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On the formation of ice on deep weakly stratified water

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ABSTRACT

Making use of the simplest possible model, we analyse the formation of ice on top of a weakly stratified ocean. Our interest is concentrated on the consumption of freshwater associated with ice formation and the dynamics of the system when the cooling continues beyond the point where further ice formation would destroy the stability of the system. We find that after an initial stage of ice formation the system will not overturn but go into a stage of development which may be called "freeze melting". This stage is characterized by increasing mixed layer depth, slowly decreasing ice thickness and small but finite stability. If the freeze melting continues for a sufficiently long time, considerably longer than required for the initial ice formation, the ice cover may be removed altogether, whereupon the stratification overturns and the fresh-water in the top layer gets lost. It is suggested that if this happens one year it will contribute to pre-conditioning the system for ice-free conditions the following year. An essential condition for the analyses, which may be put in question, is the presence of at least some wind generated turbulence and that competing mixing processes, e.g., associated with cabbeling do not become dominating. Observations from the Wedell sea give support to the conclusion that late winter conditions in this area may be well described in terms of a freeze melting stage of development.

1. Introduction

It is generally recognized that the formation of ice on deep ocean water requires the presence of a stable salinity stratification (Aagaard and Carmack, 1989). If sufficiently strong, the salinity stratification limits the thickness of the layer that has to be cooled to the freezing point before ice formation can take place. Without such a protective stratification the ocean water has to be cooled to the freezing point over the entire ocean depth which is not possible if the depth exceeds a few hundred meters.

In some ocean areas such as the Iceland sea the necessary salinity stratification is present only certain years and with varying areal extension which gives rise to strong inter-annual variations in the ice conditions as discussed by Malmberg (1972).

Of importance in areas with weak stratification is that the formation of ice consumes the salinity stratification. The fresh water contained in the initial stratification will impose an upper limit, D_{max}, on the thickness of the ice cover according to the formula

$$D_{max} < H_{f0} \tag{1a}$$

where the fresh water content H_{f0} with good approximation is given by

$$H_{f0} = \frac{1}{S(-\infty)} \int_{-\infty}^{0} (S(-\infty) - S(z))\,dz. \tag{1b}$$

$S(z)$ is the salinity in the water column before the start of the ice formation and D_{max} is an "equivalent thickness", i.e., the thickness of an equal amount of liquid water.

The magnitude of H_{f0} provides a convenient measure to judge whether the formation of sea ice will be hampered by insufficient stratification of the water column. This was early recognized by Zubov (1943) and by Weyl (1968) who pointed out the close relation between climate variations and the extent of ocean ice cover controlled by the presence of freshwater in the top layer of the ocean.

In this paper, we focus on the case when ice formation is limited by the smallness of H_{f0}. We will analyse in some detail how the system behaves when the stability is reduced to small values and the depth of the mixed layer is allowed to increase as a result of entrainment of the underlying deep water. Our ambition is to specifically analyse the behaviour of the ice and mixed layer in the case when the stability is threatened. In order to illuminate the qualitative aspects of the problem we will use the simplest possible model that can reproduce the essential features of the development.

The most extensive ice covered ocean areas on the earth are the Arctic and Antarctic oceans. In the Arctic we find a perennial ice cover with impressive thickness (3 to 5 m) and a relatively modest annual variation of the ice covered area (Coachman and Aagaard, 1974; Aagaard et al., 1981). The Antarctic ocean behaves quite differently in that almost the entire ice sheet disappears in the southern summer and that the thickness of the ice cover is modest despite extremely cold winter climate (Gordon and Huber, 1990; Foster and Carmack, 1976).

A possible contributing reason for this difference shows up when we look at the stratification in respective regions as illustrated in Fig. 1. We find

$H_{f0} \sim (5\text{–}25)$ m in the Arctic ocean,

$H_{f0} \sim (0.5\text{–}5)$ m in the Antarctic ocean.

It appears from these numbers that the weak stratification in the Antarctic ocean may provide a limiting factor for the formation of sea ice, while this is not the case in the Arctic ocean.

The importance of the fresh water content in the surface layer was early recognized by Zubov, translated to English in Zubov (1943) and by Weyl (1968) in a discussion of the relation between oceanic ice and climate.

A dramatic consequence of the weak Antarctic stratification is the occasional appearance of large areas of open water, polynyas, inside the ice cover throughout the cooling season. This phenomenon was described by Martinson et al. (1981) who also presented a model for the formation of polynyas of this type.

Essentially Martinson et al. (1981) assumed in their model that ice formation consumes the

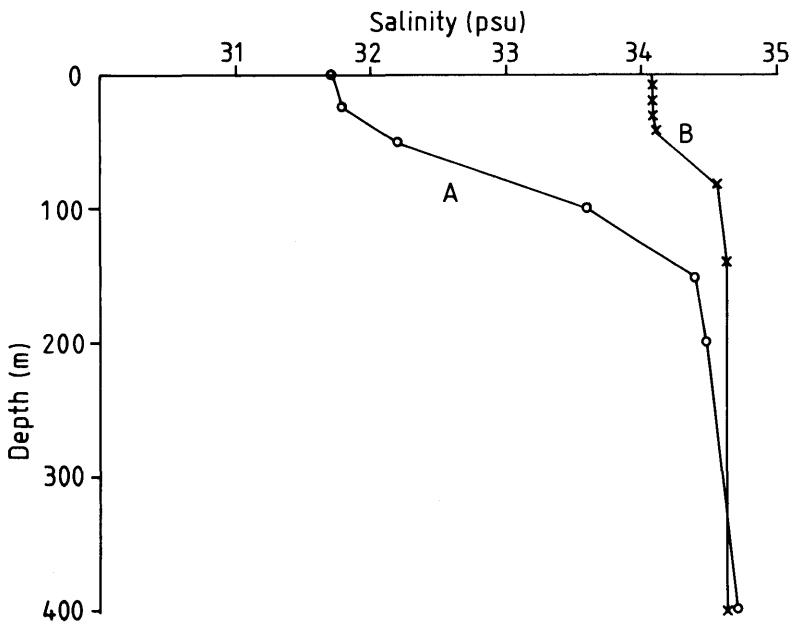

Fig. 1. Salinity stratification from the Eurasian basin in the Arctic ocean (A) and from the Weddell sea in the Antarctic ocean (B). From Aagaard et al. (1981) and NODC data bank. A: Feb. 1964. 87° N, 40° W. B: March 1970. 66.3° S, 24.6° W. The fresh water content in the stratification is approximately 15 m (A) and 1.4 m (B).

stratification until the top layer, taken to be some two degrees colder than the deep water, becomes equally dense as the deep water whereupon the water column overturns and the ice sheet is melted by the contact with the relatively warm deep water. An essential ingredient in the model presented by Martinson et al. (1981) is that the well mixed top layer has a constant prescribed thickness. In the present paper we challenge certain qualitative aspects of the model presented by Martinson et al. (1981).

In a recent paper, Martinson (1990) presents a very elaborate study with particular emphasis on the conditions in the Antarctic ocean. Martinsons study is related to the present work in that the ocean heat flux is of importance to limit the ice formation and thereby to keep the system stable. Martinson (1990) however excludes the feed back from wind forced entrainment which is the key element in the present analysis.

A great number of recent papers dealing with important aspects of the dynamics of sea ice have been presented e.g., Houssais (1988) and Häkkinen (1987) who emphasize the interaction with the deep water and Tang (1991), Röed (1984), Pollard et al. (1983), and Ikeda (1986) who reproduce successfully the ice dynamics in the marginal ice zone and the extent of the ice cover. These works, mostly more sophisticated than the present analysis, do not however shed light on the question what really happens at the critical stage when the stability under the ice is threatened by continued cooling, which is the key issue for the present analyses.

2. Model equations

Our ambition is to formulate the simplest possible model that retains the qualitative behaviour of the ice cover and the underlying mixed layer when the stability of the mixed layer becomes small. Accordingly we limit our analyses to the cooling season which we believe justifies the use of a single mixed layer with monotonically increasing depth. We assume the presence of an ice cover and that the mixed layer is kept at the freezing point by contact with the ice. We ignore the small dependence of the freezing point on salinity. Furthermore we use a linear equation of state and ignore all types of mixing or diffusion besides pure entrainment. These last approximations which exclude double diffusion and cabbeling may be questionable which will however be given some consideration in Section 7.

Our model is one-dimensional and consists of:

(1) a layer of ice with equivalent thickness D and salinity S_i;

(2) a mixed layer with thickness H, salinity S, and temperature at the freezing point T_f;

(3) a homogeneous deep water with salinity S_2 and temperature $T_2 = T_f + \Delta T$.

The "equivalent thickness" D means the thickness of an equal mass of liquid water.

We obtain the following conservation equations for ice thickness, mixed layer thickness, mixed layer salinity, mixed layer heat content, and heat content of the ice. The heat capacity of the ice cover is neglected.

$$\mathrm{d}/\mathrm{d}t(D) = F + f, \tag{2a}$$

$$\mathrm{d}/\mathrm{d}t(H) = W_e - F, \tag{2b}$$

$$H\,\mathrm{d}/\mathrm{d}t(S) = F(S - S_i) + W_e(S_2 - S), \tag{2c}$$

$$0 = W_e \rho c\,\Delta T - Q_1, \tag{2d}$$

$$0 = FL\rho + Q_1 - Q_a, \tag{2e}$$

where F is the rate of ice formation, f the rate of precipitation, W_e the rate of deepening of the mixed layer resulting from entrainment of water in the deep layer, ρ and c density and specific heat of water, L latent heat of sea ice, Q_1 heat flux from the mixed layer to the ice, and Q_a heat flux to the atmosphere. Note that (D, F, and f) represent the thickness and rate of change of thickness of equal masses of liquid water.

Furthermore W_e is assumed to be governed by an energy equation of the type (see, e.g., Stigebrandt, 1985):

$$2m_0 u_*^3 = W_e\, gH\,\Delta\rho/\rho + \gamma HB \quad \text{if} \quad W_e > 0, \tag{2f}$$

$$W_e = 0. \qquad \text{otherwise.}$$

m_0 is an empirical constant, u_* is the friction velocity, g is the acceleration of gravity, $\Delta\rho$ is the density contrast between deep water and mixed layer, B is the buoyancy flux into the mixed layer from above, and γ is an efficiency parameter given by

$$\gamma \approx 1 \quad \text{if} \quad B > 0,$$

$$\gamma \approx 0.05 \quad \text{if} \quad B < 0.$$

Furthermore, we assume that the density contrast between deep water and the mixed layer is given by the linear relation

$$\Delta\rho = \Delta\rho_S - \Delta\rho_T, \tag{2g}$$

where

$$\Delta\rho_S = \rho\beta(S_2 - S), \qquad \Delta\rho_T = \rho\alpha\,\Delta T.$$

Consistently with eq. (2f), the buoyancy flux B may be written

$$B = -Q_1\alpha g(\rho c)^{-1} - F(S - S_i)\,\beta g. \tag{2h}$$

3. Idealized description of the response to cooling

Two idealized stages of development may occur as the result of heat loss to the atmosphere.

3.1. Initial iceformation

In the beginning of the cooling season the stability is strong, which essentially prevents entrainment of heat from below. To idealize this situation W_e in eqs. (2c, d, e) is put equal to zero. We thus obtain

$$\mathrm{H}\ \mathrm{d}/\mathrm{d}t(S) = F(S - S_i), \tag{3a}$$

$$0 = FL\rho - Q_a. \tag{3b}$$

These equations describe how the top layer salinity S increases in response to the ice formation F and how F is directly related to the rate of cooling Q_a. Including the entrainment terms in the balance does not change the qualitative behaviour as long as the entrainment terms remain of second order importance. During this stage of development the heat flow is mainly from the ice formation to the atmosphere. The initial iceformation regime is illustrated in Fig. 2.

Obviously this can not go on indefinitely since S would then approach S_2 and eventually the stability of the mixed layer would be destroyed. We conclude that either F will change sign or the system will become unstable and overturn. We shall here explore the possibility that the development of the system changes into a state of net ice melt (F changes sign) and the mixed layer preserves stability even though the heat loss to the atmosphere continues with similar strength.

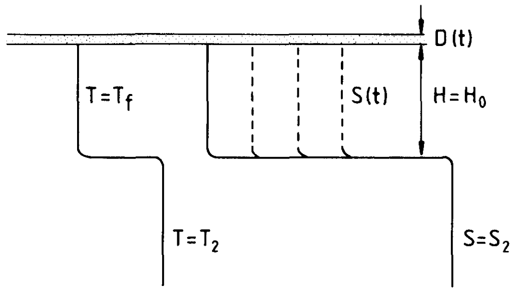

Fig. 2. The idealized initial ice formation. The temperature and the depth of the mixed layer stay constant while the salinity increases along with the ice formation.

3.2. The freeze melting regime

When S has grown large enough to make $\Delta\rho$ small, the entrainment terms in eqs. (2) become of primary importance. We assume that $\Delta\rho$ does not become negative which implies that the rate of change of S has to vanish or at least become very small. To idealize this situation we thus put $\mathrm{d}/\mathrm{d}t(S) = 0$ in eq. (2c) and obtain from eqs. (2c, d, e):

$$0 = F(S - S_i) + W_e(S_2 - S), \tag{4a}$$

$$0 = W_e\rho c\,\Delta T + FL\rho - Q_a. \tag{4b}$$

Since $W_e > 0$ the balance expressed by eq. (4a) implies that the icecover is melting ($F < 0$) at a rate large enough to keep the top layer salinity constant despite the ongoing entrainment of more saline water. Furthermore eq. (4b) implies that the heat supplied by entrainment is large enough to cover the heat loss to the atmosphere as well as the heat required for ice melting.

We do not a priori know that such a "freeze-melting" stage of developement is possible, i.e., if eqs. (4) are compatible with the constraints posed by the physics of our model, i.e., that $W_e \geqslant 0$ and $\Delta\rho > 0$. (The term "freeze melting" introduced here was criticized by one reviewer, who suggested "entrainment melting" as an alternative. The reason that the present author prefers "freeze melting" is that the rate of melting as well as the rate of entrainment is in fact determined by the rate of heat loss even though entrainment actually supplies the required heat.)

To proceed we eliminate W_e from eqs. (4) to obtain

$$FL\rho = -Q_a(1+\Sigma)/(R-1-\Sigma) \tag{5a}$$

where

$$R = \beta c(S-S_i)/(L\alpha), \qquad \Sigma = \Delta\rho/\Delta\rho_T. \tag{5b, c}$$

The new variable Σ replacing S as dependent variable in eq. (5a) must obviously satisfy the condition

$$\Sigma > 0. \tag{5d}$$

The physical constraint that $W_e \geqslant 0$ implies that eqs. (4) represent a physically possible development only if $F<0$. In view of eqs. (5) we thus obtain the condition

$$(R-1-\Sigma) > 0. \tag{6}$$

From this, we conclude that "freeze-melting" in the sense that the mixed layer salinity remains constant is possible only when $\Sigma < (R-1)$. Since we do not allow Σ to become negative we conclude that the existence of a freeze melting regime requires that

$$R > 1. \tag{7}$$

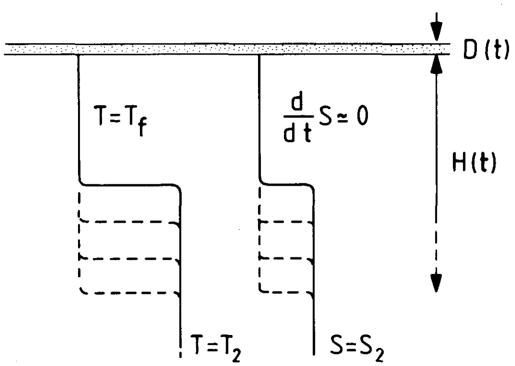

Fig. 3. Salinity and temperature structure in the freeze melting regime. The mixed layer depth, $H(t)$, increases in response to the heat loss to the atmosphere while mixed layer temperature and salinity remain essentially constant and the ice thickness slowly decreases.

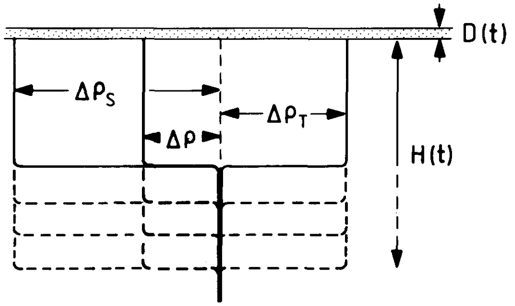

Fig. 4. Density structure in the freeze melting regime. The small density contrast $\Delta\rho$ adjusts to make entrainment suitably large to produce the heat required for heat loss to the atmosphere and the slow ice melt.

The magnitude of the thermodynamic parameter R is accordingly of crucial importance for the qualitative behaviour of the system. To estimate R we use the following values for the thermodynamic parameters for sea water and ice (Gill, 1982).

We have $S \approx 35$ psu, $S_i \approx 5$ psu, $\beta \approx 0.81 \cdot 10^{-3}$, $\alpha \approx 0.045 \cdot 10^{-3}$ (in the temperature range -2 to $+1$°C) $L \approx 250 \cdot 10^3$ Ws kg^{-1}, $c \approx 4.18 \cdot 10^3$ Ws kg^{-1} K^{-1}, which when inserted in eq. (5b) give

$$R \approx 8.8.$$

We thus find that R is considerably larger than unity, a fact of great importance, since it implies that the freeze melting stage of development is possible and that the system may protect itself from overturning at the end of the initial ice formation regime. The "freeze-melting" regime is illustrated in Figs. 3 and 4.

4. Development from "initial ice formation" to "freeze melting"

In Section 3, we found that a freeze melting regime characterized by essentially constant stability and slowly melting ice cover represents a possible state of the system. We now analyse if or how the system can change from the initial ice formation to the postulated state of freeze melting.

From eqs. (2c and d) we have

$$Q_1 = W_e \rho c\, \Delta T,$$

$$F(S-S_i) = -W_e(S_2-S) + H\, \mathrm{d}/\mathrm{d}t(S),$$

which may be entered into the expression for the buoyancy flux, eq. (2h), to give

$$B = gW_e(\Delta\rho/\rho) + gH\,\mathrm{d}/\mathrm{d}t(\Delta\rho/\rho).$$

This is introduced into the energy eq. (2f) to obtain

$$2m_0u_*^3 = gHW_e(1+\gamma)\,\Delta\rho/\rho + gH^2\gamma\,\mathrm{d}/\mathrm{d}t(\Delta\rho/\rho),$$

or

$$W_e = (1+\gamma)^{-1}\,\Sigma^{-1}(W_* - \gamma H\,\mathrm{d}/\mathrm{d}t\Sigma), \tag{8a}$$

where we have made use of the variable Σ previously defined by eq. (5c), and introduced W_*

$$W_* = 2m_0u_*^3\rho/(gH\,\Delta\rho_T) \tag{8b}$$

to represent the strength of the wind forced mixing. According to this form of the energy equation W_e becomes arbitrarily large when Σ tends to zero assuming only that W_* does not vanish altogether.

We now turn back to our continuity equations and eliminate Q_1 and F from eqs. (2c, d, e). We obtain

$$H\,\mathrm{d}/\mathrm{d}t\Sigma = W_e(R-1-\Sigma) - W_aR, \tag{9a}$$

where we have introduced W_a

$$W_a = Q_a(\Delta T\rho c)^{-1} \tag{9b}$$

to represent the rate of heat loss to the atmosphere.

Eqs. (8 and 9) impose certain important restrictions on the development of our system. Thus we find assuming that $W_a > 0$ (heat being lost to the atmosphere) and $W_* > 0$ (nonzero wind forced turbulence) that

(i) $\mathrm{d}/\mathrm{d}t\Sigma \leqslant 0$ when $\Sigma \geqslant R-1$,

i.e., continued cooling will always force Σ below the critical value $R-1$. Once below this value Σ can not climbe over it again:

(ii) $\mathrm{d}/\mathrm{d}t\Sigma > 0$ when $\Sigma \to 0$ with finite W_*,

i.e., the presence of some finite wind-forcing will guarantee that Σ remains positive even though the cooling continues.

Accordingly, we find that once the value of Σ has been forced below the value $R-1$ it will be trapped in the interval

$$0 < \Sigma < R-1. \tag{10}$$

The qualitative properties of the development imposed by eqs. (8 and 9) are illustrated in Fig. 5, where Σ has been plotted against the integrated heat loss $E(t)$ defined by

$$E(t) = \int_0^t Q_a\,\mathrm{d}t. \tag{11}$$

Eqs. (8 and 9) thus prove the main point of our analysis i.e., that the initial ice-formation does not lead to vanishing stability but rather to a new stage of development with the qualitative properties of the idealized "freeze-melting" outlined in Section 3.

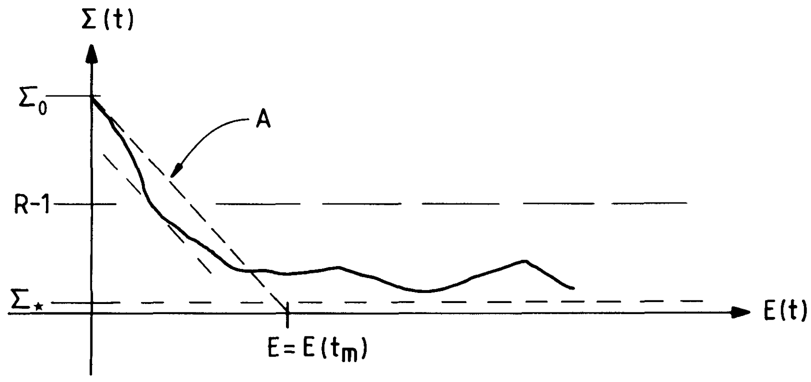

Fig. 5. Σ $(=\Delta\rho/\Delta\rho_T)$ decreases monotonically with $E(t)$ from an initial value Σ_0, until $\Sigma < R-1$, and becomes trapped in the interval $0 < \Sigma < R-1$. The dashed line A represents $\Sigma(t)$ up to the time t_m if the ice formation takes place without entrainment. Cabbeling might be important if $\Sigma \leqslant \Sigma_*$, as discussed in Section 7. For Weddell sea conditions $\Sigma_* \approx 0.4$ and $R-1 \approx 8$.

4.1. The asymptotic regime

It follows from eqs. (8 and 9) that an idealized development characterized by a pure ice formation stage followed "abruptly" by a pure freeze-melting stage corresponds to the case

$$0 < W_* \ll W_a. \tag{12a}$$

In this asymptotic case we have

$$W_e = 0 \qquad \text{for} \quad t < t_m \tag{12b}$$

$$\Sigma = 0, \qquad d/dt\Sigma = 0 \qquad \text{for} \quad t > t_m \tag{12c}$$

where t_m is the time for the shift from initial ice-formation to freeze-melting.

To avoid misunderstanding, we underline that a development such as described by eqs. (12) is asymptotic in the sense that it can be approached but never exactly realized since physical reality requires that the system retains at least some stability, i.e., Σ may be small but not vanish altogether.

Making use of eqs. (11 and 12) we easily obtain the following relations from eqs. (2) which describe the asymptotic development.

(i) For the initial ice formation when $t < t_m$ we have:

$$H(t) = H_0, \tag{13a}$$

$$L\rho\, D(t) = E(t), \tag{13b}$$

$$\rho c\, \Delta T H_0(\Sigma(t) - \Sigma_0) = -RE(t), \tag{13c}$$

where the initial conditions have been prescribed as

$$H(t=0) = H_0, \qquad \Sigma(t=0) = \Sigma_0,$$
$$D(t=0) = 0. \tag{13d}$$

The end of the initial ice formation (at $t = t_m$) occurs when $\Sigma = 0$, i.e., when

$$E(t_m) = \rho c\, \Delta T H_0 \Sigma_0 / R, \tag{13d}$$

and t_m is determined implicitly from eq. (11).

(ii) For the freeze-melting regime when $t > t_m$, we have:

$$\rho c\, \Delta T(H(t) - H_0)$$
$$= (E(t) - E(t_m))\, R/(R-1), \tag{14a}$$

$$L\rho(D(t) - D(t_m))$$
$$= -(E(t) - E(t_m))/(R-1), \tag{14b}$$

$$\Sigma(t) = 0. \tag{14c}$$

If the heat loss to the atmosphere continues sufficiently long the freeze melting process will eliminate the entire ice cover. Denoting the time when this happens by t_c we obtain from eqs. (13b and 14b)

$$E(t_c) = RE(t_m). \tag{15b}$$

In the derivation of eqs. (13 and 14), we have neglected the precipitation term in eq. (2a) and used that $D \ll H$.

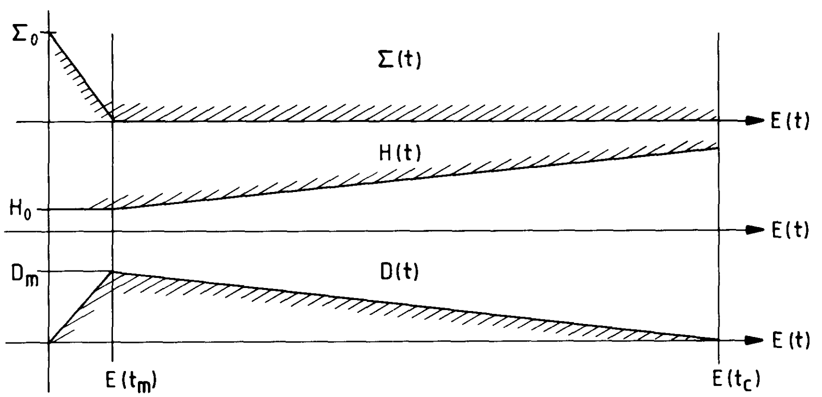

Fig. 6. The development of Σ, H, and D as functions of the integrated heat loss $E(t)$. The solid curves represent the asymptotic development. Possible real developments can be expected to fall on the shaded areas up to the maximum value of $E(t)$ that is reached during the cooling season.

The development given by eqs. (13 and 14) is illustrated in Fig. 6. Note that the asymptotic development is independent of the wind-forcing, i.e., determined completely by the heat flux $E(t)$ and appropriate initial conditions for H, Σ, and D.

4.2. Deviations from the asymptotic regime

The asymptotic regime represents the case with minimum entrainment, i.e., minimum mixed layer depth, and maximum ice cover that is possible with a prescribed rate of heat loss and certain fixed initial conditions. The state of the real system may be described in terms of the deviation from the asymptotic state. From eqs. (2) we easily obtain:

$$\rho c\,\Delta T(H-H_{\mathrm{a}}) = -L\rho(D-D_{\mathrm{a}}), \tag{15}$$

$$H(S_2-S)-H_{\mathrm{a}}(S_2-S_{\mathrm{a}}) = -(S-S_{\mathrm{i}})(D-D_{\mathrm{a}}), \tag{16}$$

where (S, H, D) and $(S_{\mathrm{a}}, H_{\mathrm{a}}, D_{\mathrm{a}})$ represent two different developments corresponding to the same forcing $E(t)$ and the same initial conditions but for different windforcing. One or the other, e.g., the development $(S_{\mathrm{a}}, H_{\mathrm{a}}, D_{\mathrm{a}})$ may be identified with the asymptotic development corresponding to sufficiently weak windforcing.

If $E(t)$ and the initial conditions are given eqs. (15, 16) make it possible to calculate anyone of the variables (H, D, S) if the deviation in one of these variables from its asymptotic counterpart is given.

A substantial deviation from the asymptotic development occurs as a result of strong winds making W_* comparable to W_{a}. However since W_* decays with increasing mixed layer depth as described by eq. (8b) we conclude that such deviations will become smaller as the mixed layer depth increases.

5. On the influence of deep water stratification

The discussion above has been based on the assumption that the deep water is completely homogeneous. This is a strong assumption and it is therefore of interest to estimate how the conclusions are modified by the presence of a modest stratification in the deep water.

There is in principle no problem to allow S_2 and T_2 in our model equations to be functions of a vertical coordinate z. The essential influence of the deep water stratification is to decrease the rate of melting in the freeze-melting process. Downward increasing salinity and decreasing temperature thereby cooperates. It is thus found that if the deep water stratification satisfies the condition:

$$H\,\mathrm{d}/\mathrm{d}z(\rho_2(z)) \geqslant \beta\rho(S_2(z)-S) \quad \text{at} \quad z=H, \tag{17}$$

where $\rho_2(z)$ is the deep water density, then continued cooling and entrainment does not require melting of the ice cover to retain a certain constant degree of stability of the mixed layer. Note that $\beta\rho(S_2(z)-S)$ is the density contrast due to salinity chosen by the system. The right hand side of eq. (17) is thus influenced e.g., by the intensity of the windforcing. Stronger windforcing will create a somewhat stronger stabilizing salinity contrast and thereby increasing somewhat the deep water stability needed to eliminate the freeze melting.

The minimum value for the deep water stratification as obtained from eq. (17) will of course correspond to the marginally stable case when $\Sigma=0$ or $\beta(S_2-S)=\alpha\,\Delta T$.

6. On the significance of the parameter *R* for the formation of a protective layer of melt water

To illustrate the general significance of the parameter R defined by eq. (5b) in Section 3 we consider a simple special case. It is well known that melting sea ice in general forms a protective layer of cold water underneath itself. See, e.g., Gade (1979) for a comprehensive discussion. The basic reason that such a mechanism operates is of course that the melt water (or rather the mixture of melt water and sea water) is buoyant relative to the underlying deep water. Let us now look at a simple case to see why (or if) this is really the case.

We consider an ice sheet with salinity S_{i} which initially is located on top of homogeneous water of temperature T_2 and salinity S_2. We assume that $T_2 > T_{\mathrm{f}}$, T_{f} being the freezing point of the sea water.

We now assume that at some later time a mixed layer with temperature T_{f}, salinity S_1, and thickness H has formed underneath the ice sheet as illustrated in Fig. 7. We assume further that the heat released by the cooling of the mixed layer

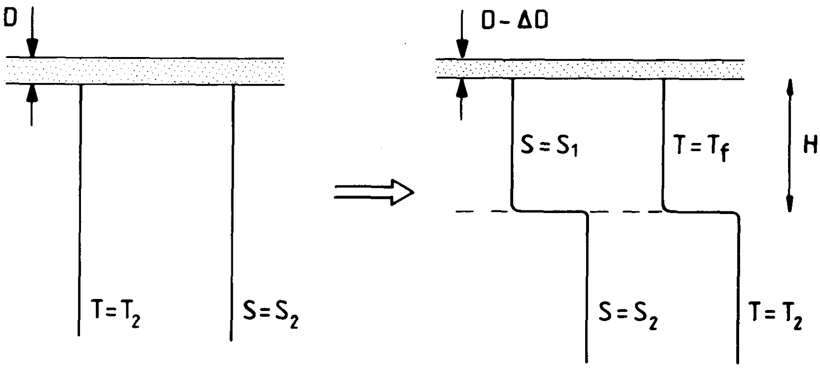

Fig. 7. An ice sheet is assumed to be placed on top of a homogeneous body of sea water, as discussed in Section 6. Turbulence generated by the ice sheet creates a mixed layer underneath the ice which is buoyant if $R > 1$, where R is the thermodynamic parameter defined by eq. (5b).

from T_2 to T_f is used entirely for the melting of ice. From continuity of volume, salt, and heat we obtain:

$$H = H_0 + \Delta D, \tag{18a}$$

$$HS_1 = H_0 S_2 + \Delta D S_i, \tag{18b}$$

$$\Delta D L \rho = \rho c H_0 (T_2 - T_f), \tag{18c}$$

where ΔD and H_0 are the thicknesses of the layers of ice and deep water consumed to form the mixed layer of thickness H (ΔD being the "equivalent" thickness as defined in Section 2).

From eqs. (18), we easily obtain:

$$(S_2 - S_1) = (S_1 - S_i)(T_2 - T_f)\, c/L. \tag{19}$$

Making use of this equation to eliminate $(S_2 - S_1)$ from the expression for the density contrast as given by eq. (2g) we obtain.

$$\Delta \rho = \rho \alpha (T_2 - T_f)(R - 1), \tag{20}$$

where R is the parameter defined by eq. (5b) and $\Delta \rho$ is defined so that $\Delta \rho > 0$ implies that the mixed layer water is lighter than the deep water.

We thus find that the mixed layer formed underneath the melting ice is buoyant relative to the deep water if and only if $R > 1$, which is the same condition as we found in Section 3 making the "freeze melting" regime possible. Accordingly the capability of the system to form a buoyant meltwater layer, protecting the ice cover from rapid melting, is related to the size of the parameter R.

It may be pointed out that the above given conclusion does not depend on a linearization of the equation of state. Accordingly the analysis in this and previous sections remains valid if the coefficient α in eq. (20) and in the definition of R represents the mean value of the thermal expansion coefficient over the interval $(T_f - T_2)$. In view of eq. (5b), defining R, and the well known fact that α increases with temperature we thus find that R will decrease with T_2. It does however take very high temperatures to make $R < 1$. For, e.g., $T_2 = 20°C$ we have $R \approx 3$, which still provides a good margin for the stability criterion. (We should add here that the nonlinear equation of state may create other problems as dicussed below in Section 7.)

We may also note that the formation of a stable melt water layer depends on the presence of some turbulent mixing generated underneath the ice. If molecular diffusion processes come into play the behaviour of the system becomes more complicated (Gade, 1979).

7. On the influence of cabbeling and diffusion in the pycnocline

In the above given analysis mixing has been described as a pure entrainment process, entraining deep water into the mixed layer, through a sharp pycnocline. We have thus neglected other types of mixing or diffusion which may give rise to heat and salt fluxes and modify the vertical structure. In particular we have not considered the possible influence of so called cabbeling arising from the non-linear equation of state. In this section we will give such phenomena some consideration and try to evaluate their importance.

It is well-known that the heat transport through a sharp interface may be quite large when the heat flow is driven from a lower to an upper layer (Turner, 1973). The justification to ignore such fluxes relies on the assumption that the pycnocline is either smoothed out or broken up in many steps by some sort of turbulent activity. It may appear that we then are trapped in a contradiction with our model which involves a sharp pycnocline. It is believed however that this is not serious since the thickness of the pycnocline may still be small compared to the mixed layer thickness.

The possible importance of cabbeling in Antarctic waters was pointed out by Foster (1972) and Foster and Carmack (1976) on the basis of observations showing steps in the hydrographic structure very close to neutral stability. The possible importance of the nonlinear equation of state was also emphasized by Fofonoff (1956) in a discussion of the relation between the properties of the Antarctic bottom and deep water.

The cabbeling phenomenon may be discussed in terms of the critical salinities S_0 and S_* defined by:

$$\rho(S_0, T_f) = \rho(S_2, T_2), \tag{21a}$$

$$(T_2 - T_f) = (S_2 - S_*)(\mathrm{d}T/\mathrm{d}S)_2, \tag{21b}$$

where $(\mathrm{d}T/\mathrm{d}S)_2$ represents $\mathrm{d}T/\mathrm{d}S$ at constant ρ evaluated in the point (S_2, T_2). With this definition S_* becomes the limiting salinity for cabbeling in the sense that ordinary mixing along a straight mixing line will produce a density maximum in the pycnocline if $S > S_*$.

Table 1. *Values of* $(S_2 - S_0)$, $(S_2 - S_*)$ *and* Σ_* *for different* T_2

T_2 (K)	$S_2 - S_0$ (psu)	$S_2 - S_*$ (psu)	Σ_*
0	0.10	0.13	0.3
1	0.17	0.24	0.4
2	0.26	0.38	0.5

Σ_* is the value of Σ for $S = S_*$ while S_0 and S_* are defined by eqs. (21).

Making use of data tabulated by Gill (1982) we obtain the numbers given in Table 1.

From Table 1, we see that there is a small but finite range for the mixed layer salinity, S, in which $S_* < S < S_0$ allowing for cabbeling to occur. In this regime we might expect the system to be vulnerable since mixing will create somewhat heavier water parcels expected to fall out of the pycnocline. One could speculate that such a process could be self amplifying and trigger a fast erosion of the mixed layer from below. As discussed below there is presently no evidence of such dramatic effects of cabbeling.

In the process of freeze melting outlined in this paper the salinity of the top layer may be pushed very close to S_0 in order to give room for the entrainment needed to keep the system stable. We thus run a risque that the mixed layer salinity at least on occasions may pass beyond S_* giving room for the cabbeling instability to operate. Accordingly it is of considerable interest to know how efficient the cabbeling process may be.

The behaviour of pycnoclines of the kind considered in this paper has been studied by McDougall (1981a, b) in great detail. McDougall performed laboratory experiments running well into the regime corresponding to $S_* < S < S_0$. Somewhat surprisingly McDougall (1981b) finds that the system does not undergo any distinct change when the system goes into the "cabbeling" regime. The fluxes through the interface remain of the same order of magnitude as expected without cabbeling i.e., with a linear equation of state. The essential effect of cabbeling was a slow upwards migration of the interface. In particular McDougall (1981b) finds no sign of a self-amplifying catastrophic development.

In a recent paper, Rudels (1991) has made quite elaborate theoretical calculations of the convective processes produced by combined double-diffusion and cabbeling with special reference to the conditions prevailing in the Antarctic ocean. The results obtained by Rudels (1991) indicates that the cross-isopycnal heat flux as well as the migration velocity of the pycnocline is of minor importance if the pycnocline is broken up in many small steps. Such step structures have been observed, e.g., by Foster and Carmack (1976).

From the work of McDougalls (1981a, b) and Rudels (1991), one is tempted to believe that cabbeling is of minor importance for our system. Some uncertainty remains however since neither author has been able to take into account the presence of turbulence of independent origin which could possibly trigger more dramatic effects.

For the time being, it appears that the most important requirement for the validity of the present model is that the pycnocline has a certain thickness, possibly broken up in small steps, since a pycnocline with a single sharp interface will give rise to an important diffusive heat flux through the pycnocline.

7.1. Inferences from hydrographic observations

In our model of freeze melting, it is important that the stability can attain small values without collapse of the mixed layer in order to trigger enough entrainment. We would thus expect to find at least some observations of mixed layer salinity in the range $S_* < S < S_0$. Such observations may exist but we have not found any solid evidence. In fact already Fofonoff (1956) claimed that observations indicate that the surface water salinity does not exceed S_*. (Fofonoff gave the estimate 34.51 psu for S_*.) Furthermore, there are winter observations showing S very close to S_*, see, e.g., the station from Maud Rise shown in Gordon and Huber (1990) or Fig. 9 in this paper. These observations could be interpreted as an indication that cabbeling is somehow important in the range of very small stability of the mixed layer.

Against this background, we may speculate that some effect related to cabbeling, e.g., enhanced heat flux, protects S from increasing above S_*. Σ would then be confined in the somewhat smaller interval (compare eq. (10) and Fig. 5).

$$\Sigma_* < \Sigma < R - 1,$$

where Σ_* is the value of Σ corresponding to $S = S_*$. We note that this interval is only marginally smaller than the interval obtained without consideration of cabbeling since $\Sigma_* \approx 0.4$ compared with $R - 1 \approx 8$.

In summary of this section, we have to admit that there remains some uncertainty concerning the behaviour of our system in the limit of extremely small stability, e.g., concerning the importance of cabbeling and enhanced diffusion through the pycnocline. For the time being we do not believe however that such processes will qualitatively effect the behaviour of the system.

8. Conclusions and implications for the Antarctic sea ice

We have analysed the development of the ice mixed layer system when the stability is threatened by ice formation. We have found that the stability of the mixed layer will settle at a small but finite value. Accordingly the system will not overturn after the initial ice formation and associated reduction of stability but go into a different regime, "freeze melting", characterized by a slowly melting ice cover and a deepening mixed layer. The buoyancy required to protect from overturning is provided by the melting ice. Even if the rate of heat loss to the atmosphere remains essentially unchanged, the rate of ice melting in this second stage of developement is typically much slower (by a factor of 5–10) than the initial rate of ice growth.

We thus conclude that the initial ice formation is followed by a prolonged melting regime rather than by overturning and abrupt melting, as assumed by Martinson et al. (1981). This conclusion depends on the size of the thermodynamic parameter R (see eq. (5b)). It so happens that the relative magnitude of the various properties of ocean water and ice are such that R is considerably greater than 1 which is the basis for the above given conclusion. This fact appears to be fundamentally important for the behaviour of sea ice in a wider context than considered here. As an example an ice sheet floating on top of moderately warm sea water will produce an insulating layer of melt water which considerably slows down further melting. Naturally this mechanism depends on the melt water being lighter than the warm ocean water which, as shown in Section 6, is a direct consequence of R being larger than 1.

The discussion in this paper relies on the assumption that molecular diffusion of salt and heat may be disregarded. This is not always the case which may be of significant importance for the dynamics of ice-melting as discussed by Gade (1979). The possibility that cabbeling could be important, though disregarded in our analyses, has been given some consideration in Section 7.

8.1. Implications for the Antarctic sea ice development

In the idealized development as described in Subsection 4.1, the time t_m represents the end of the initial ice formation while t_c represents the end of the freeze melting stage of development assuming that the cooling continues for a sufficiently long time.

The potential relevance of the analysis presented in Sections 2–5 depends on how the times t_m and t_c compares with the length (T_c) of the cooling season. We may summarize as follows:

(i) $t_m \geqslant T_c$ implies that the initial ice formation regime continues over the entire cooling season;

(ii) $t_m \leqslant T_c \leqslant t_c$ implies that the initial ice formation has time enough to finish and switch over to the freeze melting regime; the time T_c is however not long enough to allow the ice cover to be eliminated by the freeze melting;

(iii) $t_c \leqslant T_c$ implies that the ice cover is eliminated before the end of the winter; in this case we expect a deeply convecting polynya towards the end of the winters.

To calculate t_m and t_c, we need estimates of the heat flux to the atmosphere. During the (Austral) winter 1986 the research vessel Polarstern crossed the Weddell sea along the Greenwich meridian. Important observations concerning the mixed layer and ice conditions, which were taken during the cruise has been reported by Gordon and Huber (1990).

They found that the ice cover had an approximate thickness of 0.5 m towards the end of the winter. They also report that the heat flux from the deep water appears to be in approximate balance with the heat loss to the atmosphere. The mean winter time heat loss was estimated to be 41 W/m^2.

These observations are in harmony with a state of freeze melting as described in Section 3. In a freeze melting stage of development the main heat flux is from the deep water to the atmosphere since the heat required for the slow ice melting is relatively small compared with the heat loss to the atmosphere as described by eqs. (14). Given a relatively small initial fresh water content in the stratification we also expect the ice thickness in the freeze melting regime to be modest.

Gordon and Huber (1990) refer to the entrainment process as the agent that limits the ice thickness to the observed 0.5 m. Though this is evidently true we would like to propose the more complete explanation: The ice thickness is limited by the amount of available freshwater while the rate of entrainment is controlled through internal feed back, in such a way that the oceanic heat flux supplies the heat required by the atmosphere as well as the slow ice melting.

Estimates of the heat loss to the atmosphere was also given by Martinson et al. (1981) in connection with their analysis of the polynya phenomenon. It appears that the winter time heat loss varies between 200 W/m^2 (or even higher) for ice free conditions down to 20 W/m^2 with an ice cover 0.5–1 m thick. The lower number may be an underestimate since the ice cover is reported to be full of cracks and leads.

To estimate the time t_m required for the initial ice formation, we assume that the fresh water content is 1.4 m initially distributed over a 50 m thick mixed layer. We then obtain $\Sigma_0 \approx 5.8$ from the definition of Σ given in eq. (5c). This number is introduced in eqs. (13) to calculate $E(t_m)$. Assuming $\Delta T \approx 2.3$°C we obtain $E(t_m) \approx 2.9 \times 10^8$ Ws/m^2.

The heat loss will of course decrease during the ice formation because of the insulating effect of the ice cover and we estimate the mean value of Q during the time t_m to be 80 W/m^2. With these numbers we obtain $t_m \approx 42$ days. Assuming that the same mean heat flux also characterizes the freeze melting period up to complete ice removal we obtain $t_c \approx 370$ days, i.e., considerably longer than the length of the cooling season. These numbers, though based on very rough estimates, still demonstrate that we can expect the intial ice formation to be finished well before the end of the winter. On the other hand we do not normally expect the freeze melting to come to an end during the cooling season, i.e., we are normally in the regime $t_m < T_c < t_c$.

The expected qualitative features of the development of the Antarctic ice cover is illustrated in Fig. 8, where the ice thickness is plotted against time rather than against the integrated heat loss, $E(t)$. The shape of the asymptotic curve in this representation will accordingly depend, e.g., on the relation between heat loss and ice thickness.

The results presented above and illustrated in Fig. 8 are sensitive to the amount of fresh water initially available. In fact we find that the crucial times t_c and t_m vary essentially as the square of the amount of fresh water, H_{f0}, initially available in the mixed layer. This becomes obvious if we recognize that the times t_m and t_c increases with H_{f0} not only because a thicker ice cover, associated with larger H_{f0} requires a larger integrated

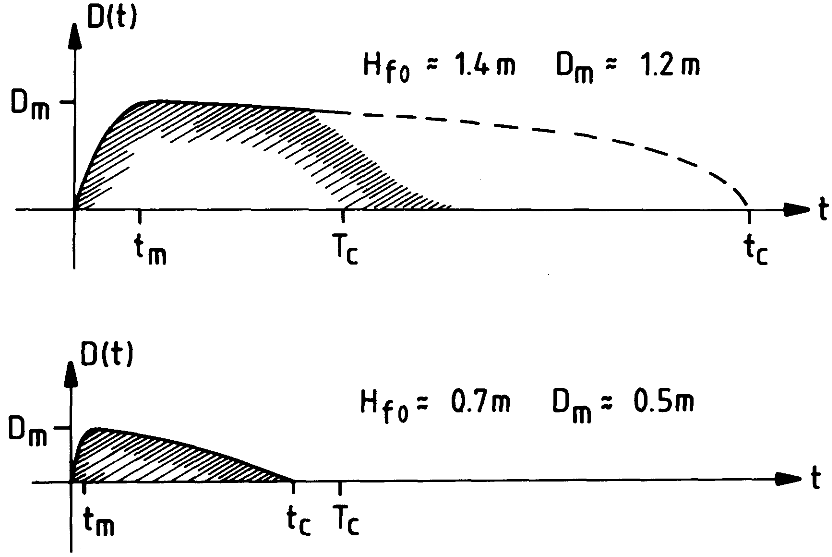

Fig. 8. Ice thickness $D(t)$ qualitatively illustrated. Solid curves represent the asymptotic development giving an upper bound for $D(t)$. T_c represents the end of the cooling season, and t_c the time for collapse of the mixed layer if the cooling season is long enough. Reducing H_{f0} may drastically reduce the time t_c.

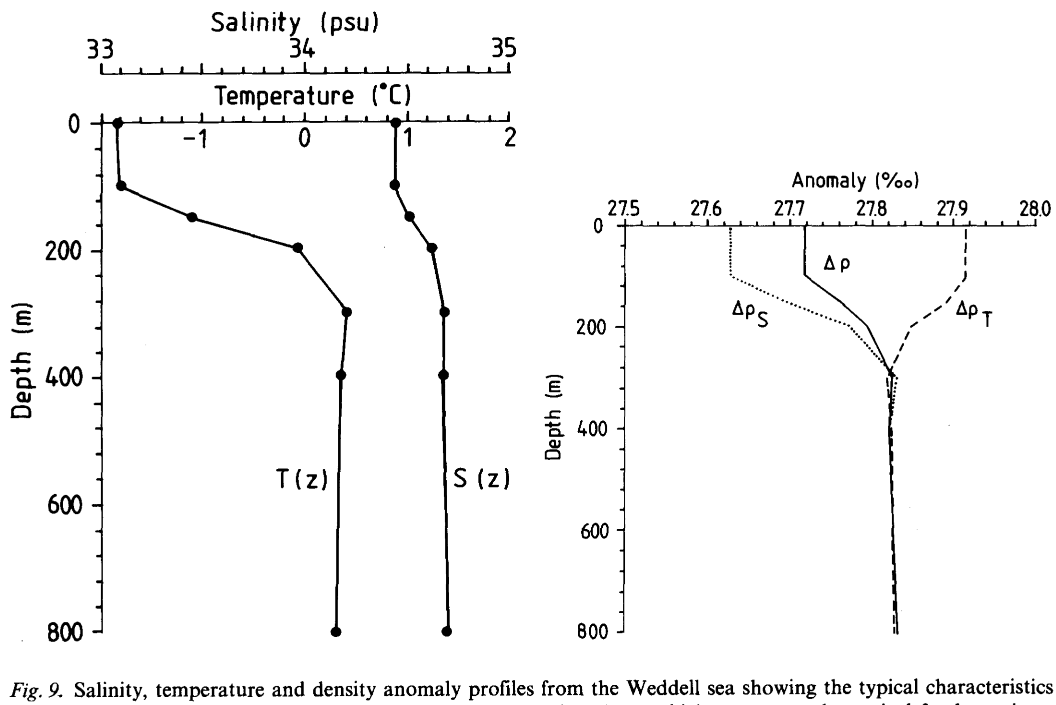

Fig. 9. Salinity, temperature and density anomaly profiles from the Weddell sea showing the typical characteristics of freeze melting (compare Fig. 4). In this case $\Delta\rho \approx \Delta\rho_T$, i.e., $\Sigma \approx 1$ which appears to be typical for late winter conditions. From the NODC data bank, Nov. 1912. 64.3° S, 36.5° W.

heat flux, but also since the rate of heat loss to the atmosphere decreases with increasing ice thickness.

Observations from the Weddell sea taken from the NODC data base consistently show a fresh water content in the upper layer in the range 1–2 m. This suggests that at least this part of the Southern ocean will in late winter be in a state showing the qualitative features of the freeze melting regime. An important characteristic of this state is its stability in the sense that internal feed back between ice melt and the rate of entrainment controls the stability of the mixed layer and the mean thickness of the ice cover. Pronounced short term variability of the ice cover which is typical for the area does not contradict this predicted feature. On the contrary it could be thanks to this basic stability that the system can keep up with the violent forcing and associated perturbations with reasonably stable mean conditions.

8.2. *On the formation of deeply convecting polynyas*

We suggest the following scenario for the occasional appearance of major polynyas like the one observed 1974–1976 (Martinson et al., 1981). Under normal conditions we do not expect the freeze melting period during late winter to be able to remove the ice cover altogether. However the margin is not large in view of the sensitivity of the length of the freeze melting period to the magnitude of H_{f0}. Assume that the water column in the fall contains an unusually small amount of fresh water, e.g., as a result of wind driven divergence of the surface water as discussed by Martinson et al. (1981). Furthermore assume that the following winter is unusually cold and dry. Then we cannot exclude the possibility that the ice cover is removed before the end of the winter. If so, the water column will overturn and the fresh water in the mixed layer will get lost.

Presumably such an event will not be noticed as a polynya when the overturning takes place since it might happen late in the cooling season. The influence on the next year situation may however be dramatic since the mixed layer will contain only the fresh water supplied by precipitation and lateral advection during the last summer. This could mean that the fresh water content could be well below 0.5 m leading to a very short ice covered period or none at all.

Some further evidence in favour of our suggestion that the late winter conditions in the Weddell sea may be described as a freeze melting stage can be found by looking at hydrographic stations taken during late winter or early spring. Fig. 9 shows one such station obtained from the NODC data base which may be compared with the postulated behaviour shown in Figs. 3 and 4.

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