Tellus (1994), 46A, 222–227
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TELLUS
ISSN 0280–6495

Correcting co-tidal charts produced by numerical models

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(Manuscript received 1 December 1992; in final form 22 June 1993)

ABSTRACT

Much of the tedious task of tuning a 2-dimensional tidal numerical model has been avoided by introducing a corrective procedure. The model is run in a time-stepping mode until a steady state is achieved, and harmonic constants determined; these are then corrected by reference to the measured values at representative ports. The technique has been successfully applied to a fine-mesh model of the English Channel. When it is extended to adjust the tidal streams, the technique fails because of the sensitivity of the computed streams to the spatial gradient of the vertical tide (especially its clockwise component). It would be better to replace the geometrically-based corrective technique by a least-squares fit to the measured data, applied to a non-time-stepping model.

1. Introduction

The classical method of operating a 2-dimensional tidal numerical model of a shelf sea is to input the vertical tide on the open boundaries, and to tune the model until the modelled tide most closely fits the observed tide at a representative selection of ports. The parameter most frequently varied in the tuning process is the coefficient of frictional drag, but since the tide on the open boundaries is often less well-known than that at the comparison ports, this too, is commonly regarded as a quantity which can be varied, within limits, in order to tune the model.

An approximate solution for the vertical tide is obtained by solving the equations of motion and continuity over several tidal cycles, and this solution is corrected by reference to the measured tide at the comparison ports.

2. The correcting mechanism

The model is run from rest and spun up in the first tidal cycle, to receive the full tidal forcing at the open boundaries in subsequent run-in cycles. Owing to demands on computer time, the tidal input in each cycle is identical: there is no attempt to run the model for the equivalent of a month or more in order to determine harmonic constants in detail. When the model is in a steady state, values of elevation E for a complete cycle are subjected to species analysis; i.e., amplitudes A_s ($s = 0, 2, 4, 6$) and ε_s ($s = 2, 4, 6$) are determined by fitting E to:

$$\begin{aligned} E = A_0 + A_2 \cos(\sigma t - \varepsilon_2) &\qquad \text{semi diurnal} \\ + A_4 \cos(2\sigma t - \varepsilon_4) &\qquad \text{quarter-diurnal} \\ + A_6 \cos(3\sigma t - \varepsilon_6) &\qquad \text{sixth-diurnal.} \end{aligned} \tag{1}$$

This is done for every point which is wet for the whole of the tidal cycle. The modelled amplitudes and phases for the points corresponding to the comparison ports are then extracted.

The harmonic constants H and g for M_2 and S_2 at the comparison ports are extracted from *Admiralty Tide Tables*, and all the values of g are reduced to the same time-zone (that of Greenwich).

Let

$$\begin{aligned} B_2 = H \text{ of } M_2, &\qquad \nu_2 = g \text{ of } M_2, \\ B_4 = H \text{ of } M_4, &\qquad \nu_4 = g \text{ of } M_4. \end{aligned} \tag{2}$$

It is assumed that these quantities are correct, and that the modelled values at the corresponding

Table 1. *Comparison of measured and modelled M_2 tide*

	Measured		Modelled		Differences		
Port	ampl. B_2 (cm)	phase ν_2 (deg.)	ampl. A_2 (cm)	phase ε_2 (deg.)	ampl. D_2 (cm)	phase δ_2 (deg.)	ratio R_2
Devonport	169	154	161	150	−8	−4	0.95
Dartmouth	142	162	134	166	−8	+4	0.94
Lyme Regis	111	178	116	173	+5	−5	1.05
Portland	60	191	63	189	+3	−2	1.05
Swanage	36	260	40	242	+4	+18	1.11
Freshwater	59	289	53	277	+4	−18	0.90
Nab Tower	143	317	142	319	−1	+2	0.99
Newhaven	226	325	218	321	−8	−4	0.96
Hastings	260	325	253	326	−7	+1	0.97
Dover	225	332	232	333	+7	+1	1.03
Boulogne	293	330	292	331	−1	+1	1.00
Dieppe	310	311	305	311	−5	0	0.98
Fécamp	268	297	267	295	−1	−2	1.00
Le Havre	262	286	258	283	−4	−3	0.98
Cherbourg	186	229	187	229	+1	0	1.01
St Peter Port	268	184	257	181	−11	−3	0.96
St Helier	337	183	328	181	−9	−2	0.97
St Servan	369	178	366	176	−3	−2	0.99
Roscoff	269	142	268	142	−1	0	1.00

Table 2. *Comparison of measured and modelled M_4 tide*

	Measured		Modelled		Differences		
Port	ampl. B_4 (cm)	phase ν_4 (deg.)	ampl. A_4 (cm)	phase ε_4 (deg.)	ampl. D_4 (cm)	phase δ_4 (deg.)	ratio R_4
Devonport	144	135	82	130	−62	−5	0.57
Dartmouth	98	103	68	072	−30	−31	0.69
Lyme Regis	104	075	174	047	+70	−28	1.67
Portland	126	030	217	020	+91	−10	1.72
Swanage	149	014	228	010	+79	−4	1.53
Freshwater	162	016	257	013	+95	−3	1.57
Nab Tower	148	354	160	353	+12	−1	1.08
Newhaven	95	242	83	235	−12	−7	0.87
Hastings	220	228	261	219	+41	−9	1.19
Dover	259	222	206	227	−53	+5	0.80
Boulogne	323	219	340	222	+17	+3	1.05
Dieppe	262	185	389	185	+123	0	1.48
Fécamp	127	118	166	127	+39	+9	1.31
Le Havre	245	075	304	081	+59	+6	1.24
Cherbourg	139	356	212	007	+73	+11	1.53
St Peter Port	75	289	67	318	−12	+39	0.88
St Helier	204	314	129	306	−75	−8	0.63
St Servan	276	279	33	277	−243	−2	0.22
Roscoff	105	144	83	153	−22	+9	0.79

points in the model are in error. Quantities R_2, δ_2, R_4 and δ_4 are defined by the equations:

$$B_2(x, y) = A_2/R_2(x, y) \quad \text{for semi-diurnal amplitude}$$

$$\nu_2(x, y) = \varepsilon_2/\delta_2(x, y) \quad \text{for semi-diurnal phase}$$

$$B_4(x, y) = A_4/R_4(x, y) \quad \text{for quarter-diurnal amplitude}$$

$$\nu_4(x, y) = \varepsilon_4/\delta_4(x, y) \quad \text{for quarter-diurnal phase} \qquad (3)$$

The values of these quantities, known as correctors, are given for the comparison ports in Tables 1, 2. Elsewhere, it is assumed that the values of the correctors vary smoothly in space, and may be determined from the known values at the comparison ports. The form assumed for the interpolated surface is not nearly so critical as if one were trying to interpolate the values of the harmonic constants themselves, as was attempted by Simon (1978). There is no need to resort to local fits of two-dimensional cubic surfaces, for example. Instead, a suitable function is a weighted average of the corrector for all ports within a certain radius R:

$$C(x, y) = \sum W_i C(x_i, y_i) \Big/ \sum W_i, \qquad (4)$$

where $C(x, y)$ represents the corrector, and W_i is the weight for the ith port, given by:

$$W_i = \begin{cases} 1/r_i - 1/R & \text{if } r_i < R \\ 0 & \text{otherwise.} \end{cases} \qquad (5)$$

3. Application to a fine-mesh model of the English Channel

A tidal numerical model of the English Channel was constructed with a mesh-size of about 3.7 km (2′ of latitude by 3′ of longitude), and run for 5 tidal cycles to achieve a steady state representing mean tides. The results for the semi-diurnal tide (Table 1) are in good agreement with the measured tide, with standard errors of 77 mm and 5.4 deg in amplitude and phase at the 19 comparison ports, for a drag-coefficient of 0.0025. Those for the quarter-diurnal tide (Table 2), despite a measure of tuning on the western boundary, show over-prediction of the amplitude A_4 by some 40%. The cause of this is obscure. Pingree

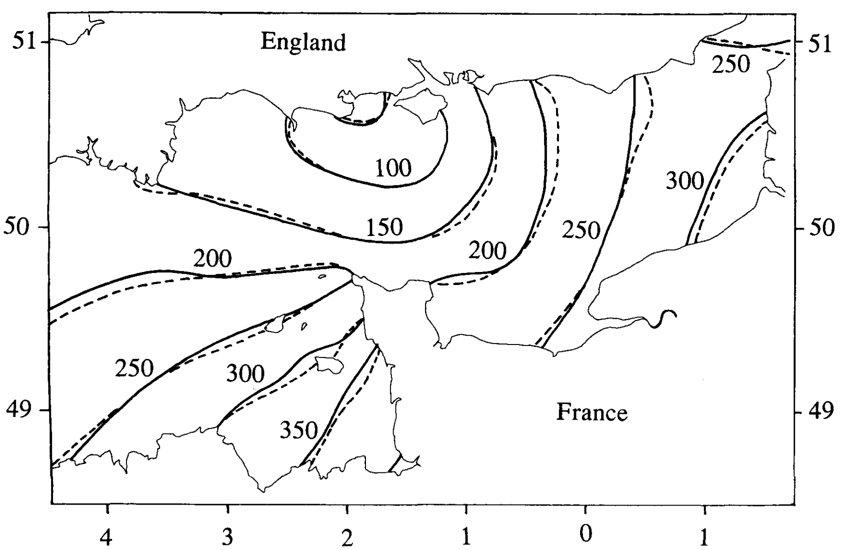

Fig. 1. Amplitude of M_2 tide (cm): -----, as produced by a numerical model; ———, after correction using the techniques described in Section 2.

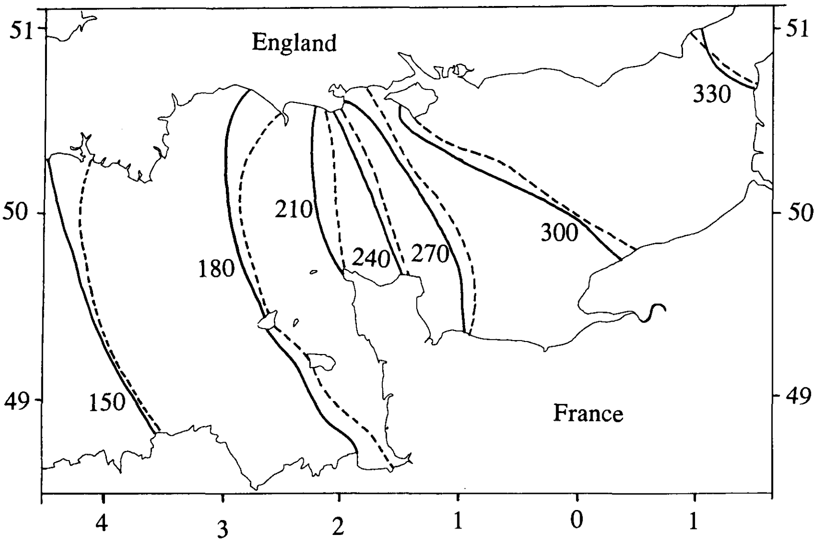

Fig. 2. Phase of M_2 tide (deg.): -----, as produced by a numerical model; ———, after correction using the techniques described in Section 2.

and Maddock (1977) implied that their original values for A_4 were also too great (without explaining why), and the only way to reduce them (once friction had been optimised) was to increase the eddy viscosity to an unnaturally high value ($10^4\ m^2/s$). This was confirmed by the author: an experimental twenty-fold increase in eddy viscosity halved the excess amplitude, but at the cost of seriously degrading the solution for the semi-diurnal tide.

The effect of applying the corrections, using $R = 60$ nautical miles (111 km) is illustrated in Figs. 1, 2 for the semi-diurnal tide. It consists of slight local adjustments to the amplitude and phase.

4. Attempts to adjust the modelled tidal streams

The principal aim of constructing the fine-mesh model of the English Channel is to produce a tidal stream atlas similar to that already published for the Cornish coast (George, 1990). This aim requires the results to be as accurate as possible, and so a corrective technique was sought for the streams similar to that used for the tides.

Correction was first applied in the time domain. After achieving a steady state, the model is run for a further 2 cycles but with the tidal elevations specified everywhere. The equation of continuity is not used, but the equations of motion are solved to give the depth-averaged tidal streams. This works only because an approximate solution for the streams has already been obtained. 2 cycles are used because the elevations at the start of the first of them do not correspond exactly to the values given by eqs. (3); 1 cycle is needed to act as a "spline" into the adjusted values. This approach was unsatisfactory: on comparison with observed data at 19 offshore locations, the errors in modelled streams were found to be greater after adjustment than before.

Correction was then attempted in the frequency domain. At a given location, the vertical tide is subjected to species analysis, given complex Fourier coefficients E_0, E_2, E_4 and E_6 for species 0, 2, 4 and 6 respectively. The horizontal tidal stream is similarly analysed, and the resulting Fourier coefficients expressed in terms of its clockwise ($-$) and anti-clockwise ($+$) components; let these be denoted by V_0, V_{2+}, V_{2-}, V_{4+}, V_{4-}, V_{6+}, V_{6-}.

The reason for using these components is that, in the frequency domain, the equations of motion for species s may be written in the form:

$$i(f+s\sigma/2)\,\boldsymbol{V}_{s+}+\boldsymbol{A}_{s+}=\boldsymbol{P}_{s+}+\boldsymbol{F}_{s+}+\boldsymbol{D}_{s+},$$
$$i(f-s\sigma/2)\,\boldsymbol{V}_{s-}+\boldsymbol{A}_{s-}=\boldsymbol{P}_{s-}+\boldsymbol{F}_{s-}+\boldsymbol{D}_{s-}, \quad (6)$$

where $\boldsymbol{A}$ refers to the advective acceleration, and $\boldsymbol{P}$, $\boldsymbol{F}$ and $\boldsymbol{D}$ are the pressure, frictional and eddy diffusive forces per unit mass, respectively.

Differentiation of these equations gives:

$$i(f+s\sigma/2)\,\Delta\boldsymbol{V}_{s+}=\Delta\boldsymbol{P}_{s+}+\Delta\boldsymbol{B}_{s+},$$
$$i(f-s\sigma/2)\,\Delta\boldsymbol{V}_{s-}=\Delta\boldsymbol{P}_{s-}+\Delta\boldsymbol{B}_{s-}, \quad (7)$$

where $\boldsymbol{B}=\boldsymbol{F}+\boldsymbol{D}-\boldsymbol{A}$.

It is assumed that the adjustments to the tidal stream (but not the stream itself) are linear and frictionless; i.e., that $\Delta\boldsymbol{B}_{s+}=\Delta\boldsymbol{B}_{s-}=0$. The adjustments become:

$$\Delta\boldsymbol{V}_{s+}=\frac{\Delta\boldsymbol{P}_{s+}}{i(f+s\sigma/2)}$$

and

$$\Delta\boldsymbol{V}_{s-}=\frac{\Delta\boldsymbol{P}_{s-}}{i(f-s\sigma/2)}. \quad (8)$$

$\Delta\boldsymbol{P}_{s+}$ and $\Delta\boldsymbol{P}_{s-}$ are found from the spatial derivatives of the corrections to the vertical tide.

Again, comparison with observed data showed a decrease in accuracy after adjustment.

5. Discussion

Normally errors in a tidal numerical model are reduced by separate optimisation of the different variables during the tuning process (e.g., Verboom et al., 1992) which tends to be time-consuming. The method presented above avoids the need for extensive tuning: it may be regarded as an elaborate exercise in interpolation between the known values of the harmonic constants at the comparison ports. The function of the model is to define approximately the shape of the co-amplitude and co-phase lines, which are then adjusted to fit the measured values.

The corrective technique would work very well for the vertical tide, if the basic assumption holds that the harmonic constants for the comparison ports are correct. It is also implicitly assumed that the model describes correctly the local differences in the tide near the ports. In practice, however, while these assumptions may be reasonable for the semi-diurnal tide, they (especially the second) break down for the quarter-diurnal tide.

Although there is a standard error associated with the measured values of the harmonic constants at the comparison ports, it still seems more logical to use these data as part of the input, rather than just for verification purposes, as did Bennett and McIntosh (1982). Such an approach is more in keeping with the philosophy of Schwiderski (1980): it could be implemented using a non-time-stepping technique such as that introduced by Snyder et al. (1979), and successfully developed by Walters and Werner (1991).

Extension of the corrective technique to the tidal streams does not produce satisfactory results. This is because the streams are driven partly by the spatial gradient of the tide. Although the corrections to the vertical tide may be relatively small, the effect on its spatial gradient, and hence on the streams, may be considerable. Typically, a 3% change in the amplitude A_2 of the semi-diurnal tide produces a 10% change in the magnitude of $\boldsymbol{P}_{2+}$ and a 60% change in that of $\boldsymbol{P}_{2-}$. This startling difference arises because, for the semi-diurnal species ($s=2$) in the latitude of the English Channel,

$$|f+s\sigma/2|\sim 250\times 10^{-6}\ \text{rad/s},$$
$$|f-s\sigma/2|\sim 30\times 10^{-6}\ \text{rad/s}.$$

Thus the adjusted stream is very sensitive to the clockwise component of the spatial gradient.

This failure emphasises the fact that the corrective technique is based on geometric rather than dynamical principles. A more effective alternative would appear to be to use a least-squares fit to the measured tidal data in the context of a non-time-stepping (frequency domain) model.

6. Conclusion

A method has been developed which adjusts the vertical tide in a time-stepping tidal numerical model so that it fits the measured tide at comparison ports. An extension of the method to correct the tidal streams fails to work, both in the time domain and the frequency domain, owing to the sensitivity of the streams to the spatial gradient of the tide. It would be better to use a least-squares fitting technique to the measured data in a non-time-stepping (frequency domain) model).

REFERENCES

Bennett, A. F. and McIntosh, P. C. 1982. Open ocean modelling as an inverse problem: tidal theory. *J. Phys. Oceanog.* **12**, 1004–1018.

George, K. J. 1990. *Tidal stream atlas of the Cornish coast.* Plymouth: Polytechnic South West.

Pingree, R. D. and Maddock, L. 1977. Tidal residuals in the English Channel. *J. Mar. Biol. Assn.* UK **57**, 339–354.

Schwiderski, E. W. 1980. Ocean tides, part II: a hydrodynamical interpolation model. *Marine Geodesy* **3**, 219.

Simon, B. 1978. *Réduction des sondes au large en Manche.* Brest: Service Hydrographique et Océanographique de la Marine, publication no. 226.

Snyder, R. L., Sidjabat, M. and Filloux, J. H. 1979. A study of tides, setup and bottom friction in a shallow semi-enclosed basin. *J. Phys. Oceanog.* **9**, 170–188.

Verboom, G. K., De Ronde, J. G. and Van Dijk, R. P. 1992. A fine-grid tidal flow and storm surge model of the North Sea. *Continental Shelf Research* **12**, 213–233.

Walters, R. A. and Werner, F. E. 1991. Nonlinear generation of overtides, compound tides and residuals. In: *Tidal hydrodynamics* (ed. B. B. Parker). New York: John Wiley & Sons, Inc. 297–320.

Printed in Belgium