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The Variation of Gravity within the Earth

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Abstract

The different methods to determine the values of gravity within the Earth are reviewed.

By using the equation

$$g = g_0 \left(\frac{r_0}{r}\right)^2 - \frac{4}{3}\,\pi G \varrho\, \frac{r_0{}^3 - r^3}{r^2}$$

and substituting the density values tabulated by Ramsay the variation of gravity has been computed (table 1 and fig. 1). The g value of 982 gal at the surface of the Earth is attained without any correction to the density distribution.

RAMSAY (1949) has recently developed a theory in which the Earth's core and mantle are assumed to be chemically identical. The discontinuity of seismic waves at a depth of 2,900 km. is interpreted in the usual way as a jump in density. But the jump in density is attributed to phase transition under pressure and not to the appearance of a new material such as iron. Olivine has a ionic structure in the mantle; in the core it is in a dense metallic state. The density figures given by Ramsay are slightly different from figures given by BULLEN (1940 and 1942) as the density values vary in the mantle from 3.29 to 6.02 and in the core from 8.92 to 18.5. The corresponding figures in Bullen's papers are 3.32 to 5.68 and 9.43 to 17.20. No gravity values have been computed from the density variation given by Ramsay, as far as it is known, and it is therefore the purpose of this paper to compute new gravity values.

The value of gravity at a distance r from the center of the Earth is given by

$$g_r = \frac{4\pi}{r^2}\, G \int_0^r \varrho_r \cdot r^2 \, dr \qquad (1)$$

where G is the constant of gravitation and ϱ_r the density at the point considered.

BENFIELD (1937) used this formula regarding the Earth as a non-rotating sphere, an assumption which was justified by the uncertainty of the density values. From $r = 0$ to 1,070 km. g_r was evaluated by using a power series of approximation for ϱ_r. Beyond this distance g_r was obtained by numerical integration in steps of 200 km or less.

STEWART (1949) has investigated theoretically the conditions under which maximum and minimum values of gravity occur. By means of equation (1), and by successive differentiation finally he obtained the equation which defines the critical values of g

$$\frac{dg}{dr} = 4\pi \left(\varrho_r - \frac{2}{3}\bar{\varrho}\right) \qquad (2)$$

where $\bar{\varrho}$ is the average density within the sphere of radius r. It is further pointed out, that if the depths and densities of the various zones are known, it is possible to evaluate exactly the positions of all maxima and minima, as well as the resultant values of gravity.

Another method was used by BULLEN (1940 and 1942). The values of gravitational force at various points are given, also, using the relation

$$g = \frac{1}{r^2} \cdot G \cdot M \qquad (3)$$

where M is the mass within a sphere of radius r concentric with the Earth, and G the constant of gravitation. Bullen proceeded this way obtaining the expression on the right-hand side in equation (3) from the following

$$\frac{d\varrho_r}{dr} = - GM\varrho_r \cdot \frac{1}{r^2}\left(\alpha^2 - \frac{4}{3}\beta^2\right) \qquad (4)$$

where ϱ_r is the density at a distance r from the Earth's center, α and β the velocities of P and S seismic waves there.

We have, however, preferred not to use equation (1) or (3) but to proceed the following way; considering the Earth is a sphere and made of concentrical strata of equal density it is shown (OLCZAK 1938) that the gravity value g has the following expression

$$g = g_0 \left(\frac{r_0}{r}\right)^2 - \frac{4}{3}\pi G \varrho \frac{r_0^3 - r^3}{r^2} \qquad (5)$$

where ϱ is assumed to be the constant density in the concentric layer with the radii r and r_0, and G the constant of gravitation. Having g on the outer face of any layer we can compute g everywhere in its interior down to its inner face. Since g is a continuous function of r, the value obtained can be used in the same way for calculation in the following layer.

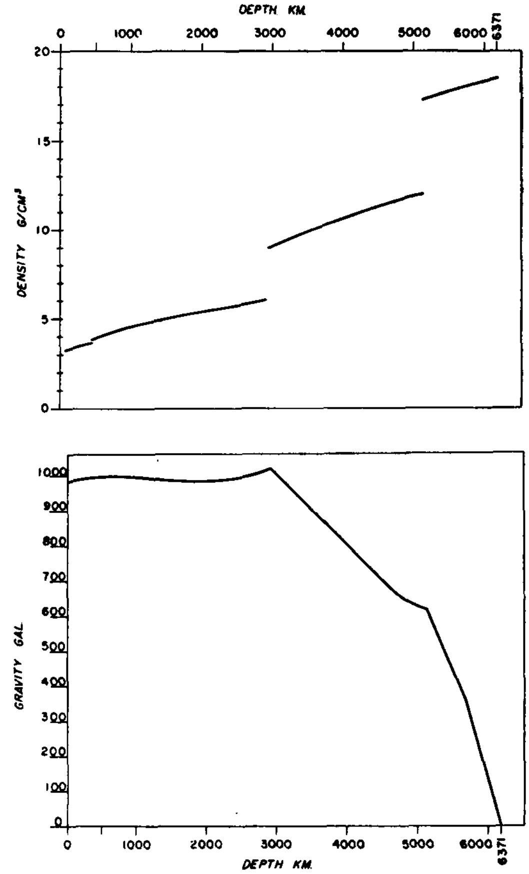

Fig. 1. The distribution of density with depth (the upper part) and the variation of gravity (the lower part).

In table 1 the depths and densities according to Ramsay are listed in the two first columns, and the following column contains the corresponding gravity values obtained from equation (5). Fig. 1 illustrates the distribution of density with depth (the upper part) and the variation of gravity (the lower part).

The value of g rises to a maximum of 999 gal at a depth of about 6—700 km. The function then falls to a minimum of 986 gal at a depth of the order of 1 800 to 2 000 km. rising to maximum (the greatest value) of 1 017 gal at the boundary between the Earth's mantle and the central core at a depth of 2 898 km.

Table 1

Depth	Density	Gravity	Depth	Density	Gravity
km	g/cm³	gal	km	g/cm³	gal
0	2.72	982	2600	5.78	999
33	3.29	985	2800	5.91	1010
100	3.39	987	2898	6.02	1017
200	3.50	990		8.92	
300	3.59	993	3000	9.09	1000
400	3.68	996	3200	9.43	965
413	3.70	996	3400	9.76	927
500	3.91	998	3600	10.06	887
600	4.11	999	3800	10.35	846
700	4.27	999	4000	10.63	804
800	4.38	998	4200	10.89	762
900	4.50	997	4400	11.14	721
1000	4.60	996	4600	11.38	682
1200	4.75	993	4800	11.61	648
1400	4.90	990	4982	11.8	626
1600	5.06	988	5125	12.0	618
1800	5.20	986		17.3	
2000	5.36	986	5700	18.0	342
2200	5.49	988	6171	18.5	0
2400	5.63	992			

The gravity values in the central core fall continuously from the boundary to the mantle and to a depth of 5 125 km. where the density value has a jump, and the gravity continues at another rate to the center where is has its zero-value.

The property used by BIRCH (1939) that g is always within 1 per cent of the value 990 gal down to a depth of about 2,500 km remains true.

While the gravity values given by Benfield were adjusted by ½ % to bring g at the surface of the Earth to the value of 982 gal and Olczak had to multiply his results by the factor 1.00572 in order to obtain 982 gal at the surface, we are attaining the value of 982 gal without any correction to the density values given by Ramsay.

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