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Truncation and use of model-coordinate data

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ABSTRACT

The advantages and disadvantages of using global analyses on pressure level surfaces versus model (sigma or hybrid) level surfaces are explored. Model levels consist of a terrain-following coordinate in the lowest levels but may gradually transition to pressure with height. One major issue is that the model surface often does not correspond with the earth's real surface. Another is that a change in horizontal resolution is not well defined because the vertical coordinate also changes in the process. However, such changes are required as comparison of analyses from one center with another or with model output requires a common vertical coordinate. Also, a reduction from high resolution with 106 waves (T106 resolution) to a moderate resolution with, say, 42 waves (T42), so as to reduce the size of datasets by a factor of about 6, is often desirable. Another issue involves the meaning and use of time averages on model surfaces and their relationship to corresponding time averages on pressure surfaces. The traditional representation on constant pressure surfaces has much to recommend it and is widely used and familiar to the community, but suffers from errors arising from interpolation to the pressure surfaces and the need to properly treat regions of those surfaces that are below ground (in regions of high topography). Spectral truncation and resolution changes are well defined on pressure surfaces. Test results are presented for cases where T106 analyses on model surfaces are (1) transformed to pressure and then truncated, (2) truncated and then transformed to pressure levels, and (3) then compared at comparable resolutions. The differences arise mainly from the ill-posed truncation on model surfaces, but also from vertical interpolations. These tests are also applied to a heat budget calculation involving nonlinear terms for one month. However, the latter comparison also brings in considerations of how best to do time averaging. Fortunately, in practical terms, differences between time-averaging approaches are quite small and probably negligible for most purposes. Other results show that truncation on model surfaces from T106 to T42 produces errors considerably larger than vertical interpolation errors. However, truncation to T63 for heat budget computations produces more acceptable, although not inconsequential, truncation errors provided that only T31 waves are retained as reasonably accurate.

1. Introduction

Global analyses of various quantities are needed to carry out comprehensive diagnostic studies. The most convenient global data sets for this purpose are the global analyses produced operationally for weather forecasting purposes (Fortelius and Holopainen, 1990; Trenberth, 1991a). These are produced using a four-dimensional data assimilation system in which multivariate observed data are combined with the "first guess" using a statistically optimum scheme. The first guess is the best estimate of the current state of the atmosphere from previous analyses produced using a numerical weather prediction model.

The global analyses are produced on model (sigma or hybrid) surfaces which consist, in simplest form, of a sigma (σ) terrain-following coordinate in which the lowest level corresponds to $p = p_s$, where p is pressure and p_s is the surface pressure and $\sigma = p/p_s$. Hybrid levels consist of sigma near the surface but with a gradual transition to pressure with height (often above about

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100 mb). At the US National Meteorological Center (NMC) the sigma coordinate is used while hybrid levels are used at the European Centre for Medium Range Weather Forecasts (ECMWF). Analyzed variables are interpolated or extrapolated onto constant pressure surfaces in the postprocessing stages using various schemes.

The nature of the interpolation scheme and whether it preserves physical relationships among variables is a critical issue which has been explored to some extent by Shen et al. (1986) and Trenberth (1991a). For the ECMWF analyses, Trenberth (1991a) shows that mass is not conserved by the analyses on pressure surfaces evidently because of the methods of postprocessing the data. It appears that the interpolation procedure used by ECMWF produces the most accurate values possible at the archived levels, but they are not representative values for finite sized layers. Further work has revealed that there are substantial problems in the vicinity of the surface of the earth and extrapolation below ground. In part these arise from the fact that the earth's surface in the analyses and the real world differ, and in part they arise from the desire to use constant pressure surfaces for variables, so that the earth's topography above mean sea level or 1000 mb is replaced by an artificial atmosphere everywhere.

At ECMWF, an enhanced "envelope orography" is used, which places the earth's surface at surface pressures up to 100 mb or more lower in value (Trenberth, 1992). At NMC a "silhouette orography" was used until 6 March 1991, when it was replaced by "mean mountains". Justification of the enhanced topographic heights is made on the grounds that the free atmosphere does not dynamically connect with air in valleys. At both centers the representation is spectral, so that the surface has extensive ripples over the ocean, arising from Gibbs phenomena.

Because of the vertical interpolation errors, it seems desirable in working quantitatively with the global analyses that diagnostic calculations should be performed on model rather than pressure surfaces (e.g., Hoerling and Sanford, 1993). However, other difficulties related to the resolution of the data set then arise. One problem is the volume of data and the desire to reduce it to manageable proportions and include only the scales where the analyses contain reasonably reliable information. For instance, the analyses at ECMWF were done at T106 resolution after May 1985 until September 1991 (more recently the resolution is T213) but it is well established that only scales up to about T42 contain information from the observations (Trenberth and Solomon, 1993).

Also, use of the analyses for many purposes is very limited because of the unique vertical coordinate. For instance, at NCAR we make widespread use of global analyses for validating model output. However, the base version of the Community Climate Model CCM2 is at T42 resolution which differs from the higher resolution in the ECMWF analyses. Moreover, even at the same horizontal T106 resolution, the surface topography differs and thus so does the vertical coordinate. *Comparison of analyses from one center with another, or with model output requires a common vertical coordinate.* Traditionally, that coordinate has been the pressure coordinate, but increasing use is being made of model coordinates at lower resolution, and effectively a truncation is performed on the data to change resolution. An example is the extended-range ensemble forecasting at NMC where the model used for the first 6 days in the forecasts has been at T126 resolution, but this is reduced to T62 to carry the forecasts out to 12 days (Toth and Kalnay, 1993). As shown below, truncation is often an ill-advised thing to do in model coordinates.

The problem is equivalent to one of changing the resolution from, say, T106 to T42 resolution on model coordinates. At the high T106 resolution, detailed topographical features can be captured that are not present in the lower T42 resolution model and the result is that the surface pressures and thus the model coordinates will contain more structure. This problem is considerably exacerbated by the use of enhanced artificial lower boundary surfaces (envelope orography at ECMWF or silhouette orography at NMC). Unfortunately, it is not a trivial problem to change resolution on model coordinates because the model coordinate itself changes so that horizontal and vertical gradients of variables become confounded.

Yet a further problem exists in working on model surfaces, and that is the process and meaning of time averaging. As a time average is performed, the surface pressure and thus the model coordinate itself also becomes time averaged. Thus the definition of a time-averaged quantity in model coordinates and its relationship to the corresponding time average in pressure coordinates is not at all clear. Any interpolation of the model-coordinate time average to pressure using the time-averaged pressure clearly ignores a contribution from the covariance between the surface pressure and the variable in question. In practice this difference is usually assumed small, but this assumption needs to be tested. In any event, such a time average cannot then be mass weighted to produce vertical integrals. Instead, it seems that it should be the layer mass-weighted variable that should be time averaged.

Consequently, this paper explores the problems of interpolation from model to pressure coordinates, appropriate methods for truncating model coordinate data, and ways of time averaging. Horizontal smoothing on pressure surfaces is well posed and methods for truncating spherical harmonics at lower resolution are described in Trenberth and Solomon (1993) and Trenberth (1992). Similar truncation is not well posed with sigma surfaces. To change resolution on sigma surfaces, it appears to be necessary to first interpolate to pressure, then truncate, and finally interpolate back to sigma. Ironically, the main advantage of working in model coordinates to avoid vertical interpolation errors is immediately lost.

This paper presents the main new results from Trenberth et al. (1993). The latter documents in detail new algorithms for interpolating from model coordinates to pressure coordinates that have been implemented in the CCM postprocessor. For the most part, these methods are identical to those in place at ECMWF as this helps facilitate comparison of model output with analyses. The exception is a new algorithm for sea level pressure computation. Below we summarize the results of changing resolution by truncating fields in both pressure and model coordinates for both individual fields and for diagnosed fields involving higher order products. Results will be presented for truncations of T106 data to T42 resolution. The problems revealed are qualitatively the same at other resolutions. Results for truncations to T63 resolution are given in Trenberth et al. (1993) but only a summary of the magnitudes is given here.

2. Truncation on model surfaces

The global analyses used for illustration here are from the ECMWF on model hybrid coordinates although the issues are of importance in all global atmospheric data sets. A comprehensive description of the ECMWF analyses is given in Trenberth (1992). The hybrid level scheme at ECMWF is described by Simmons and Burridge (1981) and Simmons and Strüfing (1983).

2.1. *Model versus pressure coordinates*

As noted in the introduction, a change in resolution of the analyses is often desirable but is not easily achieved on model surfaces because as the resolution changes, the vertical coordinate also changes. The problem can be illustrated by considering the horizontal gradients of any quantity A on both σ and p surfaces

$$\left.\frac{\partial A}{\partial s}\right|_p = \left.\frac{\partial A}{\partial s}\right|_\sigma + \left.\frac{\partial A}{\partial \sigma}\frac{\partial \sigma}{\partial s}\right|_p, \tag{1}$$

where s is x or y. For

$$\sigma = \frac{p}{p_s}, \qquad \frac{\partial \sigma}{\partial s} = -\frac{\sigma}{p_s}\frac{\partial p_s}{\partial s}.$$

Consequently, the horizontal gradients or structure of the field A on p surfaces is made up of horizontal gradients of A on σ surfaces plus contributions from the product of vertical gradients of A and horizontal gradients of p_s. The latter term is very large near high orography so the nonlinear term becomes important as long as there are vertical gradients in the field A near orography.

2.2. *Tests for changes in resolution: individual fields*

For 1 January 1991, a number of tests have been made using the hybrid archive from ECMWF at T106 resolution. In the first case, the fields are extracted from the archive on the hybrid surfaces at full resolution, interpolated or extrapolated to p surfaces and then spectrally truncated to lower resolution, T63 or T42 for purposes of illustration (see Trenberth and Solomon, 1993 for spectral truncation on constant pressure surfaces). This is referred to as the "HPTxx" (Hybrid interpolated to p then Truncated to Txx) field. In the second case, the latter two steps are reversed, so it is referred to as the "HTxxP" case. The full fields on the hybrid surfaces are first truncated to lower resolution (T63 or T42), then vertically interpolated to p surfaces, and finally truncated again.

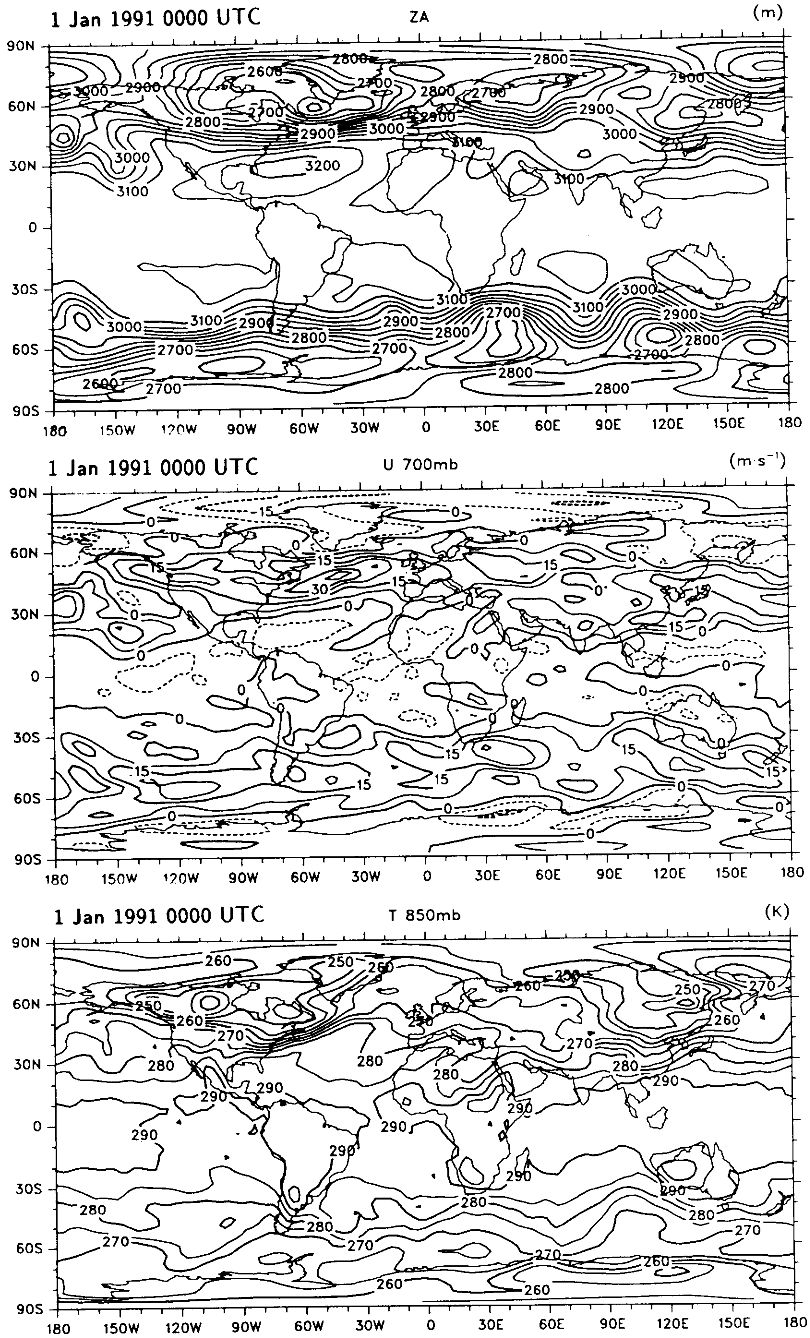


Fig. 1. Fields of z at 700 mb, contour interval 50 m (top), u at 700 mb, contour interval 7.5 m s^{-1} (middle), and temperature at 850 mb, contour interval 5 K. The fields are for 1 January 1991 and have been interpolated from the original hybrid level archive to pressure levels and then truncated (HPT42) at T42.

The vertical interpolation typically adds structure on the grid scale so that an additional truncation is necessary to make the fields consistent with the new resolution for comparison with the first case.

Note that in all cases in dealing with p_s the field $\ln p_s$ is the preferred variable, and all truncations were made using spherical harmonic series of the latter, not p_s.

Figs. 1–4 show the results for geopotential height z and zonal wind u at 700 mb and temperature T at 850 mb. Because of the nonlinear term in (1), the simple reversal in order of operations does not give the same result. The correct solution is the HPT case and the difference is a measure of how large the errors introduced by truncating directly on the hybrid surfaces are. The results are given for T42 truncation and the total field is shown (Fig. 1) along with the differences at T42 resolution (Fig. 2) and at T21 resolution (Fig. 3). Fig. 4 shows the corresponding spectra for all three fields giving the power versus total wavenumber n. In all cases, the largest errors are at the truncation resolution at wavenumber 42 and are physically located in the immediate vicinity of the mountains. The typical magnitudes of differences are 25 m (range -70 to $+80$) for z, 1.5 m s^{-1} (range -4 to $+5$) for u and 1.5°C (-5 to $+8$) for T at T42, and 6 m for z, 0.5 m s^{-1} for u and 0.5°C for T at T21. At T63 resolution (not shown), typical magnitudes of differences are 20 m (range -50 to $+60$) for z, 1 m s^{-1} (range -2 to $+3$) for u and 2°C (range -5 to 10) for T. The errors at T42 are larger on all scales and spectra reveal that the error in the smallest scales is about the same regardless of the truncation (i.e., the errors at wave 63 are about the same as those at wave 42 in the corresponding T63 and T42 truncations). The acceptability of these errors will depend on application.

3. Time averages on model surfaces

Given any variable x on model surfaces x_η or interpolated to pressure x_p, we define the time average by an overbar

$$\bar{x} = \frac{1}{N} \sum_{n=1}^{N} x_n. \qquad (2)$$

(1) This can be performed on pressure surfaces to give $\bar{x}_p$, or

(2) it may be performed on model (η) surfaces to give $\bar{x}_\eta$, in which case it applies on $\bar{\eta}$ surfaces presumably, and could be interpolated to p using these redefined surfaces, or

(3) it can factor in a mass weighting by recognizing that the level value is also used as representative of the layer $\Delta\eta$. In this latter case the mass weighting is proportional to

$$x_\eta \frac{\partial p}{\partial \eta} \Delta\eta = x_\eta \, \Delta p, \qquad (3)$$

where Δp is the thickness of the layer and we have ignored the $1/g$ factor in the mass.

In this latter case we can define a time average $\tilde{x}_\eta$

$$\tilde{x}_\eta = \frac{\overline{x\,\Delta p}}{\overline{\Delta p}}. \qquad (4)$$

This has the advantage that it can be directly related to any layer or column integral quantity by weighting it by $\overline{\Delta p}$ which is not a function of time. Again we can interpolate results to p using the time average coordinate.

$\overline{x_\eta}$ and $\overline{x_p}$ are fundamentally different and do not relate to one another. $\overline{x_\eta}$ ignores covariances with p_s. For p coordinates, a Δp weighting can be added at any point in the calculation as Δp is constant, so layer or column integrals are easily achieved. From this standpoint $\tilde{x}_\eta$ from (4) would seem to be preferable to $\bar{x}_\eta$.

Technically, therefore, the meaning of a time average relates to the coordinate system in which it is obtained and it may not be possible to strictly relate the time averages in one coordinate with those in another. Practically, however, we can ask how large the differences are between the three definitions given above. Perhaps fortunately, the differences among the three are quite small. This is illustrated below for the month of January 1991 for 0000 UTC every second day for the month, so that there are 15 values going into the monthly mean fields.

Fig. 5 shows, for 850 mb, differences between $\bar{T}_\eta$ and $\bar{T}_p$, i.e., the time average of T in hybrid coordinates, interpolated to p, versus the time average of T in pressure coordinates, with the result truncated at T31 resolution. In the bottom panel is shown the corresponding difference between $\bar{T}_\eta$ and $\tilde{T}_\eta$. All differences are small and range up to

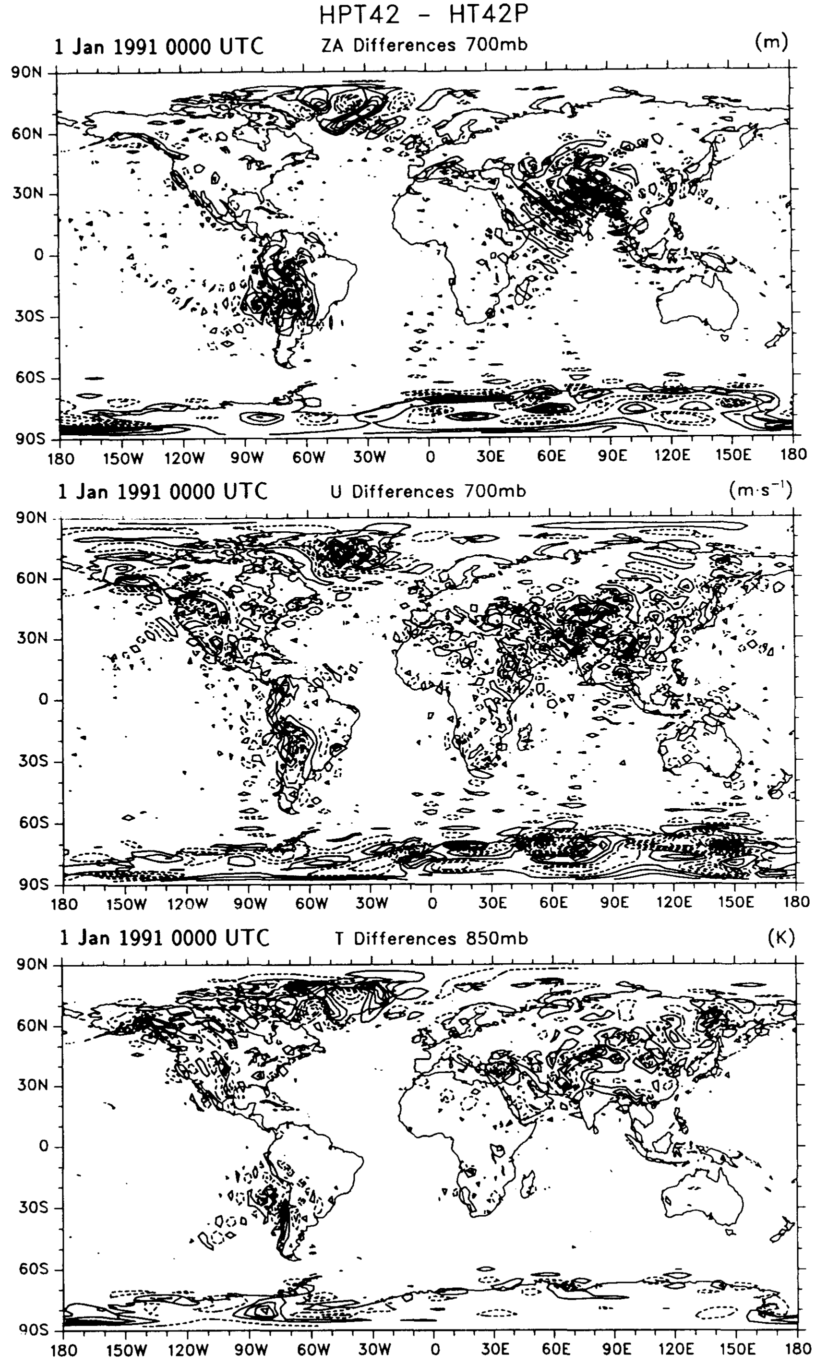


Fig. 2. Difference fields of z at 700 mb, contour interval 5 m (top), u at 700 mb, contour interval 0.5 m s^{-1} (middle), and temperature at 850 mb, contour interval 1 K. The fields are for 1 January 1991 and are differences HPT42–HT42P, where HT42P refers to hybrid, truncated at T42, then interpolated to pressure and HPT42 refers to hybrid, interpolated to pressure, then truncated at T42. Contours are chosen so that the zero contour is not plotted.

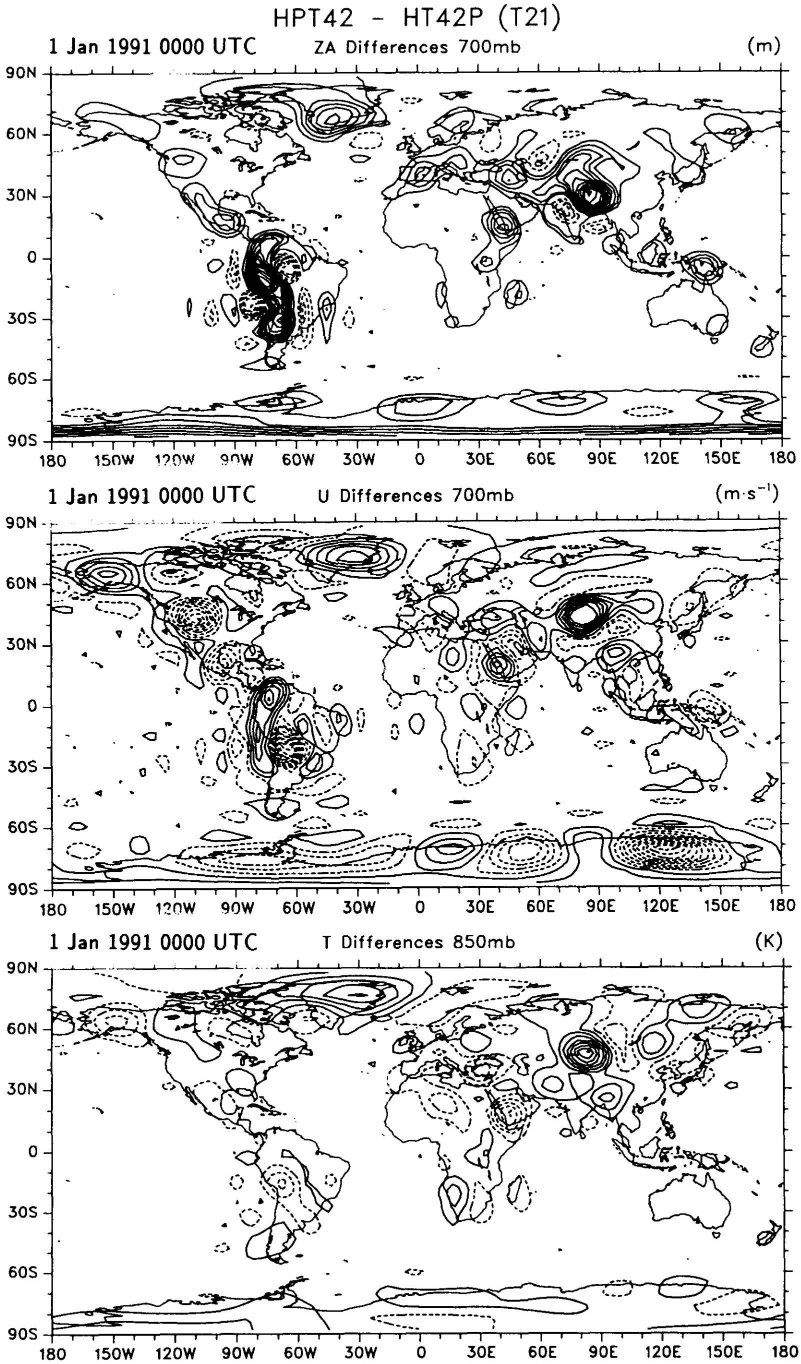


Fig. 3. As in Fig. 2 except the results have been truncated at T21 and the contour intervals are 1 m, 0.1 m s^{-1} and 0.2 K.

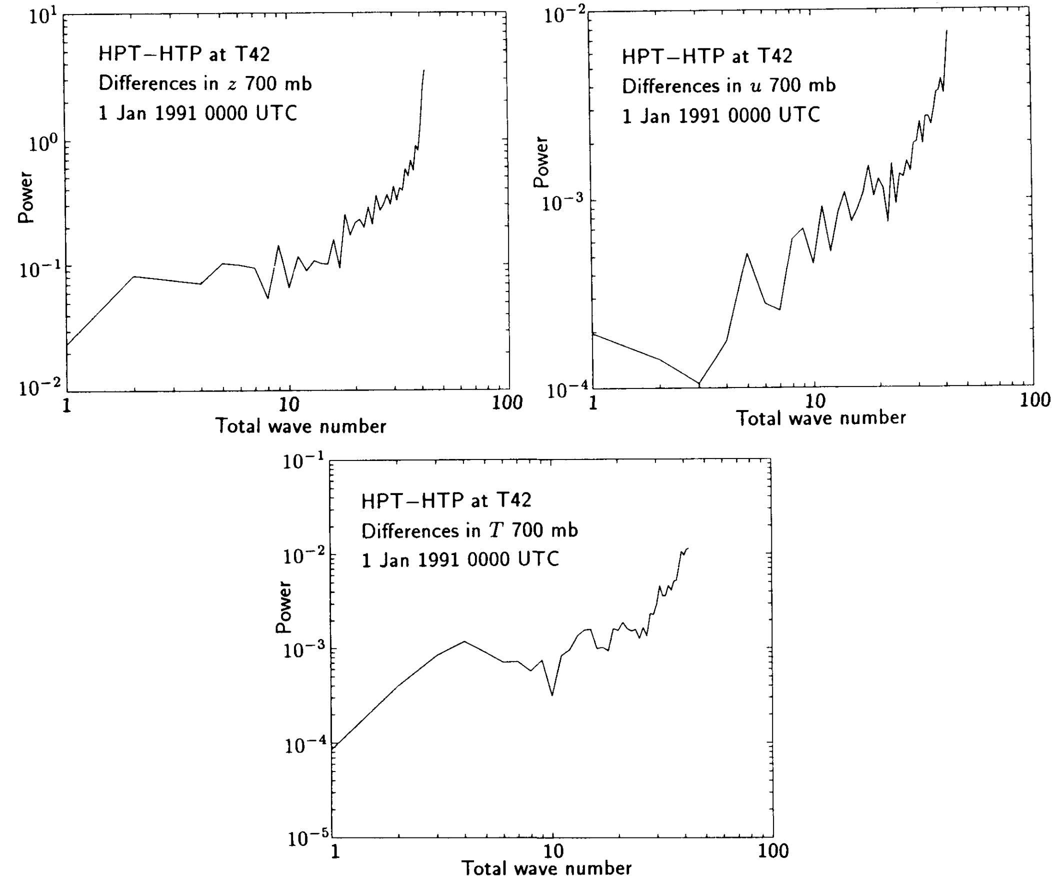


Fig. 4. Spectra of the difference fields shown in Fig. 2. The coordinates are logarithmic and the abscissa includes points up to wave 42.

± 0.3 K in the top panel and ± 0.1 K in the bottom panel. These results are typical of other levels in the vertical. Similarly, for geopotential height (not shown), differences between the methods of time averaging are small, typically about 1 m at 500 mb and with a maximum value of -3 m near Iceland for the Δp weighting case (cf. Fig. 5).

Fig. 6 shows the corresponding differences for u at 700 mb (v fields are given in Trenberth et al., 1993). The contour interval is 0.05 m s^{-1} and values range up to about ± 0.2 m s^{-1}. The patterns are interesting, however, and relate to the covariance of u with p_s that arise because of storm tracks (Trenberth, 1991b). Equivalent differences at other levels are mostly smaller than at 700 mb.

The conclusion from this section is that while the meaning of a time average in different coordinates is not the same and it is not always possible to relate the time average in p to that in η, in practical terms the differences are quite small and probably negligible for most purposes.

4. The heat budget: tests for changes in resolution

To further test the importance of the truncation errors for budget applications, it was deemed necessary to determine whether the effects might become magnified through nonlinear terms. Accordingly, a test has been made for the thermodynamic equation. The tests are made for January 1991. Because of the difficulty in handling T106 data, the test uses analyses from only 0000

UTC every 2nd day for the month, so that there are 15 values going into the monthly mean fields.

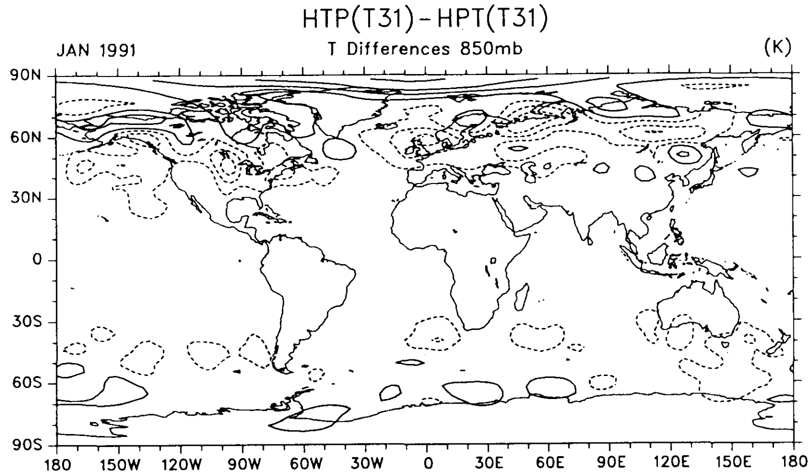
In p coordinates the thermodynamic equation may be written as:

$$\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{v} T + T\delta - \omega \frac{\partial T}{\partial p} + \kappa \frac{\omega T}{p} + \frac{Q}{c_p}, \quad (5)$$

where $\delta = \nabla \cdot \mathbf{v}$ is the divergence and other notation is standard. Note the first two terms on the right-hand-side (rhs) together constitute the horizontal advection term, but are written in this form for purposes of evaluation using spectral methods.

In hybrid η coordinates, where $p(\eta, p_s) = A(\eta) + B(\eta)\, p_s$ the thermodynamic equation takes the form

$$\left(\frac{\partial T}{\partial t}\right)_\eta = -\mathbf{v} \cdot \nabla_\eta T - \dot{\eta} \frac{\partial T}{\partial \eta} + \kappa \frac{\omega T}{p} + \frac{Q}{c_p}, \quad (6)$$



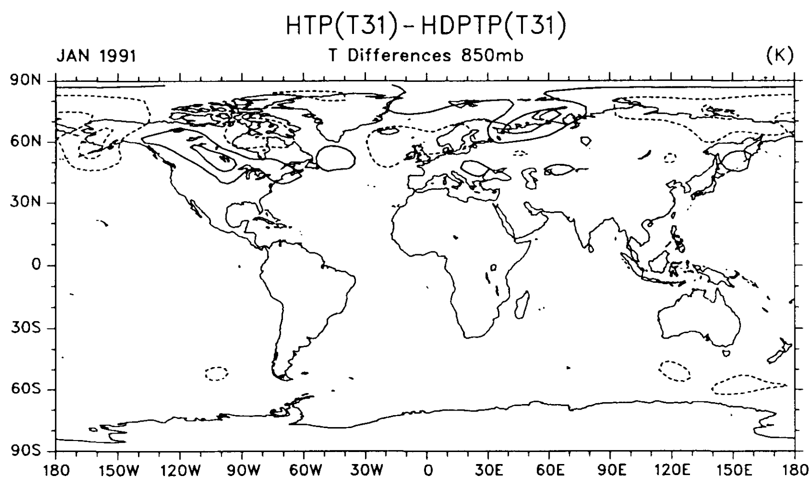


Fig. 5. At 850 mb for January 1991, differences between $\overline{T}_\eta$ (the time average of T in hybrid coordinates, interpolated to p) and $\overline{T}_p$ (the time average of T in pressure coordinates) with the result truncated at T31 resolution (top), and the corresponding difference between $\overline{T}_\eta$ and $\tilde{T}_\eta$ (which has the Δp weighting) (bottom). The contour interval is 0.1 K (top) and 0.05 K (bottom) with negative values dashed and contours chosen so the zero contour is not plotted.

and this is written

$$\left(\frac{\partial T}{\partial t}\right)_\eta = -\nabla_\eta \cdot \mathbf{v}T + T\delta - \dot{\eta}\frac{\partial p}{\partial \eta}\frac{\partial T}{\partial p} + \kappa\frac{\omega T}{p} + \frac{Q}{c_\mathrm{p}}, \tag{7}$$

see Hack et al. (1993). It is important to note that while several terms look very similar, (e.g., $\nabla \cdot \mathbf{v}T$) the divergence operator in the first case, eq. (5), is on constant pressure surfaces, and in the second case, eq. (7), is on the hybrid surfaces. The same applies to the time tendency terms and δ.

The vertical advection term is the product of a vertical velocity ($\dot{\eta}\,\partial p/\partial\eta$) and the vertical derivative of a field ($\partial T/\partial p$). The fields are defined at layer midpoints k, where k is the vertical index. The vertical velocity is defined in terms of vertical integrals of fields and is taken to interfaces at levels $k + 1/2$. The vertical derivatives are also naturally taken to interfaces, so the product is formed there

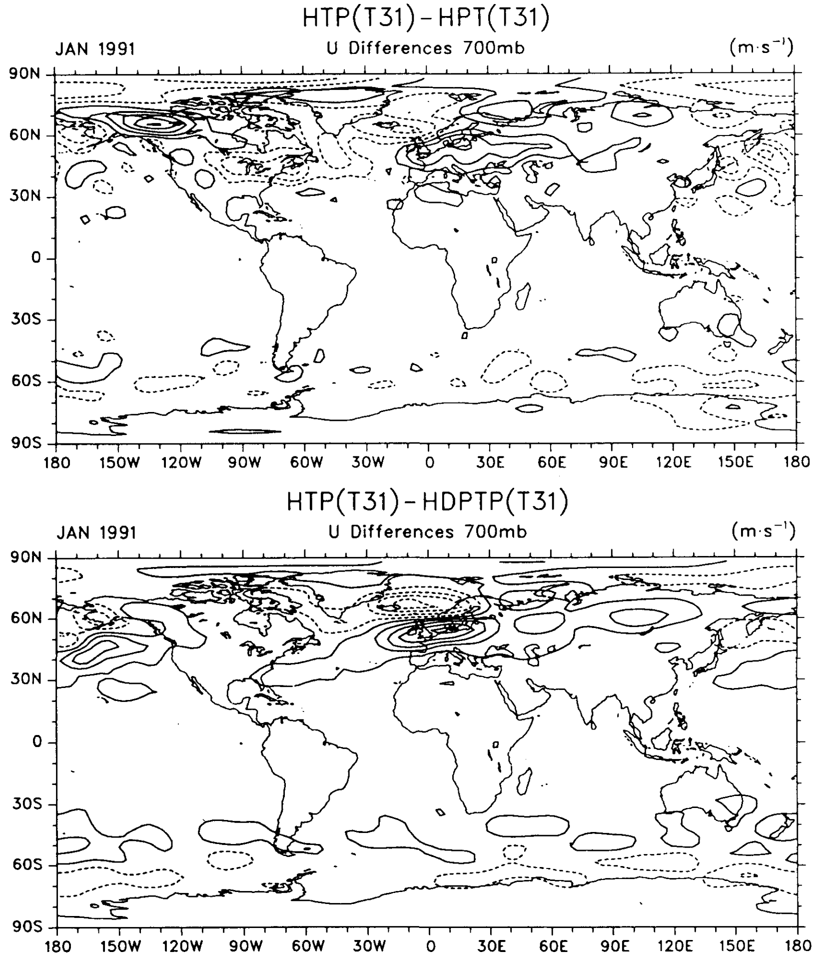


Fig. 6. At 700 mb for January 1991, differences between $\bar{u}_\eta$ (the time average in hybrid coordinates, interpolated to p) and $\bar{u}_\mathrm{p}$ (the time average in pressure coordinates) with the result truncated at T31 resolution (top), and differences between $\bar{u}_\eta$ and $\tilde{u}_\eta$ (the time average of u weighted with Δp in hybrid coordinates, interpolated to p) with the result truncated at T31 resolution (bottom). The contour interval is 0.05 m s^{-1} with negative values dashed and contours chosen so the zero contour is not plotted.

and the adjacent interface values of the products are averaged to give a midpoint value. It is the definition of the average which must be correct in order to conserve kinetic energy under vertical advection. Thus, as noted in Hack et al. (1993), the finite difference form of the vertical advection term in (7) is written as

$$\left(\dot{\eta}\frac{\partial p}{\partial \eta}\frac{\partial T}{\partial p}\right)_k=\frac{1}{2\,\Delta p_k}\left[\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{k+1/2}(T_{k+1}-T_k)+\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{k-1/2}(T_k-T_{k-1})\right], \tag{8}$$

$$\Delta p_k=p_{k+1/2}-p_{k-1/2}. \tag{9}$$

Accordingly, for comparison purposes, the formulation of the corresponding term in p coordinates $\omega\,\partial T/\partial p$ is given as

$$\omega\frac{\partial T}{\partial p}=\frac{1}{4\,\Delta p_k}\left[(\omega_{k+1}+\omega_k)(T_{k+1}-T_k)+(\omega_k+\omega_{k-1})(T_k-T_{k-1})\right], \tag{10}$$

so that we have averaged the ω values at levels above and below to give mid-point values. It is also worth noting that for consistency, we have recomputed ω. Although values were provided by ECMWF, we find significant differences, mostly in the smaller scales and especially around orography. On examining the postprocessing formulae used at ECMWF it is not clear that the archived values are correct. Some of the differences may arise from use of p_s in the postprocessing whereas ln p_s is the variable used by the model and in our processing.

We have computed the three terms, (i) the advection term, consisting of the first two terms on the rhs of each equation, (ii) the vertical advection terms as given in (8) and (10), (iii) the $\kappa\omega T/p$ terms and (iv) their sum, as an approximation to Q/c_p. All terms are converted into units of K/day. All the terms are presented in Trenberth et al. (1993) but here we will present results only for the sum.

Several different heat budget calculations have been performed to test the impact of interpolation and truncation.

(1) The full calculation in model coordinates (T106 hybrid), with the results interpolated to p using a ln p interpolation each day, then time averaging on p surfaces, and finally truncation to T31.

(2) T106 hybrid, interpolated to p, then do the calculation in p coordinates, time average, then truncate results to T31.

(3) T106 hybrid, truncate to T42, calculate results, interpolate to p, time average, then truncate to T31.

(4) T106 hybrid, interpolate to p, truncate to T42, calculate results, time average, then truncate to T31.

(5) T106 hybrid, calculate results at full resolution, time average, interpolate to p, then truncate to T31.

(6) T106 hybrid, calculate results at full resolution, weight by Δp, time average, interpolate to p, then truncate to T31.

In addition, corresponding results were generated with truncations at T63. There is no single "correct result". All interpolations to p are at 50 mb vertical resolution, so the latter is not an issue. All results are finally truncated to T31 so that the larger scales of interest are emphasized. Result (1) is one version of truth but suffers from interpolation of the results to p. Result (2) shows the most accurate result that can be achieved using p coordinates. The difference between (1) and (2) reveals the effects of vertical interpolation at different times in the calculation and for extrapolating below ground.

Result (4) versus result (2) are both in p coordinates where truncation is well defined, so that effects of truncation can be quantified. The results should be little changed unless nonlinear interactions of high wavenumbers project onto the lower wavenumbers. Result (3) versus result (1) tests the impact of the ill-posed truncation in hybrid coordinates at T42 on results at T31 resolution.

Results (5) and (6) are alternatives to result (1) where the time averaging is performed on η surfaces. From Section 3, we anticipate that the ordering of the time averaging will not make a large difference, and this has been confirmed, so those results are not presented.

The total diagnosed diabatic heating at 700 mb is given in Fig. 7 and for 300 mb in Fig. 8. Differences are large at 700 mb near high mountains where extrapolation below ground has occurred. At 300 mb differences range from −2 to 3 K/day. A pattern is apparent in the difference field at all

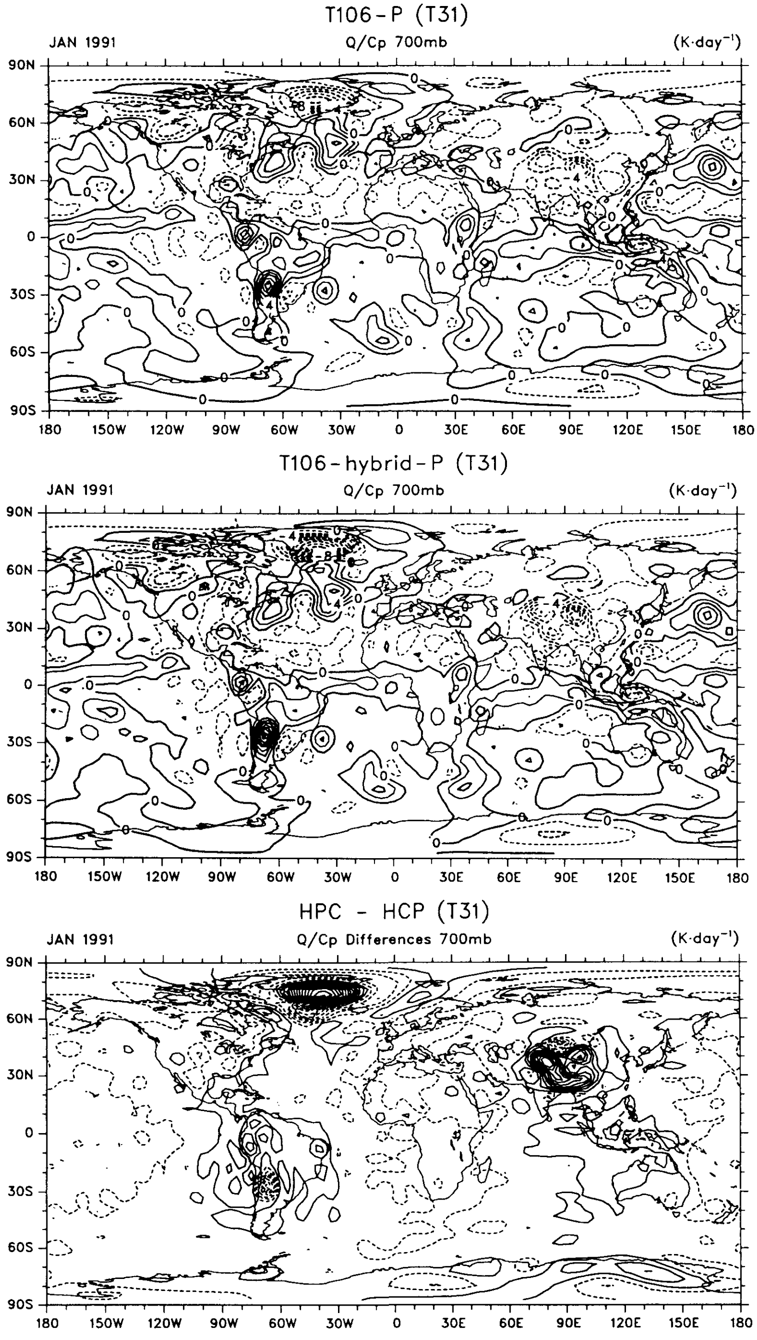


Fig. 7. For January 1991 at 0000 UTC Q/c_p at 700 mb from p coordinates (top), hybrid coordinates (middle) and their difference (bottom) truncated at T31. Units are K/day and the contour interval is 2, 2, and 0.5 K/day, respectively. The first contour is ± 0.25 for the latter and negative values are dashed.

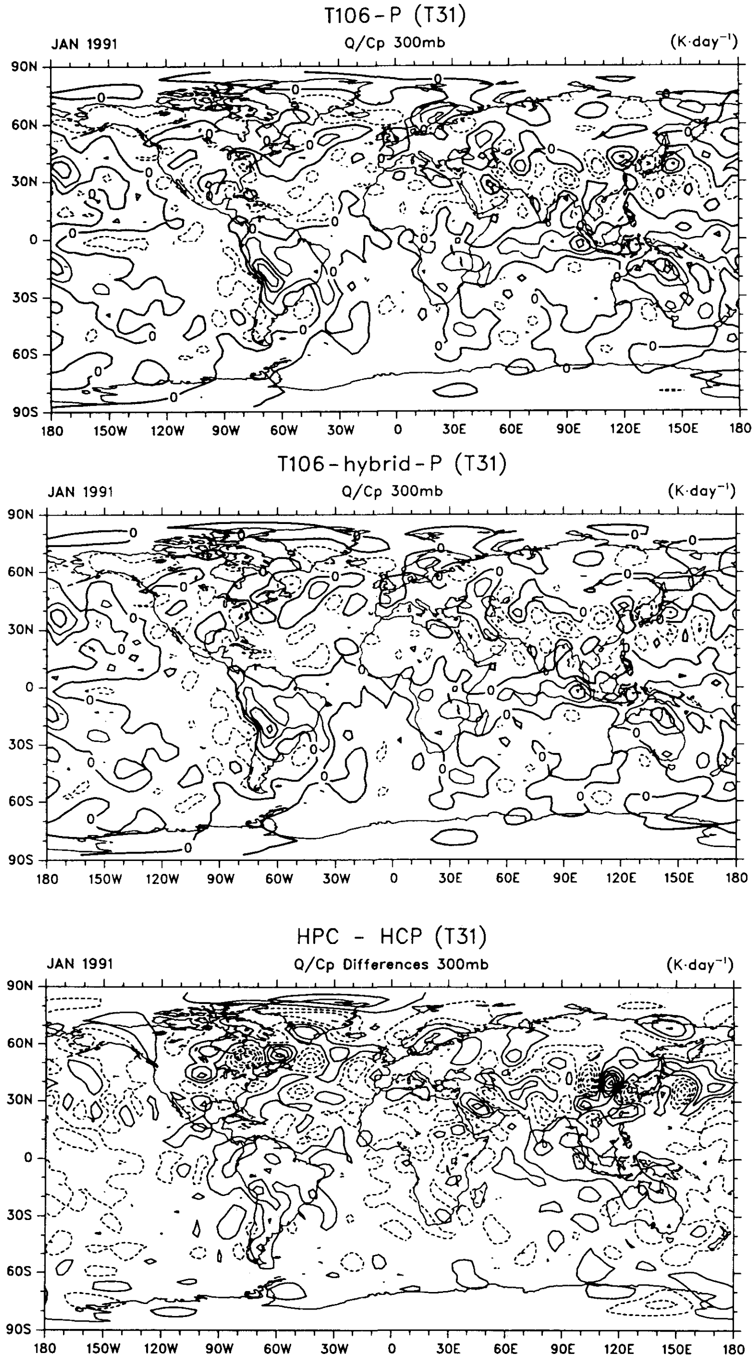


Fig. 8. As in Fig. 7 but for 300 mb. Q/c_p with the contour interval 2, 2, and 0.5 K/day, respectively. The first contour is ± 0.25 for the latter and negative values are dashed.

levels with a wave 2 structure emerging (mostly negative differences over the Pacific and Africa, but positive differences over South America and Asia). This structure comes mostly from the vertical advection terms. The main problem, as addressed in considerable detail in Trenberth et al. (1994), is that while the continuity equation is exactly satisfied in the model coordinates, it is no longer satisfied in p-coordinates. Imbalances arise from independent interpolation of $\mathbf{v}$ and ω onto pressure surfaces, and problems of dealing with the lower boundary and substituting an artificial atmosphere below ground. Diurnal pressure tendencies associated with the semidiurnal tide and the timing and distribution of observations are a factor in both coordinate systems because we use only 0000 UTC analyses. At the lower boundary, $\dot{\eta} = 0$ in model coordinates, but in p-coordinates $w_s = \partial p_s/\partial t + \mathbf{v}_s \cdot \nabla p_s$ and neither term on the right-hand side is zero. The time average of the first term at 0000 UTC gives the diurnal pressure tendency which is dominated by the semidiurnal tide and thus has a wave 2 structure. The second term is the orographic influence. Both distort the mass

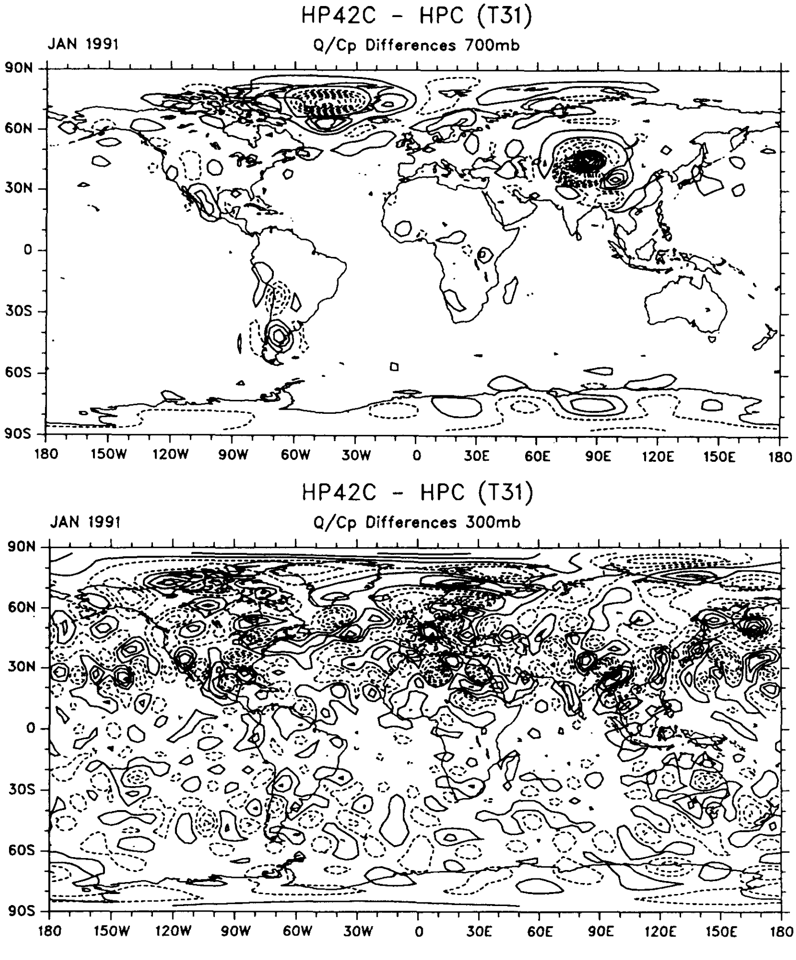


Fig. 9. Effects of truncation in p coordinates to T42 resolution before performing the heat budget. Shown at T31 resolution is the difference from the top panel in Fig. 7 at 700 mb (top) and the difference from the top panel of Fig. 8 at 300 mb (lower). The contour interval is 0.5 and 0.1 K/day with the first contour is ± 0.25 and ± 0.05, respectively, and negative values dashed.

balance and thus the heat budget in p-coordinates, evidently more than in η coordinates.

Otherwise in Figs. 7 and 8, the differences are quite noisy and largest near mountains, even in the free atmosphere. Thus the lower panels in Figs. 7 and 8 show the magnitude of errors from vertical interpolations compounded by mass imbalances. Zonal averages (not shown) reveal differences of ~0.1 K/day.

In pressure coordinates, Fig. 9 shows the effects of truncation at T42 of each field before the terms are evaluated. Shown at T31 resolution are the differences from the top panels in Fig. 7 at 700 mb and Fig. 8 at 300 mb. These differences can only arise from nonlinear effects of waves shorter than T42 projecting onto lower wavenumbers. At 300 mb, differences are at most 0.5 K/day with fairly small scale structure and typical values of 0.1 to 0.2 K/day. At 700 mb, differences are similar in magnitude over the oceans but an order of magnitude larger near mountainous areas where extrapolation below ground is a factor.

Fig. 10 shows the result of the calculation with the fields first truncated at T42 in hybrid coor-

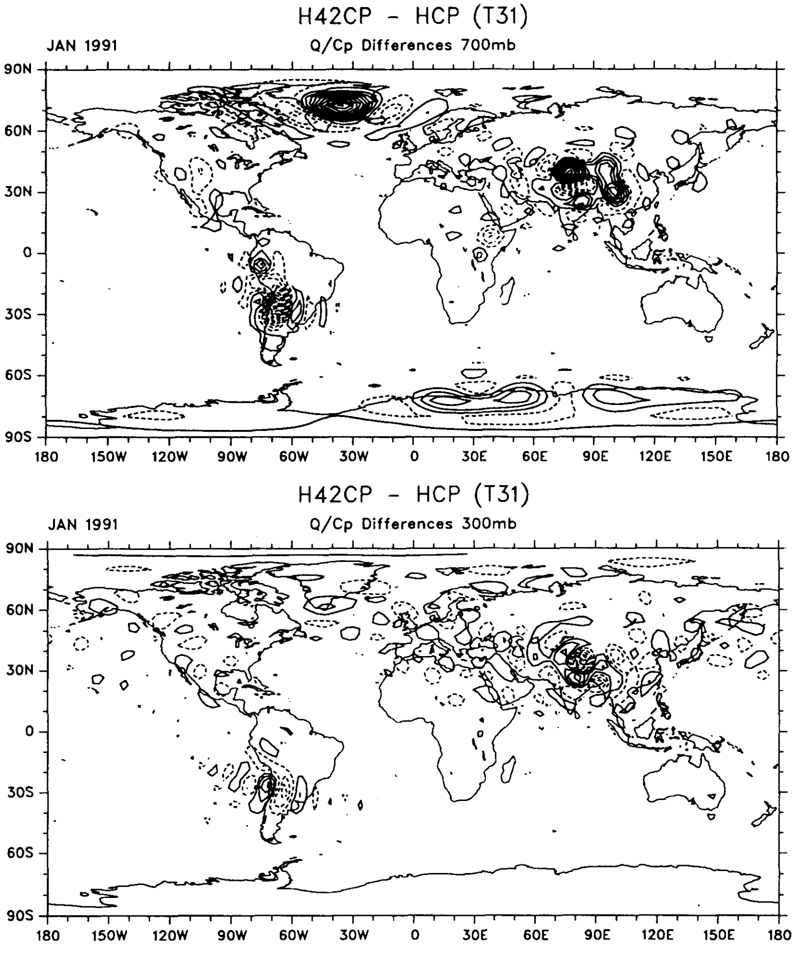


Fig. 10. Effects of truncation in hybrid coordinates to T42 resolution before performing the heat budget. Shown at T31 resolution is the difference from the middle panel in Fig. 7 at 700 mb (top) and the difference from the middle panel of Fig. 8 at 300 mb (lower). The contour interval is 1.0 and 0.5 K/day and the first contour is ± 0.5 and ± 0.25, respectively, and negative values dashed.

dinates. As noted earlier, this is ill posed because of the coordinate change (e.g., see Fig. 2). Shown in each panel are differences from the full calculation in the second panels of Figs. 7 and 8. As expected, differences are greatest in the vicinity of mountains but propagate out from those areas. Differences range from −3.25 to 1.75 K/day at 300 mb and −12.5 to 11.5 K/day at 700 mb. Such large magnitudes are judged unacceptable.

When the truncation is performed at T63 instead (not shown), errors are reduced at both levels and range from −0.63 to 0.63 at 300 mb and −8 to 5 K/day at 700 mb, and are confined to the high mountain areas. These errors are comparable to or less than the discrepancies arising from vertical interpolation and are therefore judged to be more acceptable. However, compared with the total fields (Figs. 7, 8) the errors are still large. Accordingly, for acceptable results at T31 resolution using hybrid coordinates it seems necessary to retain at least T63 resolution in the computation or else truncation errors become unacceptably large. On the other hand, a case can be made that comparable or better accuracy can be achieved in p coordinates at T42 resolution (compare Fig. 9 with bottom panels of Figs. 7 and 8).

Finally, the effect of time averaging on model versus p coordinates (cases 5 and 6) (not shown) reveals maximum differences of about 0.5 K/day with some links clearly present with the topography, and thus the coordinate system, but all differences are quite small compared with those in the previous figures.

5. Conclusions

Hoerling and Sanford (1993) have previously evaluated estimates of atmospheric heating using residual techniques, as we have here, and they compared use of model (in this case sigma) and pressure coordinates. They do not say how they handled the time averaging. They used the older CCM algorithms for interpolating all fields to p and showed results only for vertical integrals and zonal means, both of which are greatly impacted by how the below ground part is handled and the lower bound of integration for the vertical integral, and how the results are interpolated to p-levels for the zonal means. Moreover, they only show fields truncated at wave 10, so that influences of mountains are spread throughout the globe. Such influences are clearly seen coming out from Greenland and the Himalayan complex, especially at lower vertical resolution integrations using p coordinates. Their results reveal the benefit of using a high (50 mb) vertical resolution for these calculations.

New datasets are becoming available on the model levels, minimizing any loss of information for subsequent analysis and diagnosis of the atmosphere. However, the volumes of data are a major obstacle so that changes in resolution prove desirable. Moreover, for many purposes there is a need for a common coordinate system which is clearly best achieved using pressure coordinates. As we have noted, changes in resolution are not well posed mathematically in model coordinates because the coordinate system itself also changes. We have shown that very large errors can arise in a thermodynamic equation budget for one month if the model data are truncated to T42 resolution for producing results at T31 resolution, but that truncation to T63 resolution produces more acceptable but still large errors. No matter what method is used, errors arise from interpolation to p surfaces. However, there seems to be a lot to recommend continued use of the traditional representation on constant pressure surfaces provided sufficient vertical resolution is retained. Here we used a 50 mb vertical resolution. The main disadvantage is the potential contamination of regions outside of the below-ground areas when fields are used in calculations such as application of the divergence operator, although there are methods for dealing with this (Trenberth et al., 1994). In addition, vertical integrals are more readily performed in model coordinates.

We have also noted that the meaning of a time average in different coordinates is not the same and it is not always possible to relate the time average in p to that in η, but in practical terms, the differences are quite small and probably negligible for most purposes.

Another outstanding problem with data on model levels is the fact that the model surface does not correspond with the earth's real surface. Nevertheless, overall the advantages in working in the model coordinate system are the ability to handle the lower boundary properly although any changes in resolution somewhat compromise accuracies, especially on small scales.

The results here have implications for processing and archival of model data. Reanalysis of global data sets is underway at both NMC and ECMWF, and the primary archives will be on pressure surfaces at 2.5° resolution. To avoid aliasing, it is important that the resolution of the global analyses should be reduced to be compatible with this scale of grid, which has not always been done in the past (Trenberth and Solomon, 1993), but spectral truncation to an appropriate resolution should *not* be done in model coordinates.

Another example is the change in resolution from T126 to T62 at NMC in the midst of an extended-range forecast (Toth and Kalnay, 1993) which probably introduces substantial small scale perturbations that should be accounted for in evaluating the results.

Generally, truncation of fields of global variables in model coordinates should be avoided.

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