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Some comments on the barotropic flow through the Danish Straits and the division of the flow between the Belt Sea and the Öresund

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ABSTRACT

The flow between the Baltic Sea and the Kattegat takes two routes, the Belt Sea and the Öresund. The division of the flow between the Belt Sea and the Öresund is discussed. The currently widely used estimate is based on results from a model for the barotropic flow through the Öresund and current measurements in the Belt Sea. By using a recently established, thoroughly calibrated model for the barotropic flow in the Öresund and existing models for the total flow through the Danish Straits, it is found that the mean division of the instantaneous flows through the Öresund and the Belt Sea is 2:8, rather than the at present generally accepted estimate 3:8. The division of the net outflow from the Baltic is an unsolved problem, due to the difficulty to determine the net flow separately in the straits. By using the models and some qualitative arguments it appears that the division of the net outflow may be approximately 1:1. Some comments on and results related to the difficulty to determine the net flow from current measurements or models of the barotropic flow are also given.

1. Introduction

The Baltic Sea is connected to the Kattegat through the Belt Sea and the Öresund, together called the Danish Straits (Fig. 1). The connection through the Belt Sea is the larger of the two, with a mean depth of 13 m and a sill depth to the Baltic of 18 m. The mean depth in the Öresund is 11 m and the sill depth is 8 m. The narrow passages in the Belt Sea have vertical cross-sectional areas of order 300 000 m^2, while the narrow cross-sectional areas in the Öresund are about 60 000 m^2. The fresh water surplus of the Baltic causes a long-term net outflow through the Danish straits of approximately 15 000 $m^3\ s^{-1}$. The instantaneous flow, into and out of the Baltic, often exceeds the net outflow by one order of magnitude. The large instantaneous flows are mainly driven by the sea level difference between the Baltic and the Kattegat (Stigebrandt, 1983).

The mean division of the instantaneous flow between the Öresund and the Belt Sea was estimated by Jacobsen (1980) to 3:8, i.e., 27 percent of the flow takes the route through the Öresund. This appears to be a generally accepted estimate, used in various studies (Omstedt 1987, Stigebrandt 1992). The mean division of the instantaneous flow was estimated by Jacobsen by comparing the mean absolute flow rate in the two straits. The flow rate in the Belt Sea was determined from current measurements, while the flow rate in the Öresund was calculated from a model for the barotropic flow. The model was calibrated against current measurements from two weeks. However, the verification in Jacobsen (1980) suggested that the flow rate was overestimated by the model. This should have given an uncertainty in the estimate of the division, which Jacobsen did not comment upon. The currently accepted estimate of the division represents an overestimation of the

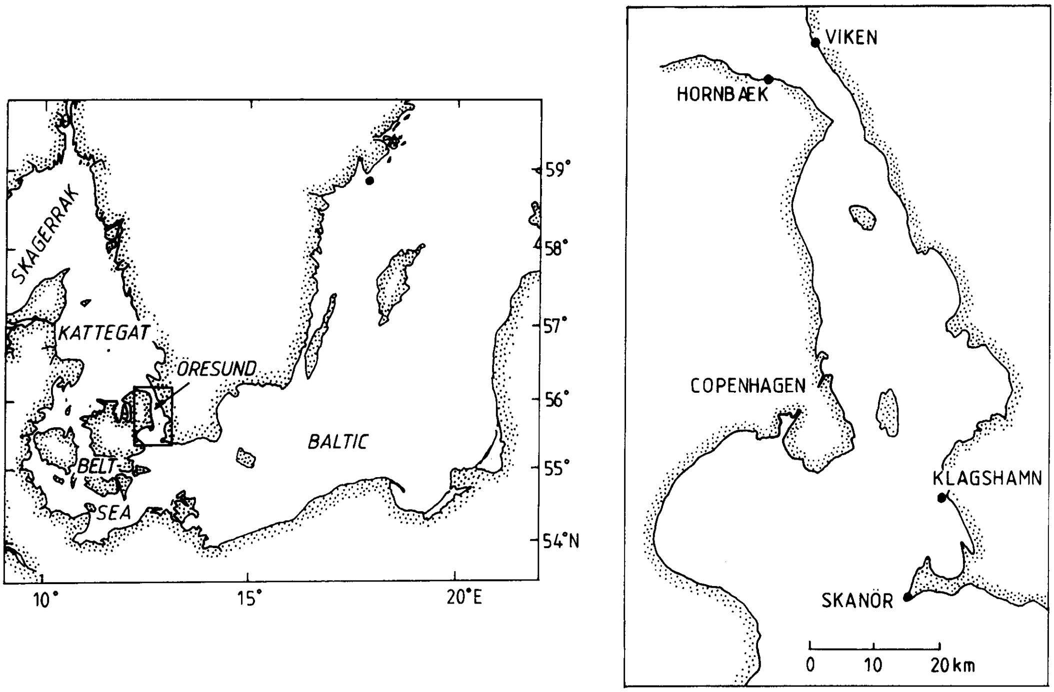

Fig. 1. Map of the area.

flow rate through the Öresund. The present paper gives a somewhat different estimate of the mean division. Using results from a recent study of the barotropic flow rate in the Öresund (Mattsson 1995, hereafter referred to as M95) and various estimates of the flow through the Danish Straits, it is found that a correct division should be close to 2:8. This means that as a mean approximately 20 percent of the instantaneous flow between the Baltic and the Kattegat takes the route through the Öresund.

The division of the net outflow between the straits can not be expected to be equal to the mean division of the instantaneous flow. The division of the instantaneous flow is mainly determined by the flow resistance properties of the straits with respect to barotropic flow. The division of the net flow, however, should be influenced by, for instance, differencies in baroclinic conditions between the straits. Determining the division of the net outflow is a difficult problem, related to the problem of determining the net flow. The most certain estimate (this far) of the net flow is based on the water balance of the Baltic. However, this estimate contains no information of the division of the flow among the routes. Considering that the net flow is a small difference between two large numbers, it is clear that estimates of the net flow determined from models of the flow in the straits, or current measurements, are likely to be uncertain. In this study it is qualitatively argued that more than 20 percent of the net flow should pass through the Öresund, which is also suggested by results from the model in M95. The results are as expected very uncertain, and the difficulties in determining the net flow from models or current measurements are also discussed.

2. Models and data sets

The barotropic flow through the Danish Straits has commonly been modeled as proportional to the square root of the sea level difference between the connected water bodies. The flow resistance is assumed to be due to frictional effects and effects of contraction — expansion, and thus being proportional to the flow rate squared. In M95, the barotropic flow through the Öresund was carefully studied. It was found that a better correspondence between observed and modelled flow rate was achieved by extending the existing model with a term linear in the flow rate.

The commonly used quadratic model for the barotropic flow is

$$\Delta\eta - A = K_f Q|Q|, \tag{1}$$

where $\Delta\eta$ is the sea level difference between the Kattegat and the Baltic, Q is the flow rate and K_f a flow resistance coefficient (cf. Stigebrandt 1980, Jacobsen 1980, Omstedt 1987). Outflow from the Baltic is counted positive. A covers effects of wind stress, baroclinic pressure differences and rate of change. A also includes a constant related to the difference between the mean geoid and the local height systems of the sea level stations. In M95, it was found that this model systematically overestimates small flow rates and underestimates large flow rates in the Öresund. A better correspondence between observed and modelled flow rate was achieved by adding a linear dependence to the model

$$\Delta\eta - A = K_l Q + K_q Q|Q|. \tag{2}$$

The values of K_f, K_q, K_l and A were determined by a least-squares fit using a data set with 1,802 simultaneous observations of sea level difference and estimated flow rate. This data set will be referred to as the calibration data set. Using the sea level difference between Skanör and Hornbaek, the least-squares fit gave $K_f = 2.03 \times 10^{-10}$ s^2 m^{-5}, $K_q = 1.29 \times 10^{-10}$ s^2 m^{-5} and $K_l = 0.32 \times 10^{-5}$ s m^{-2}. In an investigation of the flow resistance in connection with the planning of a fixed link across the Öresund (DHI/LIC/SMHI, 1992), K_f was estimated to 2.00×10^{-10} s^2 m^{-5} using the sea level difference between Skanör and Viken. This means that their estimate of the flow rate are equal to the present. The uncertainty in K_f, K_q and K_l is approximately 10%, mainly due to uncertainty in the estimated flow rate. A was incorporated in $\Delta\eta$ to give a corrected sea level difference $\Delta\eta' = \Delta\eta - A$, i.e., A is determined by the requirement of zero flow rate when the corrected sea level is zero. Models (1) and (2) are plotted in Fig. 2.

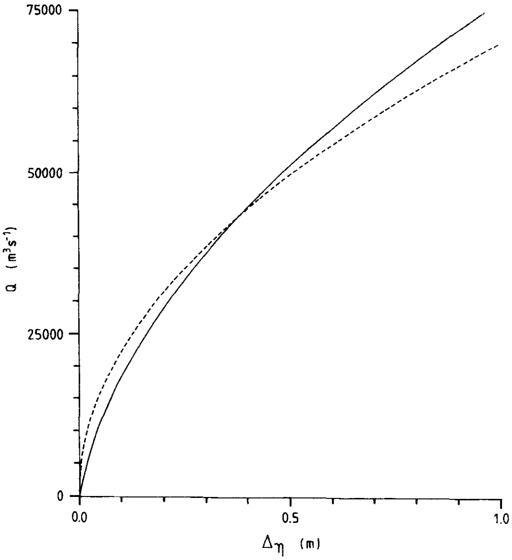

Fig. 2. Flow rate (Q) versus sea-level difference ($\Delta\eta$) according to models (1) (dotted line) and (2) (solid line).

In M95 it was proposed that the linear term was due to rotational effects in a way similar to the geostrophic control derivation (Toulany and Garrett, 1984). Following their approach, but using a quadratic fricion law, the along-strait frictional balance may be expressed (see Fig. 3)

$$\eta_2 - \eta_3 = \eta_4 - \eta_5 = \frac{c_D L}{g\, H^3\, W^2}\, Q^2, \tag{3}$$

where L is the length, W is the width and H the depth of the strait, c_D the drag coefficient, g the gravitational acceleration and Q is the flow rate. The cross-strait geostrophic balance may be expressed as:

$$\eta_2 - \eta_4 = \eta_3 - \eta_5 = \frac{f}{gH}\, Q, \tag{4}$$

where f is the Coriolis parameter.

The fundamental step is the basin-strait coupling. Toulany and Garrett (1984) assumed that the highest sea level in the strait is equal to the upstream basin sea level and the lowest level in the strait is equal to the downstream basin sea level, i.e., $\eta_1 = \eta_2$ and $\eta_6 = \eta_5$, which gives

$$\Delta\eta = \eta_2 - \eta_5 = \frac{f}{gH}\, Q + \frac{c_D L}{g H^3 W^2}\, Q^2. \tag{5}$$

Whitehead (1986) included the Bernoulli setdown (but excluded friction). The coupling between the upstream basin and the strait is the sum of a non-rotating and a rotating part, $\eta_1 - \eta_4 =$

$(\eta_1-\eta_{nr})+(\eta_{nr}-\eta_4)$ where the non-rotating part is

$$(\eta_1-\eta_{nr})=\frac{1}{2gH^2W^2}Q^2 \tag{6}$$

and the rotating part is

$$(\eta_{nr}-\eta_4)=\frac{1}{2}\frac{f}{gH}Q. \tag{7}$$

The factor 1/2 in the rotating part corresponds to a geostrophic tilt that is distributed such that the sea level is depressed over half the width of the strait compared to the non-rotating solution. This basin-strait coupling results in the expression

$$\Delta\eta=\frac{1}{2}\frac{f}{gH}Q+\left(\frac{1}{2gH^2W^2}+\frac{c_DL}{gH^3W^2}\right)Q^2. \tag{8}$$

Accordingly, the size of the linear term differ by a factor 2 between the two proposed basin-strait couplings.

The Öresund may be regarded as composed of two constrictions, the narrow strait in the north and the shallow sill in the south, and an interjacent basin with relatively small velocities. The flow resistance is mainly confined to the two constrictions. Thus, the size of K_l should correspond to $\alpha(f/g)\,(1/H_s+1/H_n)$, where index $_{s\,(n)}$ refers to the southern (northern) constriction and α should be either 1/2 or 1. Inserting appropriate values ($f=1.2\times10^{-4}$, $g=9.8$, $H_s=5$, $H_n=10$), K_l should fall in the interval 0.18–0.37×10^{-5} s m^{-2}.

The two basin-straits couplings can be combined, however, which was not accounted for in M95. Using the second coupling, and with cross-strait geostrophy, the sea level at the right hand side in the strait may formally become higher than the upstream sea level ($\eta_2>\eta_1$). This happens for $Q<fW^2H$. Obviously, this is not possible. By simply requiring (cf. Whitehead, 1986) that $\eta_2=\eta_1$ for $Q<fW^2H$, one has

$$\begin{aligned}\Delta\eta&=\frac{1}{2}\frac{f}{gH}Q+\left(\frac{1}{2gH^2W^2}+\frac{c_DL}{gH^3W^2}\right)Q^2\\ &\quad Q\geqslant fW^2H,\\ \Delta\eta&=\frac{f}{gH}Q+\frac{c_DL}{gH^3W^2}Q^2\\ &\quad Q<fW^2H.\end{aligned} \tag{9}$$

This means that K_l in the Öresund should correspond to $(f/g)\,(1/H_s+1/H_n)$ for $Q<19\,200$ m^3 s^{-1}, $(f/g)\;(1/H_s+1/(2H_n))$ for $19\,200$ m^3 s$^{-1}\leqslant Q<135\,000$ m^3 s^{-1} and $(f/g)\;(1/(2\,H_s)+1/(2\,H_n))$ for $Q\geqslant135\,000$ m^3 s^{-1}. Flow rates less than 19 200 m^3 s^{-1} occur approximately 40 percent of the time and flow rates in the interval 19 200–135 000 m^3 s^{-1} occur approximately 60 percent of the time. Thus K_l should correspond to 0.40 $(0.37\times10^{-5})+0.60(0.31\times10^{-5})+0.00$ $(0.18\times10^{-5})=0.33\times10^{-5}$ s m^{-2}, which is very close to the empirically found 0.32×10^{-5} s m^{-2}.

Properties of the sea-water exchange will in this study be derived from a data set consisting of 11 years (1977–1987) of daily mean sea level from Viken, Klagshamn and Landsort (Fig. 1), and the monthly mean fresh water flux to the Baltic Sea by runoff from land (Bergström and Carlsson, 1993). This data set will be referred to as the test data set. The sea level difference between the Baltic and the Kattegat driving the flow through the Öresund is estimated by the sea level difference between Klagshamn and Viken. Having no observations of the flow rate, it is not possible to empirically determine A for the test period. Ekman and Mäkinen (1991) and Ekman (1994) has developed a height system, NH 60, which is intended to reflect the level of the mean geoid in the Baltic Sea area. The sea levels in the test data set are given in NH 60. Assuming that NH 60 is accurate, the constant related to the difference between the local height systems and the mean geoid vanish from A for the test period. In M95 it was found that effects of local wind stress and rate of change vanish from the mean picture, and thus not influence A. However, the baroclinic pressure difference constitutes an almost constant contribution to the forcing, i.e., to A, due to a practically constant salinity difference between the surface waters of the Baltic and the Kattegat. The effect can be estimated by $A=H/2\;\beta\;\Delta\;S=0.01$ m, where $H=5$ m is the mean sill depth, $\beta=8\times10^{-4}$ psu^{-1} the salinity expansion coefficient and $\Delta S=5$ psu the mean salinity difference between Klagshamn and Viken.

2.1. *Differencies between the models*

The solutions for the flow rate from expressions (1) and (2) are:

$$Q_1=\frac{\Delta\eta'}{|\Delta\eta'|}\sqrt{\frac{|\Delta\eta'|}{K_f}}, \tag{10}$$

$$Q_2=\frac{\Delta\eta'}{|\Delta\eta'|}\left(\sqrt{\frac{|\Delta\eta'|}{K_q}+\frac{1}{4}\frac{K_l^2}{K_q^2}}-\frac{1}{2}\frac{K_l}{K_q}\right). \tag{11}$$

The defect of the quadratic model can be perceived by recognizing that the derivative of Q_1 with respect to $\Delta\eta'$ is infinite at $\Delta\eta'=0$. The inability of model (1) to correctly account for low flow rates, i.e., overestimating them, is compensated for, by the least-squares fit, by underestimating high flow rates.

An approximate expression for the ratio of the mean absolute flows calculated by the models is

$$\frac{\overline{|Q_1|}}{\overline{|Q_2|}} \approx \left[1 - \frac{1}{2}\frac{K_l}{K_f}\left(\frac{1}{\overline{|Q_1|}} - \frac{1}{Q_e}\right)\right]^{-1}. \tag{12}$$

The mean absolute flow given by model (1) is

$$\overline{|Q_1|} = \frac{\overline{\sqrt{|\Delta\eta'|}}}{\sqrt{K_f}}, \tag{13}$$

and Q_e is a flow rate defined by the least-squares fit,

$$|Q_e| \equiv \frac{K_l}{K_f - K_q}. \tag{14}$$

Q_e is the flow rate for which the two models are equal (Fig. 2). The magnitudes of the resistance coefficients determined by the least-squares fits give $Q_e = 43\,200$ m^3 s^{-1} and $K_l/K_f = 15\,800$ m^3 s^{-1}. With $\overline{\sqrt{|\Delta\eta'|}} = 0.385$ m$^{1/2}$ for the calibration period, expression (12) gives a ratio of 1.12.

For the test period $\overline{\sqrt{|\Delta\eta'|}} = 0.362$ m$^{1/2}$, based on the daily mean sea level difference between Klagshamn and Viken. The models (1) and (2) for the flow through the Öresund are calibrated using instantaneous observations of the flow rate and the sea level difference between Skanör and Hornbaek. To use models (1) and (2) on the test period, it is necessary to (i) relate the instantaneous sea level difference to the daily mean sea level difference and (ii) to relate the daily mean sea level difference between Skanör and Hornbaek to the daily mean sea level difference between Klagshamn and Viken. Using data from the calibration period, the relation is estimated to $\overline{\sqrt{|\Delta\eta'|}} = 0.362/0.83 = 0.436$ m$^{1/2}$ to be used with models (1) and (2) for the test period. $\overline{\sqrt{|\Delta\eta'|}} = 0.436$ m$^{1/2}$ gives $\overline{|Q_1|} = 30\,600$ m^3 s^{-1} and expression (12) gives a ratio of 1.08. It may be concluded that the quadratic model overestimates the mean absolute flow in the Öresund by approximately 8%.

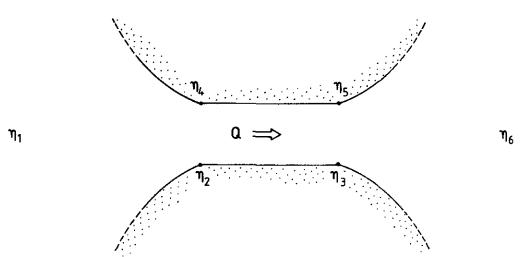

Fig. 3. The sea level (η) at various positions in and outside a strait with flow rate Q.

3. The mean division of the flow between the Danish Straits

The total flow through the transition area between the Baltic and the Kattegat is divided between the Öresund and the Belt Sea (Fig. 1). The total flow, Q_T, thus has two contributions, $Q_T = Q_{Ö} + Q_B$, corresponding to the parallell straits. Q_T is either directly or indirectly estimated. Direct estimates are made from current measurements in the straits or by models of the barotropic flow. Indirect estimates are made by regarding volume changes in the Baltic, using the water balance (outflow positive)

$$\frac{dV}{dt} = Q_0 - Q_T, \tag{15}$$

where V is the volume and Q_0 the fresh water supply to the Baltic.

The mean division of the flow between the routes was estimated by Jacobsen (1980) to 3:8, respectively, i.e., $\overline{|Q_{Ö}|}/(\overline{|Q_{Ö}|} + \overline{|Q_B|}) = 3/11 = 0.27$. Based on estimates given by Jacobsen, the ratio of the mean absolute total flow to the sum of the mean absolute flows in the straits is $\overline{|Q_T|}/(\overline{|Q_{Ö}|} + \overline{|Q_B|}) = 0.94$. The reason for the ratio being less than one is that the flow occasionally has opposite directions in the straits. Thus $\overline{|Q_{Ö}|}/\overline{|Q_T|} = 0.29$. These ratios were established using flow rates estimated from current measurements in the Great Belt and the Little Belt, and a quadratic model for the flow rate in the Öresund. The quadratic model was calibrated using the sea level difference between Klagshamn and Copenhagen and flow rates estimated from measurements by current meters located between Viken and Hornbaek.

Stigebrandt (1980) has calibrated a quadratic model for the total flow, yielding a resistance coefficient $K_{f,T} = 0.115 \times 10^{-10}$ s^2 m^{-5}. The model was calibrated to give a best-fit between observed and calculated horizontal mean sea level of the Baltic. The ratio using the quadratic models will then be $\overline{|Q_{\ddot{O},1}|}/\overline{|Q_{T,1}|} = \sqrt{K_{f,T}}/\sqrt{K_{f,\ddot{O}}} = 0.24$. It is noticable that the verification of the quadratic model in Jacobsen (1980) showed that his quadratic model gave 20% higher flow rates than observed, thus also giving $\overline{|Q_{\ddot{O}}|}/\overline{|Q_T|} = 0.29/1.2 = 0.24$. However, using the previous result $\overline{|Q_{\ddot{O},1}|}/\overline{|Q_{\ddot{O},2}|} = 1.08$ gives $\overline{|Q_{\ddot{O},2}|}/\overline{|Q_{T,1}|} = 0.22$. Consequently, it can be estimated that $\overline{|Q_{\ddot{O}}|}/(\overline{|Q_{\ddot{O}}|} + \overline{|Q_B|}) = 0.22 \times 0.94 = 0.21 \pm 0.01$, where the uncertainty is due to the uncertainty in $K_{f,\ddot{O}}$, $K_{q,\ddot{O}}$ and $K_{l,\ddot{O}}$.

Indirect estimates of the total flow rate during the test period can be computed using the water balance. Volume changes are estimated by sea level changes at Landsort times the horizontal surface area of the Baltic, $A_h = 3.7 \times 10^{11}$ m^2. Using daily mean sevel, the resulting flow rate corresponds to the daily mean total flow rate. Models (1) and (2) calibrated against the daily mean flow rate are used to estimate the daily mean flow rate in the Öresund during the test period. Monthly mean flow rates are formed from the daily means. The monthly mean flow rates in the Öresund estimated from model (2) are plotted against the monthly mean total flow rates in Fig. 4. A straight line is fitted to the data, and the slope of the straight line is 0.20. Using a quadratic model for the total flow and calibrating to give best-fit between observed and calculated sea level at Landsort, i.e., a similar calibration as Stigebrandt's (1980), gives $K_{f,T} = 0.105 \times 10^{-10}$ s^2 m^{-5}. This gives $\overline{|Q_{\ddot{O}}|}/(\overline{|Q_{\ddot{O}}|} + \overline{|Q_B|}) = 0.20$.

Simultaneous estimates of the flow rate in the Great Belt are available for 1683 observations of the flow rate in the Öresund from the calibration period. The flow rate in the Great Belt is calculated from ADCP-data, using an algorithm described by Møller and Pedersen (1993). The division of the flow between the Öresund, the Great Belt and the Little Belt is according to Jacobsen (1980) 3:7:1, i.e., $\overline{|Q_{\ddot{O}}|}/\overline{|Q_{GB}|} = 3/7 = 0.43$. Based on the present estimates of the flow rates, $\overline{|Q_{\ddot{O}}|}/\overline{|Q_{GB}|} = 0.30 \approx 2/7$ (Fig. 5), suggesting a division of 2:7:1. It can be concluded that the mean division of the flow between the Öresund and the Belt Sea should be 2:8, rather than the previous estimate 3:8.

4. The division of the net outflow and the uncertainty in flow estimates

The fresh water surplus of the Baltic Sea gives a positive mean total flow and a positive mean sea level difference between the Southwestern Baltic and the Kattegat. The mean total flow during the test period determined indirectly from

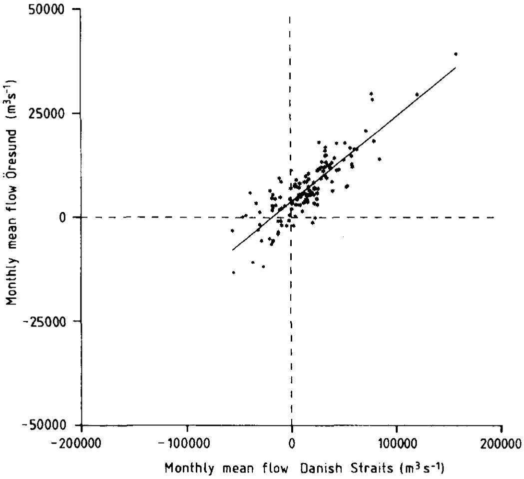

Fig. 4. The monthly mean flow rates in the Öresund estimated from model (2) versus the monthly mean total flow rates through the Danish Straits.

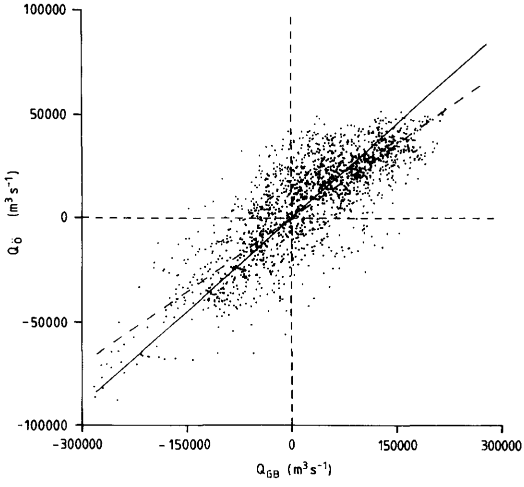

Fig. 5. 1683 simultaneous estimates of the flow rate in the Great Belt (Q_{GB}) and the Öresund ($Q_{\ddot{O}}$). The solid line corresponds to $Q_{\ddot{O}} = 0.30\ Q_{GB}$. Linear regression gives $Q_{\ddot{O}} = 0.24\ Q_{GB}$ (dashed line).

the water balance is $\overline{Q_T}=\overline{Q_0}-A_h\ \Delta h/T=14\,800$ $m^3\ s^{-1}$. The flow determined from volume changes gives no information of the division of the flow between the straits. The division of the net flow has never been safely determined. The reason is that direct estimates of the net flow are very sensitive to uncertainty in the transformation of current measurements to flow rate estimates, if current measurements are used, or to uncertainty in the correction term A, if a model for the barotropic flow is used.

The uncertainty in the net flow estimated from current measurements can be assessed by considering the procedure by which the current measurements are transformed to flow rate estimates. Generally, velocity data from each current meter is multiplied by a vertical cross-sectional area to give a flow rate and the flow rates related to each current meter are summed to give the estimated total flow rate. The mean flow is given by

$$\bar{Q}=\overline{\int_A v\,\mathrm{d}A}\approx\int_A \bar{v}\,\mathrm{d}A=\sum_{i=1}^{N}A_i\bar{v}_i+\sum_{i=1}^{N}\int_{A_i}\bar{v}-\bar{v}_i\,\mathrm{d}A_i, \tag{16}$$

where A is the total cross-sectional are, N is the number of current meters, and $\bar{v}_i$ and A_i are the mean velocity and the cross-sectional area, respectively, at the ith current meter. $\Sigma_{i=1}^{N}\int_{A_i}\bar{v}-\bar{v}_i\,\mathrm{d}A_i$ is the error when the flow rate is estimated by $\Sigma_{i=1}^{N}A_i\,\bar{v}_i$. The uncertainty in the estimate is thus related to the size of the error.

The uncertainty can be derived under some general assumptions: The current is measured at m evenly spaced stations across the strait. At each station i, $i=1\ldots m$, n_i current meters are placed at equal vertical distance. The flow rate per unit width, $\bar{q}_i$, is calculated at each station, and equal weight is given to each current meter j, $j=1\ldots n_i$. Next the flow rate is calculated by the sum of the $\bar{q}_i$, and equal weight is given to each station. The total uncertainty is given by the integral over the cross-sectional area of the uncertainty at each point. It is assumed that the uncertainty vanish at each current meter (no measurement errors), and that the uncertainty increases linearly to a maximum midway between any two neighbouring current meters j and $j+1$ where it equals $\frac{1}{2}|\bar{v}_j-\bar{v}_{j+1}|$. Likewise, the uncertainty in $\bar{q}$ increases linearly to a maximum midway between any two neighbouring stations, given by $\frac{1}{2}|\bar{q}_i-\bar{q}_{i+1}|$ plus the uncertainty at stations i and $i+1$.

These are reasonable assumptions, provided that the grid of current meters is not too sparse. Under these conditions the total uncertainty can be calculated, and is given by

$$\sum_{i=1}^{m}\alpha_i\frac{1}{4n_i}+\beta\frac{1}{2m}, \tag{17}$$

where

$$\alpha_i\approx\frac{A_i\,|\bar{v}_{\max}-\bar{v}_{\min}|_i}{\bar{Q}_{\mathrm{est}}},\quad \beta\approx\frac{W|\bar{q}_i|_{\max}}{\bar{Q}_{\mathrm{est}}},$$

$$\bar{Q}_{\mathrm{est}}=\sum_{i=1}^{m}\sum_{j=1}^{n_i}A_i\bar{v}_j \tag{18}$$

for the case with homogeneous flow and simple topography. Thus $\sum_{i=1}^{m}\alpha_i$ and β are numbers greater than but of order one. α_i and β increase for complicated topography and for stratified conditions, i.e., particularly in the Belt Sea.

Expression (17) may be a reasonable general assessment of the uncertainty in the net flow estimate, if n and m are not too small. For small n and m, the assumed distribution of the uncertainty may not be valid. In this case it may be predicted on basis of the expected flow conditions, e.g., by using theoretical current profiles. The total uncertainty will certainly not be smaller than what is given by expression (17), however. For large n and m, theoretical considerations are less useful. However, it is indicative that even for a dense $n\times m=5\times 10$-grid of current meters, the uncertainty may be of order 10%.

The uncertainty in estimates of the net flow calculated from the models for the barotropic flow is mainly caused by uncertainty in the correction terms A. The net flow through the Öresund during the test period is $\overline{Q_{\ddot{o},2}}=7800\ m^3\ s^{-1}$, calculated from model (1) and using $\overline{Q_{\ddot{o},1}/Q_{\ddot{o},2}}=1.08$, $A=0.01$ m and $K_{f,\ddot{o}}=2.03\times10^{-10}\times0.83^2=1.39\times10^{-10}\ s^2\ m^{-5}$. The calculation is very sensitive to the uncertainty in A. Numerically it is found that $(\overline{Q_{\ddot{o},2}})^{-1}\ \partial(\overline{Q_{\ddot{o},2}})/\partial A=-21\ m^{-1}$. An uncertainty in A of 0.01 m thus causes an uncertainty of about 20% in the estimate of the net flow. For the mean absolute flow, the corresponding number is $(\overline{|Q_{\ddot{o},2}|})^{-1}\ \partial(\overline{|Q_{\ddot{o},2}|})/\partial A=-1.0\ m^{-1}$, i.e., much less sensitive to uncertainty in A. Uncertainty in A is due to uncertainty in the estimated baroclinic component of the forcing, and uncertainty in the height system (the difference between local height system and the mean geoid). The uncertainty in the net flow through the Öresund due to uncertainty in the resistance parameters is approximately 5%.

Consequently, the mean flow can only be accurately determined indirectly from the water balance, provided that the fresh water surplus is accurately known. Still, it should be possible to predict some qualitative aspects of the division of the net flow. While the mean division of the flow is mainly determined by the flow resistance of the straits, the division of the net flow is influenced by the baroclinic properties of the transition area. Without baroclinic effects, the ratio of the net flow through the Öresund to the total net flow, $\overline{Q_{\ddot{O}}}/\overline{Q_T}$, should be equal to $\overline{Q_{\ddot{O}}}/\overline{Q_T} = 1/5$. However, the corrected sea level difference, $\Delta\eta' = \Delta\eta - H/2\ \beta\ \Delta S$, related to the barotropic forcing through the Belt Sea is less than the corrected sea level difference through the Öresund. The forcing sea level difference, $\Delta\eta$, is equal for the flow through the Öresund and the Belt Sea, but the greater depth makes the correction term, $H/2\ \beta\ \Delta S$, larger for the Belt Sea. Thus, the ratio of the net flow through the Öresund to the total net flow should be greater than the ratio of the corresponding mean absolute flows, i.e., $\overline{Q_{\ddot{O}}}/\overline{Q_T} > 1/5$.

$\overline{Q_{\ddot{O}}}/\overline{Q_T} = 1/5$ may be regarded as a lower bound for the division, applicable for strong mean barotropic forcing. An upper bound may be provided by considering the case with no variable barotropic forcing. The main pycnocline level in the Kattegat is at a depth of ~ 15 m, and in the Belt Sea and the Öresund at a depth of ~ 10 m. The baroclinic exchange in the Öresund is hampered by the sill, the sill depth being only 8 m. Due to the shallowness of the Öresund, the baroclinic exchange should be mainly confined to the Belt Sea. However, the sea level difference associated with the baroclinic exchange in the Belt Sea can drive a barotropic outflow through the Öresund.

The correction term for the Belt Sea can be estimated by $H/2\ \beta\ \Delta S = 0.09$ m, using a mean sill depth in the Belt Sea $H = 12$ m and a salinity difference $\Delta S = 18$ psu. The corresponding number for the Öresund is $H/2\ \beta\ \Delta S = 0.03$ m using a mean sill depth in the Öresund $H = 5$ m and a salinity difference $\Delta S = 15$ psu. In a steady-state situation with no fluctuating barotropic transport and assuming no net exchange through the Belt Sea, it is assumed that the sea level difference would be equal to the baroclinic forcing through the Belt Sea, $\Delta\eta = 0.09$ m. This is almost sufficient to suppress baroclinic exchange through the Öresund, and model (2) can be used to estimate the flow. A corrected sea level difference through the Öresund $\Delta\eta' = 0.09–0.03 = 0.06$ m gives $Q_{\ddot{O},2} = 12\,500\ \mathrm{m^3\ s^{-1}}$, i.e. $\overline{Q_{\ddot{O}}}/\overline{Q_T} = 0.85$. (It is noticable that model (1) would give $Q_{\ddot{O},1} = 17\,200\ \mathrm{m^3\ s^{-1}}$.)

Simply assuming that, in the absence of variable barotropic forcing, the total net flow takes the route through the Öresund (i.e., $\overline{Q_{\ddot{O}}}/\overline{Q_T} = 1$), one has $1/5 < \overline{Q_{\ddot{O}}}/\overline{Q_T} < 1$. $\overline{Q_{\ddot{O}}}/\overline{Q_T}$ approaches $\overline{|Q_{\ddot{O}}|}/\overline{|Q_T|} = 1/5$ if the mean barotropic forcing is strong, and $\overline{Q_{\ddot{O}}}/\overline{Q_T}$ approaches 1 if the mean barotropic forcing is weak. The net flow through the Öresund during the test period is estimated to $\overline{Q_{\ddot{O},2}} = 7800\ \mathrm{m^3\ s^{-1}}$, which gives $\overline{Q_{\ddot{O},2}}/\overline{Q_T} = 0.53$. A simple guess for the division of the net flow thus is $\overline{Q_{\ddot{O}}}/\overline{Q_T} = 1/2$.

5. Conclusions

The presently used division of the mean absolute flow through the Öresund and the Belt Sea, 3:8, is based on an overestimation of the flow rate through the Öresund. By using new estimates of the flow rate through the Öresund, and comparing to estimates of the total flow rate and the flow rate through the Great Belt it is found that the division should be 2:8.

Using a recently calibrated model for the barotropic flow rate through the Öresund, and considering the baroclinic exchange, it is also argued that the long-term net flow through the Öresund may be approximately equal to the net flow through the Belt Sea, i.e. a considerably larger ratio than the ratio of the mean absolute flows.

Considering the uncertainty in the estimates of the net flow determined from current measurements or models for the barotropic flow, the division of the net flow is very uncertain and remains an unsolved problem. For the same reason, the possibility to determine the fresh water balance of the Baltic Sea to a high accuracy from direct estimates of the sea water exchange must be regarded as rather poor.

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