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Baroclinic instability and induced air-sea heat exchange

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ABSTRACT

A very simplified model of baroclinic waves in a saturated environment, in the presence of heat fluxes at the bottom boundary is studied, along the lines of Emanuel et al. The model uses the linearized geostrophic momentum approximation and two levels in the vertical, and it is solved separately in the updraft and downdraft regions, allowing for different static stabilities. The use of a slantwise convectively neutral updraft constitutes a parameterization of upright and slantwise convective processes. The boundary fluxes of heat are represented in the form of a linearized draw law for equivalent potential temperature. The results, which include faster growth, higher translational speed and destabilization of shorter scales of motion, are discussed in relation to a previous numerical study of mid-latitude explosive cyclones. A linear dependence of deepening rates on heating rate is also obtained and related to the "normal mode" character of the solutions sought.

1. Introduction

In the recent paper by Emanuel et al. (1987) (hereafter referred to as EFT), a version of the classical 2-level model (see, e.g., Pedlosky, 1979) was proposed, which allowed the use of different values of the stability parameter in the regions of upward and downward vertical motion. The model was devised in order to simulate the behavior of baroclinic waves in a saturated environment, in which all upward motions lead to condensation of water vapor and consequent release of latent heat. The assumption was made there that the large-scale effect of the condensation process was to reduce the effective static stability of the updraft to the point of conditional symmetric neutrality. This assumption was presented in detail in EFT, and in Emanuel (1988), and we will not discuss it here except by saying that this procedure constitutes a parameterization of the convective processes that cannot be explicitly included in the model. It is the purpose of this paper to examine the effect of adding a heat flux from the lower boundary to a linear version of the 2-level, 2-dimensional, semi-geostrophic model of EFT. This is intended to simulate the behavior of developing baroclinic waves over the ocean, in the environment characteristic of the "explosive cyclones" of Sanders and Gyakum (1980). A numerical study of the same matter was performed with a primitive-equation non-hydrostatic model (Fantini, 1990a) and we will here discuss the *qualitative* similarities in the results of the two models. We are stressing the word "qualitative" since a number of drastic simplifications are necessary in the present work. We believe nonetheless that conclusions of relevance to the problem can be drawn by comparison of the two approaches.

2. The model

2.1. The momentum equations

In the geostrophic momentum approximation (e.g., Hoskins, 1975) the momentum equations are written

$$\begin{cases}(\partial_t + u\partial_x + v\partial_y + w\partial_z)\,u_g - f_0 v_a = 0,\\ (\partial_t + u\partial_x + v\partial_y + w\partial_z)\,v_g + f_0 u_a = 0.\end{cases} \tag{1}$$

Consider a steady solution $\bar{u}(z)$, $\bar{\theta}(y,z)$ satisfying $f_0\bar{u}_z = -(g/\theta_0)\,\bar{\theta}_y$ and y-independent perturbations to this mean state (which implies $u'_g = 0$). The linearized equations for the perturbations are then

$$\begin{cases} \bar{u}_z w = f_0 v_a, \\ (\partial_t + \bar{u}\partial_x) v_g + f_0 u_a = 0. \end{cases} \tag{2}$$

Hydrostatics and continuity are written

$$\begin{cases} f_0 v_{gz} = (g/\theta_0)\,\theta_x \\ u_{ax} + w_z = 0. \end{cases} \tag{3}$$

We introduce a stream-function ψ such that $u_a = -\psi_z$ and $w = \psi_x$ and restrict ourselves to the two-level system, in which the meridional wind component v is known on two levels in the vertical, and w and θ only at mid-level (see Fig. 1). With this simplification equations (2) and (3) in non-dimensional form become

$$\begin{cases} (\sigma + (1 - u_0)\,\partial_x)\, v_1 + \psi = 0, \\ (\sigma - (1 + u_0)\,\partial_x)\, v_2 - \psi = 0, \\ \theta_x = v_1 - v_2. \end{cases} \tag{4}$$

The dimensional variables (now denoted by an asterisk) are:

$$x^* = Lx, \qquad \sigma^* = \frac{U}{L}\sigma, \qquad \psi^* = Uh\psi,$$

$$v^* = Lf_0 v, \qquad \theta^* = \frac{\theta_0}{F}\theta,$$

with $F = gh/(f_0^2 L^2)$ and L a length scale to be specified below. We have already assumed a time dependence $\exp(\sigma t)$ and set ourselves in a reference frame moving with the speed of the mid-level basic wind plus u_0. The reason for this choice will become clear later on.

2.2. *Thermodynamics*

It was shown in Emanuel (1986) that the relation

$$\Gamma_m (\mathrm{d} \ln \theta_e)_p = \Gamma_d (\mathrm{d} \ln \theta)_p \tag{5}$$

holds between the differentials at constant pressure of dry and moist entropy for a saturated parcel of air. Here θ is the potential temperature, θ_e the equivalent potential temperature, $\Gamma_d = -g/c_p$ the dry adiabatic lapse rate and Γ_m the moist pseudo-adiabatic lapse rate. In our system (5) can be approximated as

$$\frac{\Gamma_m}{\theta_{e0}} (\mathrm{d}\theta_e)_{z=\mathrm{const}} \simeq \frac{\Gamma_d}{\theta_0} (\mathrm{d}\theta)_{z=\mathrm{const}},$$

which allows the thermodynamic equation for saturated motion $\mathrm{d}\theta_e/\mathrm{d}t = \dot{Q}/c_p$ to be written

$$(\partial_t + u\partial_x + v\partial_y)\,\theta + w \frac{\Gamma_m}{\Gamma_d} \frac{\theta_0}{\theta_{e0}} \theta_{ez} = \frac{\Gamma_m}{\Gamma_d} \frac{\theta_0}{\theta_{e0}} \frac{\dot{Q}}{c_p}, \tag{6}$$

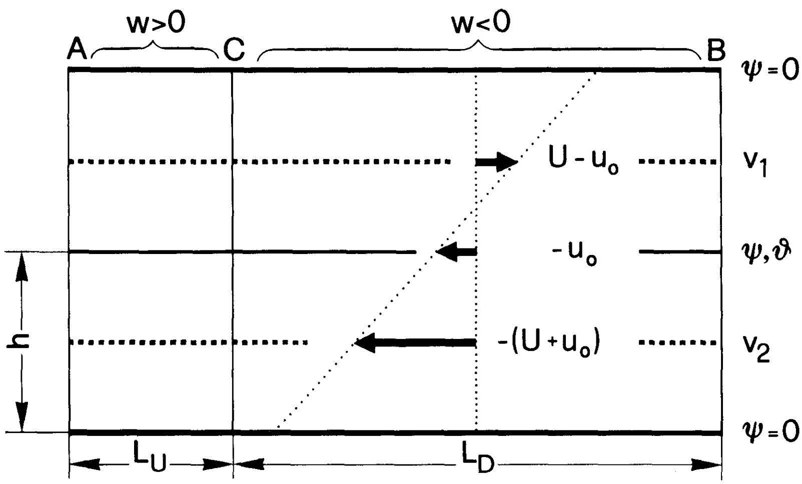

Fig. 1. Sketch of the model, showing the levels at which the variables v, ψ, θ are known, the basic wind profile, and the horizontal extent of updraft and downdraft regions. The equations are solved separately in the two regions and then matched at points A = B and C.

while the thermodynamic equation for unsaturated motion remains

$$(\partial_t + u\partial_x + v\partial_y + w\partial_z)\,\theta = \frac{\dot{Q}}{c_p}. \tag{7}$$

Here and above $\dot{Q}$ are all the sources of heat other than the release of latent heat of condensation which is already accounted for by the use of θ_e. Specifically we want to write it as a heat flux from the lower boundary. In the traditional "bulk aerodynamic" formulation the sensible plus latent heat flux from the sea surface (see, e.g., Ooyama, 1969) is written

$$F_0 = c_D\,|\mathbf{u}|^{\text{air}}\,c_p(\theta_e^{\text{sea}} - \theta_e^{\text{air}}), \tag{8}$$

where the superscript "sea" refers to air in contact with the sea surface, "air" to some distance (usually 10 m) above sea level and c_D is a (dimensionless) drag coefficient. Using Eq. (2) in (6) and (7), and linearizing we get:

$$(\partial_t + \bar{u}\partial_x)\,\theta + v_g\bar{\theta}_y + wq^* = H^*,$$

where

$$q^* = \frac{\theta_0}{g}\bar{q}_d = \frac{\bar{\theta}_y\bar{u}_z}{f_0} + \bar{\theta}_z,$$

when (7) is considered, and

$$q^* = \frac{\Gamma_m}{\Gamma_d}\frac{\theta_0}{\theta_{e0}}\left(\frac{\bar{\theta}_{ey}\bar{u}_z}{f_0} + \bar{\theta}_{ez}\right) = \frac{\Gamma_m}{\Gamma_d}\frac{\theta_0}{f_0}\bar{q}_e,$$

if (6) holds. H^* will be defined below with the aid of (8). As shown in Fig. 1, the thermodynamic equation is only needed at mid level; then we can disregard the vertical variability of Γ_m and consider q^* a piecewise constant function of x whose value is determined by the sign of the vertical velocity w. We will in fact use the form (6) whenever $w > 0$ and (7) if $w < 0$. The non-dimensional form of the thermodynamic equation is now

$$(\sigma - u_0\partial_x)\,\theta - (v_1 + v_2) + q\psi_x = H, \tag{9}$$

where $q = q^*/\bar{q}_d$ and the choice $L = (h/f_0)(g\bar{q}_d/f_0)^{1/2}$ has been made, which is the form of the Rossby radius of deformation in the GM approximation.

We now consider our (x, z) domain divided in two parts (see again Fig. 1): one (Region 1) where $w > 0$, and one (Region 2) with $w < 0$. Then

$$q = \begin{cases} 1 & \text{in region 1} \\ r < 1 & \text{in region 2.} \end{cases}$$

The use of $r \to 0$ constitutes the application of the slantwise convective adjustment hypothesis referred to in the Introduction. The problem formulated by (4) and (9) with $H = 0$ was solved in EFT (also see Fantini, 1990b for the linearized version). We now introduce the heating term H^* in a form derived from (8). Consider that the heat exchanged with the ocean will remain in the boundary layer unless some mechanism more efficient than turbulent diffusion can transport it upward: this role is played by convective activity in the updraft region and we here assume that no such mechanism is present in the downdraft. Since we know the potential temperature only at one point located in the middle troposphere we assume that only in the "moist" updraft region is the thermodynamic equation affected by diabatic heating. In the real world it is of course possible to have convection, dry or moist, anywhere if the latent and sensible heat input at the bottom boundary is large enough to create instability. For consistency reasons, we should therefore make sure that we are not in a range of parameters which would, in the real world, lead to widespread convection in the area of large-scale downdraft, which the model in unable to account for. This is a limitation on the size of the air-sea temperature difference. An obvious consideration in static conditions would be that the thermodynamic sounding in the downdraft region should not become unstable. However the imposed air-sea temperature difference is only used to force the fluxes, and does not immediately translate in air temperature. The behavior we wish to parameterize here is that shown in the numerical experiments of Fantini (1990a): the air in the boundary layer is slowly heated up and a circulation develops which advects warm air towards the updraft region, where it is lifted, convectively or otherwise, while the meridional wind advects cooler air in the downdraft regions, thus maintaining the air-sea temperature difference that drives the whole circulation. In Fantini (1990a) experiments were run with a maximum of 30 K air-sea potential temperature difference and they all showed this kind of circulation. As all the results shown in Section 3 are obtained for $\overline{\Delta\theta_e}$ lower than that value we are confident that this parameterization holds.

We further assume, consistently with the 2-level discretisation, that the heat received from the sea is uniformly distributed in the troposphere, i.e. that the heat flux decreases linearly from the value F_0 at $z=0$ to zero at the top of the model $z=2h$, which is taken to coincide with the tropopause. Then

$$H^* = \frac{c_D}{2h} |\mathbf{u}|^{\text{air}} (\theta_e^{\text{sea}} - \theta_e^{\text{air}}).$$

We now consider a base state with zero velocity at the surface and a constant air-sea equivalent temperature difference $\overline{\Delta\theta_e}$. We remark here that the whole model has been set up having in mind a situation in which boundary layer air is being heated, and not cooled, so we will limit our considerations to positive $\overline{\Delta\theta_e}$. The partition of $\overline{\Delta\theta_e}$ into a sensible and a latent heat component, which determines how much of the input of energy goes into warming the boundary layer and how much is instead saved for later release in the updraft, is here fixed by the formulation used for moist thermodynamics (eq. (5)) to whichever value is correct for saturated air at the temperature of the sea surface. Therefore, a situation in which the air in contact with the sea is sub-saturated and the fluxes have a larger component of latent heat, is only reproduced in those aspects which pertain to the total amount of heat supplied. Qualitatively, the updraft is reinforced by release of latent heat, so it could be argued that the saturated case is the one of least growth. Our parameter Γ_0 measures the heat received in mid-troposphere in the updraft, rather than that received in the boundary layer, so in a first approximation the sub-saturated case could be described by our results with a higher Γ_0.

The perturbation form of the diabatic heating term is

$$H^* = \frac{c_D}{2h} |\mathbf{u}'|_{z=0} \overline{\Delta\theta_e}.$$

We have written the equations of motion in moving coordinate frame but it is understood that in the system in which the sea is at rest the basic-state surface wind is zero so that the thermal forcing does not enter the equation for the basic state.

The term $|\mathbf{u}'|$ is not easily tractable unless it has just one component that does not change sign, in which case it becomes linear. Since this is the only case we can actually solve, and as we are here mostly interested in the qualitative changes that this form of diabatic heating can cause on the eigensolutions of a baroclinic instability problem, rather than in accuracy of details, we impose $|\mathbf{u}'| = v_2$. This approximation can only in part be justified on physical grounds, as $v_a = 0$ at the surface, being proportional to w (see eqs. (2)), and $u_g = 0$ by the 2D construction of the model; but u_a might be of the same order as v_g with the scaling that we are using. However, this heating is only active in the updraft, which is a fairly narrow region of high w and therefore, by continuity, a zero of u_a (relative to the storm). The consideration of the total wind should then add uniformly to the heat flux (the storm is moving relative to the sea) but not change its spatial structure. Since heat is applied at mid-level this term should be viewed as an integral measure of the heat received anywhere in the boundary layer, which is proportional to the wind (and represented in this linear normal mode study by the amplitude of any one of the wind components) and then advected towards the center of the storm. The spatial distribution of this heating within the updraft has then a relatively minor importance at this stage although one result of this model (to be discussed in the next section) is an increased westward phase speed, which is very likely a consequence of this choice, and would not appear with a different description of the heat fluxes.

We now introduce the diabatic heating in (9) and write the thermodynamic equation as

$$(\sigma - u_0 \partial_x)\,\theta - (v_1 + v_2) + q\psi_x - \Gamma v_2 = 0, \tag{10}$$

where

$$\Gamma = \begin{cases} 0 & \text{where} \quad \psi_x < 0, \\ \Gamma_0 \equiv \dfrac{1}{2} \dfrac{g}{f_0 U} \dfrac{c_D}{\theta_0} \dfrac{\Gamma_m}{\Gamma_d} \overline{\Delta\theta_e} & \text{where} \quad \psi_x > 0. \end{cases}$$

The constant Γ_0 is of order 0.1 times $\overline{\Delta\theta_e}$ for ordinary atmospheric values of the parameters.

An equation for ψ can now be derived from (10) and (4):

$$(1-u_0^2)\,q\psi_{xxxx}+2\sigma u_0 q\psi_{xxx}+(2(1+u_0^2)+\Gamma(1-u_0)-\sigma^2 q)\,\psi_{xx}+\sigma(\Gamma-4u_0)\,\psi_x+2\sigma^2\psi=0, \tag{11}$$

which is valid in the interior of the updraft and downdraft regions, with the respective values of q and Γ. The solutions are linear combinations of four exponentials, whose coefficients have to be determined by application of the matching conditions at the interfaces, which we will now derive.

The physical conditions are the continuity of the wind component across the interface, and of pressure. Since the interface is vertical the normal wind component is $u_a=-\psi_z$, which for the 2-level becomes the requirement

$$\psi|_{-\varepsilon}^{+\varepsilon}=0. \tag{12}$$

(We have here temporarily assumed an interface located at $x=0$ and an $\varepsilon\to 0$.) If we now integrate the momentum equations in (4) across the interface we get:

$$[\sigma p_1+(1-u_0)\,v_1]_{-\varepsilon}^{+\varepsilon}+\int_{-\varepsilon}^{+\varepsilon}\psi=0,$$

$$[\sigma p_2-(1+u_0)\,v_2]_{-\varepsilon}^{+\varepsilon}-\int_{-\varepsilon}^{+\varepsilon}\psi=0.$$

Then the requirements

$$p_1|_{-\varepsilon}^{+\varepsilon}=p_2|_{-\varepsilon}^{+\varepsilon}=0$$

can be expressed as

$$[(v_1-v_2)-u_0(v_1+v_2)]_{-\varepsilon}^{+\varepsilon}=0,$$

$$[(v_1+v_2)-u_0(v_1-v_2)]_{-\varepsilon}^{+\varepsilon}+2\int_{-\varepsilon}^{+\varepsilon}\psi=0. \tag{13}$$

Since the continuity of pressure cannot depend on the frame of reference we have to require

$$[v_1-v_2]_{-\varepsilon}^{+\varepsilon}=0 \tag{14}$$

$$[v_1+v_2]_{-\varepsilon}^{+\varepsilon}=0 \tag{15}$$

separately in order to satisfy the first of (13) for any u_0. Then the 2nd becomes just $\int_{-\varepsilon}^{+\varepsilon}\psi=0$ which we can express by integrating eq. (11) or, more simply get an equivalent condition from the thermodynamic equation (10):

$$[q\psi_x-\Gamma v_2]_{-\varepsilon}^{+\varepsilon}=0. \tag{16}$$

If we now express v_1 and v_2 in terms of ψ and its derivatives (from eqs. (4)) and set $\Gamma=0$ we can recover the conditions used in EFT.

We have until now reasoned as if the interfaces were located at the point where w changes sign, and this is indeed what we want to obtain, but we have yet to impose any condition to force the eigensolutions to do so. In EFT, where condition (16) was simply $[q\psi_x]_{-\varepsilon}^{+\varepsilon}=0$, it was easy to argue that this should be replaced by $\psi_x=0$ at both interfaces, since we wanted ψ_x to change sign and $q\psi_x$ to be continuous. In the present case the argument is a little more involved. Let

$$w^{(1)}(x_0)=w_{\mathrm{I}}\geqslant 0,$$

$$w^{(2)}(x_0)=w_{\mathrm{II}}\leqslant 0,$$

$$v_2^{(1)}(x_0)=v_{\mathrm{I}}\geqslant 0$$

at an interface located at $x=x_0$ (the superscript 1(2) indicates the eigensolution in the updraft (downdraft) region). Then condition (16) gives

$$rw_{\mathrm{I}}-\Gamma_0 v_{\mathrm{I}}=w_{\mathrm{II}}\leqslant 0.$$

So, we want $\Gamma_0 v_{\mathrm{I}}\geqslant rw_{\mathrm{I}}\geqslant 0$. We then introduce a factor δ such that:

$$rw_{\mathrm{I}}=\delta\Gamma_0 v_{\mathrm{I}}, \qquad w_{\mathrm{II}}=(\delta-1)\,\Gamma_0 v_{\mathrm{I}},$$

with $0\leqslant\delta\leqslant 1$. When $\delta=0$, then $w_{\mathrm{I}}=0$ and $w_{\mathrm{II}}<0$. When $\delta=1$ then $w_{\mathrm{I}}>0$ and $w_{\mathrm{II}}=0$. The discontinuity of w at the interface is given by

$$w_{\mathrm{I}}-w_{\mathrm{II}}=\left(\frac{\delta}{r}-\delta+1\right)\Gamma_0 v_{\mathrm{I}}.$$

We cannot say a priori what its value is, because v_{I} is part of the eigensolution, but we note that for $0<r\leqslant 1$ and $0\leqslant\delta\leqslant 1$ the term in parenthesis is always positive, so the vertical velocity will have a discontinuity at the interface unless $v_{\mathrm{I}}=0$.

We now replace condition (16) with the two separate conditions

$$r\psi_x^{(1)}[A, C] = \delta_{[A,C]}\Gamma_0 v_2^{(1)}[A, C], \tag{16i}$$

$$\psi_x^{(2)}[A, C] = (\delta_{[A,C]} - 1)\Gamma_0 v_2^{(1)}[A, C], \tag{16ii}$$

where δ is allowed to have different values δ_A, δ_C at the two interfaces A ($\equiv B$) and C of Fig. 1. We then have a total of 10 matching conditions ((12), (14), (15), (16i), (16ii) for each of the two interfaces) and 8 coefficients. The solubility condition of the 10 by 8 homogeneous system will determine three eigenvalues. We solve in the next section for the growth rate σ, the ratio of the width of updraft and downdraft regions λ, and the phase speed of the eigenmodes u_0. As we wrote the equations in a frame of reference with a mean wind $-u_0$ solutions with a real σ, i.e., non-traveling in this system, have a positive phase speed u_0 over the mid-level wind.

We regard as free parameters the width of the downdraft region L_D, the values r and Γ_0 which appear in eq. (11) when written for the updraft region, and the constants δ_A, δ_C. The latter two are perfectly arbitrary as long as they are between 0 and 1. In the results presented in the next section the most unstable solution is always the one with $\delta_A = \delta_C = 0$ and the least unstable the one with $\delta_A = \delta_C = 1$. Since the width of the updraft changes when the δ's change we can think of these parameters as arbitrarily choosing the point where the matching is performed, within an interval of allowed widths determined by the other, physical, conditions. In a work by Lilly (1960) a similar technique was applied to the study of convection and the same problem arose. The growth rate was there found after arbitrarily picking the width of the updraft.

3. Results and discussion

Fig. 2 shows the behavior of the three eigenvalues σ, λ, u_0 when the heating parameter Γ_0 is increased, at fixed downdraft width $L_D = 3.6$ and $r = 0.1$. The value of L_D was chosen as close to the most unstable throughout the parameter range discussed here and in the previous papers. The value of r was chosen as representative of the behavior in the neighborhood of the singularity both for the present model and for the primitive-equation case (see Fantini, 1990b). For all three eigenvalues, the spread due to the choice of δ_A and δ_C is shown. At fixed Γ_0, we always find the largest growth rate, the largest width of the updraft and the largest value of phase speed obtainable by changing δ_A and δ_C associated with $\delta_A = \delta_C = 0$.

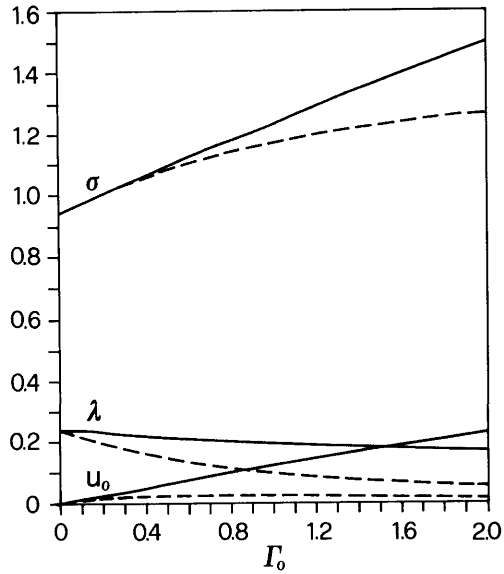

Fig. 2. Eigenvalues σ, λ, u_0 of the SG two-level model with heat flux at the lower boundary, versus heating parameter Γ_0, for $r = 0.1$, $L_D = 3.6$. Solid lines are the most unstable solution ($\delta_A = \delta_C = 0$). Dashed lines are the least unstable case ($\delta_A = \delta_C = 1$).

The results show an increase of growth rate σ and phase speed u_0 with the heating parameter Γ_0. So far as the phase speed is concerned the result could be expected as the thermal forcing was here assumed proportional to v_2 which is maximum slightly ahead of the perturbation in the θ field. By construction, one level is representative of the thermodynamics of the whole troposphere, so the simulated wave cannot change phase with height and adjust differently to this thermal forcing; it can only be translated solidly ahead of its unforced position.

Fig. 3 shows the three eigenvalues, for selected values of Γ_0, versus the width of the downdraft L_D. Only the "most unstable" situation $\delta_A = \delta_C = 0$ is considered from now on. The dry case

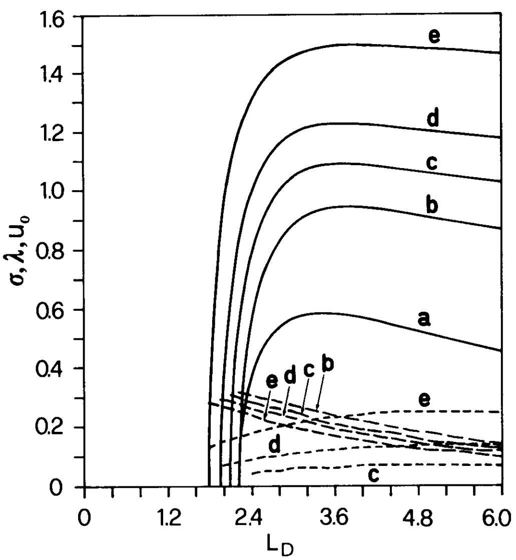

Fig. 3. Eigenvalues σ, λ, u_0 versus L_D for various values of the heating parameter Γ_0 for the most unstable case ($\delta_A = \delta_C = 0$). Solid lines: growth rate σ. Long dash: ratio $\lambda = L_U/L_D$ of the width of updraft to downdraft regions. Short dash: relative phase speed u_0. The labels identify the environmental parameters as follows: (a) $r = 1$, $\Gamma_0 = 0$; dry case; (b) $r = 0.1$, $\Gamma_0 = 0$; EFT case; (c) $r = 0.1$, $\Gamma_0 = 0.5$; (d) $r = 0.1$, $\Gamma_0 = 1$; (e) $r = 0.1$, $\Gamma_0 = 2$.

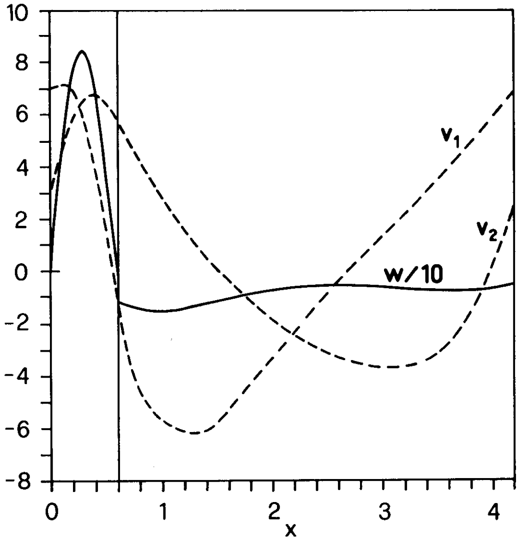

Fig. 4. Normalized eigensolution for $L_D = 3.6$, $r = 0.1$, $\Gamma_0 = 2$, $\delta_A = \delta_C = 0$. Eigenvalues for this mode are $\sigma = 1.5$, $\lambda = 0.17$, $u_0 = 0.23$. Only vertical velocity w and meridional geostrophic wind v are shown.

$r = 1$ is also shown for comparison. The curves of σ have the same general form detected in EFT but reach higher values; the short wave cut-off is moved to shorter waves, and, since an increase of Γ_0 also causes a slight decrease of the updraft width, the destabilisation of short waves is more effective than it might appear from the σ curves alone. The phase speed appears to increase with increasing Γ_0. The form of the eigensolution for $L_D = 3.6$ on curve (e) of Fig. 3 is shown in Fig. 4. Notice in particular the following.

(i) The discontinuity in w at the interfaces. As discussed in Section 2, it is not possible to avoid this discontinuity in the present model.

(ii) The double minima of w in the downdraft, suggestive of a return flow concentrated at the edges of the updraft, rather than being spread out uniformly over the whole region.

(iii) The degree of asymmetry introduced between the meridional wind components at upper and lower levels, stronger than in any of the results with no heat input, which can be ascribed to stretching (shrinking) of vortex tubes at lower (upper) level, by the new, stronger, updraft.

Finally Fig. 5 is a contour plot showing lines of constant growth rate σ on the (Γ_0, r) plane, which exhibit a remarkably linear behavior.

As we were principally motivated in this work by observations of the so called "explosive cyclones" (Sanders and Gyakum, 1980) it should be noted that the increase in growth rate is in

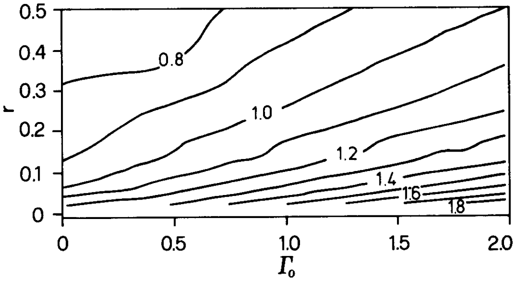

Fig. 5. Contours of constant growth rate σ for $L_D = 3.6$, $\delta_A = \delta_C = 0$ on the (Γ_0, r) plane.

accord with the observed features, and so would be a destabilisation of shorter scales of motion. As far as the phase speed is concerned not all the documented observations are in agreement on this point, but at least Sanders (1986) reports a faster translational speed for faster deepening events.

We have already referred to the numerical study by Fantini, 1990a (hereafter referred to as F) which provided the basis for the formulation of this model. We wish to further point out that, as shown in Fig. 4, meridional wind in the updraft region is advecting warm air, so that the heat input underneath region 1 will soon be reduced and shut off. It was shown in F that in this phase heat is still fed into the boundary layer in the area of large-scale downdraft, and advected into the center of the storm by low level winds. The convergent circulation at low levels can here be inferred from the strong updraft shown in Fig. 4, and it is this source of heat that provides the justification for the use of a constant Γ in the updraft. The ability of the perturbations to extract energy, in the form of sensible and latent heat, from the boundary layer by means of their own secondary circulation accounts for the faster growth obtained.

In the comparison with the results of F two facts stand out.

(i) An almost linear relation between σ and Γ, of which there is also some observational evidence (Davis and Emanuel, 1988), is reproduced in both studies, despite the vastly different treatment of the thermodynamics.

(ii) The phase speed in the numerical study has a behavior opposite to the one obtained here. In F the phase speed was reduced below its "unforced" value at the beginning of the explosive phase. This coincided with the formation of an "hurricane" type of structure which was mostly confined to the lower half of the troposphere and so more sensitive to the low-level wind. It was shown in F that this non-baroclinic, mesoscale structure was instrumental in the transport of energy from the boundary layer into the mid-troposphere.

This organisation is unnecessary here as the heat source term is already written in the thermodynamic equation at mid-level. In doing so we have parameterized all of the intermediate steps, and assumed that it is the baroclinic updraft (strengthened by latent heat release) that effects the necessary vertical transport of heat once convergence in the boundary layer has provided a continued source of heat below the updraft itself. It may then be possible that, e.g., an enhanced "baroclinic" updraft in the interaction with a pre-existing upper-level trough could perform the same role played by the 2D "hurricane" in the numerical model of F. Then the phase speed and the other dynamic characteristics of the system would be determined by the action, in competition or cooperation, of those two mechanisms.

More problematic is the linear relation between deepening rate (σ here) and heating rate (Γ_0 here), which appears to be a rather robust feature of the boundary-flux-induced development, being present in this very idealized study as well as in numerical simulations (F) and observational studies (Davis and Emanuel, 1988). The models seem to behave as if the air-sea temperature difference $\Delta\theta_e$, which appears linearly in the expression for the heat flux (e.g., eq. (10) here) were applied directly to perturbation kinetic energy, bypassing the details of the circulation. To account for this result in different models would have to conjecture that the presence of a thermal forcing at the lower boundary can give rise to a well defined configuration of flow (which may be the "hurricane" phase of F, or other as yet undiscovered) which then acts as a "transmission belt" between the heat source and the baroclinic wave, maintaining the same efficiency over a wide range of intensities of the source. While we do not see any way to test this conjecture at the present time, we want to point out that an element that this study and F have in common is that they are both "normal mode" studies: the present work in the ordinary sense of assuming a separate time dependence $\exp(\sigma t)$ for all variables; and the numerical study of F by letting the simulation evolve until the fastest exponential growth could take over. As a possible mode of interpretation, we can think of what we observe in the numerical model (and in some real cases) as a "most unstable" solution in the normal-mode sense, i.e., that the time evolution only changes the amplitude of the perturbation and not its spatial structure. This spatial structure is the most efficient in the transport of heat out of the boundary layer and it is independent of the intensity of the external heat source, because any other structure would imply a less efficient energy conversion and thus a slower growth of perturbation energy. All the perturbation quantities that appear in the drag law for heat transfer (either the full form with wind-dependent drag coefficient c_D used in F, or the linearized form, eq. (10) above, used here) could then be invariant when $\overline{\Delta\theta_e}$ is changed, in the same way as e.g. a quantity like the slope of trajectories (appropriately scaled) on the meridional plane is the same for the most unstable baroclinic wave for any value of environmental parameters.

A study of the initial value problem could shed more light on how this structure is achieved. Unfortunately the present model is inadequate for this purpose: the superposition of two or more normal modes like those found here is not necessarily a consistent solution of the problem because it may change the sign of w. There is a residual non-linearity in the model, given by the coupling of the static stability with the sign of the vertical velocity, which makes the superposition principle invalid. It was also shown in F that an initial value problem which starts with an infinitesimal perturbation has to go through an amplitude threshold before the input of energy at the air-sea interface can be effectively used to strengthen the storm. The present model is an idealization of the mature phase of this process, and should not be used to study its inception. The solution found here should only be regarded as the asymptotic configuration of the system, after the most unstable exponential mode has had the time to become dominant, and provided that no change in the environment occurs in the meantime.

4. Summary

Through the addition of a heat flux at the lower boundary, in very idealized form, to the model of EFT we have achieved an increase of the growth rate and phase speed of unstable baroclinic waves. While the increased phase speed may be an artifact of the parameterization chosen (see Section 2), the faster deepening of the model's perturbations is a manifestation of their ability to organize warm and moist boundary layer air in a convergent circulation which continuously feeds a strong mesoscale updraft. Comparison with the numerical study of F leads us to suggest that different mechanisms (of which the "hurricane" phase of F is one) may be able to transport the heat received at the sea surface outside the boundary layer, and make it available for conversion into kinetic energy of the baroclinic wave. The dynamical characteristics of the storm would then be determined by which mechanism is present, or predominant at any given time. A "normal mode" behavior of these mechanisms, in the sense of a separated space and time dependence, may help explain the apparent linear relation between deepening rate and heating rate, but surely more sophisticated studies than the present one are needed in this direction.

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