

Energy of Rossby waves as a part of global atmospheric oscillations

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ABSTRACT

A data set of 4 years of daily ECMWF global operational analyses was projected onto normal modes which were constructed by the solution of the Laplace tidal equations including the effect of mean zonal flow and the vertical structure equation. A time-series analysis was set up to identify free Rossby waves and to quantify the energy distributions of free propagating, forced propagating, standing and quasi-stationary waves. 18 modes could be identified or evidence of their existence could be stated. The energy partition of free propagating waves varies from 43 % in the case of the 16-day-wave down to about 5 % in the case of higher indexed modes. The pure westward propagating modes are seen to be the more consistent with theory the higher their energy is.

1. Introduction

Since the pioneering work of Haurwitz (1940a, b) on solving the barotropic vorticity equation solutions to this equation has been regarded as free propagating waves and forced standing waves.

Dikii and Golitsyn (1968) computed the Rossby-normal modes as solutions of an isothermal atmosphere on a rotating sphere taking the divergence into account. The phase velocities they computed agree remarkably well with those of westward propagating perturbations which were analysed by Eliassen and Machenhauer (1965) for the northern hemisphere, and also for the global scale (Eliassen and Machenhauer, 1969). Thus these early results supported Haurwitz's results.

Forecast models have normal modes too. A projection of geopotential height and wind fields onto basic functions barely provides a realistic estimation of the energy of the normal modes since there is no distinction of free propagating and forced stationary waves. This is suspected to affect the forecast skill (Tribbia and Madden,

1988). Lambert and Merilees (1978) showed that the misrepresentation of standing waves as propagating waves can lead to significant prediction errors. Similarly, Daley et al. (1981) described the detriments of incorrectly captured external Rossby waves. Finally, Tribbia and Madden (1988) assumed, that "at least some of the prediction errors of large-scale weather are thought to result from improper partitioning of energy between forced stationary fields and free traveling or normal modes of the forecast models". An erroneous estimation of stationary waves could be detrimental for climate and circulation models since they are thought to exert a profound influence on long term wind regimes, storm tracks, and blocking.

There are two different approaches to the problem:

(i) An energetical approach with a statistical survey, which separates the atmospheric variability, its energy and energy fluxes and other parameters in terms of spatial distribution or wavenumbers.

(ii) An analytical approach, which identifies single atmospheric oscillations in terms of a

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physical model and observes their individual evolution.

Since the introduction of the energy cycle by Lorenz (1955) numerous articles have been concerned with energy partition and energy conversion in terms of spectral distribution or similar tools supplied by approximation theory. It has not been the intention of these studies to prove the real existence of such oscillations as three dimensional physical events with predictable evolution. Early investigations of this kind made use of latitude dependent Fourier-coefficients (Saltzman, 1957) and later (Baer, 1972; Burrows, 1976; Chen and Wiin-Nielsen, 1978) of spherical harmonics on selected levels.

A vertical analysis was often set up by Empirical Orthogonal Functions (EOF) (e.g., Obukhov, 1960; Holmström, 1963). Due to their orthogonality and cheap computability, the basic functions or basic vectors are well suited to estimate energy partitions, but they are not solutions of a physical model, which intends to describe the fluid mechanics of the atmosphere. Nevertheless these basis systems are able to verify a variety of important statistical features of the atmosphere, such as the energy cascade subject to the s^{-3} -law of the zonal wavenumber s (Chen and Wiin-Nielsen, 1978). A comprehensive investigation of the atmospheric energy cycle over a long time period has been performed for example by Arpe et al. (1986).

Laplace's tidal equations are used in simplified global models of the atmosphere. Therefore the Hough functions are a better tool to represent the energy partition with more physical relevance. These functions make sure that there is a realistic dependency of mass field and winds. Further, since ultralong waves are affected by divergent motions, solutions of linearized primitive equations usually are better solutions than those of the nondivergent vorticity equation. In addition, solving the vertical structure equation, the atmospheric energy can be projected onto the basic functions (or normal modes) of a simplified spectral model. With this approach, for example Kasahara (1980) and Tanaka (1985) confirmed the s^{-3} -law.

Studies mentioned so far restricted their attention to energy partitions and did not verify the existence of the modes by analysing their temporal evolution in time.

From the viewpoint of the second approach, the objection may be raised, that projection onto normal modes of a model can by no means prove the physical existence of these modes. Madden and Julian (1972) enhanced the above mentioned investigations of Eliassen and Makenhauer (1965, 1969) and analysed a 5-day-wave and a 16-day-wave (Madden, 1978), which were believed to be identical with two symmetric zonal wavenumber 1 Hough-modes. The latter mode was found to have a 12-day period. Leith (1974) stressed that forecast models in general tend to over-estimate the propagation velocity of long planetary waves, which contain the largest portion of eddy kinetic energy. Solving Laplace's tidal equations including mean zonal flow, Kasahara (1980, 1981) derived the modes' retardation theoretically. Model calculations performed by Salby (1981) appear to confirm this finding.

Ahlquist (1982) succeeded in identifying eight additional modes and stated hints of the existence of another six. Lindzen et al. (1984) analysed time series of normal mode coefficients on a seasonal basis, after filtering out eastward propagating components and found that they agreed with Laplace's tidal theory.

The intention of this study is to address the following questions.

- Which energy partition of the projected mode is in accordance with Laplace's tidal theory and how does the remaining part divide into standing, quasistationary, biased retrogressing and eastward progressing waves?
- What is the life time and temporal evolution of the Rossby normal modes?

Another aspect of the present work is to extend the results of Ahlquist (1982) and to identify additional normal-mode global Rossby waves in the atmosphere.

The paper is organized as follows. In Section 2, the construction of the three dimensional basic functions is presented. The Section 3 describes the projection onto those basic functions is described and the temporal analysis method applied is thereafter. The data sources and data structures are given in Section 4, and the results are presented in Section 5 for both a spectral analysis and a resynthesize of the filtered time series. In Section 6 the results are summarized and suggestions for future work are given.

2. Basis functions

This section outlines the model equations and their solution. For a more comprehensive survey the reader is referred to the papers cited below. The present section follows a modified approach described by Kasahara (1980), Kasahara and Puri (1981) and Daley (1981). Here Hough-functions as solutions of the Laplace's tidal equations were taken as a linear combination to shape the horizontal profiles and to infer the temporal behaviour as they are influenced by zonal mean winds.

The equations of motion, the thermodynamic equation and the mass continuity equation read:

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{k} \times f \mathbf{V} + \nabla \Phi = -\mathbf{k} \times (\mathbf{k} \cdot \nabla \times \mathbf{V}) \mathbf{V} - \frac{1}{2} \nabla \mathbf{V} \cdot \mathbf{V} - \omega \frac{\partial \mathbf{V}}{\partial p}, \quad (1)$$

$$\frac{\partial^2 \Phi}{\partial t \partial p} + \frac{R \gamma \omega}{p^2} = \frac{R}{p} \mathbf{V} \cdot \nabla T - \frac{R Q}{c_p p}, \quad (2)$$

$$\frac{\partial \omega}{\partial p} + \nabla \cdot \mathbf{V} = 0, \quad (3)$$

where T is the temperature, R the gas constant, c_p the specific heat at constant pressure, and p_s the surface pressure. The potential temperature is $\Theta = T(p_s/p)^{R/c_p}$, the vertical velocity $\omega = dp/dt$, and the static stability parameter is

$$\gamma = -\frac{p T_0}{\Theta} \frac{\partial \Theta}{\partial p} = \frac{R T_0}{c_p} - p \frac{\partial T_0}{\partial p},$$

where p_s is the surface pressure and T_0 is the global mean temperature of level p . We apply the vertical boundary conditions $w = dz/dt = 0$ at the surface p_s and $\omega = 0$ at the top of the atmosphere $p = 0$.

If we assume a basic state in rest equations (1) and (3) using spherical polar coordinates give:

$$\frac{\partial u}{\partial t} - 2\Omega v \sin \varphi + \frac{1}{a \cos \varphi} \frac{\partial \Phi}{\partial \lambda} = 0, \quad (4)$$

$$\frac{\partial v}{\partial t} + 2\Omega u \sin \varphi + \frac{1}{a} \frac{\partial \Phi}{\partial \varphi} = 0, \quad (5)$$

$$\frac{\partial \omega}{\partial p} + \frac{1}{a \cos \varphi} \left(\frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \varphi} (v \cos \varphi) \right) = 0. \quad (6)$$

The variable ω can be eliminated by differentiating the linearized adiabatic form of eq. (2) with respect to p and introducing (6) to give

$$\frac{\partial^2}{\partial t \partial p} \left(\frac{p^2}{R \gamma} \frac{\partial \Phi}{\partial p} \right) - \frac{1}{a \cos \varphi} \left[\frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \varphi} (v \cos \varphi) \right] = 0. \quad (7)$$

A vertical structure and zonal wavenumber decomposition may be obtained by defining

$$\begin{pmatrix} u(\lambda, \varphi, p, t) \\ v(\lambda, \varphi, p, t) \\ \Phi(\lambda, \varphi, p, t) \end{pmatrix} = \begin{pmatrix} \hat{u}^s(\varphi) \\ \hat{v}^s(\varphi) \\ 2\Omega \hat{\Phi}^s(\varphi) \end{pmatrix} \times \exp[i(s\lambda - 2\Omega \sigma t)] \cdot G(p). \quad (8)$$

A vertical structure equation

$$\frac{\partial}{\partial p} \left(\frac{p^2}{R \gamma} \frac{\partial G}{\partial p} \right) + \frac{G}{g \tilde{H}} = 0, \quad (9)$$

and the horizontal structure equations in terms of latitude dependent variables $\hat{u}^s$, $\hat{v}^s$, $\hat{\Phi}^s$ are

$$\begin{aligned} i s \hat{u}^s &= -\hat{v}^s \sin \varphi + \frac{i s \hat{\Phi}^s}{a \cos \varphi}, \\ i s \hat{v}^s &= \hat{u}^s \sin \varphi + \frac{\partial \hat{\Phi}^s}{a \partial \varphi}, \\ i s \hat{\Phi}^s &= \frac{g \tilde{H}}{4 \Omega^2 a \cos \varphi} \left(i s \hat{u}^s + \frac{\partial}{\partial \varphi} (\hat{v}^s \cos \varphi) \right). \end{aligned} \quad (10)$$

$1/g \tilde{H}$ is the separation constant, where g is the gravitational constant and $\tilde{H}$ is the equivalent height.

The solution of (9) performed by a Galerkin approach is discussed in the appendix of Kasahara (1984). Its approximation in terms of a linear combination of Legendre polynomials

$$G(\sigma) = \sum_{j=0}^{J-1} a_j P_j(\sigma) \quad (11)$$

results in an eigenvalue problem to determine the coefficients a_j ,

$$\sum_{j=0}^{J-1} M_{ij} a_j = \Lambda a_i, \quad i = 0, \dots, J-1 \quad (12)$$

of a symmetric matrix M .

The equivalent heights are given by the eigenvalues Λ through

$$\Lambda = \frac{RT_{00}}{gH_m} = \frac{H_{00}}{H_m}. \quad (13)$$

We focus our interest on the barotropic modes, which have an equivalent height of about $H_m = 9663$ m.

The introduction of a zonal flow requires an extension of the linearized shallow water equations. Including the terms due to the spherical coordinates the equations may be written as:

$$\begin{aligned} \frac{du}{dt} - \left(2\Omega \sin \varphi + \frac{u}{a} \tan \varphi \right) v \\ + \frac{g}{a \cos \varphi} \frac{\partial h}{\partial \varphi} = 0, \end{aligned} \quad (14a)$$

$$\begin{aligned} \frac{dv}{dt} + \left(2\Omega \sin \varphi + \frac{u}{a} \tan \varphi \right) u \\ + \frac{g}{a} \frac{\partial h}{\partial \varphi} = 0, \end{aligned} \quad (14b)$$

$$\frac{dh}{dt} + \frac{h}{a \cos \varphi} \left(\frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \varphi} (v \cos \varphi) \right) = 0. \quad (15)$$

Following Kasahara (1980), the dependent variables may be expressed by zonal means denoted by overbars and primed perturbation quantities as follows

$$\begin{aligned} u(\lambda, \varphi, t) &= \bar{u}(\varphi) + u'(\lambda, \varphi, t), \\ v(\lambda, \varphi, t) &= v'(\lambda, \varphi, t), \\ h(\lambda, \varphi, t) &= \bar{h}(\varphi) + h'(\lambda, \varphi, t). \end{aligned} \quad (16)$$

The transcription of these equations in vector notation is

$$(L^s + iN^s - i\sigma I) \hat{w}^s = 0. \quad (17)$$

Upon setting

$$\alpha_m = \frac{\sqrt{gH_m}}{2a\Omega}, \quad \bar{\omega}(\varphi) = \frac{\bar{u}(\varphi)}{2a\Omega \cos(\varphi)}$$

and

$$\bar{H}(\varphi) = \frac{\bar{h}(\varphi)}{H_m},$$

the linear operator L is defined

$$L^s = \begin{pmatrix} 0 & -\sin \varphi & \frac{i s \alpha_m}{\cos \varphi} \\ \sin \varphi & 0 & \frac{\alpha_m d}{d\varphi} \\ \frac{i s \alpha_m}{\cos \varphi} & \frac{\alpha_m d[(\) \cos \varphi]}{\cos \varphi d\varphi} & 0 \end{pmatrix}, \quad (18)$$

and the operator N collecting the zonal flow dependent terms

$$N^s = \begin{pmatrix} \bar{\omega} s & i \left(2\bar{\omega} \sin \varphi - \frac{d\bar{\omega}}{d\varphi} \cos \varphi \right) & 0 \\ -i 2\bar{\omega} \sin \varphi & \bar{\omega} s & 0 \\ \frac{s \alpha_m (\bar{H} - 1)}{\cos \varphi} & i \alpha_m \left(\frac{d\bar{H}}{d\varphi} + \frac{(\bar{H} - 1) d[(\) \cos \varphi]}{\cos \varphi d\varphi} \right) & \bar{\omega} s \end{pmatrix}. \quad (19)$$

In the case of $\bar{\omega} \equiv 0$ the Laplace's tidal equations are obtained with the Hough vector functions

$$\Theta_i^s(\varphi) = \begin{pmatrix} U_i^s(\varphi) \\ -i V_i^s(\varphi) \\ Z_i^s(\varphi) \end{pmatrix}$$

as solutions. The method of solution is discussed by Longuet-Higgins (1968) and Kasahara (1976).

In order to determine the solutions $\bar{\Theta}_i^s(\varphi)$ of the eqs. (17), again the Galerkin method is applied with the Hough vector functions $\Theta_i^s(\varphi)$ taken as the basic functions. Implementation of the linear combination

$$\bar{\Theta}_{sl}(\varphi) = \sum_{l=0}^{\infty} c_l^s \Theta_l^s(\varphi) \quad (20)$$

in (17) yields a new eigenvalue problem to determine the coefficients c_l^s and the frequencies $\bar{\sigma}_l^s$. The modified Hough vector functions $\bar{\Theta}_{sl}(\varphi)$ are calculated by truncating the sum with index $l_R = 24$ for the Rossby modes and $l_G = 6$ for both eastward and westward gravity modes.

Fig. 1 shows as an example of the horizontal structure of one-half of the zonal wavenumber $s = 1$ Hough vector functions which will figure prominently later on. An evident feature of the normal modes illustrated here is the increasing

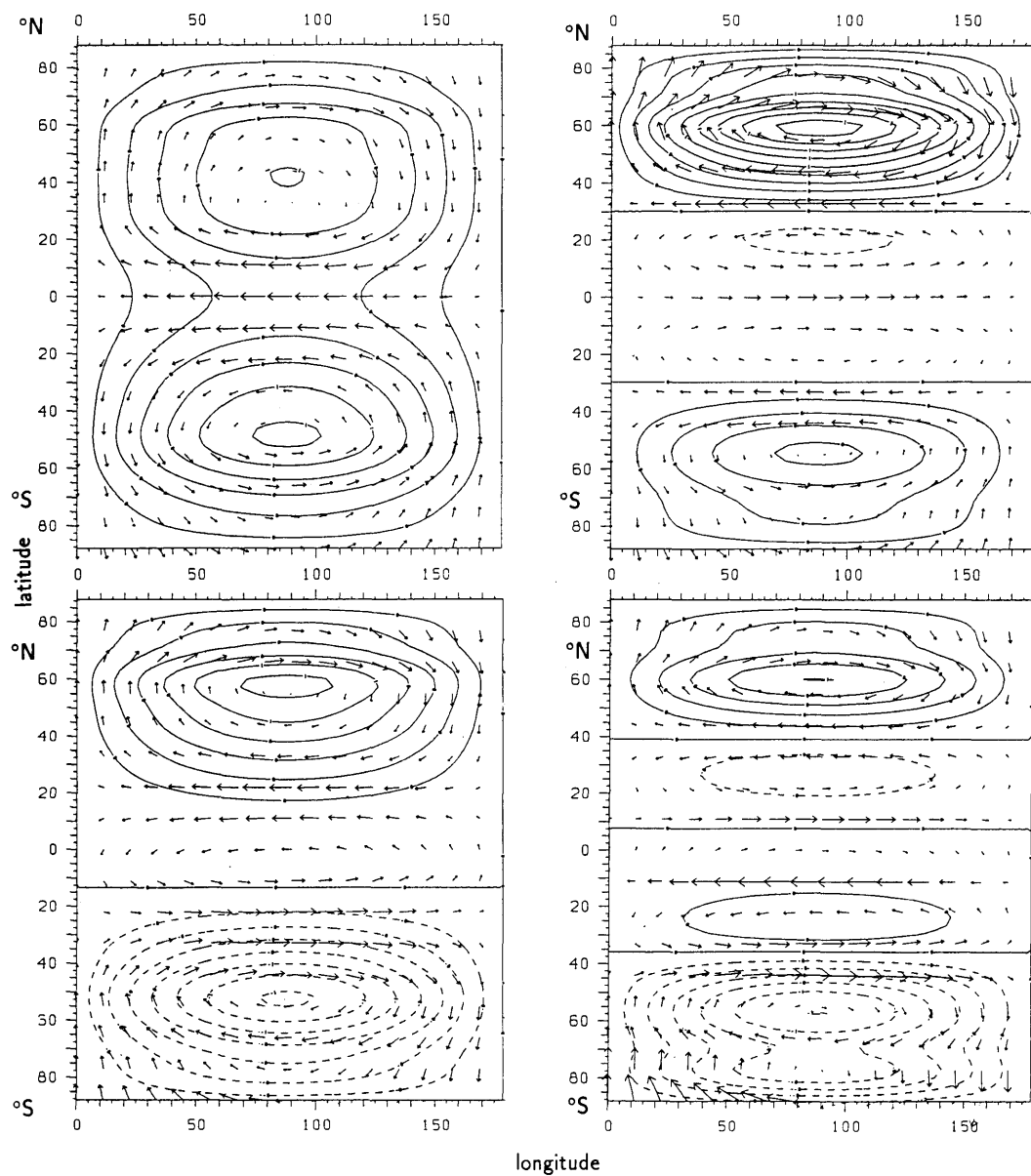


Fig. 1. Horizontal structures and wind vectors of the modified Hough modes due to mean zonal winds for January 1984. Plain projection of semiwaves on a longitudinal hemisphere. v -components are symmetric and geopotential and u -components are skew symmetric with respect to the longitudinal boundaries. 5-day-wave ($s = 1, l_R = 1, m = 0$) (top left) and 10-day-wave ($1, 2, 0$) (bottom left), 16-day-wave ($1, 3, 0$) (top right) and ($1, 4, 0$) (bottom right).

correspondence to a geostrophic regime with growing index sum $l_R + s$ and poleward-increasing latitude. The simultaneously decreasing absolute value of the Rossby number $\varepsilon = \tilde{\sigma}_l^s/f$ is advanced as an explanation.

3. The projection of data

Following Ahlquist (1982) we restricted the investigations only to barotropic modes. The Hough transformation integral may be expressed as

$$W_{sl_R}(p, t; m) = \int_{-\pi}^{\pi} \left[S_m \begin{pmatrix} \mathcal{U}_s(\varphi, p, t) \\ \mathcal{V}_s(\varphi, p, t) \\ \mathcal{W}_s(\varphi, p, t) \end{pmatrix} \right] \times \tilde{\Theta}_{sl_R}(m) \cos \varphi \, d\varphi \quad l_R = 0, 1, \dots, 5,$$

where

$$S_m = \begin{pmatrix} (gH_m)^{-1/2} & 0 & 0 \\ 0 & (gH_m)^{-1/2} & 0 \\ 0 & 0 & (gH_m)^{-1} \end{pmatrix}.$$

Then the total transformation is

$$w_{slm} = \frac{\sqrt{gH_m}}{2\pi} \int_{p_s}^0 \int_{-\pi}^{\pi} \int_0^{2\pi} S_m \times \begin{pmatrix} u(\lambda, \varphi, \tilde{\sigma}, t) \\ v(\lambda, \varphi, \tilde{\sigma}, t) \\ \Phi(\lambda, \varphi, \tilde{\sigma}, t) \end{pmatrix} \tilde{H}_{sl_R}(m) \times G_m(p) \cos \varphi \, d\lambda \, d\varphi \, dp, \quad (21)$$

and $\tilde{H}_{sl_R} = \tilde{\Theta}_{sl_R} \exp(i(s\lambda - 2\Omega\tilde{\sigma}t))$. Dealing only with barotropic modes, m always equals zero.

The complex coefficients of the normal modes solely depend on time. Because of the low Nyquist frequency the Kelvin waves $l_{EG} = 0$ and the mixed Rossby-gravity wave $l_R = 0$ of the wavenumbers $s = 1$ and 2 are not depictable by the direct approach presented here. The bivariate time analysis scheme applied here follows the methods proposed by Hayashi (1971, 1977, 1982) and Straus and Shukla (1981). These references are concerned with the presentation of analysis tools

aimed at diagnosing the composites of atmospheric waves in terms of eastward and westward traveling waves and standing oscillations.

Since only zonal propagation occurs, subscript l is dropped for the sake of neatness, and the time series may be expanded in real terms as

$$w'_s(t) = \sum_{s=1}^{\infty} (C_s(t) \cos(s\lambda) + S_s(t) \sin(s\lambda)), \quad (22)$$

where the time dependent coefficients are substituted as follows

$$C_s(t) = \sum_{q=1}^{\infty} (A_{s,\omega_q} \cos(\omega_q t) + B_{s,\omega_q} \sin(\omega_q t)),$$

$$S_s(t) = \sum_{q=1}^{\infty} (a_{s,\omega_q} \cos(\omega_q t) + b_{s,\omega_q} \sin(\omega_q t)).$$

$w'_s(t)$ may be set equal to

$$w'_s(t) = \sum_{q=1}^{\infty} (W_{s,\omega_q} \cos(s\lambda + \omega_q t + \Lambda_{s,\omega_q}^W) + E_{s,\omega_q} \cos(s\lambda - \omega_q t + \Lambda_{s,\omega_q}^E)), \quad (23)$$

with W_{s,ω_q} and E_{s,ω_q} as coefficients for westward and eastward moving waves, respectively. Hence we have a new set of equations, where the left hand sides may be obtained by Fourier transformation of the cosine and sine coefficients $C_s(t)$ and $S_s(t)$.

$$A_{s,\omega} = +W_{s,\omega} \cos \Lambda_{s,\omega}^W + E_{s,\omega} \cos \Lambda_{s,\omega}^E \quad (24a)$$

$$B_{s,\omega} = -W_{s,\omega} \sin \Lambda_{s,\omega}^W + E_{s,\omega} \sin \Lambda_{s,\omega}^E \quad (24b)$$

$$a_{s,\omega} = -W_{s,\omega} \sin \Lambda_{s,\omega}^W - E_{s,\omega} \sin \Lambda_{s,\omega}^E \quad (24c)$$

$$b_{s,\omega} = -W_{s,\omega} \cos \Lambda_{s,\omega}^W + E_{s,\omega} \cos \Lambda_{s,\omega}^E \quad (24d)$$

It is at this point that we can infer two possible ways to separate the waves components into westward and eastward traveling parts or standing oscillations:

(1) reconstruction of the time series by applying a Fourier synthesis to the coefficients of the propagation direction of interest, that is, in our case, the pure westward traveling waves, or

(2) calculation of the power spectra of standing and traveling parts.

The first approach will first be performed upon solving equations (24) for the four quantities $W_{s,\omega}$,

$E_{s,\omega}$, $\Lambda_{s,\omega}^W$, $\Lambda_{s,\omega}^E$ and filtering out the standing parts: For the cases $W_{s,\omega} > E_{s,\omega}$, $W_{s,\omega}$ is split into a traveling part and a standing part

$$W_{s,\omega} = W'_{s,\omega} + \bar{W}_{s,\omega} \quad (25)$$

in the way that the right hand side of (23) is the sum of

$$\begin{aligned} & (W'_{s,\omega} + \bar{W}_{s,\omega}) \cos(s\lambda + \omega t + \Lambda_{s,\omega}^W) \\ & + E_{s,\omega} \cos(s\lambda - \omega t + \Lambda_{s,\omega}^E) \\ & = W'_{s,\omega} \cos(s\lambda + \omega t + \Lambda_{s,\omega}^W) \\ & + 4S_{s,\omega} \cos\left(s\lambda + \frac{\Lambda_{s,\omega}^W + \Lambda_{s,\omega}^E}{2}\right) \\ & \times \cos\left(s\lambda + \frac{\Lambda_{s,\omega}^W - \Lambda_{s,\omega}^E}{2}\right), \end{aligned}$$

where $\bar{W}_{s,\omega} = E_{s,\omega} = S_{s,\omega}$, when $W_{s,\omega} > E_{s,\omega}$. The recalculation of the left-hand side of (24) in terms of the primed coefficients $W'_{s,\omega}$ provides us with the Fourier coefficients $A'_{s,\omega}$, $B'_{s,\omega}$, $a'_{s,\omega}$, and $b'_{s,\omega}$ for the westward traveling parts. For example

$$A'_{s,\omega} = \begin{cases} W'_{s,\omega} \cos \Lambda_{s,\omega}^W & \text{if } W_{s,\omega} > E_{s,\omega} \\ 0 & \text{if } W_{s,\omega} \leq E_{s,\omega} \end{cases}$$

The second approach follows several papers of Hayashi (1971, 1977, 1982) and results in power spectra of the entire time interval of concern. Therefore it depicts no individual events but gives a general modal behaviour and energetic partition.

The power spectra, quadrature spectrum and cospectrum may be defined as

$$P_\omega(C_s) = \frac{1}{2}(A_{s,\omega}^2 + B_{s,\omega}^2),$$

$$P_\omega(S_s) = \frac{1}{2}(a_{s,\omega}^2 + b_{s,\omega}^2),$$

$$Q_\omega(C_s, S_s) = \frac{1}{2}(B_{s,\omega}a_{s,\omega} - A_{s,\omega}b_{s,\omega}),$$

$$K_\omega(C_s, S_s) = \frac{1}{2}(A_{s,\omega}a_{s,\omega} - B_{s,\omega}b_{s,\omega}).$$

Hayashi (1977) showed that, after some manipulations, the standing part of a wave is discernible by

$$ST_{s,\omega}(w') = \sqrt{K_\omega^2(C_s, S_s) + \frac{1}{4}(P_\omega(C_s) - P_\omega(S_s))^2}.$$

and the pure progressive (+) and retrogressive (−) part can be inferred from

$$P_\omega(w_\pm^*) = P_{s,\pm\omega} - \frac{1}{2}ST_{s,\omega}(w'),$$

where the retrogressive part stands for the Rossby waves.

4. The global data set

The data set of global operational analyses was extracted from the archives of the European Weather Centre (ECMWF) in Reading. The daily data used were geopotential height, temperature, vorticity and divergence at 13 standard pressure levels: 1000, 850, 700, 500, 400, 300, 250, 200, 150, 100, 70, 50, and 30 hPa in terms of spherical harmonics at 00 UTC. A comprehensive description of the analysis procedure can be found in Arpe et al. (1986) as well as Trenberth and Olsen (1988a, b). The data assimilation scheme is described by Shaw et al. (1987).

In 1983 a very strong El-Niño event occurred. The eruption of El Chichón in 1982 may have produced a stratospheric dust veil optically thick enough to influence the atmosphere. Another El-Niño event took place in 1987/88. In order to minimize the effects of these possible influences, a four year time interval was selected spanning 1 January 1984 to 31 January 1987, which provides a time series of 1461 days.

The data of geopotential height and the velocity components were coded in a T63 spherical harmonic coefficient data set at the 13 levels. The inferred Fourier coefficients of the wavenumbers $s = 1$ to 4 were evaluated at 64 Gaussian latitudes. To perform the vertical projection, 24 Gaussian levels were evaluated from 13 levels.

The basic functions were computed for each month's mean zonal wind conditions.

The computational work was done on a Cray-2S of the Max-Planck-Institute of Meteorology in Hamburg, Germany.

5. Analysis of free global oscillations

5.1. Identification of Rossby modes

The comparison of the analysed evolutions of the modes with the propagation speed proposed

by Laplace's tidal theory gives a basis for judging whether the modes exist. The time series analysis mentioned above gives the possibility of finding not only the propagation frequency but also classifying the waves' energy into standing and traveling parts by means of a power spectrum. Thus it is possible to state approximately the portion of wave energy which is (a) traveling waves conformable with Laplace's theory, (b) traveling waves with speed or direction other than predicted by the theory, and (c) standing waves. In terms of this classification very low frequency standing waves can be treated as quasistationary waves.

Figs. 2 to 8 show power spectra of the barotropic modes based on a time series of four years length and 1460 degrees of freedom. It is smoothed

by a 5-point moving averaging procedure. The abscissa scales the frequency in units of cycles/day. Negative (positive) values denote a westward (eastward) traveling direction. The ordinate shows a nondimensional spectral energy density. The solid line marks the distribution of the traveling waves, whereas the dotted line describes the same for the standing waves. Following the idea of standing waves as being coherent but antiparallel progressing, they are plotted symmetrically with respect to zero frequency with one half of their densities on each side of zero. The theoretically determined frequencies are marked by 13 horizontal bars showing the influence of the zonal winds of each pressure level by their mean position with respect to the abscissa. The lengths of the bars

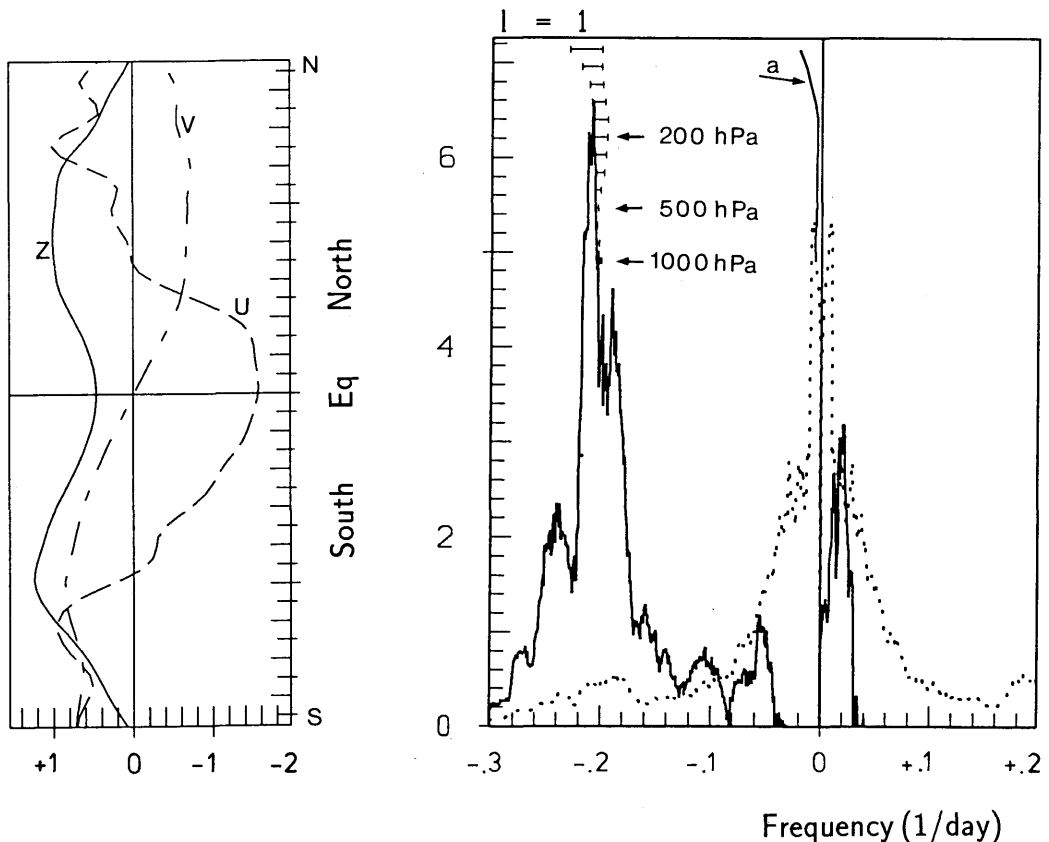


Fig. 2. Left panel: profiles of the wave's Hough components with mean zonal wind conditions for January 1984. (See text for a comprehensive description.) Right panel: Power spectrum of the barotropic 5-day wave (1, 1, 0), traveling waves solid line, standing wave dotted line. Negative abscissa values denote westward traveling, positive abscissa values denote eastward traveling waves.

denote the spread of the theoretical frequency during the observation period. The curve indicated by arrow "a" in Fig. 2 shows the minimum frequency gained by the maximum easterly wind of each level in order to show the limits of the discrete matrix spectra. The left side panel presents the 500 hPa modified Hough mode profiles for January 1984 zonal mean flow conditions as a quick reference.

Fig. 2 displays the barotropic 5-day wave of zonal wavenumber 1 and meridional index $l_R = 1$. It is the lowest indexed Rossby mode of our analysis. The diagram shows a nearly complete agreement between the analysis of the normal mode time series and theoretical eigenvalues of the different levels marked by the horizontal bars. This again confirms the observations of Madden and Julian (1972). There are nearly no variations of the

theoretical frequencies with increasing height, i.e., the influence of the altitudinally changing zonal mean winds is still very small. Although the total amount of energy is comparatively low, the signals at odds with the theoretically expected positions are only very faint. There is an exception placed by a low frequency eastward travelling wave.

The u and z components of the 10-day-wave in Fig. 3 ($l_R = 2$) have an antisymmetric structure. The positions of the bars of the theoretical frequencies bend towards lower frequencies at the upper troposphere. This Doppler effect results from strong zonal winds at that height. Nevertheless there is a quite good agreement between theoretical and observed frequencies. Daley and Williamson (1985) pointed out that the 10-day-wave and the 16-day-wave may have rather similar profiles,

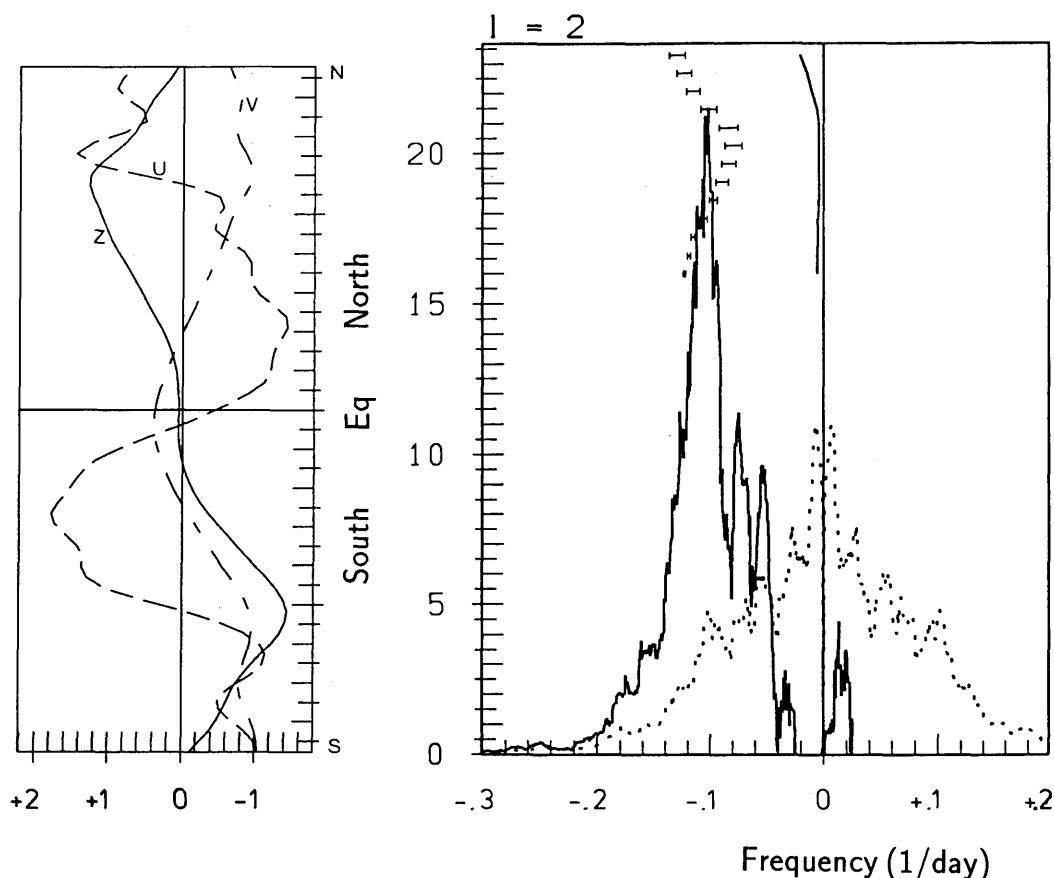


Fig. 3. Power spectrum of the barotropic 10-day-wave (1, 2, 0). Otherwise as in Fig. 2.

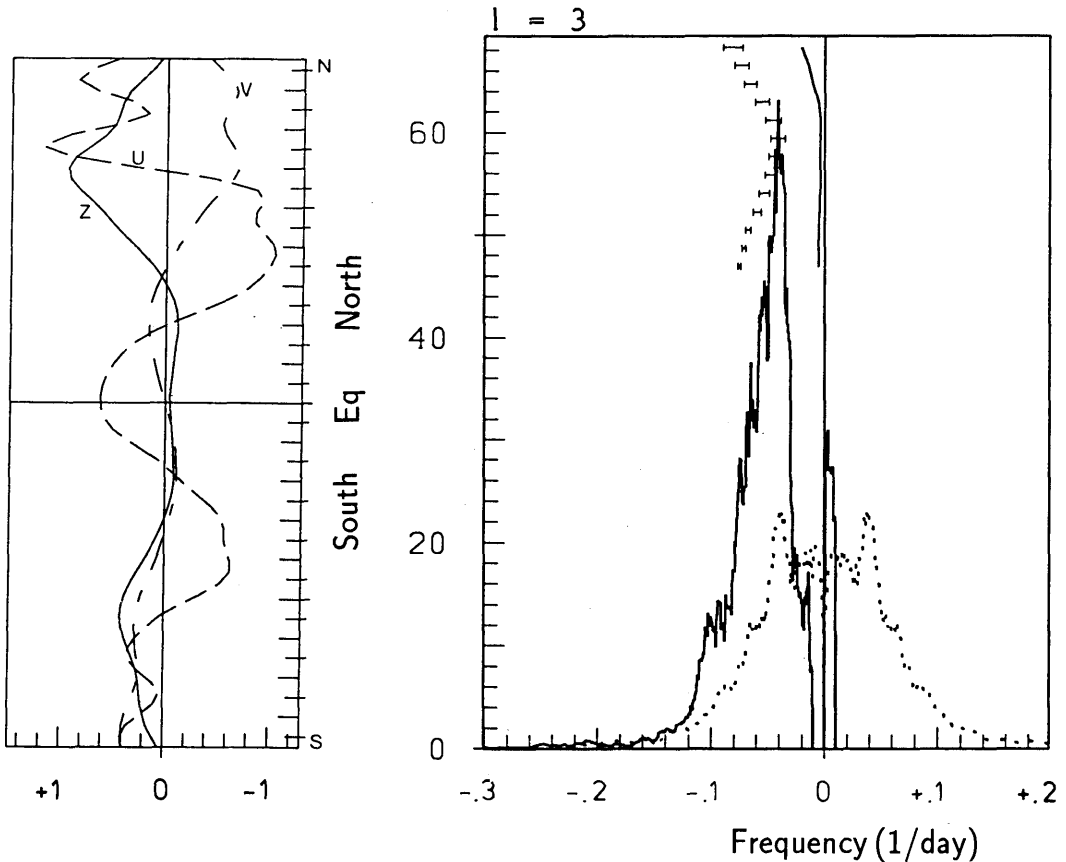


Fig. 4. Power spectrum of the barotropic 16-day-wave (1, 3, 0). Otherwise as in Fig. 2.

regarding only one hemisphere. The modes' amplitudes on the other hemisphere are held small by zonal flow modulation; this might give rise to a loss of orthogonality of the modified Hough modes and hence fails to separate the energetic partition exactly. By that we explain the secondary maxima around -0.1 cycles per day, which belong to a slower wave, probably to the 16-day-wave. All together the standing fraction of the 10-day-wave is relatively larger compared to the 5-day-wave, whereas the red noise portion is smaller.

The meridional profiles and the temporal behavior of the 16-day-wave were the first to make the conjecture that it was an eigenmode of Laplace's tidal theory (Eliassen and Machenhauer, 1969; Madden and Julian, 1972), but the theoretical period of 12.3 days for a resting basic state deviated too much from observed 14–18 day

period for the theory to be universally accepted as an explanation. As noted before, the difference in period is due to the omission of the zonal mean flow effects. Fig. 4 still reveals a tendency of the mode to be a little bit slower than predicted theoretically. It is the 16-day-wave which has the biggest amount of energy propagating with the theoretical speed. There are no secondary maxima worth mentioning. The fraction of the standing waves' energy is small.

Looking at the total energy distribution of the barotropic modes only in terms of their meridional indices the maximum total energy is gathered by the modes $l_R = 4$ and $l_R = 5$ in Fig. 5 (Kasahara, 1981; Tanaka, 1985); compared to that the 16-day-mode has a smaller total energy, but much energy in its westward propagating part. The frequency diagrams in Fig. 5 also show strong signals in the

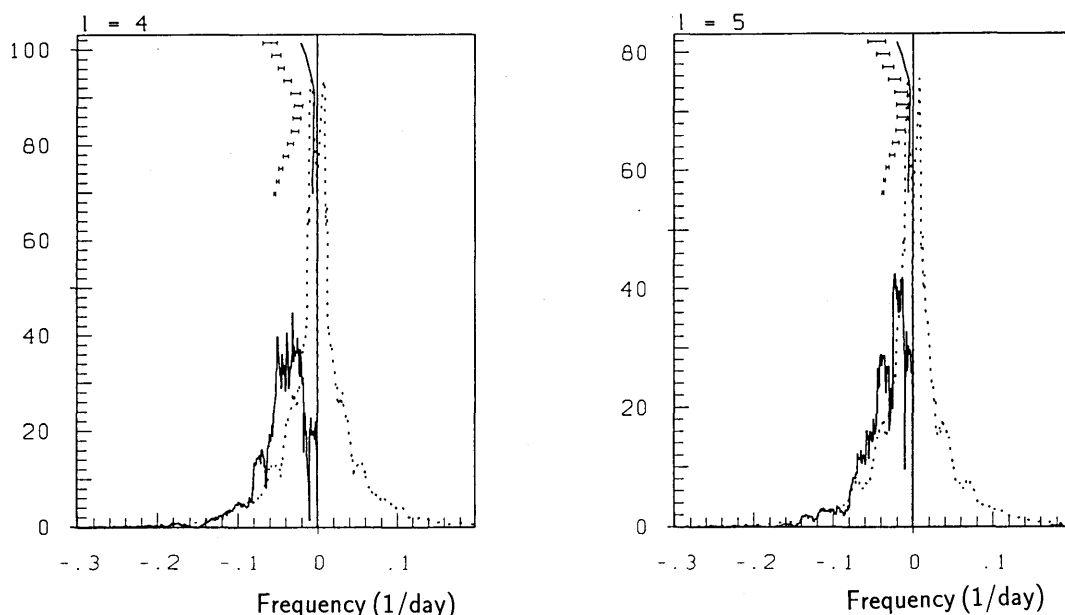


Fig. 5. Power spectra of the barotropic modes (1, 4, 0) and (1, 5, 0). Otherwise similar to Fig. 2.

theoretically predicted band, but there is more energy in the low frequency standing waves. The theoretical frequencies of the $l_R=5$ -mode are Doppler shifted close to the continuous spectrum frequency marked by the curved line just to the left of the zero frequency line. Nevertheless its maximum peak is too significant for the identification to be neglected.

The power spectra of the barotropic modes of wavenumber $s=2$ are displayed in Fig. 6. The $l_R=1$ mode has a broad maximum with a peak just at the theoretically expected position. The frequencies of the wavenumber 2 modes are more sensitive to the zonal winds. The upper tropospheric maximum of westerlies causes an extension of the periods. With increasing meridional index the theoretically expected periods approach the critical value of the continuous spectrum. Moreover the horizontal bars show that the variances of the zonal winds and thus the variances of the theoretical frequencies increase with height. The analysed maxima for $l_R=2$ and $l_R=3$ are in reasonable agreement with their predicted values. The $l_R=4, 5$ modes have frequencies too low to be identified properly. Due to the standing wave component their total energy is considerably higher than the wavenumber-1 modes mentioned before.

The frequencies of each of the wavenumber-3 theoretical modes (Fig. 7) vary over an interval of more than 0.1 cycle/day with respect to elevation. The Yanai wave ($l_R=0$, mixed Rossby-gravity mode) has an average frequency of about 0.4 cycle/day and is the first mode we identified by daily data time series: a remarkable part of the traveling wave's energy is explained by this mode. Again, for all modes standing waves contain more energy than the traveling components. The meridional modes $l_R=2$ and 3 have peaks in the expected positions too, but beginning with index $l_R=3$ the identification becomes difficult due to the vicinity of the continuous spectra frequencies and the increasing red noise.

The observed features of wavenumber-3 modes are seen to be further enhanced when looking at the oscillations of zonal wavenumber-4 (Fig. 8). The distorting effect of the zonal winds increases the modes' deceleration. Albeit with a blurred view there are still signals to identify as eigenmodes. The Yanai wave as well as the $l_R=1$ mode can be seen clearly, the latter is emphasized by a relatively high spectral peak. The theoretical frequency bands of the higher indexed modes broaden more and more so that peaks so obscured are not reliable mode indicators.

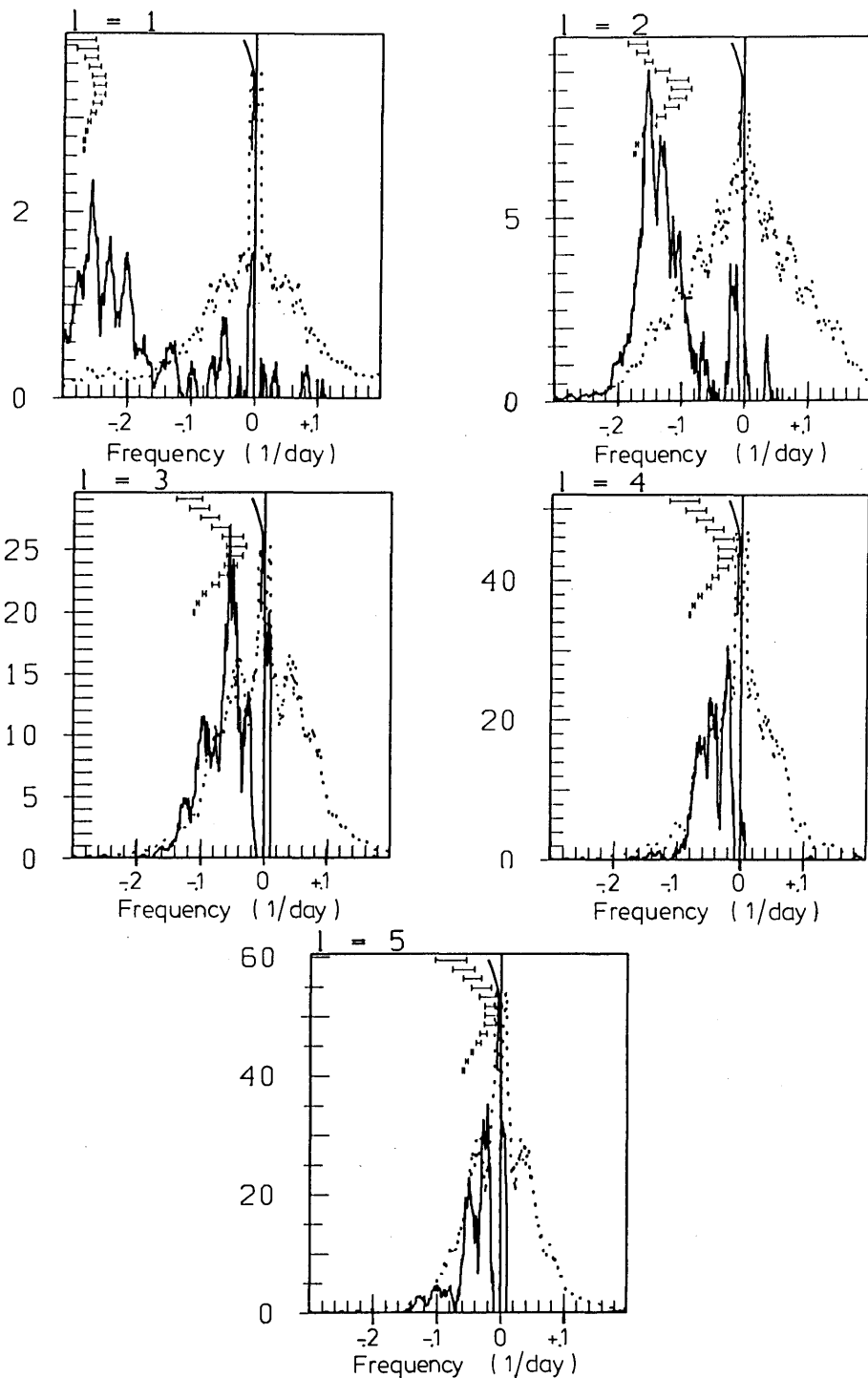


Fig. 6. Power spectra of the barotropic modes of zonal wavenumber 2 and meridional indices $l_R = 1$ to $l_R = 5$. Otherwise similar to Fig. 2.

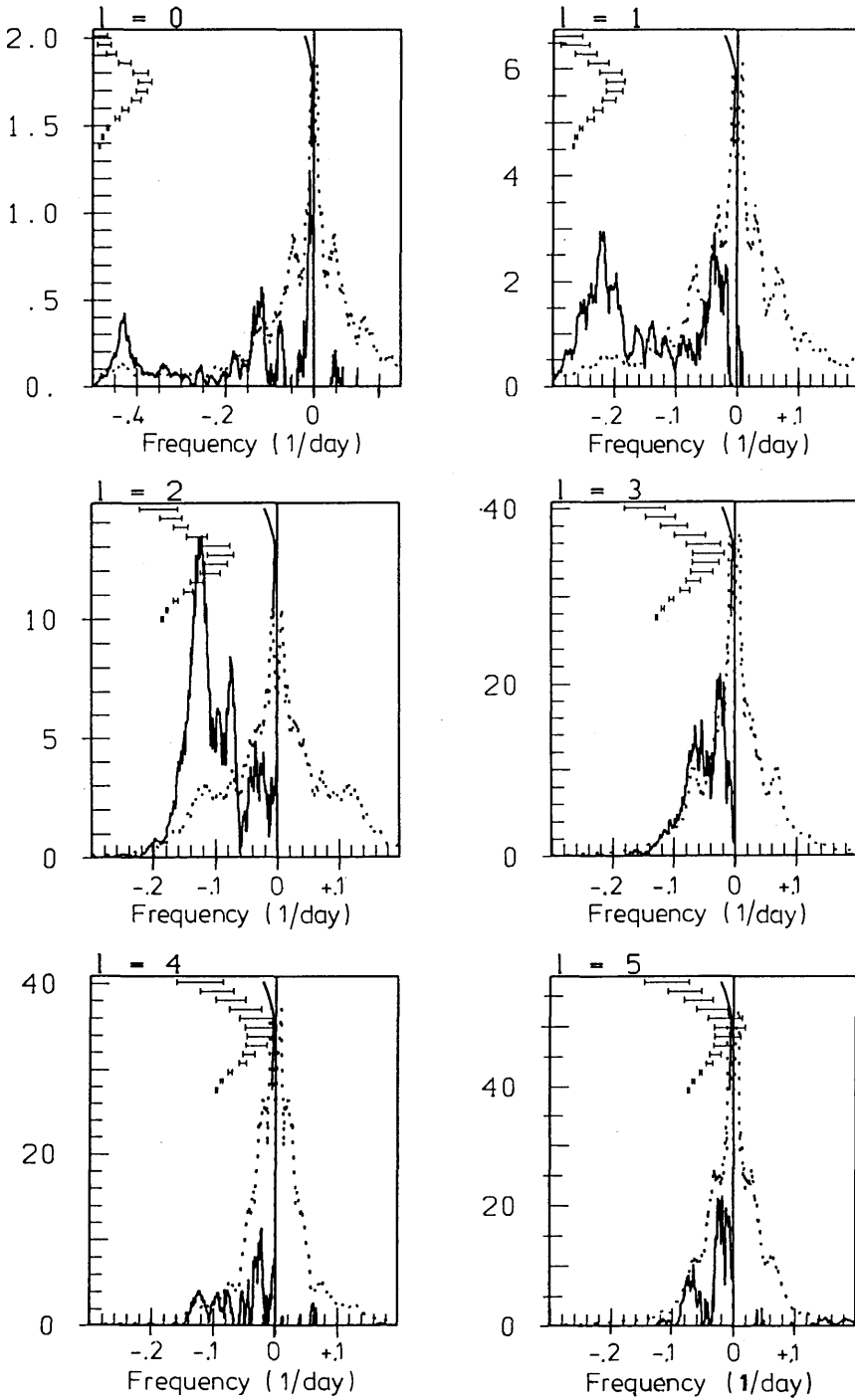


Fig. 7. Power spectra of the barotropic modes of zonal wavenumber 3 and meridional indices $l_R=0$ to $l_R=5$. Otherwise similar to Fig. 2.

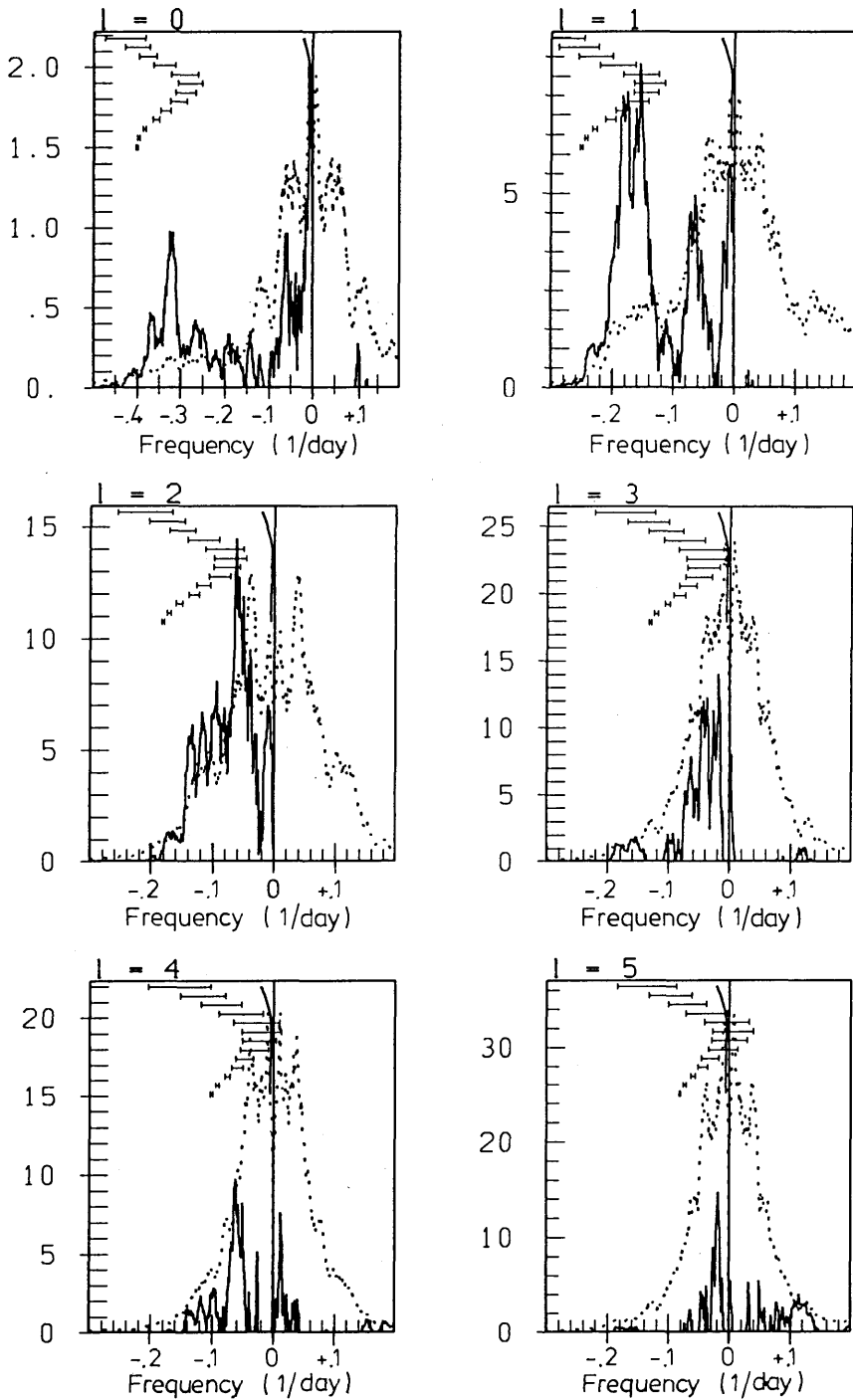


Fig. 8. Power spectra of the barotropic modes of zonal wavenumber 4 and meridional indices $l_R = 0$ to $l_R = 5$. Otherwise similar to Fig. 2.

For the benefit of comparison with the results of Ahlquist (1982) the essential deviations of the present paper to his analysis are summarized as follows:

(1) He made no distinction between standing and traveling waves.

(2) He included the mean zonal flow in terms of a seasonal mean.

(3) He approximated the modes' vertical structure by an adjusted Lamb wave profile, fitting the reference heights 850, 500 and 200 hPa.

(4) His length of time series consists of 1200 consecutive days of twice-daily NMC global tropospheric analyses.

With those differences in mind, a significance test similar to Ahlquist (1982) was set up (see also Bloomfield (1976) for details). By a moderate contraction of the reference windows, the resolution of low frequency peaks could be slightly improved. In this way 11 modes passed the significance test; another 7 modes were identified subjectively following Ahlquist (1982). Table 1 shows a summary of the identified 18 modes in terms of a separation into standing and traveling waves, which is noted to be the essential difference to the time series analysis applied by Ahlquist (1982). The existence of all of his detected 14 modes could be confirmed. The modes $(s, l_R) = (1, 4)$, $(1, 5)$, $(3, 3)$ and $(4, 4)$ could be identified by us additionally.

The histograms of Fig. 9 summarize the energy distribution findings in terms of: (1) standing waves; (2) eastward traveling waves; (3) westward traveling waves, (a) with a frequency in conformity with Laplace's theory, (b) with a frequency lower or higher than predicted by the shallow water theory.

For each zonal wavenumber $s = 1, \dots, 4$ the energy of the meridional modes $l_R = 1, \dots, 5$ is represented by columns. The broad hatched bases represent the standing waves' energy. Their width symbolizes the equipartition into coherent eastward and westward traveling waves of the same amplitude. This means in integral form

$$\mathcal{E}_{ST} = \int_{0 < \sigma < \infty} E_{ST}(\sigma) d\sigma.$$

The superposed left column denotes the pure westward traveling partition. It is partitioned:

Table 1. *Significance of identification of the barotropic modes*

	$l_R =$	0	1	2	3	4	5
$s = 1$	F		+++	+++	+++	++	+
	o		+++	+	+	?	—
2	F		++	+++	++	++	++
	o		++	++	+	++	++
3	F	++	+++	+++	+	?	?
	o	+	+++	+++	—	—	—
4	F	+	++	+	?	+	—
	o	+	++	+	—	—	—

F: after filtering out the standing part of westward moving modes (present paper), o: without that filter (Ahlquist, 1982). Signal strength: +++ highly significant, ++ significant, + moderate, ? uncertain, — no signal.

The upper black rectangle represents the energy partition, which complies with the Laplace's tidal theory. To measure this, it was integrated along the interval $I_R = \min_{a,b}(I_R^a, I_R^b)$, where I_R^a, I_R^b is defined by:

(a) the unification of the theoretical frequencies and their standard deviations $\sqrt{\mathcal{V}}$ of all levels i ,

$$I_R^a = \bigcup_i (\sigma_i - \sqrt{\mathcal{V}(\sigma_i)}, \sigma_i + \sqrt{\mathcal{V}(\sigma_i)})$$

$$i = 1, 2, \dots, 13,$$

or,

(b) following Ahlquist (1982), by an interval of 0.1 cycles/day length centered around the mean of the theoretical frequencies $\mathcal{E}(\sigma_i)$

$$I_R^b = (\mathcal{E}(\sigma_i) - 0.05d^{-1}, \mathcal{E}(\sigma_i) + 0.05d^{-1}).$$

By this procedure the energy partition of Rossby waves according to the tidal theory is extracted by

$$\mathcal{E}_R = \int_{I_R} E_R(\sigma) d\sigma.$$

The lower white strip of the left column displays the energy of those traveling waves which do not comply with the theoretical frequencies by

$$\mathcal{E}_{WT} = \int_{\substack{-\infty < \sigma < 0 \\ \sigma \neq I_R}} E_{WT}(\sigma) d\sigma.$$

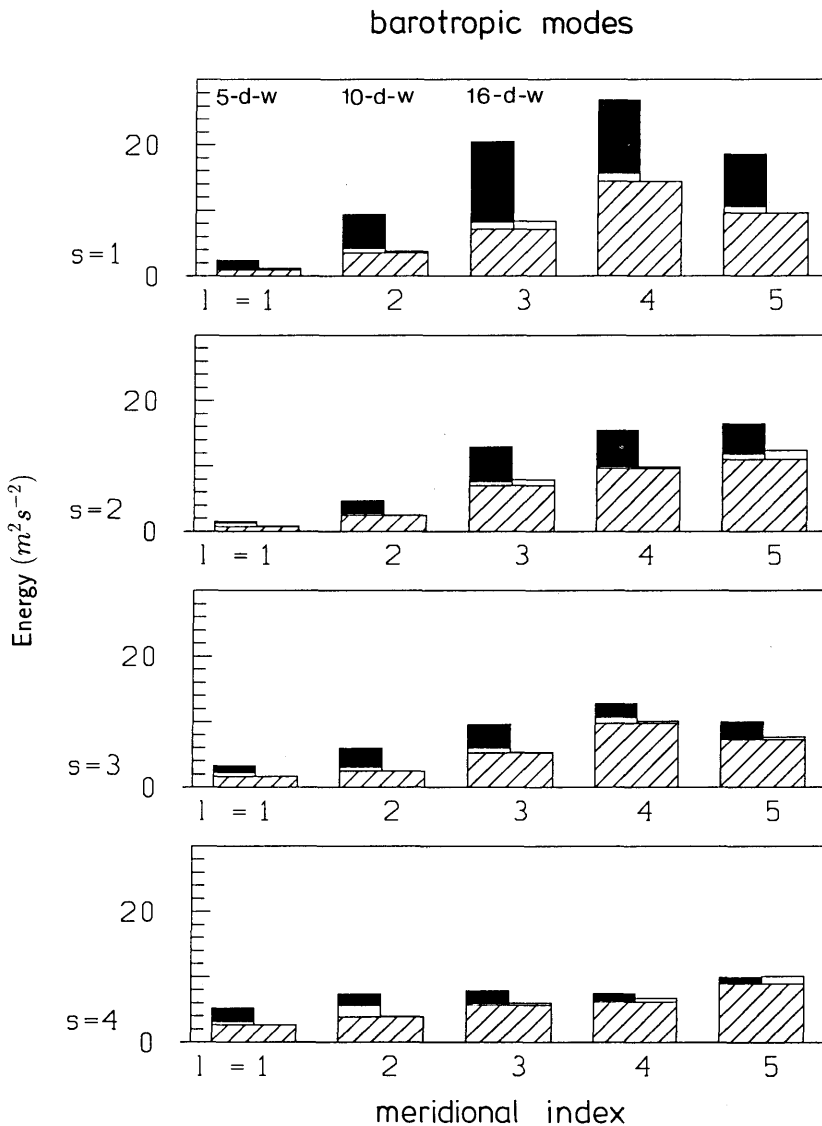


Fig. 9. Histograms of the energy partition of the barotropic modes. Hatched rectangles: Standing waves; left columns, black: westward propagating Rossby waves propagating at frequency complying with theory; left column, white rectangle: westward propagating Rossby waves propagating at frequency not complying with theory; white columns, right: energy of eastward traveling waves.

In the same way, the eastward traveling Rossby waves are represented by the right column as a result of the integral

$$\mathcal{E}_{ET} = \int_{0 < \sigma < \infty} E_{WT}(\sigma) d\sigma.$$

It is evident that the standing waves comprise the biggest part of the total wave energy. This is even true for the 5-day-wave, which shows the most significant signal of all power spectra. At a first look it is readily seen from the histograms that the modes' total energy may be truly represented

only by a fraction in terms of the Laplace's tidal theory. Looking again at the 5-day-wave, there is nearly no energy propagating with frequencies deviating from the theory. The 10-day-wave, which possesses much more energy, has a similar feature. The 16-day-wave ($s = 1$, $l_R = 3$), one of the first identified in the atmosphere (Madden, 1978), exhibits the largest amount of propagating energy in terms of the theory; but even in this case the standing waves part is even larger, whereas traveling energy nonconforming to theory remains negligibly small.

The waves just mentioned have nearly the same proportion of standing and traveling energy, but this is not true for the modes indexed as (1, 4) and (1, 5), which have the most total energy. The result from the histogram complies with Kasahara and Puri (1981) and Tanaka (1985). For these modes a considerable amount of energy is due to the standing waves component. The consistent with theory traveling parts are comparably large, but they do not exceed the 16-day-wave amounts.

With increasing zonal wavenumber, the traveling part conformable with theory becomes relatively smaller compared to the standing part. Traveling parts deviating from the expected frequencies remain small in most cases.

Table 2 summarizes the percentages of the free propagating Rossby wave energy and standing oscillations compared to the total energy of the projected barotropic modes. As depicted by histograms (Fig. 9), the 16-day wave shows up prominently; the westward traveling component

contains 43% of the total energy. Generally the standing wave energy increases with meridional index.

It is of interest to extend the applied analyses to baroclinic modes. For an unbounded isothermal atmosphere it is known that only barotropic free modes can exist. Chapman and Lindzen (1970) showed that even a sufficiently realistic standard atmosphere cannot sustain true baroclinic normal modes. Nevertheless we applied tentatively the power spectrum method to identify baroclinic modes. But significant peaks only appeared for the same frequency bands as for the barotropic modes with the same horizontal indices s and l_R and decisively not at their very own frequencies. As expected no observational evidence of free baroclinic modes was found.

5.2. Time series of westward traveling components

Here we describe the reconstruction of purely westward traveling Rossby waves and analyse individual episodes to examine their existence, strength and temporal evolution.

5.2.1. Energy- and phase diagrams. Our graphical presentation follows Lindzen et al. (1984). Before comparing his results with ours it should be stressed that Lindzen et al. (1984):

- (1) evaluated data only at the 500 hPa level;
- (2) analysed seasonally limited time series of twice-daily data;
- (3) did not remove the westward propagating portions of the standing waves.

Table 2. *Estimated percentage of free Rossby waves (upper lines: fR) and standing oscillations related to the mode's total energy (lower lines: sW); rowwise for zonal wavenumber s , columnwise for meridional index l_R*

$l_R =$		0	1	2	3	4	5
$s = 1$	fR		38	40	43	27	28
	sW		52	54	50	70	68
2	fR		13	29	26	22	16
	sW		64	68	67	76	76
3	fR	23	21	34	25	9	15
	sW	76	66	58	71	86	82
4	fR	27	27	16	14	8	5
	sW	72	66	69	82	89	89

Figs. 10–12 display both phase and energy, which is defined by the square of $W'_{s,\omega}$ in formula (25). The abscissa represents consecutive days for a selected interval from the time series. All diagrams comprise an interval of about four months. The left ordinate scales the wave energy, which is depicted by the solid curve. The second result of the time series analysis is the wave phase and is represented by the right hand side ordinate. For the sake of neatness the phase angle interval is doubled to a $[-2\pi, 2\pi]$ interval, so two identical phase trajectories run parallel to each other as dotted lines separated by 2π . An ascending phase line denotes a positive rotation in the complex plane which means a westward propagation in terms of geographical coordinates. The average theoretical phase velocity is displayed by an ascending

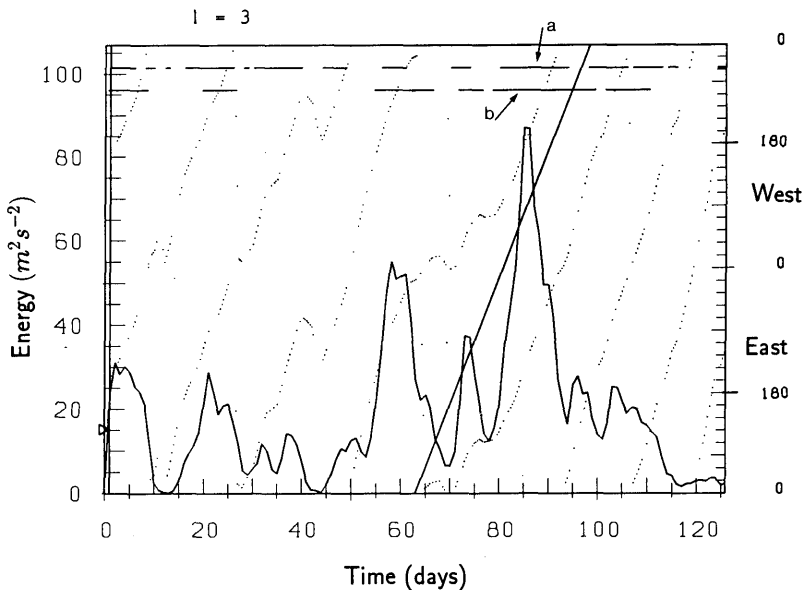


Fig. 10. Energy and phase diagram of the 16-day-mode (1, 3, 0) during the month January to April 1984 on abscissa for consecutive days. See text for further explanation.

straight line originating in the middle of the abscissa. Thus, ideally, the analysed phase trajectory should run parallel to that slope of reference. During days when this is seen to be true, a line is drawn at the upper side of the diagram (e.g., the line marked by "a" in Fig. 10). Close below there is a similar line drawn in the case when the pure westward traveling wave's energy exceeds its quadrennial mean value ("b"). This value is marked by a triangle on the left ordinate.

The 5-day-wave

In terms of its power spectrum the 5-day-wave showed the most significant relative signal strength, lack of noise and consistency with the Laplace's tidal theory. The top panel in Fig. 11 presents an example of a time interval with remarkable irregularities during days 230 to 270 (i.e., August/September 1984). Beginning with day 187 a long-lived and later on very strong episode occurs, only interrupted by two small phase irregularities about at days 192 and 202. The longest episode ever observed during the period studied runs up to 39 days beginning with day 977 and ending with day 1016 (Fig. 11 bottom panel). Its phase trajectory depicts about seven revolu-

tions. Phase leaps or irregularities are seen only to be associated with small energy transitions. Estimating a mean life time of the 5-day-mode, the result depends quite sensitively on the exclusion of very short lasting episodes. To assess quantitatively a mean life time of the mode, a threshold value of four days of regular propagation was defined arbitrarily. This results in a mean life time of about 11.6 days for the 5-day-mode.

On visual inspection of the 5-day-wave's energy curve (Fig. 11a, b) the sequence of the peaks suggests that the mode might be subject to a 5 to 7-day excitation mechanism. In order to evaluate this observation an analysis in terms of an energy density spectrum was performed; Fig. 14a verifies this assumption by displaying a broad maximum marked by an arrow. The significance of the peak is confirmed by a 95% confidence test. (In Fig. 14 the bandwidth is labelled as "BW".)

Seasonal variances could not be investigated due to the short time series.

The 10-day-wave

There is a somewhat different behaviour when looking at the antisymmetric 10-day wave. Propagation velocities consistent with theory only

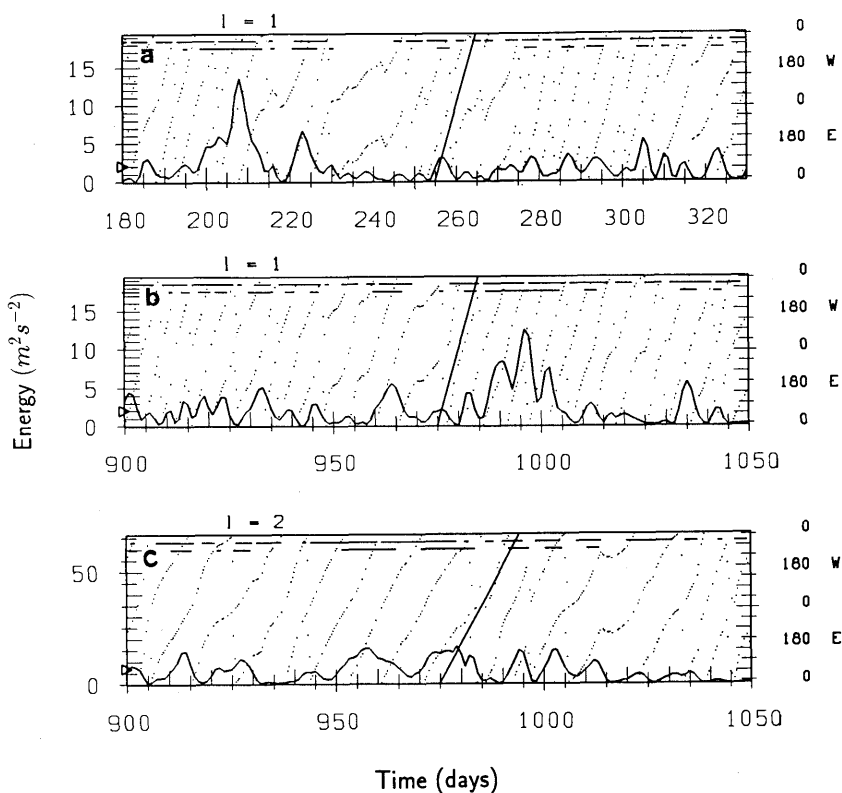


Fig. 11. Energy and phase diagram of the 5-day-mode (1, 1, 0), (a) during the month July to October 1984 with exceptional low energy episode around day 240 and regularly occurring phase leaps during energetic minima (days 280 to 330), and (b) with a long lived episode during September 1986 (approximately day 1000). (c) Energy and phase diagram of the 10-day-mode (1, 2, 0) during a long-lived episode in August 1986.

reliably occur during significant episodes. Phase leaps always arise during a transition between stronger events. The mode's longest life time episode took place in 1986. It lasted for 37 days. The mean life time according to the definition set up before is 8.7 days. There is only a slight tendency for regular excitation at about 18 days period as indicated by the peaks in the energy density spectrum (Fig. 14b).

The 16-day-wave

Having the largest amount of propagating energy, the August 1985 episode (around day 600 in Fig. 12a) has the highest energy of the entire period of investigation. The theoretically expected phase velocity and the actually analysed propagation speed are in full agreement. The longest event

took place in April 1986 and lasted about 30 days, i.e., two revolutions. Again the disparities between Laplace's tidal theory and observations are smaller the more the energy of the wave increases.

The modes with meridional indices 4 and 5

Although generally having the largest amounts of total energy, the free energy of traveling waves of wavenumber-1 modes with $l_R = 4$ and $l_R = 5$ is less than that of the $l_R = 3$ mode. Nevertheless there are episodes where some peaks have the magnitude of the 16-day-wave. Fig. 12b presents an example of a maximum in August 1986 (i.e., day 987) for $l_R = 5$. Again during these and other high energetic episodes, analyses of the phase propagation and theoretical speed match fairly well. This will be shown to hold also for the following modes.

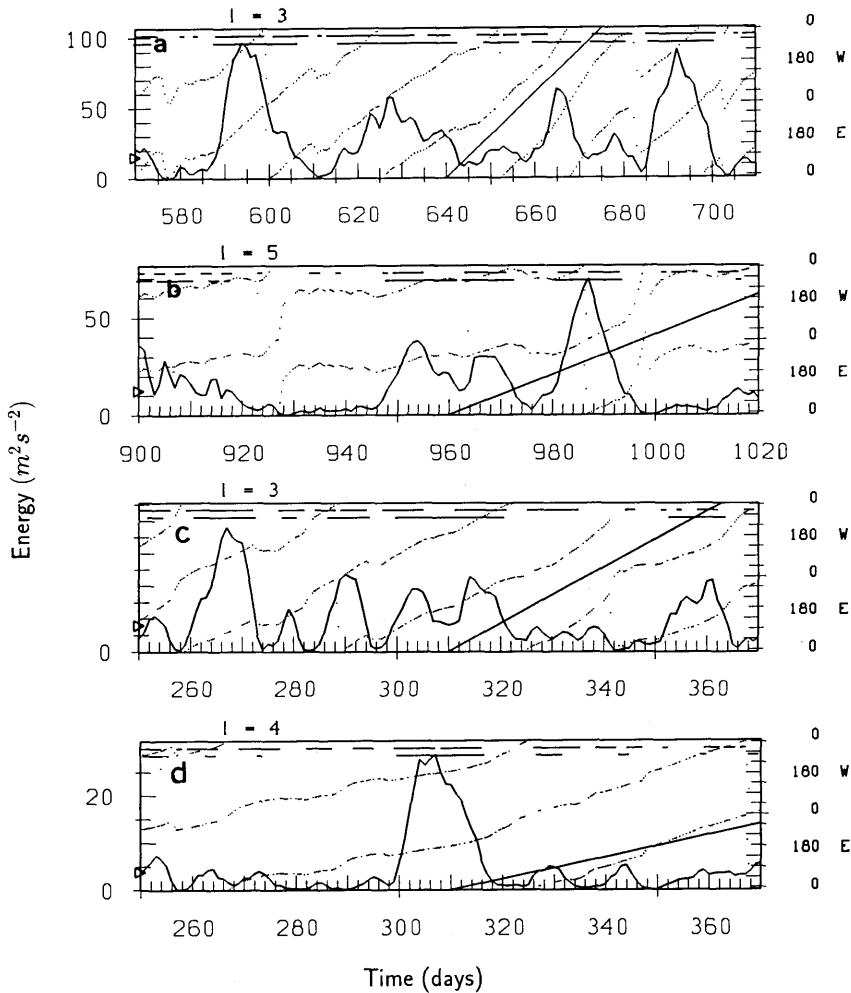


Fig. 12. (a) The 16-day-wave with its most energetic episode during the observation period (about day 600) and high energetic episodes of the barotropic modes (1, 5, 0) (b) during August 1986, (c) (2, 3, 0), and (d) (4, 4, 0), during October to December 1984.

The modes of zonal wavenumber 2, 3 and 4

As mentioned before, the increasingly strong Doppler shifts of the theoretical frequencies of higher indexed modes obscures the signal and thus impedes the distinction between free and forced oscillations (Salby, 1981). The energy phase diagrams serve as an alternative way of identification. Fig. 12c, d depicts examples for the modes (2, 3, 0) and (4, 4, 0) during the northern winter 1984/85. Additionally to the correlation of the occurrence of theoretical match and high energy

episodes, the life time of those events increase with wavenumber as demonstrated by energy curves and phase trajectories. Thus the capability to conserve energy grows with the index sum $l_R + s$.

5.2.2. Summary of special features. Some notable findings repeatedly obtained so far during the analysis of the single time series are now subjected to an investigation of a more general validity. Lindzen et al. (1984) were the first to state the modes' increasing agreement with the Laplace's tidal theory when its energy is large. The

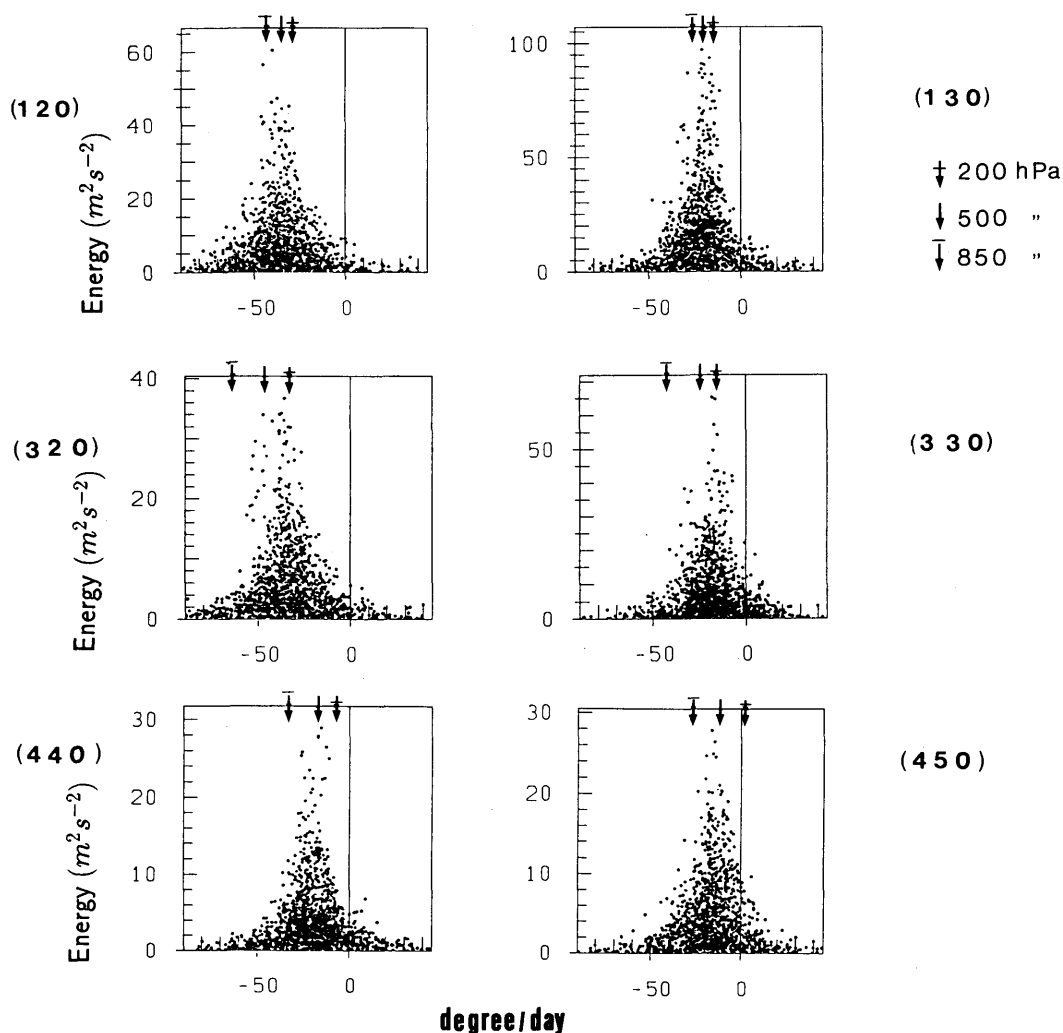


Fig. 13. Scatter diagrams of barotropic modes (1, 2, 0), (1, 3, 0), (3, 2, 0), (3, 3, 0), (4, 4, 0), and (4, 5, 0). A dot denotes the daily propagation in degrees at the abscissa versus the energy of the mode at the ordinate. Westward propagation has a negative sign.

scatter diagrams in Fig. 13 display the energy of a wave, as computed from Figs. 10 to 12, versus its daily phase progression. As representatives of the barotropic modes, some modes for zonal wavenumbers 1, 3 and 4 were selected for illustration. The abscissa shows propagation speed in degrees per day. Negative values denote westward propagation. The ordinate stands for the energy as before. At the top of the diagrams the theoretical speeds for the 850 hPa, 500 hPa, and 200 hPa

levels are marked. To confirm the above mentioned observation, the scattered dots are expected to align under the theoretical values when their energy increases. Most of the diagrams show this property. Thus, the more energy a mode has, the greater agreement with theory.

Another observation suggests that the life times of the normal modes increase with growing index sum $l_R + s$. As demonstrated by Fig. 12d there are long-lived modes which show consistency with

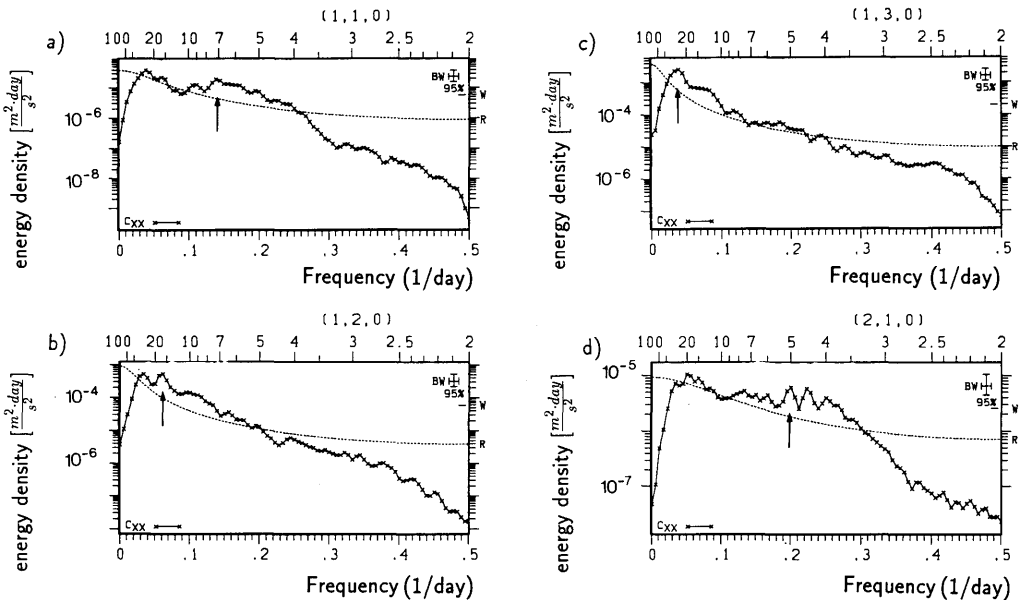


Fig. 14. Energy density spectra of Rossby modes. Lower abscissa frequency [1/day], upper abscissa period [day], ordinate undimensional energy, spectra were computed by applying a Hanning-window-filter. (a) 5-day-wave (1, 1, 0) upper left panel, (b) 10-day-wave (1, 2, 0) lower left, (c) 16-day-wave upper right, and (d) mode (2, 1, 0) lower right.

theory only for short periods. Since this is believed to be due chiefly to forced oscillations, the life time, as a criterion, should be distinguished in terms of undisturbed free propagation episodes as proposed by theory versus slightly forced deviations. The first criterion results in Table 3.

The 5-day wave (1, 1) is the most long-lived. Most of the other waves have life times varying a little around 8 days, which seems to be rather short compared to about $1\frac{1}{2}$ revolutions found by Ahlquist (1985). This should be due to the more rigorous criterion posed here, which interrupts a

mode's life time by the occurrence of small phase velocity deviations, even if the mode resumes propagation in a theoretically consistent way shortly afterwards.

Therefore Table 4 was compiled counting those episodes which are moderately affected by external forcing, i.e., the criterion of interrupt ignores periods of 1 day of stationary or eastward moving propagation or small phase leaps. In some cases the life times of the lower indexed modes ($s \leq 1$)

Table 3. Mean life span in days of the barotropic modes ($m=0$) for free oscillations

	$l_R=0$	1	2	3	4	5
$s=1$		11.6	8.7	9.2	7.6	6.6
2		9.2	10.8	8.6	8.1	6.7
3	5.7	8.7	9.6	8.8	6.7	7.4
4	7.1	8.8	8.4	6.9	7.6	8.8

Meridional indices l_R (columns) versus wavenumbers s (rows). (Episodes with less than 4 days duration are neglected.)

Table 4. Mean life span in days of the barotropic modes ($m=0$) including moderately degraded oscillations

	$l_R=0$	1	2	3	4	5
$s=1$		17	19	22	14	12
2		12	13	18	17	16
3	7	14	17	23	21	19
4	9	11	19	25	28	18

Meridional indices l_R (columns) versus wavenumbers s (rows). (Episodes with less than 4 days duration are neglected.)

Table 5. *Typical excitation periods of barotropic modes (in days), from energy density spectra (Fig. 14)*

	$l_R = 0$	1	2	3	4	5
$s = 1$		7	18	25?	—	—
2		5	8	14?	—	—
3	2.5	6	8	15?	9	—
4	4	7	—	—	—	—

or $l_R \leq 1$) appear to be doubled, when compared to Table 3. In general there is a tendency, that with increasing index sum $l_R + s$ the life time is enhanced up to a factor close to four compared to Table 3.

A third item of this section addresses the question of a hypothetical regular excitation period. The filtered time series, that is the squared amplitude of the pure westward propagating waves as plotted by the solid lines in Figs. 10 to 12, were analysed by energy density spectra (Fig. 14). The results are presented in Table 5. There is a slight tendency to slow down the excitation frequency with increasing index sum $l_R + s$, which is however connected with decreasing significance. Thus only the meridional modes $l_R = 0$ and 1 have significant signals which are worth mentioning. Nevertheless it is well known that nonlinear energy exchanges are one of the prime agents of modal activity which cannot be properly analysed by the time series method applied in this paper.

6. Summary and conclusions

It was the intention of the present study to evaluate what portion of the total Rossby wave energy projected onto the eigenmodes of the Laplace's tidal equations is due to free propagating oscillations and what portions are due to standing waves, quasistationary waves, and other waves not propagating in accordance with the theory. The basic functions, forming the horizontal and vertical profiles, are determined by the solutions of Laplace's tidal equations and the vertical structure equation. In generalizing the tidal model, the construction of the basic functions took into

account the zonal winds to implement a realistic retardation of the propagation speed.

By projecting global fields of geopotential height and winds from ECMWF operational analyses a four year time series from 1 January 1984 through 31 December 1987 of daily normal mode coefficients was obtained. A time series analysis separated wave energy into pure westward and eastward traveling waves as well as standing oscillations. After investigating the modes of the first four zonal wavenumbers and six meridional indices, fourteen signals of eigenmodes could be identified with hints of four additional signals (Table 1). An energy partition revealed that free oscillations account for a considerable part of the barotropic energy that project onto the first three meridional indices of zonal wavenumber one. In the case of the 16-day-wave, approximately 43% of the energy belongs to the wave which propagates westward very close to the theoretical frequency. The energy of free oscillations decreases with increasing wavenumber (Fig. 9). Since the sensitivity of the theoretical wave speed to the zonal winds increases with the wavenumber, a separation into free and forced oscillations is increasingly difficult with increasing wavenumber sum $l_R + s$. Nearly the entire energy of high index modes $l_R + s$ is contained in standing waves.

Filtering out standing and eastward traveling waves makes it possible to study westward traveling waves as they episodically appear. The higher the energy of the modes the greater is the agreement of the modes with the theoretical propagation speed (Fig. 13). During the period of concern, the maximum life time of a wave lasts up to 40 days; the average life time is about 8 to 10 days. A regular excitation period could only be stated concerning modes with meridional indices zero and one with 2.5 to 7 days.

7. Acknowledgements

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REFERENCES

- Ahlquist, J. E. 1982. Normal-mode global Rossby waves: Theory and observations. *J. Atmos. Sci.* 39, 193–202.
- Ahlquist, J. E. 1985. Climatology of normal mode Rossby waves. *J. Atmos. Sci.* 42, 2059–2068.
- Arpe, K., Brankovic, C., Oriol, E. and Speth, P. 1986. Variability in time and space of energetics from a long series of data produced by ECMWF. *Contr. Atm. Phys.* 59, 321–355.
- Baer, F. 1972. An alternate scale representation of atmospheric energy spectra. *J. Atmos. Sci.* 29, 649–664.
- Bloomfield, P. 1976. *Fourier Analysis of Time Series*. Wiley, 258 pp.
- Burrows, W. R. 1976. A diagnostic study of atmospheric spectral kinetic energetics. *J. Atmos. Sci.* 33, 2308–2321.
- Chapman, S. and Lindzen, R. S. 1970. *Atmospheric tides, thermal and gravitational*. Gordon and Breach, 200 pp.
- Chen, T.-C. and Wiin-Nielsen, A. C. 1978. On nonlinear cascades of atmospheric energy and enstrophy in a two dimensional spectral index. *Tellus* 30, 313–322.
- Daley, R. 1981. Normal mode initialisation. *Rev. Geoph. Space Physics* 19, 450–468.
- Daley, R. and Williamson, D. 1985. The existence of free Rossby waves during January 1979. *J. Atmos. Sci.* 42, 2121–2141.
- Daley, R., Tribbia, J. and Williamson, D. 1981. The excitation of large-scale free Rossby waves in Numerical Weather Prediction. *Mon. Weath. Rev.* 109, 1836–1861.
- Dikii, L. and Golitsyn, G. 1968. Calculation of Rossby wave velocities of the earth's atmosphere. *Tellus* 17, 220–238.
- Eliassen, E. and Machenhauer, B. 1965. A study of fluctuations of the atmospheric flow patterns represented by spherical harmonics. *Tellus* 17, 220–238.
- Eliassen, E. and Machenhauer, B. 1969. On the observed large-scale atmospheric wave motion. *Tellus* 21, 149–166.
- Haurwitz, B. 1940a. The motion of atmospheric disturbances on the spherical earth. *J. Mar. Res.* 3, 35–50.
- Haurwitz, B. 1940b. The motion of atmospheric disturbances on the spherical earth. *J. Mar. Res.* 3, 254–267.
- Hayashi, Y. 1971. A generalised method of resolving disturbances into progressive and retrogressive waves by space Fourier and Time cross-spectral analyses. *J. Met. Soc. Japan* 49, 125–128.
- Hayashi, Y. 1977. On the coherence between progressive and retrogressive waves and a partition of Space-Time power spectra into standing and traveling parts. *J. Appl. Met.* 16, 368–373.
- Hayashi, Y. 1982. Space-time spectral analysis and its applications to atmospheric waves. *J. Met. Soc. Japan* 60, 156–171.
- Hollingsworth, A., Arpe, K., Tiedke, K., Capaldo, M. and Savijärvi, H. 1980. The performance of a medium-range forecast model in winter-impact of physical parameterization. *Mon. Wea. Rev.* 108, 1736–1773.
- Holmström, I. 1963. On a method for parametric representation of the state of the atmosphere. *Tellus* 15, 127–149.
- Kasahara, A. 1976. Normal modes of ultralong waves in the atmosphere. *Mon. Wea. Rev.* 104, 669–690.
- Kasahara, A. 1980. Effect on zonal flows on the free oscillations of a barotropic atmosphere. *J. Atmos. Sci.* 37, 917–929.
- Kasahara, A. 1981. Corrigendum. *J. Atmos. Sci.* 38, 2284–2285.
- Kasahara, A. 1984. The linear response of a stratified global atmosphere to tropical thermal forcing. *J. Atmos. Sci.* 41, 2217–2237.
- Kasahara, A. and Puri, K. 1981. Spectral representation of the three dimensional global data by expansion in normal mode functions. *Mon. Wea. Rev.* 109, 37–51.
- Lambert, S. J. and Merilees, P. E. 1987. A study of planetary wave errors in a spectral numerical weather prediction model. *Atmos. Ocean* 16, 197–211.
- Leith, C. E. 1974. Spectral statistical-dynamical forecast experiments. *Proc. Intern. Symp. Spectral methods in numerical weather prediction*, H. C. Ørstedt Institutet, Denmark, 445–467.
- Lindzen, R. S., Straus, D. M. and Katz, B. 1984. An observational study of large scale atmospheric Rossby waves during FGGE. *J. Atmos. Sci.* 28, 145–161.
- Longuet-Higgins, M. S. 1968. The eigenfunctions of Laplace's tidal equations over a sphere. *Phil. Trans. Roy. Soc. London A* 262, 511–607.
- Lorenz, E. N. 1955. Available potential energy and the maintenance of the general circulation. *Tellus* 7, 157–167.
- Madden, R. A. and Julian, P. R. 1972. Further evidence of global-scale 5-day pressure waves. *J. Atmos. Sci.* 29, 1464–1469.
- Madden, R. A. 1978. Further evidence of traveling planetary waves. *J. Atmos. Sci.* 35, 1605–1618.
- Madden, R. A. and Speth, P. 1989. The average behavior of large-scale westward traveling disturbances evident in the Northern Hemisphere geopotential heights. *J. Atmos. Sci.* 46, 3225–3239.
- Obukhov, A. M. 1960. The statistically orthogonal expansion of empirical functions. *Izv. Geophys. Ser.* no. 3, 288–291.
- Rossby, C. G. 1939. Relations between variations in the intensity of the zonal circulation of the atmosphere and the displacements of the semipermanent centers of action. *J. Mar. Res.* 2, 38–55.
- Salby, M. 1981. Rossby normal modes in nonuniform background configurations, II.: Equinox and solstice conditions. *J. Atmos. Sci.* 38, 1827–1840.
- Saltzman, B. 1957. Equations governing the energetics of large scale of atmospheric turbulence in the domain of wavenumbers. *J. Meteorology* 14, 513–523.
- Shaw, D., Lönnberg, P., Hollingsworth, A. and Unden, P. 1987. Data assimilation: The 1984/85 revisions of

- the ECMWF mass and wind analysis. *Quart. J. Roy. Met. Soc.* 113, 533–566.
- Straus, D. and Shukla, J. 1981. Space-time spectral structure of a GLAS general circulation model and a comparison with observations. *J. Atmos. Sci.* 38, 902–917.
- Tanaka, H. 1985. Global energetics analysis by expansion into three-dimensional normal mode functions during the FGGE winter. *J. Met. Soc. Japan* 63, 2, 180–200.
- Trenberth, K. E. and Olsen, J. G. 1988a. *ECMWF global analyses 1979–1986: circulation statistics and data evaluation*. NCAR Tech. Note NCAR/TN-300 + STR., 94 pp.
- Trenberth, K. E. and Olsen, J. G. 1988b. An evaluation and intercomparison of global analyses from the National Meteorological Center and the European Center for Medium Range Weather Forecasts. *Bul. Amer. Meteor. Soc.* 69, 1047–1057.
- Tribbia, J. J. and Madden, R. A. 1988. Projection of time-mean geopotential heights onto normal Hough modes. *Met. Atmos. Phys.* 38, 9–21.