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Mixing in the ocean produced by tropical cyclones

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ABSTRACT

Tropical cyclones induce entrainment in the upper ocean, and the associated mixing may cool the sea surface significantly. In this paper, the significance of mixing produced by tropical cyclones in the Atlantic is considered. Observed hydrography and tropical cyclone data are used to force an ocean model, which predicts changes in the temperature field due to wind-generated entrainment. The knowledge of the temperature field is used to translate the mixing into volume flow and heat flux, across isotherms. A calculation of the mean, cross isothermal volume flow produced by Atlantic tropical cyclones is presented. Tropical cyclones mix water in the temperature interval between 18°C and 29°C. Below 24°C, the mixing moves water masses towards increasing temperature, and vice versa above 24°C. The strength of this flow is about 1×10^6 m^3 s^{-1}. Turbulent heat-transport is associated with the mixing, and the diffusive heat-flux across the 24°C isotherm is estimated to be 4×10^{12} W. Feedback between tropical-cyclone activity and the area of surface water warmer than 26.5°C is discussed. This feedback may attenuate large-scale sea-surface temperature anomalies, and its time-scale is estimated to be about 10 years.

1. Introduction

About 80 tropical cyclones develop over the oceans each year. These storms, known as nature's most destructive weather events, move over the tropical oceans at speeds of about 6 m s^{-1}. Tropical cyclones force a vigorous response in the ocean; surface wave heights in excess of 20 meter and upper-ocean current-strengths of 1 m s^{-1} are common. Entrainment is induced by the currents in the surface mixed layer, and the temperature of the ocean surface may be decreased by several degrees Celsius (e.g., Price, 1981). Near the storm centre, the mixing can homogenise the ocean down to depths below 100 m. Tropical cyclones leave in their wake a pattern of decreased sea-surface temperature, which persists about a week and is readily detected in satellite images (Stramma et al., 1986).

This work is an effort to determine the significance of entrainment produced by tropical cyclones for the oceanic mixing. A central issue is to translate mixing into cross isothermal volume flow (Walin, 1982), which describes quantitatively how water masses are moved between different temperature bands by mixing (or by air-sea heat exchange). The cross isothermal volume flow is an additive quantity and it can be determined directly from the temperature field. Furthermore, the formation of water masses at a specific temperature and the heat flux across isotherms are obtained readily from the cross isothermal volume flow. A merit of this approach is that the mean, cyclone-related cross isothermal volume flow can be obtained by adding mixing contributions from different storm events.

Our goal is to calculate the time average of the cross isothermal volume flow produced by Atlantic tropical cyclones. As a first step, the response of the thermal stratification to wind-forced entrainment is calculated for a large number of tropical cyclones. To this end we use a simple mixed-layer model, observed hydrography, and storm data. The information of the changes in the temperature field is then translated into cross isothermal volume flow.

We evaluate the significance of tropical-cyclone mixing by comparing our result with independent studies of water mass formation and heat-flux across isotherms in the tropical Atlantic. The present knowledge of these processes comes mainly from studies of air-sea heat exchange. Niiler and Stevenson (1982), for example, analyse the heat budget of tropical warm-water pools which are enclosed by the sea surface and an isothermal surface at depth. On the average, the advective heat transport vanishes since water is entering and leaving the pools at the same temperature. The heat flux through the ocean surface in such a pool has to be balanced by turbulent-diffusive heat transport through its isothermal boundary at depth; and the heat is transported essentially downward (Niiler and Stevenson, 1982). The sea-surface heat flux has been calculated as a function of the surface temperature for the different ocean basins by Andersson et al. (1982). This quantity is equal to the diffusive heat-transport across isotherms in a steady state, and its first and second derivative with respect to temperature are proportional to the cross isothermal volume flow and the rate of water-mass formation, respectively.

Mixing in the ocean is the result of intermittent turbulent events which are localised in space and time; breaking internal waves may serve as a prototype example of such an event. From a coarse-grain perspective, the turbulent events act to enhance the effective diffusivity and to transport heat down the large-scale temperature gradient, that is, mainly downward. Mixing produced by tropical cyclones is one of the processes which sustain the mean, diffusive heat-transport in the tropical ocean. In the present work, we do not address the important question of how the combined effect of small-scale mixing and large-scale mixing maintains the mean thermal-stratification. Some aspects of this issue are considered by Rhines (1988), who discusses how perturbations on the stratification due to localised mixing adjust under gravity in a rotating system. In the adjustment process, the effect of mixing is dispersed laterally, which strives to keep isothermal surfaces nearly horizontal.

Observations reveal that tropical cyclones form only where the sea-surface temperature exceeds 26.5°C (e.g., Anthes, 1982), and it is plausible that an increase in this area involves also an increase in the number of cyclones. Higher cyclone-activity results in enhanced cooling of the surface layer and acts to reduce the area of warm surface-water; the negative feedback is obvious. As a final topic, we discuss briefly if tropical cyclones may influence the distribution of sea surface temperature.

The principal results of this study are delineated in figures 7 and 8, and discussed in Section 6. Sections 2–5 introduces some definitions and describe the method of the work.

2. Mixing and cross isothermal volume flows

2.1. *Basic definitions*

Following Walin (1982) we define the following quantities that describe the thermal state of the ocean:

$T_0(\boldsymbol{x}, t)$: temperature field in the ocean;

R_T: subregion in ocean where $T_0(\boldsymbol{x}, t) \leqslant T$, where T is a parameter that will be used as an independent variable;

$V(T, t)$: volume of water in R_T;

$G(T, t)$: volume flow through the isothermal surface $T_0(\boldsymbol{x}, t) = T$, G is counted positive when directed into R_T.

The subregion R_T is limited by the isothermal surface T, and in general, by parts of the sea surface and the sea bottom. For sea water continuity of volume implies, to high accuracy, continuity of mass. If volume flows through the sea surface are neglected, then volume conservation in R_T can be expressed as

$$\frac{\partial V(T, t)}{\partial t} = G(T, t). \tag{2.1}$$

The cross isothermal volume flow is an additive quantity since it is related to changes in volume, which are additive in the Boussinesq approximation.

Conservation of heat in the region R_T can be formulated as (Walin, 1982)

$$\frac{\partial}{\partial t}\int^{T} cT' \frac{\partial V(T', t)}{\partial T'}\,\mathrm{d}T' = cTG(T, t) + F(T, t) + Q(T, t), \tag{2.2}$$

where c is the heat capacity per unit volume. The first term on the right-hand side in eq. (2.2) represents the advective heat flux. $F(T, t)$ represents the diffusive heat flux through the isothermal surface T. (Diffusive is here used in a broad sense, which embraces molecular as well as turbulent phenomena.) $Q(T, t)$ is the net heat flux across the parts of the ocean surface where the temperature is lower than T. The fluxes are counted positive when directed in to R_T. If we differentiate eq. (2.2) with respect to T and use eq. (2.1), we obtain Walin's (1982) relation

$$cG(T, t) = -\frac{\partial F(T, t)}{\partial T} - \frac{\partial Q(T, t)}{\partial T}. \tag{2.3}$$

By defining the quantities

$$v(T, t) \equiv \frac{\partial V(T, t)}{\partial T}, \qquad g(T, t) \equiv \frac{\partial G(T, t)}{\partial T}, \tag{2.4a, b}$$

we may express the volume of water in the ocean with a temperature in the interval $(T, T+\mathrm{d}T)$ as $v(T, t)\,\mathrm{d}T$. The formation of water in the temperature interval $(T, T+\mathrm{d}T)$ is given by $g(T, t)\,\mathrm{d}T$. Water masses are produced when g is positive and consumed when g is negative.

2.2. *Cross isothermal volume flow produced by a moving storm*

In the Atlantic, the average speed of tropical cyclones is 6 m s^{-1}. As they move over the ocean the intensity and the form of their wind fields vary. At a fixed point, however, the time variation of the wind field is usually dominated by the movement of the storm. For most tropical cyclones, it is therefore a good approximation to regard the response of the ocean as steady in a frame of reference that moves along with the storm. (The steady response of the ocean to a moving storm is, kinematically, similar to the stationary wave pattern generated by a ship in uniform motion.)

When a storm produces a steady oceanic response, the cross isothermal volume flow, $G(T, t)$, can be obtained as

$$G(T, t) = U(A_1(T) - A_2(T)). \tag{2.5}$$

Here, U is the velocity of the storm. The functions $A_1(T)$ and $A_2(T)$ are defined as the area of water colder than T, in two vertical cross-track slices, which are fixed relative to the storm. Here $A_1(T)$ is a slice behind the storm, and $A_2(T)$ is a slice in front of the storm, see Fig. 1 and eq. (A8) in

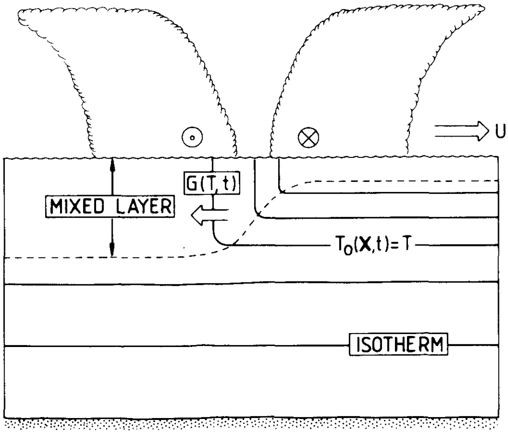

Fig. 1. Illustration of the oceanic response to a moving tropical cyclone. Entrainment occurs beneath the storm and the mixed layer deepens. The mixing is associated with a cross isothermal volume flow, $G(T, t)$, which can be obtained as $G(T, t) = U[A_1(T) - A_2(T)]$. Here $A_1(T)$ and $A_2(T)$ represent the area of water colder than T in two cross-track sections, broad enough to cover the storm induced mixing, which are located behind and ahead of the storm, respectively. An example of a section where $A_1(T)$ is computed is shown in Fig. 3; $A_2(T)$ is computed in a similar section, ahead of the storm, where the temperature field is given by the initial stratification.

Appendix A. We have neglected currents in the ocean in eq. (2.5). A general formula for calculating $G(T, t)$ is derived in Appendix A. The exact expression, eq. (A6), shows that the error introduced in eq. (2.5) is of order u/U, where u is a typical velocity of the currents in the wake of the storm.

Eq. (2.5) has a simple interpretation. The temperature field in the ocean, which is schematically delineated in Fig. 1, appears stationary relative to the moving cyclone. Water is thus continuously penetrating the nearly vertical isotherms in the mixed layer. The volume flow across an arbitrary isotherm is determined by the vertical area of that isotherm, that is, $A_1(T) - A_2(T)$, times the speed of the storm, U.

3. The data sets and ocean model

3.1. *The data sets and hurricane wind fields*

We use two data sources: the data set Atlantic Basin Best Track Documentation (Landsea, 1993), which contains position, central pressure and maximum sea level wind speed (given every sixth hour) of Atlantic tropical cyclones from 1886 to 1992. The ocean stratification is taken from the "Levitus database"; note that climatological stratification is used, which makes it impossible to account for situations where one storm crosses a fresh wake of another storm.

An impediment to the calculation is that the "Best Track" data does not contain information on the spatial form of the wind fields. No such data set is currently available (Landsea, personal communication). Therefore, we use the analytical, hurricane wind-profile of Holland (1980)

$$v(r) = v_{\max}(\{(R/r)^{B} \exp[1-(R/r)^{B}] + (r/R)^2 C^2\}^{1/2} - (r/R)\,C), \tag{3.1a}$$

where r is the radial distance from the storm centre, $v_{\max}$ is the maximum wind-speed, and R is the radius of maximum wind. The parameters B and C are defined as

$$B \equiv (v_{\max})^2\, \rho_{\mathrm{air}} \exp(1)/\Delta p, \qquad C \equiv fR/(2v_{\max}), \tag{3.1b, c}$$

where Δp is the pressure deficit in the storm centre and f is the Coriolis parameter.

To apply eq. (3.1), we need a method to determine the radius of maximum wind, R. Observations (Weatherford and Gray, 1988) and theoretical studies (Emanuel, 1986) suggest that no simple relationship exists between, on the one side, the radius of maximum wind, and on the other side, the central pressure deficit or the maximum wind speed. To make progress we use Weatherford and Grays' (1988) average relation between the eye-wall radius and the central pressure, and we place the radius of maximum wind 7.5 km outside the eye wall; the radius of maximum wind is usually found 3–10 km outside the eye wall (Emanuel, personal communication). Fig. 2 illustrates the pressure-radius relation. The study of Weatherford and Gray (1988) is based on Pacific tropical cyclones, but we assume that it is valid also for Atlantic storms.

3.2. *The model of the oceanic response*

The response of the ocean to moving tropical cyclones has been studied thoroughly by, for example, Price (1981) and Greatbatch (1984). Their work show that the mixed layer response can be estimated by a local model. The accuracy of the local description increase with the storm speed, and it is expected to be good when the storm moves past a fixed point within a few inertial periods.

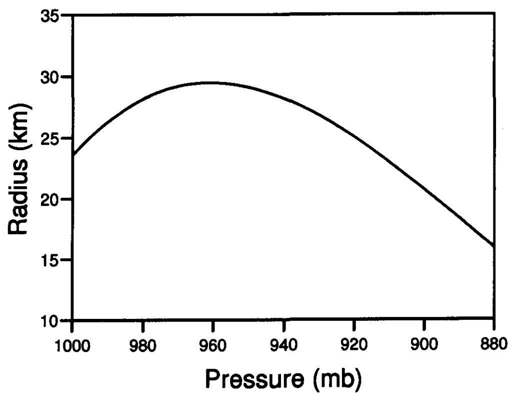

Fig. 2. Empirical, average relation between the radius of maximum wind and the central surface-pressure (Weatherford and Gray, 1988). Note that the variability of the radius of maximum wind is considerably larger than suggested by the average relation; the radius of maximum wind of a tropical cyclone may exceed 100 km.

In the forthcoming computations, the initial state of the ocean and the wind fields of the cyclones are uncertain to some degree. Therefore, it is unmotivated to use a sophisticated description of the ocean dynamics. We use the one-dimensional mixed-layer model described by Price et al. (1978). Heat and fresh-water fluxes through the sea surface are of secondary importance in the present context (Price, 1981), and they are consequently ignored. In Appendix B, the equations that govern the mixed layer are summarised.

4. The cross isothermal volume flow produced by a moving tropical cyclone

Before we address the integrated effect of cyclone mixing, it is instructive to discuss the cross isothermal volume flow produced by a single tropical cyclone. Here, we consider a storm of hurricane intensity (surface winds exceeding 33 m s^{-1}) and a tropical storm (surface winds exceeding 18 m s^{-1}). The wind fields, the speeds, and the positions of the

Table 1. *Mean values of some tropical cyclone parameters; TS denotes tropical storms and HR denotes hurricanes*

Storm class	Storm days	Position	Storm speed (m s^{-1})	Wind speed (m s^{-1})	Central pressure (mb)
TS 83–87	83	60°W 30°N	6.0	23.7	998
HR 83–87	32	62°W 31°N	6.0	40.1	976
TS 88–92	98	51°W 27°N	6.3	24.0	999
HR 88–92	74	55°W 30°N	5.5	43.0	973

storms are chosen to represent a mean hurricane and a mean tropical storm in the Atlantic during the period 1988–1992. The mean storm in each category is defined by the mean values of the storm parameters, see Table 1.

Beneath a moving tropical cyclone, the wind forces entrainment, and cold water is thereby mixed up in the surface layer. Behind the storm, the mixed layer depth is increased and the sea-surface temperature is lowered. Using information on the wind field, the speed of the storm, and the oceanic stratification, we calculate the temperature field in the wake of the cyclone (see Appendix B). Fig. 3 shows a cross-track slice of the temperature field behind a mean Atlantic hurricane. Knowledge of the temperature field is used to determine the

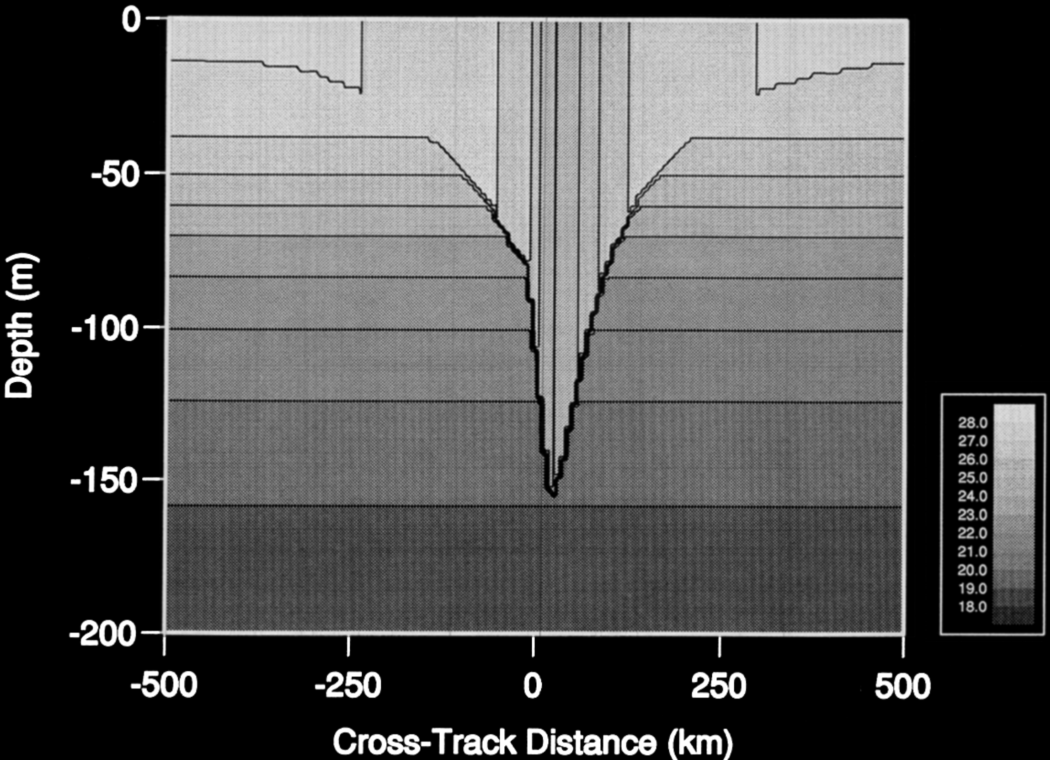

Fig. 3. Cross-track temperature field in the wake of a mean Atlantic hurricane (1988–1992), which is defined by the parameters in Table 1. The initial stratification, assumed to be horizontally uniform over the storm domain, is visible in the periphery. Entrainment alone has produced the changes in the temperature field. In reality this temperature distribution would be distorted by upwelling and internal waves, which are neglected in the present model.

cross isothermal volume flow according to eq. (2.5). We use a local model that neglects advection, which implies that the temperature field is "frozen" as soon as entrainment ceases. In reality, and in a nonlocal model, internal waves in the wake of the storm distort the temperature field (e.g., Price, 1981), but the cross isothermal volume flow can still be calculated from the velocity and the temperature field in two cross-track sections (see eq. (A6)).

Fig. 4 shows the cross isothermal volume flows produced by a mean hurricane and a mean tropical storm in the Atlantic. Recall that since air-sea heat exchange is ignored in the our model, the calculated cross isothermal volume flow is produced entirely by turbulent mixing. The hurricane-induced mixing spans between 19°C and 27°C. The limits are set by the initial mixed layer temperature and the temperature of the initial stratification at the maximum penetration depth of entrainment. Below 24°C, where $G(T, t)$ is negative, the mixing moves water masses towards higher temperature, and above 24°C water masses are moved towards lower temperature. The hurricane-forced entrainment produces water masses in the temperature interval between 23°C and 27°C, see eq. (2.4b). These water masses are

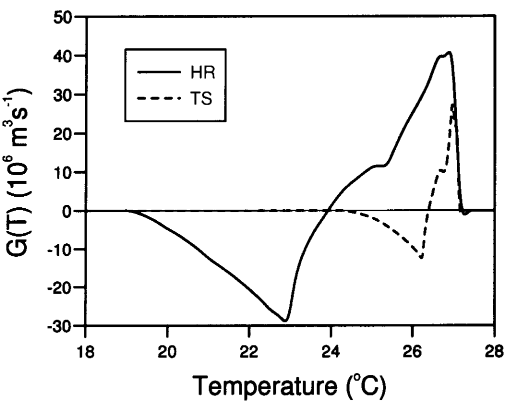

Fig. 4. Cross isothermal volume flow produced by a moving tropical cyclone. The result is obtained by calculating the changes in the stratification due to entrainment and using eq. (2.5). Two cases are illustrated: a mean Atlantic hurricane (HR) and a mean Atlantic tropical storm (TS) in the period 1988–1992, see Table 1. Where $G(T, t)$ is negative the mixing moves water masses towards higher temperature, and vice versa where G is positive.

found in the wake of the storm. The cross isothermal volume flow produced by the tropical storm has similar features, but it is confined in a more narrow temperature band. This merely reflects that entrainment is weaker for lower wind speeds.

5. The average cross isothermal volume flow produced by tropical cyclones in the Atlantic Ocean

5.1. Definitions

We consider mixing produced by Atlantic tropical storms (winds exceeding 18 m s^{-1}) and hurricanes (winds exceeding 33 m s^{-1}). Two successive 5-year periods are investigated. The first period, 1983–1987, is characterised by a low storm-activity, while the latter period, 1988–1992, has more storms (Gray, 1990). We exclude the storms in the Caribbean sea and the Mexican Gulf to avoid situations where the storms, partly or completely, move over land. Mean values of some relevant storm parameters are listed in Table 1. From the "Best Track" data, 6-h average values of position, storm speed, wind speed and central pressure are taken. Each storm class (class refers to storm category and period) is associated with a number, say N, of 6 h average storm observations. We assume that the cyclones produce a steady oceanic response throughout each six hours period. The cross isothermal volume flow associated with the nth observation, say $G_\gamma(T, n)$ (the lowered index refers to the class), is calculated from eq. (2.5).

An ensemble average (or an arithmetical mean value) of the cross isothermal volume flows for a storm class is defined as

$$\langle G_\gamma(T) \rangle \equiv \frac{1}{N} \sum_{n=1}^{N} G_\gamma(T, n). \tag{5.1}$$

The time averaged cross isothermal volume flow can be obtained as

$$\overline{G_\gamma(T)} = v_\gamma \langle G_\gamma(T) \rangle, \tag{5.2}$$

where v_γ is the relative storm frequency, which is defined as the number of storm days (i.e., the sum of the life-times of each storm) divided with the total number of days in the period.

5.2. An approximation of the ensemble average

From experience, we find that a good approximation of the ensemble average can be obtained from a limited number of the storm events. This is demonstrated in Fig. 5 which displays an approximation of the ensemble average, eq. (5.1), calculated by randomly selecting M events of the total N events. Hurricanes (1988–1992), for which $N = 295$, are chosen and the ensemble average is calculated for $M = 20$, $M = 40$, and $M = 60$. The difference between 40 and 60 storm events is small, and we will henceforth approximate the ensemble average using 60 randomly selected events. Calculations performed for the other storm classes show similar features.

5.3. On the rôle of the radius of maximum wind

The study of Weatherford and Gray (1989) indicates a fairly large scatter around the average pressure-radius relation. For a given central pressure, there are thus storms with larger as well as with smaller radius of maximum wind than predicted by the average pressure-radius relation, see Fig. 2. To shed light on how this may affect our results, we make two computations in which the pressure-radius relation is modified. Hurricanes (1988–1992) are considered and the ensemble averages are calculated from the same storm observations. In the first calculation, the radius of maximum wind is increased with a factor of 1.25 for all storms, and in the second it is decreased with a factor of 0.75. Fig. 6 illustrates the results. An increase of the radius of maximum wind, which also increases the radius of the cyclone, yields more vigorous mixing and consequently a stronger cross isothermal volume flow.

This modification of the pressure-radius relation has a nearly linear effect on the ensemble average. A calculation based on the mean pressure-radius relation may give a reasonable result, since the contributions from those cyclones that are larger or smaller than the mean storm tend to average out. However, this conclusion requires that the radius of maximum wind is not correlated with other parameters (e.g., the sea surface temperature) that determine the cross isothermal volume flow.

5.4. Results

The time average of the cross isothermal volume flow are obtained according to the following scheme.

(1) Select a random sample of 60 cyclone events from the "Best Track" data and pick the initial stratification from Levitus database.

(2) Specify the cyclone wind fields according to eq. (3.1) and the pressure-radius relation (Fig. 3).

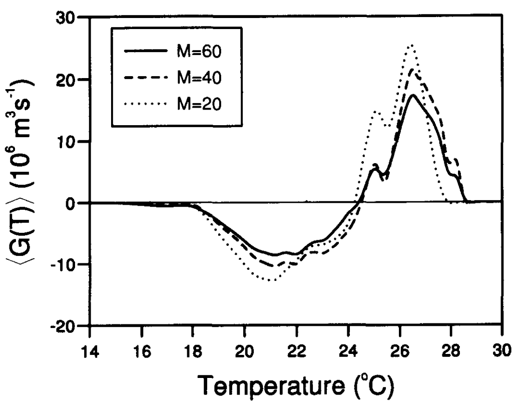

Fig. 5. An approximation of the ensemble average of the cross isothermal volume flow, eq. (5.1), obtained by randomly selecting a limited number, say M, of the available storm observations. Atlantic hurricanes (1988–1992) are considered, for which there are totally 295 observations. All ensemble averages are henceforth approximated from 60 observations.

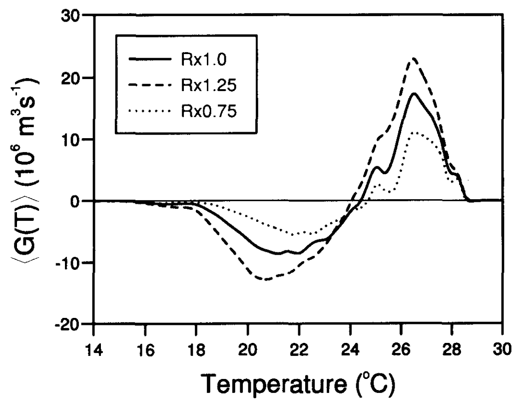

Fig. 6. The effect on the ensemble average of altering the radius of the cyclones. Relative to the normal case (see Fig. 3), the radius of maximum wind is in the one case increased with a factor of 1.25 for all cyclones, and in the other decreased with a factor of 0.75. For comparison, the normal case is illustrated.

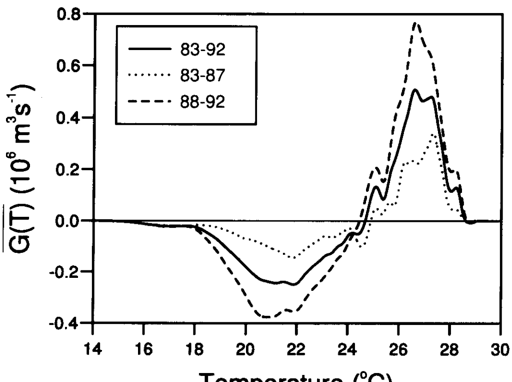

Fig. 7. The time average of cross isothermal volume flow generated by tropical cyclones in the Atlantic; this flow is due entirely to wind-forced entrainment in the surface mixed-layer. Positive values indicate flow towards decreasing temperature. The formation of water masses is given by the derivative of the cross isothermal volume flow with respect to temperature (eq. (2.4b)). Results are displayed for the periods 1983–1987, 1988–1992, and 1983–1992.

(3) Calculate the mixed layer response (see Appendix B) and determine the cross isothermal volume flow (eq. (2.5)) for each cyclone event.

(4) Add the cross isothermal volume flows and apply eq. (5.2).

Fig. 7 shows the time averaged, cross isothermal volume flow produced by tropical cyclones in the Atlantic. Results are displayed for the periods 1983–1987, 1988–1992, and 1983–1992. The strength of the time averaged, cross isothermal volume flow differs roughly by a factor of two between the two 5-year periods. During the latter period there were a larger number of intense hurricanes and also more storm days (Gray, 1990). For both periods, the cross isothermal volume flow is dominated by hurricane produced mixing.

6. Discussion

6.1. Formation of water masses

The result of tropical-cyclone mixing is to move water masses in and out of certain temperature intervals, a feature which is described quantitatively by the mean cross isothermal volume flow. Fig. 7 shows that tropical-cyclone mixing consumes water masses in the temperature bands below 22°C and above 27°C, respectively. Turbulent stirring mixes these water masses, and as a result water masses are produced in the temperature interval between 22°C and 27°C.

If the ocean is in a statistically steady-state, then the mean formation of water masses at any temperature has to vanish globally. In the ocean, two processes force formation of water masses: Mixing, which strives to reduce temperature contrasts; and heat fluxes through the sea surface, which act to increase temperature contrasts by heating at high temperatures and cooling at low temperatures. Calculations for the North Atlantic presented by Andersson et al. (1982) and by Speer and Tziperman (1992) suggest that sea-surface heat fluxes, in broad terms, produce water masses at temperatures above 27°C and below 10°C or lower, respectively, and remove water masses from the intermediate temperature band. A spectrum of oceanic mixing-processes, which are distributed widely in space and over different temperatures, compensates the formation forced by air-sea heat exchange.

In the Atlantic, the formation due to sea-surface heat fluxes exceeds significantly the cyclone-induced formation. Our calculation suggests that tropical-cyclone mixing produce water in the temperature band between 22°C and 27°C at a rate of about $1 \times 10^6\ \mathrm{m^3\,s^{-1}}$; while according to the above cited calculations, surface heat-fluxes yield a consumption of a rate between $20 \times 10^6\ \mathrm{m^3\,s^{-1}}$ and $35 \times 10^6\ \mathrm{m^3\,s^{-1}}$ in the same temperature band.

The water masses produced by cyclone mixing (between 22°C and 27°C) are found in the wake of the storms. Initially, the water masses in the wake communicate with the atmosphere. As time evolves, air-sea heat exchange and mixing in the surface layer are expected to transform these water masses, mainly towards increasing temperature (e.g., Stramma et al., 1986; Tandon and Garrett, 1994). Above 27°C, the cyclone-induced mixing consumes surface water in a temperature band where surface heat-fluxes force, on the average, a huge production of water-masses. Near-surface mixing and air-sea heat exchange can accordingly compensate the cyclone-induced formation, more or less, locally in temperature band above 22°C.

By contrast, the water-masses consumed below 22°C are not likely to be compensated locally as a result of air-sea heat exchange. These water masses are located beneath the average mixed-layer depth and are therefore isolated from the atmosphere. Mixing in the upper thermocline or surface heat-fluxes and mixing in the outcrop region of the isotherms around 20°C may reproduce these water masses, but this work gives no indication of which process, if any, that dominates. Reproduction by mixing in the thermocline is likely since air-sea heat exchange mainly acts to remove water masses from this temperature band. However, compensation due to air-sea heat exchange cannot be ruled out: The calculation of Andersson et al. (1980) indicates some production of water between 17°C and 22°C; Speer and Tziperman (1992) find an alternating pattern of production and consumption in the same temperature band. Here it is important to stress that the formation due to tropical-cyclone mixing is a small component in a global pattern of water-mass formation. Therefore it may be difficult to isolate processes which compensate the cyclone-induced formation.

Tropical-cyclone mixing produces a mean cross isothermal volume flow, towards increasing temperature, at 22°C with a strength of about $0.5 \times 10^6\ \mathrm{m^3\,s^{-1}}$. It is interesting to compare this figure with estimates of the cross isothermal circulation in the North Atlantic thermocline. Sarmiento (1986), using a general circulation model, computed the upwelling across the 22°C isothermal surface between the Equator and 20°N to $5 \times 10^6\ \mathrm{m^3\,s^{-1}}$. As another reference, Wunsch (1984) calculated the upwelling rate in the equatorial Atlantic (between 15°S and 15°N) to $10\text{–}20 \times 10^6\ \mathrm{m^3\,s^{-1}}$.

6.2. *The diffusive heat flux*

Large pools of warm near-surface water are encountered in the tropics. For example, the volume of water enclosed by the sea surface and the 25°C isothermal surface can be regarded as a pool with nearly horizontal bottom and vertical sides. A huge amount of heat enters these pools through the sea surface and is transported downward by turbulent diffusion (Niiler and Stevenson, 1982). As a first approximation, the long-term mean state of the warm-water pools (and also of the whole ocean) can be regarded as steady. At this level of description the mean cross isothermal volume flow through any isotherm vanishes, see eq. (2.1). Heat conservation (eq. (2.2)) for the long-term mean state of the ocean reduces consequently to

$$\overline{F(T)} + \overline{Q(T)} = 0, \tag{6.1}$$

where the overbar denotes a time average. The physics implied by eq. (6.1) is simple (Walin, 1982; Niiler and Stevenson, 1982): The heat flux into the part of the ocean where the surface temperature exceeds T, that is, $-\overline{Q(T)}$, is balanced by a downward diffusive heat flux across the isothermal surface T, that is, $-\overline{F(T)}$. (Recall that the fluxes are counted positive when directed into the region colder than T.)

It is difficult to estimate the diffusive heat-flux directly, but the function $\overline{Q(T)}$ has been calculated for the different ocean basins by Andersson et al. (1982). Niiler and Stevenson (1982) computed the heat supply in the Atlantic ocean to those regions where the sea-surface temperature exceeds 24, 25, and 26°C, respectively. They arrived at: $-\overline{Q(24)} = 1.9 \times 10^{14}$ W, $-\overline{Q(25)} = 2.0 \times 10^{14}$ W, and $-\overline{Q(26)} = 1.3 \times 10^{14}$ W.

A large number of intermittent mixing events contribute to the mean, diffusive heat-flux $\overline{F(T)}$. The contribution from tropical-cyclone mixing is easily determined since air-sea heat exchange is ignored in our calculations, that is, $Q(T, t)$ is zero. We obtain therefore the contribution to the mean, diffusive heat-flux from the cyclones by integrating the mean cross isothermal volume flow (see eq. (2.3)):

$$\overline{F_\gamma(T)} = -c \int^{T} \overline{G_\gamma(T')}\, \mathrm{d}T', \tag{6.2}$$

where the index refers to the storm class.

Fig. 8 shows the mean, diffusive heat-flux which is produced by tropical cyclones in the periods 1983–1987 and 1988–1992. For comparison, the form of the total, diffusive heat-flux in the North Atlantic (Andersson et al., 1982) is illustrated; note that the total heat-flux is down-scaled with a factor of 100! The estimate of Andersson et al. (1982) exceeds that of Niiler and Stevenson (1982) (who use a different data set) roughly by a factor of 4. Nevertheless, both calculations suggest that the contribution from tropical-cyclone mixing only constitutes a few percent of the total, diffusive heat-flux. This indicates that cyclone-induced entrainment plays only a minor role for the heat balance of the warm-water pools in the Atlantic. On the other hand, tropical cyclones may be of importance on regional scales: They are not for-

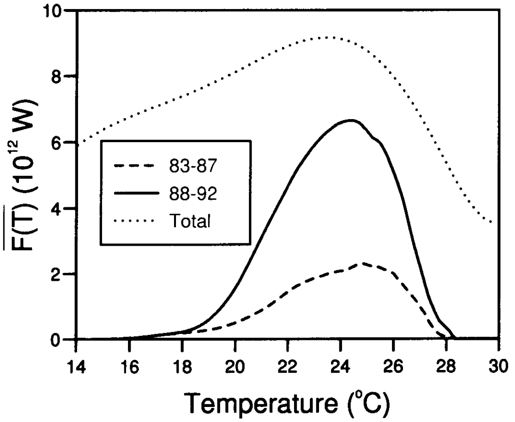

Fig. 8. The time average of diffusive heat-flux, across isotherms, produced by tropical cyclones in the Atlantic. The form of the total, diffusive heat-flux (Andersson et al., 1982) is indicated by the dotted line. Note that the total, diffusive heat-flux has been scaled down by a factor of 100!

med in the southern Atlantic, and in the northern Atlantic the cyclone activity is concentrated to the western part of the basin.

If entrainment produced by tropical cyclones can be ruled out, which mixing processes are then sustaining the diffusive heat-flux in the upper tropical ocean? Wind-induced entrainment, tropical cyclones excluded, is not a likely candidate; its penetration is too shallow to reach, for example, the 25°C isothermal surface in the interior of the warm-water pool. Mixing in the equatorial region (where the thermocline is thin and shallow and large velocity shears are encountered) and mixing due to breaking internal-waves are probably main candidates. Tropical cyclones can be of significance in this context since they pump a significant amount of energy into the oceanic internal-wave field (Nilsson, 1995).

6.3. *Temporal and geographical variability*

We have considered the Atlantic tropical cyclones in the period from 1983 to 1992. A natural question is to what extent our results are representative for different periods and other ocean basins.

In the Pacific there develops on the average six times more tropical cyclones than in the Atlantic (Gray, 1979). The heat input to waters warmer than 26°C is eight times greater in the Pacific than in the Atlantic (Niiler and Stevenson, 1982). Roughly, the ratio between the heat input and the number of cyclones is equal in the two basins. If the Pacific tropical cyclones produce a mean cross isothermal volume flow, which is similar to the one produced by Atlantic cyclones (but with an amplitude that is larger in proportion to the number of storms) then tropical cyclones ought to be equally important for the heat balance of warm-water pools in the two basins.

The cyclone activity exhibits a significant multidecadal variability (Gray, 1990). Periods of high activity are alternated by relatively quiescent ones. We have chosen a period of ten years that begins with 5 years of low activity and ends with a period of higher activity. The results displayed for these two successive 5-year periods are expected to shed light on how the cyclone-induced mixing varies between periods of different storm activity; see Figs. 7 and 8.

6.4. *Sea-surface temperature and tropical-cyclone activity*

The multidecadal variation in tropical-cyclone activity is correlated with anomalies in the global ocean-atmosphere circulation (Gray, 1990). For example, low cyclone activity coincides with periods of lowered sea-surface temperatures in the northern Atlantic, and vice versa for high activity. Variability in the annual number of cyclones may merely be a side effect of the large-scale circulation anomalies. On the other hand, tropical cyclones may relax perturbations on the distribution of sea surface temperature since they form only where the surface temperature exceeds 26.5°C (Anthes, 1982). The cyclone activity is, almost certainly, positively correlated with the area of surface-water warmer than 26.5°C, say $S(26.5°, t)$. An increase in $S(26.5°, t)$ involves more cyclones and thereby enhanced cooling of the oceanic surface-layer. Similarly, a decrease in $S(26.5°, t)$ yields reduced cooling of the upper ocean due to reduced cyclone-activity.

We make here a crude estimate of the time scale associated with this negative feedback. Suppose that the Atlantic at present is in a statistically steady state and consider departures from the mean state of $S(26.5°, t)$ and the mean, relative cyclone-frequency ν

$$S(26.5°, t) = S_0 + S'(t), \tag{6.3}$$

$$\nu(t) = \nu_0 + \nu'(t). \tag{6.4}$$

The time, t, should be interpreted as a "slow" time variable which averages out seasonal variability.

Tropical cyclone formation is the result of a finite amplitude instability (Emanuel, 1986); igniting disturbances are amplified over the warm ocean, due to a wind-induced increase of the air-sea heat exchange. The annual number of potential igniting-disturbances is relatively constant (Landsea, 1993). We propose therefore the following relation between the relative cyclone-frequency and the surface-temperature distribution:

$$\nu(t) = \alpha S_0 + \beta S'(t), \tag{6.5}$$

where α and β are constants. The variability of $\nu(t)$ affects the cross isothermal volume flow produced by tropical cyclones

$$G_{\mathrm{TC}}(T, t) = G_0(T) + G'(T, t). \tag{6.6}$$

Here $G_0(T)$ is the mean, cyclone-related flow at present, which is shown in Fig. 7. We assume that a change in the number of cyclones alters the strength of the cross isothermal volume flow but preserves its form, that is, we write eq. (6.6) as

$$G_{\mathrm{TC}}(T, t) = G_0(T) + G_0(T)(\nu'(t)/\nu_0). \tag{6.7}$$

View the volume of water warmer than 26.5°C as a pool that is characterised by its mean depth H_0 and its area $S(26.5°, t)$. In the steady mean-state, the volume flow across the 26.5°C-isothermal surface vanishes. Accordingly, the cyclone-related perturbation on the cross isothermal volume flow, that is, $G'(26.5°, t)$, acts to change the volume of the pool. The strongest feedback is achieved when the cyclone mixing affects only the area of the pool. We do not claim that this is the case, nor do we attempt to describe the link between mixing and volume adjustment of the pool. However, this assumption gives a lower bound of the feedback time-scale. We can now relate volume changes of the pool to the cyclone-related perturbation on the cross isothermal volume flow

$$H_0 \frac{\mathrm{d}S'(t)}{\mathrm{d}t} = -\frac{\beta S'(t)}{\alpha S_0} G_0(26.5°). \tag{6.8}$$

The equation describes a relaxation of perturbations on $S(26.5°, t)$ with an e-folding decay time, say τ, equal to

$$\tau = (\alpha/\beta)\, S_0 H_0 (G_0(26.5°))^{-1}. \tag{6.9}$$

A rough estimate gives:

$$S_0 \simeq 10^{13}\ \mathrm{m}^2, \qquad H_0 \simeq 100\ \mathrm{m}, \qquad G_0(26.5°) \simeq 10^6\ \mathrm{m}^3\,\mathrm{s}^{-1}. \tag{6.10}$$

We then arrive at

$$\tau = (\alpha/\beta)\ 30\ \text{years}. \tag{6.11}$$

For simple physical reasons, β ought to be larger than α: Between 5°S and 5°N no tropical cyclones are formed, and the large area where cyclones do not form has to be accounted for in the proportionality (between area and cyclone activity) expressed by α. We conjecture that the ratio α/β lays between 1 and $\frac{1}{5}$, which gives values of τ between 6 and 30 years. It is important, however, to recognise that more efficient restoring mechanisms, with shorter time scales, may be in operation when the surface temperature is perturbed from its climatological value. For example, the study of Stramma et al. (1986) shows that surface-temperature anomalies in the wake of cyclones are relaxed within a few weeks. Here we are considering water volumes with depths of about 100 m, and the time required for such deep water columns to reach equilibrium with the atmosphere is rather months or years than weeks (Barnett, 1978). Therefore, tropical cyclones may to some extent influence the distribution of sea-surface temperature; but we stress that the present analysis is tentative.

7. Concluding remarks

Although the present computation is connected with several sources of uncertainty we believe that it provides a useful, qualitative picture of mixing produced by tropical cyclones in the Atlantic. The results indicate that tropical-cyclone mixing plays a minor role for the heat balance of the tropical ocean and the global balance of water-mass formation. However, when more reliable information on the cyclone wind-fields becomes available it is motivated to further study this topic.

Our application illustrates the general principles of how mixing is quantified in terms of cross isothermal volume flow. Earlier related studies

(Andersson et al., 1982; Speer and Tziperman, 1992) have focused on cross isothermal, or cross isopycnal volume flows, forced by air-sea exchange of heat and fresh water. Complimentary information is obtained by studying mixing in the same way. Clever use of this idea might be a valuable tool for studying the relation between small-scale mixing and large-scale circulation in the ocean.

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9. Appendix A

The cross isothermal volume flow for moving storms

A general expression for cross isothermal volume flow of a travelling storm is derived here. The function $V(T, t)$, see Subsection 2.1, can be expressed as

$$V(T, t) = \iiint_{\mathrm{R}} \mu(T - T_0(\boldsymbol{x}, t))\, \mathrm{d}V, \qquad \text{(A1)}$$

where μ is a unit step function and $\mathrm{d}V$ is the element of volume. R denotes the region of interest, e.g., the Atlantic. The storm mixing is confined to a limited region and we define a region, say D, of fixed horizontal geometry, which remains aligned with the storm's velocity. The boundary of D, ∂D, is a vertical surface that extends from the sea surface to the sea floor. We introduce a new coordinate system, $\boldsymbol{x}'$, with origin in the storm centre, say $\boldsymbol{x}_\mathrm{c}(t)$, and with the positive x-axis aligned with the storm's velocity:

$$\boldsymbol{x}' = \boldsymbol{x} - \boldsymbol{x}_\mathrm{c}(t). \qquad \text{(A2)}$$

In this frame of reference the boundary of D is fixed relative to the storm. The cross isothermal volume flow produced by the storm is given by the *material* rate of change of $V(T, t)$ within D:

$$G(T, t) = \frac{D}{\mathrm{D}t} \iiint_D \mu(T - T_0(\boldsymbol{x}', t))\, \mathrm{d}V'. \qquad \text{(A3)}$$

By using Lebnitz's theorem we obtain

$$G(T, t) = \frac{\partial}{\partial t} \iiint_D \mu(T - T_0(\boldsymbol{x}', t))\, \mathrm{d}V' + \iint_{\partial D} \mu(T - T_0(\boldsymbol{x}', t)) \left(\frac{\mathrm{D}\boldsymbol{x}'}{\mathrm{D}t}\right) \cdot \mathrm{d}\boldsymbol{A}', \qquad \text{(A4)}$$

where $\mathrm{d}\boldsymbol{A}'$ is the element of area on ∂D. $\mathrm{D}\boldsymbol{x}'/\mathrm{D}t$ denotes the velocity of a material fluid element measured the new coordinate system

$$\left(\frac{\mathrm{D}\boldsymbol{x}'}{\mathrm{D}t}\right) = \boldsymbol{u}' - \hat{\boldsymbol{x}}'\, |\boldsymbol{U}| - \boldsymbol{\Omega} \times \boldsymbol{x}', \qquad \text{(A5a)}$$

where $\boldsymbol{u}'$ is the fluid velocity relative to the earth fixed frame of reference, but here expressed with the unit vectors of the storm fixed system. Here $\boldsymbol{U}$ and $\boldsymbol{\Omega}$, the angular velocity of the storm fixed coordinate system, are defined as

$$\boldsymbol{U} \equiv \frac{\mathrm{d}\boldsymbol{x}_\mathrm{c}}{\mathrm{d}t}, \qquad \boldsymbol{\Omega} \equiv \left(\boldsymbol{U} \times \frac{\mathrm{d}\boldsymbol{U}}{\mathrm{d}t}\right) |\boldsymbol{U}|^{-2}. \qquad \text{(A5b, c)}$$

In eq. (A4), the first term on the right-hand side describes the local rate of change of the volume distribution in D, while the second term describes the advective rate of change. When the oceanic response is steady and the storm travels along a straight path, the first in term eq. (A4) and $\boldsymbol{\Omega}$ are zero. This gives

$$G(T, t) = \iint_{\partial D} \mu(T - T_0(\boldsymbol{x}')) \times (\boldsymbol{u}' - \hat{\boldsymbol{x}}'\, |\boldsymbol{U}|) \cdot \mathrm{d}\boldsymbol{A}'. \qquad \text{(A6)}$$

If and we use a rectangular domain, D, and neglects the fluid velocity $\boldsymbol{u}'$, then we recover eq. (2.5)

$$G(T, t) = |\boldsymbol{U}|\, (A_1(T) - A_2(T)), \qquad \text{(A7)}$$

where

$$A_1(T) - A_2(T)$$

$$= \iint_{\partial D} \mu(T - T_0(\boldsymbol{x}'))(-\hat{\boldsymbol{x}}' \cdot \mathrm{d}A'). \tag{A8}$$

Note that the contributions from the two sides that are parallel to the storm track cancel due to symmetry; they are located far from the storm where the ocean is unaffected by the mixing. Eq. (A8) has simple interpretation. $A_1(T)$ is the area of water colder than T in a cross track section behind the moving storm. The interpretation of $A_2(T)$ is similar, except that this cross track section is located in front of the storm. The two cross track sections are of equal width and are broad enough to cover the storm induced entrainment.

10. Appendix B

The equations describing the mixed layer

Initially, the ocean is laterally homogeneous and at rest. The initial temperature and salinity field are given by $T_i(z)$ and $S_i(z)$. The mixed layer variables are the depth, $H(x, y, t)$, the temperature, $T(x, y, t)$, the salinity, $S(x, y, t)$ and the horizontal velocity, $\boldsymbol{u}(x, y, t)$. These variables are governed by:

$$\frac{\partial(\boldsymbol{u}H)}{\partial t} + \boldsymbol{f} \times (\boldsymbol{u}H) = \boldsymbol{\tau}, \tag{B1}$$

$$\frac{\partial H}{\partial t} = W_e, \tag{B2}$$

$$\frac{\partial T}{\partial t} = (T_i(z = -H) - T)\frac{W_e}{H}, \tag{B3}$$

$$\frac{\partial S}{\partial t} = (S_i(z = -H) - S)\frac{W_e}{H}. \tag{B4}$$

Here $\boldsymbol{f}$ is the Coriolis parameter times the vertical unit vector. The wind stress (kinematical units), τ, is given by Smith's (1980) formula. The entrainment velocity, w_e, is parametrised according to the critical Richardson-number criteria (Pollard et al., 1973). A bulk Richardson number is defined as

$$\mathrm{Ri} \equiv \frac{gH\,\Delta\rho}{\boldsymbol{u}^2\rho_0} \quad \text{or} \quad \mathrm{Ri} \equiv \frac{gH^3\,\Delta\rho}{(\boldsymbol{u}H)^2\,\rho_0}, \tag{B5}$$

where $\Delta\rho$ is the density difference across the base of the mixed layer. The Richardson number cannot decrease below a critical value, say $\mathrm{Ri_c}$,

$$\mathrm{Ri} \geqslant \mathrm{Ri_c}. \tag{B6}$$

We use $\mathrm{Ri_c} = 0.65$ (Price et al., 1978). Eqs. (B2)–(B4) yield

$$T(H) = \frac{T(t=0)\,H(t=0)}{H} + \frac{1}{H}\int_{H(t=0)}^{H} T_i(z = -H')\,\mathrm{d}H', \tag{B7}$$

$$S(H) = \frac{S(t=0)\,H(t=0)}{H} + \frac{1}{H}\int_{H(t=0)}^{H} S_i(z = -H')\,\mathrm{d}H'. \tag{B8}$$

The pressure dependence of ρ is neglected, which implies that $\Delta\rho$ is a function of H only. Eqs. (B5) and (B6) relate H to $\boldsymbol{u}H$:

$$\mathrm{Ri_c}(\boldsymbol{u}H)^2_{\max} = \frac{gH^3\,\Delta\rho(H)}{\rho_0}, \tag{B9}$$

where we have defined

$$(\boldsymbol{u}H)^2_{\max} \equiv \max[(\boldsymbol{u}H(x, y, t'))^2], \quad t' \in (-\infty, t). \tag{B10}$$

This definition is necessary since the mixed layer depth does not diminish when $(\boldsymbol{u}H)^2$ decreases. The mixed layer depth, H, can be determined from eq. (B9), since $(\boldsymbol{u}H)^2$ can be obtained from eq. (B1) by a straight forward numerical integration.

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