairs of u and v given in column 14 of table 3 are smaller than the corresponding ones for 31° N except at 2,000 feet, and actually become slightly negative at the highest three levels.

5. Some meteorological implications

(a) From time to time various estimates have been made of the strength of mean meridional circulations which would be needed in order to transport the net amount of angular momentum across given latitude circles prescribed by mean surface torques, on the assumption that the entire flux is of the first species. Two difficulties attend this process. In the first place accurate estimates of the requirements due to surface torques are difficult to make. In the second place there is no objective method for specifying the functional dependence of $\overline{[v]}$ on elevation (pressure). It is of interest to substitute the total flux as measured by studies of the present kind for the surface torque requirements, and to employ a variational method to specify $\overline{[v]}$ as a function of pressure. In effect one thus seeks the *least intense* meridional circulation which could bring about the stated transport T. This procedure was suggested to the writers by Messrs. E. N. Lorenz

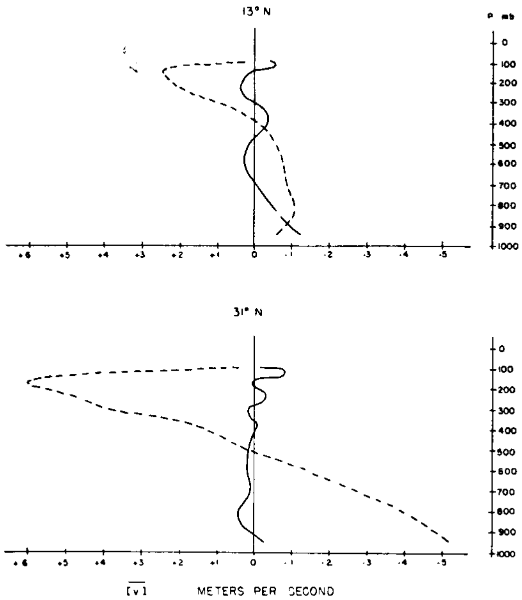

Fig. 1. Mean meridional circulations in m/sec at 13° N and at 31° N as given by data (full curves). Weakest mean meridional circulations capable of transporting total amount of angular momentum given by dashed curves.

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The process takes the form of finding the solution of the variational problem

$$\delta \int_b^a \overline{[v]}^2 \, dp = 0$$

subject to the side conditions that

$$\int_b^a \overline{[v]} \, dp = 0; \quad \int_b^a \overline{[u]} \; \overline{[v]} \, dp = KT$$

Here standard atmosphere pressuresar e used, a and b being the values at 2,000 and 55,000 feet respectively. The dependence of $\overline{[u]}$ on p is that given by the data and K is a constant. The result is simple to state qualitatively. The "least square" cell so obtained has the same shape as the vertical profile of $\overline{[u]}$, but displaced relative to the origin in order to satisfy the first side condition which stipulates that there shall be no net transport of air. The amplitude is of such a magnitude as to satisfy the second side condition, i. e., to provide the specified momentum transport.

The profiles of $\overline{[v]}$ so obtained are given by the dashed curves in Fig. 1, together with the actually observed profiles (full curves), at 13° and 31° N. The contrast between the two sets of curves is obvious and probably cannot be ascribed to poor observational material.

It would of course be possible to impose additional side conditions upon the problem stipulating, for example, that the mean meridional circulation should transport stated amounts of energy of several forms. However, such added restrictions would not lead to less vigorous circulations in the least square sense.

(b) According to what has been said under (1) of the previous section about 23×10^{25} gm cm^2 sec^{-1} represents the average rate at which angular momentum is supplied to the zone between 13° and 31° N by surface torques. Recent estimates of such surface torques made by PRIESTLEY (1951b) from surface stresses yield a value of about 15×10^{25} gm cm^2 sec^{-1}.

The average zonal component of the surface stress for the belt as derived from the present figures is approximately 0.53 dynes cm^{-2}, which is somewhat larger than the estimate used by Priestley.

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