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# Linear predictions and diagnosis of time-averages in a two-level model

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ABSTRACT

We study the exact anomaly model of a simple two-level GCM. Linear predictions made from the initial state and a posteriori forcings are described. From the initial state alone, a linear barotropic model is superior to dependent persistence forecasts. Even greater dependent and independent skill is obtained from a linear baroclinic model, especially at short time scales. Inclusion of an empirical linear prediction for surface temperature provides only a marginal improvement, despite the significant effect that the surface temperature field has a posteriori on the atmospheric anomalies. This marginal improvement is related to the skill of the linear model in predicting the surface temperature which is the same as for predicting the atmospheric anomalies. Inclusion of an empirical parameterization for the transient eddies provides a significant improvement to the linear models. This parameterization method is also used to parameterize the baroclinic forcings in a barotropic model. The heavily parameterized barotropic model shows a significant improvement, and is comparable to the parameterized linear baroclinic model, especially for long term forecasts.

## 1. Introduction

In 1977 and 1979, the Group of Long-Range Numerical Weather Forecasting in Beijing China proposed and built a quasi-steady-state anomaly model. This model was developed mainly for making monthly and seasonal weather forecasts. (See also Chao et al., 1982). The model has shown significant skill in some cases (Miyakoda and Chao, 1982) and until more elaborate numerical weather prediction models can be brought to bear on long range prediction problems, this model will probably remain as one of the principal dynamical tools used by the Group of Long-Range Numerical Weather Forecasting for making long range predictions.

There are a number of efforts underway to improve the physical parameterizations in Chao's model, centered mainly about the prediction of the evolving surface boundary conditions. Other potentially important forcings are the transient eddy fluxes which are presently only crudely parameterized by dissipation terms in the model (see Chao et al., 1986) as suggested by several studies (e.g. Roads, 1989; Lau, 1979; Youngblut and Sasamori, 1980). However, some studies (e.g. Nigam et al., 1986, 1988; Valdes and Hoskins, 1989) have also shown that the role of these flux terms are not exclusively dissipative, and they may be important to the overall balance. It is our intention here to evaluate the impact of the transient eddy flux divergences and then propose an empirical parameterization scheme for them in a linear prediction model.

A number of parameterizations for the transient eddies has been proposed in the past with baroclinic adjustment being one of the most recent (Green, 1970; Stone, 1972; Lindzen and Farrel, 1980). The essence of this adjustment parameterization is that the atmosphere always stays at the edge of a stable or marginally baroclinic unstable state. A thorough review of the theory

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has been documented by Hoskins (1983). Several difficulties with this appealing baroclinic adjustment process are the neglect of meridional variation (barotropic instability for one); the need to specify the static stability; and the arbitrary vertical redistribution of the adjustment.

Reinhold and Pierrehumbert (1982) extended the classical baroclinic instability theory and proposed a quasi-linear scheme in which the effects of the transient eddies are parameterized by the most unstable eigenmode of the quasi-equilibrium background state. Vautard and Legras (1988) evaluated Reinhold and Pierrehumbert's hypothesis using their simple two-level channel model (Vautard et al., 1988). They found that the quasi-linear theory parameterized well the behavior of the eddies when the background circulation is mainly zonal flow (weak quasi-stationary wave). However, the quasi-linear scheme failed to describe the transient effect in a blocked background flow (strong wavy basic state). Accordingly, Vautard and Legras (1988) proposed an alternative empirical (statistical) parameterization method based upon a scale separation. They partitioned the circulation into a large-scale (low wavenumber) part and a small-scale part, and use the former to empirically parameterize the effects of the latter. We do not make this scale separation here. It is our feeling that variability in all waves is related to the structure in all waves and hence, any parameterization must included the entire spectral structure.

Another unique aspect of this paper is that we treat the monthly and seasonal prediction problem as a linear initial as well as a boundary value problem. The methodology here is similar to Roads (1990), except the solutions are obtained from integrating an implicit linear operator rather than from the expansion of eigenfunctions. Since linear steady-state models, linearized around a fixed basic state, have commonly been used with some success as a diagnostic tool to explain complicated time average behavior in the real world (e.g., Lin, 1982; Jacqmin and Lindzen, 1985; Chen and Trenberth, 1989) and nonlinear models (e.g. Nigam et al., 1986, 1988; Roads, 1989), we might suspect that the atmosphere time averaged anomalies can also be related to a linear operator in a prognostic fashion. It is these aspects of the problem that we address here using the idealized model described by Roads (1989).

The advantage to working with this model is that the equations are sufficiently simple that we can derive, exactly, the anomaly equations; this can only be done approximately in the real world. These anomaly equations can then be used to assess the relative contributions from the modeled surface boundary conditions, the nonlinear transient eddy fluxes as well as the linear atmospheric dynamics that provide the basic framework for the Chao et al., (1982) model.

In Section 2, the barotropic and baroclinic anomaly equations used for this investigation are described. Section 3 explicitly describes the linear prediction method. Forecasts and diagnostics of the idealized data set are then analyzed and compared to persistence forecasts in Section 4. Attempts to further improve the forecasts are described in Section 5 and conclusions are given in Section 6.

## 2. Anomaly equations

The anomaly equations are derived from the two-level quasi-geostrophic model described by Roads (1989). Any variable, such as the streamfunction, can be separated by

$$A = \bar{A} + A', \tag{1}$$

where $\bar{A}$ represents the long term mean calculated as the average of the 1825-day control simulation from Roads' model and $A'$ represents the instantaneous anomaly from this model climatology. Only daily values are archived (by previous choice) in this data set which will hereafter be denoted as "observations".

The equations governing the time-independent climatology, which will be used as the basic state of the linear prediction models, are

$$J(\overline{\psi_i}, \overline{q_i}) = F_{\mathrm{B}} \mp F_{\mathrm{T}} - \bar{J}(\psi_i', q_i') - D_i\overline{\psi_i} \pm 2\lambda^2(K_{\mathrm{T}} + K_{\mathrm{R}})\bar{\tau} \mp 2\lambda^2 K_{\mathrm{T}}\gamma\bar{T}. \tag{2}$$

where $i$ refers to the upper ($i = 1$) or lower ($i = 3$) level and the upper sign refers to the upper level. Note that the model has incorporated a surface temperature, T, with an exchange coefficient, $K_{\mathrm{T}}$, of order 5 days$^{-1}$. The interested reader should refer to Roads (1987, 1988) for a detailed description of the model and the symbols; however, most of the notation is conventional.

The balance in the above equation was examined first as a validation of the anomaly models to be described below. It was found necessary to include a residual term on the RHS in order to get an exact balance; this residual forcing was about two orders of magnitude smaller than any other individual term. It is presumably caused by the a posteriori diagnoses of the transient eddies from coarse temporal resolution of daily samples.

Let us now consider the anomaly potential vorticity equations, which can be written as

$$\frac{\partial q_1'}{\partial t} + J(\overline{\psi_1}, q_1') + J(\psi_1', \bar{q}_1) + K\nabla^6\psi_1' - 2\lambda^2(K_T + K_R)\tau' = -J_{t1}' - 2\lambda^2 K_T \gamma T', \tag{3}$$

$$\frac{\partial q_3'}{\partial t} + J(\overline{\psi_3}, q_3') + J(\psi_3', \bar{q}_3) + K\nabla^6\psi_3' + K_s\nabla^2\psi_3' + 2\lambda^2(K_T + K_R)\tau' = -J_{t3}' + 2\lambda^2 K_T \gamma T'. \tag{4}$$

$J_{ti}'$ $(i = 1, 3)$ are anomaly transient eddy fluxes and are defined as

$$J_{ti}' = J'(\psi_i', q_i') = J(\psi_i', q_i') - \bar{J}(\psi_i', q_i'). \tag{5}$$

The other terms are

$$\overline{q_1} = \nabla^2\overline{\psi_1} - 2\lambda^2\bar{\tau} + \sin y,$$

$$\overline{q_3} = \nabla^2\overline{\psi_3} + 2\lambda^2\bar{\tau} + \sin y + h,$$

$$q_1' = \nabla^2\psi_1' - 2\lambda^2\tau',$$

$$q_3' = \nabla^2\psi_3' + 2\lambda^2\tau',$$

$$\tau = (\psi_1 - \psi_3)/2.$$

The anomaly equations derived here are the same as the linear model derived previously by Roads (1989). The only difference is that (as in Roads, 1990) the time derivative term is retained.

Even simpler anomaly equations can be derived from the barotropic vorticity equation, which is simply the vertical average of the above two potential vorticity equations. However, complications arise in that the climatology is now the barotropic climatology and additional forcing terms representing the baroclinic forcing of the barotropic motions must be included. Explicitly

$$\frac{\partial \nabla^2\psi_B'}{\partial t} + L\psi_B' = F_R, \tag{6}$$

where

$$\psi_B = (\psi_1 + \psi_3)/2,$$

$$L\psi_B' = J(\overline{\psi_B}, \nabla^2\psi_B') + J(\psi_B', \nabla^2\overline{\psi_B} + \sin y + \tfrac{1}{2}h) + K\nabla^6\psi_B' + \tfrac{1}{2}K_s\nabla^2\psi_B', \tag{7}$$

$$F_R = -J_B' - J_\tau, \tag{8}$$

$$J_B' = J'(\psi_B', \nabla^2\psi_B') = J(\psi_B', \nabla^2\psi_B') - \bar{J}(\psi_B', \nabla^2\psi_B'), \tag{9}$$

$$J_\tau = J'(\tau', \nabla^2\tau') - \tfrac{1}{2}K_s\nabla^2\tau' + J(\tau', \nabla^2\bar{\tau} - \tfrac{1}{2}h) + J(\bar{\tau}, \nabla^2\tau'), \tag{10}$$

$$J'(\tau', \nabla^2\tau') = J(\tau', \nabla^2\tau') - \bar{J}(\tau', \nabla^2\tau'). \tag{11}$$

Note that the term $J_\tau$ represents the influence of baroclinic process from both linear and nonlinear transient process.

To summarize, we have now described the time dependent anomaly equations for both the baroclinic and barotropic cases used in this investigation. In the following section, we derive the linear prediction equations for forecasts of time averages and linear diagnostics in this study.

## 3. Linear prediction models

The daily anomaly equations described above in Section 2 are now converted into a model for the prediction of time averages. The integral solution of equation (6) can be written as

$$\psi_B'(t_1) = \psi_B'(0) - \nabla^{-2}\int_0^{t_1} (L\psi_B' - F_R)\,dt, \tag{12}$$

Since we are interested here only in forecasts of, time averages, $\hat{\psi}_B$, where

$$\hat{\psi}_B = \frac{1}{T_2}\int_0^{T_2} \psi_B'(t_1)\,dt_1. \tag{13}$$

and $T_2$ is the averaged time period, we must integrate eq. (12) once more to give

$$\hat{\psi}_B = \psi'_B(0) - \frac{1}{T_2}\int_0^{T_2}\int_0^{t_1} \nabla^{-2}(L\psi'_B - F_R)\,dt\,dt_1. \tag{14}$$

The above equation can be diagnosed to find the relative contributions of each term on the RHS of the above equation to the "observed" time-averaged anomaly.

However, we also wish to go further to derive an implicit linear method for time-averaged anomaly predictions. To that purpose we define an additional anomaly. This other anomaly, $\psi''_B$, is the daily deviation of the daily anomaly from a particular time averaged anomaly, $\hat{\psi}_B$, and so it follows

$$\psi'_B = \hat{\psi}_B + \psi''_B. \tag{15}$$

Note that

$$\hat{\psi}''_B = 0, \tag{16}$$

just as

$$\overline{\psi'_B} = 0.$$

Substituting eq. (15) into (14), an implicit solution for $\hat{\psi}_B$ can now be written as

$$\hat{\psi}_B = \left(1 + \frac{T_2}{2}\nabla^{-2}L\right)^{-1} \times \left\{\psi'_B(0) - \frac{1}{T_2}\int_0^{T_2}\int_0^{t_1} \nabla^{-2}(L\psi''_B - F_R)\,dt\,dt_1\right\}. \tag{17}$$

In essence, the time-averaged anomaly is affected by the linear operator acting on the initial state and the forcings and deviations from the time averaged anomaly that develop antecedent to the initial state. The major difference between this implicit linear solution and the exact linear solution shown by Roads (1990) is the inclusion of $\psi''_B$. That is, the implicit linear operator is only an approximation for the time average of the linear operations since the linear variations $L\psi''_B$ are included in the forcing.

If finite differences are used to express the time integral, a forward difference approximation for eq. (17) gives

$$\hat{\psi}_B = [1 + \tfrac{1}{2}\Delta t(Q-1)\nabla^{-2}L]^{-1} \times \left\{(1 - \Delta t\,\nabla^{-2}L)\,\psi'_B(0) + \Delta t\,\nabla^{-2}F_R(0) + \frac{\Delta t}{Q}\nabla^{-2}\sum_{n=1}^{Q-1}(Q-n)[F_R(n\,\Delta t) - L\psi''_B(n\,\Delta t)]\right\}, \tag{18}$$

where $T_2 = Q\,\Delta t$.

A similar procedure can be used to obtain the time averaged difference equations for the baroclinic model and these are given in the Appendix.

There are a number of simplified prediction models that can be derived from (18). For example:

(a) The simplest model is a persistence model given by

$$\hat{\psi}_B = \psi'_B(0). \tag{18a}$$

This method is compared with the various linear models used here.

(b) When only the first term on RHS in (18) is retained for the forecast, i.e.

$$\hat{\psi}_B = [1 + \tfrac{1}{2}\Delta t(Q-1)\nabla^{-2}L]^{-1} \times (1 - \Delta t\,\nabla^{-2}L)\,\psi'_B(0), \tag{18b}$$

then this model is denoted as "modified persistence". This simplified model is called modified persistence because the linear operator, $L$, modifies the initial state in accordance with linear dynamics. The linear operator given in eq. (7) includes the effects of dynamical processes such as mean advection, topography, rotation of the earth and frictional dissipation. This is a potentially important model since it can be readily used operationally (if a skillful linear model for nature can in fact be constructed) providing an initial state is given. This model is equivalent to the linear model described by Roads (1990).

(c) A forecast which uses the first two terms on the RHS of (18) is denoted as the "initial forcing" model, i.e.

$$\hat{\psi}_B = [1 + \tfrac{1}{2}\Delta t(Q-1)\nabla^{-2}L]^{-1} \times \{(1 - \Delta t\,\nabla^{-2}L)\,\psi'_B(0) + \Delta t\,\nabla^{-2}F_R(0)\}. \tag{18c}$$

The initial forcing is separated from the full forcings because some parts of the forcing, the surface temperature anomaly that develops prior to the desired prediction time, for instance, is known a priori. However, in practice, it is difficult to evaluate all aspects of this forcing except insofar as it may be related to the initial state.

(d) A forecast which includes the rest of the forcings is denoted by the term "full forcings". Explicitly,

$$\hat{\psi}_{\mathrm{B}} = [1 + \tfrac{1}{2}\Delta t(Q-1)\nabla^{-2}L]^{-1} \times \left\{(1 - \Delta t\,\nabla^{-2}L)\,\psi'_{\mathrm{B}}(0) + \Delta t\,\nabla^{-2}F_{\mathrm{R}}(0) + \frac{\Delta t}{Q}\nabla^{-2}\sum_{n=1}^{Q-1}(Q-n)\,F_{\mathrm{R}}(n\,\Delta t)\right\}. \tag{18d}$$

In practice, these additional forcings, $F_{\mathrm{R}}(n\,\Delta t)$, are only known a posteriori. However, we can use this model to analyze their relative importance; analyses of their behavior may yield some clue as to methods of parameterizing them.

(e) Finally, the forecast is denoted as an ideal forecast if all terms of (18), including $\psi''_{\mathrm{B}}$, are included. In addition, a residual forcing is also included. A residual forcing is needed since the other forcings are being calculated only from daily samples. This ideal forecast is called "ideal" because it includes all a priori, a posteriori, and residual terms. This final model is used to verify the previous simplified models for consistency.

In summary, the above linear prediction models can be used to analyze the relative contributions of the initial state and the developing boundary forcings. The skill of these linear prediction models are described below.

## 4. Forecasts and diagnostics

All variables are expanded in spherical harmonics and are triangularly truncated at the two dimensional wavenumber $N = 21$, and the zonal wavenumber $M = 20$ with only odd harmonics included. Forecast skill is assessed by an ensemble correlation which is defined as

$$\rho = \frac{\langle \hat{\psi}_{\mathrm{o}}\hat{\psi}_{\mathrm{p}}\rangle}{\langle \hat{\psi}_{\mathrm{o}}^{2}\rangle^{1/2}\langle \hat{\psi}_{\mathrm{p}}^{2}\rangle^{1/2}}. \tag{19}$$

$\hat{\psi}_{\mathrm{o}}$ is the "observed" time-averaged anomaly streamfunction, $\hat{\psi}_{\mathrm{p}}$ is the prediction from the corresponding linear prediction models (eqs. 18a–d), and the brackets denote an ensemble average over all cases. The ensemble hemispheric mean of the anomalous variables (which is approximately zero) has been removed. All the quantities in (19) are also a function of two-dimensional wavenumber, $n$, and zonal wavenumber, m; these indices are suppressed for simplicity. Obviously, the $\rho$ of an ideal forecast, the forecast made by including every term in (18), should be 1.

### *4.1. Forecasts from initial values*

Ensemble means of non-overlapping averages are calculated from the first 5-year data set described by Roads (1989). One day forecasts include 1800 members in the ensemble whereas 90 day forecasts include only 20 members in the ensemble. Fig. 1 shows the ensemble hemispheric mean pattern correlations as a function of averaging time for the forecasts of the barotropic and baroclinic streamfunction predicted by the linear prediction models. In this figure and others shown below, curves labeled "P", "M", "I", "F" and "ideal" denote "persistence", "modified persistence", "initial forcing", "full forcings" and "ideal" forecasts, respectively. Fig. 1a shows the barotropic streamfunction predicted by the barotropic model. Figs. 1b and 1c are the barotropic and baroclinic components of streamfunction predicted by the baroclinic model. From Figs. 1a and 1b, one can easily compare the prediction ability of both models. Persistence forecasts are identical in both cases because no dynamical processes are taken into account in these forecasts. In general, "modified persistence" forecasts are better than pure persistence forecasts for any range, especially in the baroclinic model. The ensemble skill of monthly "modified persistence" forecasts increase from 0.26 for pure persistence forecasts to 0.42 (0.28) in the baroclinic (barotropic) models. For seasonal forecasts, the modified persistence forecasts are not significantly better than persistence forecasts in the barotropic model but do show a slight improvement in the baroclinic model. The "initial forcing" forecasts are even better, especially for short term averages, but show little improvement over "modified persistence" at the seasonal time scales. The skills of the baroclinic component of streamfunction shown in Fig. 1c,

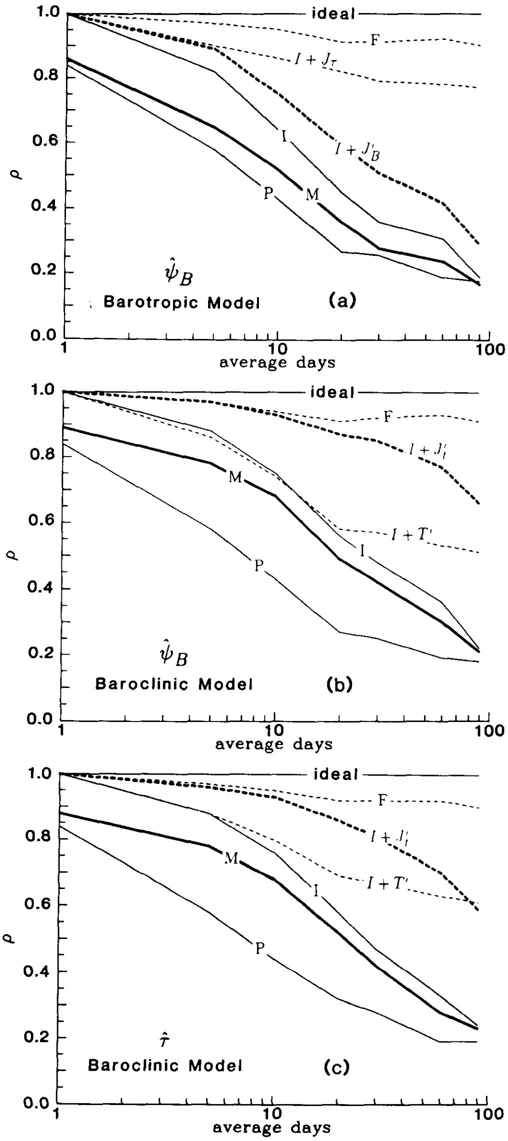


*Fig. 1.* The mean pattern correlation of the streamfunction for time averaged forecasts 1–93 days predicted by (a) the barotropic model; and (b) the barotropic component and (c) the baroclinic component predicted by the baroclinic model. "P" denotes persistence, "M" modified persistence, "I" initial forcing, "F" full forcings, and "ideal" ideal forecasts. "$I + J'_B$" and "$I + J_\tau$" stand for forecasts with initial forcing plus barotropic anomaly transient eddy and baroclinic component $\tau$ terms forcings, $\tau$ for barotropic streamfunction, respectively. "$I + J'_t$" and "$I + T'$" are the forecasts with specified baroclinic transient eddy flux divergences and surface temperature anomalies, respectively.

predicted by the "modified persistence" and "initial forcing" models have similar behavior as that of the barotropic component. The curves marked with "F", "$I + J_\tau$", "$I + J'_B$", "$I + J'_t$" and "$I + T'$" will be discussed later.

Fig. 2 shows the two dimensional wave number spectra of the pattern correlation of various forecasts by the barotropic model. At shorter time scales (Figs. 2a and 2b), "modified persistence" and "initial forcing" forecasts are only marginally better than pure persistence for all wave numbers. At longer time scales (Figs. 2c and 2d), persistence forecasts become less skillful in the long waves while both "modified persistence" and "initial forcing" forecasts are slightly improved. Overall, the skill of short waves quickly approach a level with little skill past 10 days.

Fig. 3 is the pattern correlation spectra for the barotropic component of the streamfunction predicted by the baroclinic linear prediction model (A12). Again the persistence forecast skill is the same distribution as shown in Fig. 2. The large differences between "P" and "M" curves indicate that, for intermediate time scales, 10 days to monthly, "modified persistence" forecasts in the baroclinic model outperform those by the barotropic model for the short as well as the long waves.

### *4.2. Forecasts from a posteriori forcings*

Also shown in Figs. 1–3 are the effects of the a posteriori developed forcings. In Fig. 1 the "full forcings" forecasts are shown equally skillful in both models and the extremely good skill for all time scales indicates the negligible importance of $\psi''_B$, which is due to the implicit linear approximation. This large skill justifies our use of the implicit time averaging method and also the sampling of transient effects using daily data. The forecasts skills exceed 0.9 for all wavenumers in both barotropic and baroclinic models, as demonstrated in Figs. 2–3. Note that the "full forcings" model is different from the "initial forcing" model by the a posteriori ("unknown") forcings involving $F_R(n\,\Delta t)$ (see (18d)). Compared to those by the "modified persistence" and "initial forcing" forecasts, the most significant contribution of the "unknown" forcing is the improvement of the skills in the short waves and the ultra-long waves for longer time periods.

In Fig. 1a, the curves labeled with "$I + J'_B$" "$I + J_\tau$" show the forecasts made by the barotropic

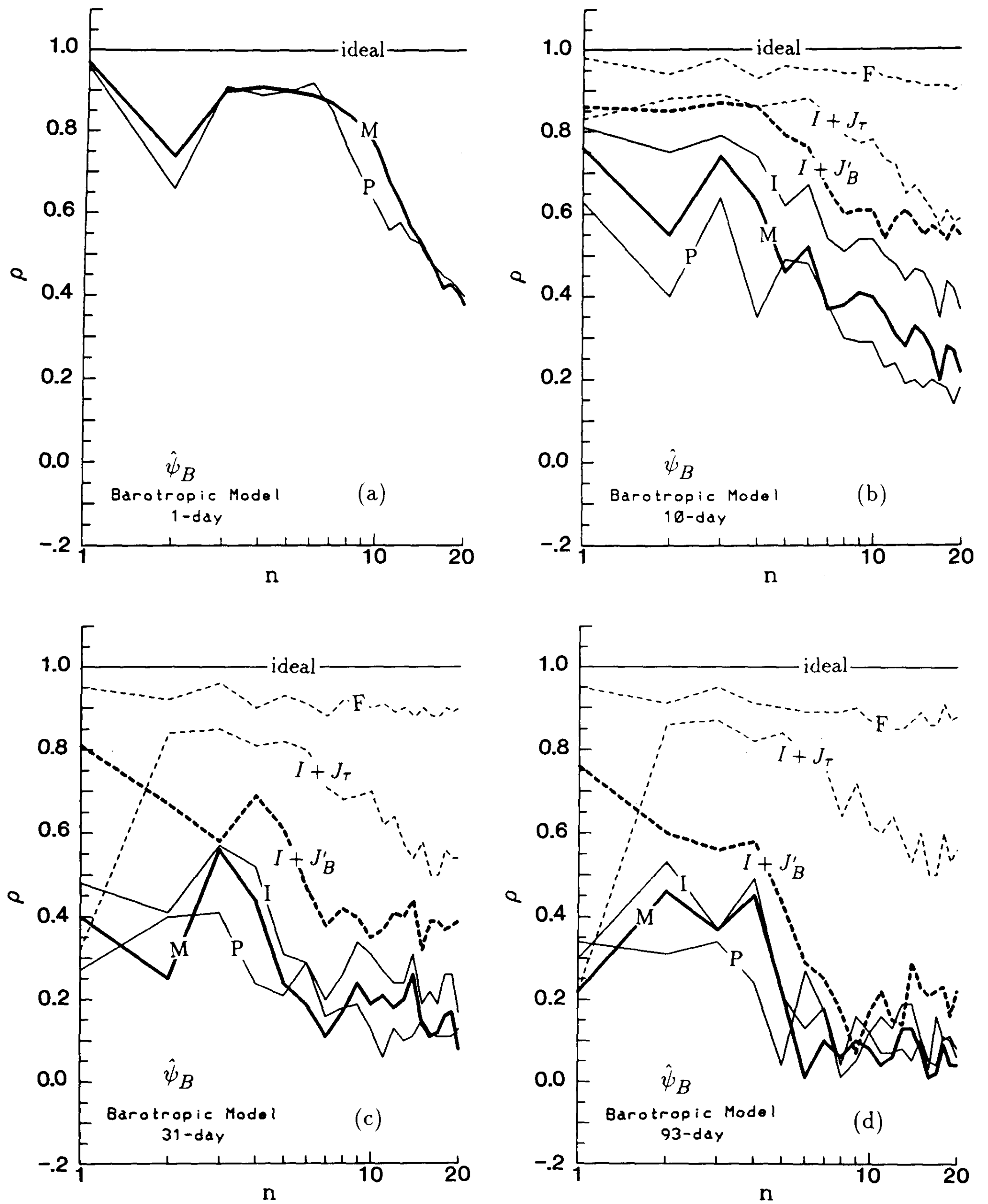


*Fig. 2.* Forecast pattern correlation spectra as a function of two dimensional wave number for (a) 1-day, (b) 10-day, (c) 31-day and (d) 93-day averaging times in the barotropic model. Notations are the same as those in Fig. 1.

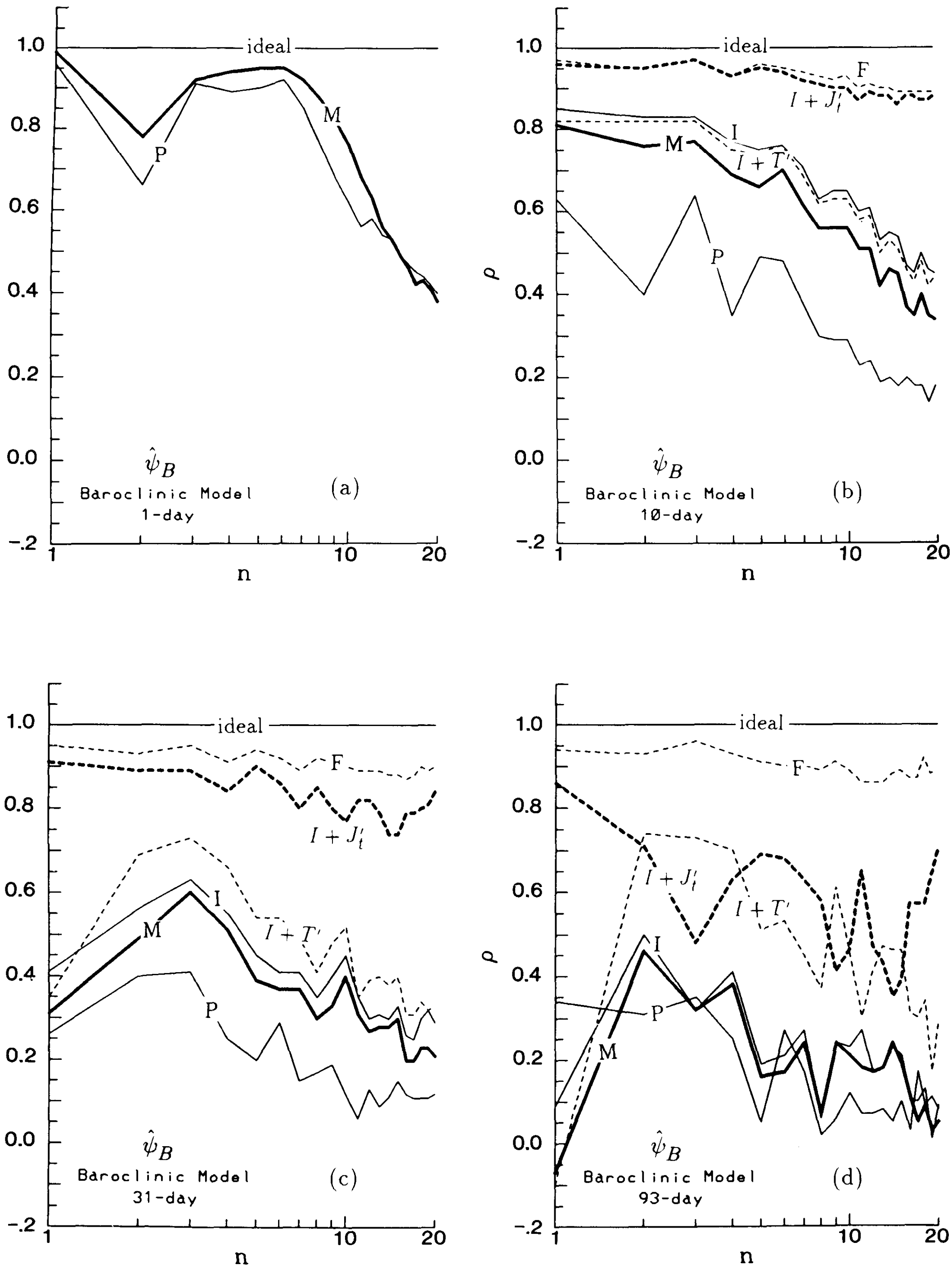


*Fig 3.* As in Fig. 2, except for the baroclinic model.

"initial forcing" model with the additional forcing of barotropic transient flux transfer, term (9), and the baroclinic processes, term (10), respectively. In other words, only the $J'_B$ or $J_\tau$ term is retained in the full forcings forecast model for the corresponding curves. These curves shows that the baroclinic process terms are much more important than the barotropic transient eddy fluxes, especially for longer time average forecasts. However, in Fig. 2, the wavenumber breakdown demonstrates that the barotropic transient eddy fluxes have a greater contribution than the baroclinic processes for the extreme long wave ($n = 1$) and this contrast becomes even more evident for longer time averages.

In the baroclinic model the "unknown forcings" consist of the surface temperature and the baroclinic transient eddy fluxes. The relative importance of surface temperature can be seen in Figs. 1b and 1c by comparing the two curves "$I$" and "$I + T'$". Curve "$I + T'$" is the forecast by the "initial forcing" model with the extra forcing of the a posteriori developing surface temperature anomaly. As may be seen, the surface temperature forcing provides a negligible improvement for short-range (shorter than two week) forecasts but becomes more important at longer time scales. A striking difference between Figs. 1b and 1c is that the additional skill provided by the temperature forcing is much greater for the baroclinic component than the barotropic component. From the correlation spectra in Fig. 3, the surface temperature seems to contribute more to all waves with $n > 1$ for the barotropic component. However, this surface temperature forcing only contributes to the long waves ($n < 7$) for the baroclinic component (not shown).

The effect of the baroclinic transient flux convergences are shown by the curves labeled with "$I + J'_t$" in Figs. 1b–c and 3. In Figs. 1b and 1c, the transient fluxes have an even larger effect than the surface temperature. In fact, the forecasts have very high skills for all time averages. These better skills in the barotropic component for seasonal time scales are predominantly contributed, as revealed in Fig. 3d, at $n = 1$, which is the wave for the zonal mean only and is the largest scale in the model. It is interesting to see from Figs. 2 and 3 that although for $n = 1$ the skill for all forecasts, "$P$", "$I$", "$M$" and "$I + T'$", drops rather quickly once the time scale is longer than a month, the transient momentum forcing seems to be the leading contributor in returning the skill to the level given by the "full forcings" model. The transient heat flux (not shown) also demonstrates similar characteristics for the forecast skill of the baroclinic component. In Fig. 3, note that the transient forcings are the most skillful term contributing to the total skill ("$F$") for all time scales and for all spatial scales. Therefore, in practice, a better prediction model can be implemented only when the transient flux convergences can be adequately parameterized. This aspect will be explored further in the next section.

In summary,

(1) "Modified persistence" forecasts have higher skill than the persistence, and if the "initial forcing" forecasts are included the skills are even higher. This may be useful for eventually developing linear predictions.

(2) The linear baroclinic operator has more skill than the linear barotropic operator.

(3) The surface temperature strongly contributes to the forecasts at monthly scales and longer, especially for the baroclinic component.

(4) A posteriori forcings crucially determine the skill of forecasts beyond a week, and are dominated by the transient eddy flux divergences.

## 5. Forecast improvement

Among the linear prediction models in the previous section, one could only really choose, in practice, the "modified persistence" and "initial forcing" model to make an actual forecast. Although the linear operator provides additional skill above pure persistence, the improvements are severely limited for long-range forecasts. It is our intention here to describe efforts to improve this extended-range skill.

### 5.1. *Equivalent barotropic operator*

It has been suggested that the appropriate level to apply a barotropic model should be higher than the traditional 500 mb (Held, 1983). Simmons et al. (1983) also used a 300 mb climatological mean streamfunction in the barotropic model to explain the teleconnection responses. To test this, we used the upper level (250 mb) climatological streamfunction in place of the $\overline{\psi_B}$ (500 mb) in operator $L$ of eq. (7). The modified persistence forecast demonstrated the same skill for forecasts shorter than 30 days. But the skill increases from the 0.17 level to 0.21 when using the higher level basic state for seasonal forecasts. Basically, the effect is not very significant for this two-level model.

### 5.2. *Effect of surface temperature*

As indicated in a previous section, the monthly and seasonal forecasts could be improved substantially by specifying surface temperature. Two methods of including surface temperature were tested.

First, the initial surface temperature is persisted throughout the forecast. The hypothesis here is that boundary forcings usually tend to be more persistent than the atmospheric components. However, when the persistent surface temperature is added to the "initial forcing", the resulting forecasts are even worse for both short and long term cases (not shown). It is speculated that the use of highly varying land temperatures may be one cause contributing to this quick deterioration.

Another method simply uses a coupled atmosphere-surface model, as described in Roads (1989), the anomaly surface temperature equation is

$$\frac{\partial T'}{\partial t} + K\nabla^4 T' = K_g\left(\frac{\tau'}{\gamma} - T'\right), \qquad (20)$$

where through $\tau$ and $T'$ the surface equation is coupled with the baroclinic atmospheric model. By combining (3), (4) and (20), a new (and larger) system is formed. The coupled linear prediction model is described in more detail in the Appendix (A13). Since surface temperature now becomes a variable to be solved, only the anomaly baroclinic transient eddy fluxes remain as unknown forcings, (besides linear operation on the $\psi''$ and $T''$ components and the residual forcing).

Fig. 4 shows the barotropic and baroclinic components of streamfunction and surface temperature predicted by "modified persistence" (curves labeled with "M") and "initial forcing" forecasts ("I") in the coupled model. For comparison purposes, the results predicted by the non-coupled model (thin lines) are also reproduced from Fig. 1. In the previous section, we have shown that the predictions made by specifying the observed surface temperature are significantly improved for monthly and seasonal cases. As

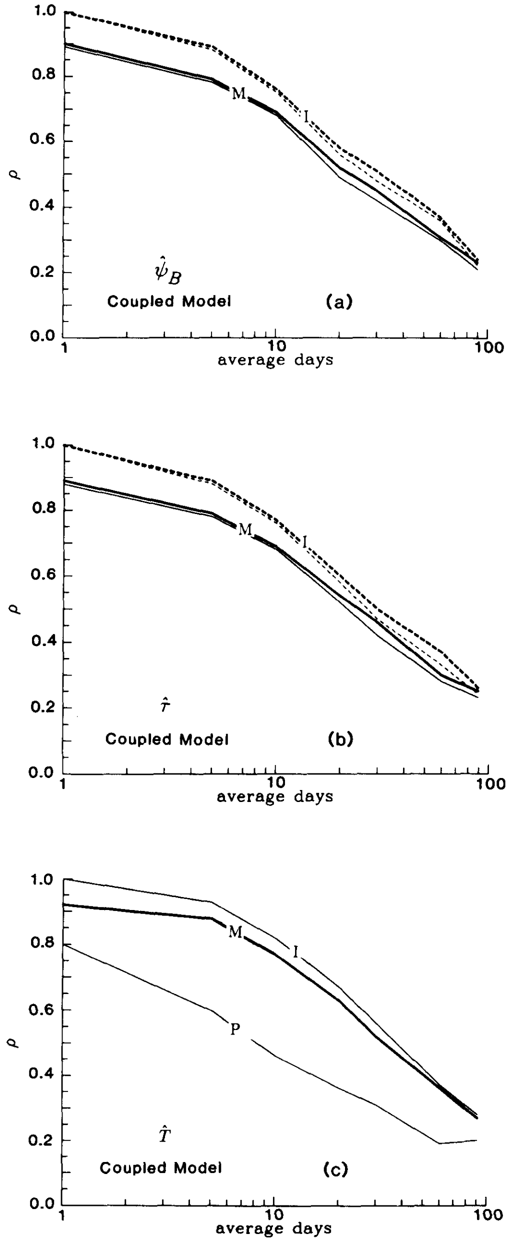


*Fig. 4.* Forecast skills (correlations) for the (a) barotropic and (b) baroclinic streamfunctions, and (c) surface temperature by the coupled linear prediction model (thick solid and dashed curves). The curves labeled by "P", "M", and "I" have the same representation as those in Fig. 1. The thin lines in (a) and (b) are reproduced from Figs. 1b and 1c for those forecasts using a non-coupled model.

revealed in Figs. 4a and 4b, however, the skills of modified persistence (heavy solid lines) and "initial forcing" (heavy dashed lines) forecasts in the coupling model are only marginally better than they are in the non-coupled model (thin lines). This result is similar to the results from by Miller and Roads (1990). The skill of predicting surface temperature here (see Fig. 4c) is about the same as found for predicting atmospheric streamfunction. Although the surface temperature forecasts are actually improved over the persistence, the improvement in streamfunction forecasts is still very limited. There is clearly a large difference between specifying and predicting surface temperature for long-range forecasts.

*5.3. Parameterization of unknown forcings*

In addition to the surface temperature, we have shown that the anomalous transient eddy fluxes are an important forcing in the linear models. Since this forcing is an unknown term involving transient activity (and baroclinic processes for barotropic models; see (8)–(11)), one can, in practice, only learn about it a posteriori. However, we can improve the forecast skill by parameterizing these eddies. It has been suggested that transient momentum and heat fluxes play mainly dissipative roles for the time mean flow (see Lau, 1979; Youngblut and Sasamori, 1980; Roads, 1987). The simplest way to parameterize these effects, therefore, is to use a Newtonian or Laplacian type of damping term acting on the streamfunctions which is a common practice in linear models. Such parameterizations were completely unsatisfactory in this study when constant (or scale weighted) dissipation times were chosen. Apparently the transient fluxes force as well as dissipate. Other studies (cf. Nigam et al., 1986, 1988; Valdes and Hoskins, 1988) also suggest a similar effect from the transient eddies. Since the transient activity is organized by the time mean flows (Opsteegh and Vernekar, 1982; Hendon, 1986), the relationship between the transient fluxes and the anomaly streamfunctions is probably not a constant or a Laplacian operator but a more complicated spatial dependent variable.

In an empirical effort to develop an appropriate parameterization we assume

$$F_a = \alpha\psi. \tag{21}$$

Here, $F_a$ represents the full unknown forcings that

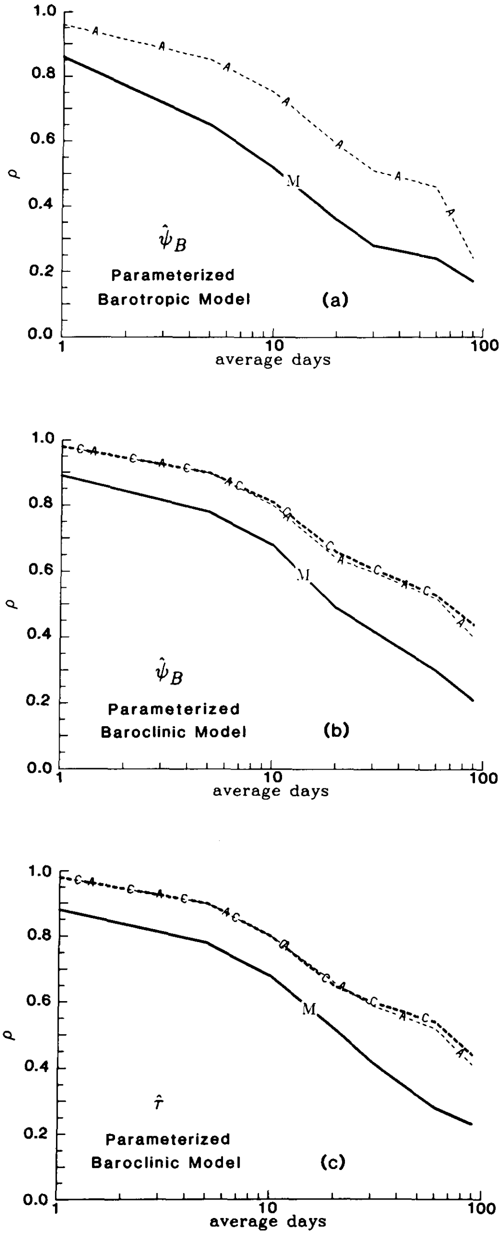


*Fig. 5.* Forecast skills with parameterized forcings in (a) the barotropic model; and the (b) barotropic and (c) baroclinic components predicted by the baroclinic models. "A" denotes the forecasts with parameterized forcings in both models. "C" is the prediction by the coupled model with parameterized baroclinic transient eddy. The modified persistence forecasts ("M") are also reproduced from Fig. 1 for comparison purposes.

are either the transient eddy fluxes in the coupled model, or the sum of the transient eddies and surface temperature forcings in the baroclinic model, or the sum of the barotropic transient eddy and the baroclinic processes, $J_\tau$, in the barotropic model. By using the 1825 daily "observations", the matrix, $\alpha$, is obtained by means of a least square fit:

$$\alpha = \langle F_a \psi^T \rangle \langle \psi \psi^T \rangle^{-1}. \tag{22}$$

The superscript "T" in (22) stands for the transpose of the vector and brackets are the ensemble average over (1800) cases. The vector $\psi$ represents the full model state, its length is 231 for the barotropic case, 462 for the baroclinic case and 693 for the coupled model. This parameterization is quite skillful (artificially) with a correlation of about 0.86 (0.91) between the parameterization calculated by the "observed" streamfunction and the actual forcings for the barotropic (baroclinic) cases. This scheme parameterizes all forcings, not just transient forcing; for example, we can parameterize baroclinic effects in a barotropic model. Also, our parameterization applies to all waves rather than just those of planetary scale.

With this parameterization, the barotropic linear prediction model to be compared with the modified persistence model is

$$\hat{\psi}_B = [1 + \tfrac{1}{2}\Delta t(Q-1) L_p]^{-1} \times \{(1 - \Delta t L_p)\, \psi'_B(0)\}, \tag{23}$$

where $L_p$ is the parameterized linear operator defined as

$$L_p = \nabla^{-2}(L - \alpha). \tag{24}$$

The corresponding baroclinic parameterized model in the baroclinic case and the coupled parameterized model are given in Appendix (A.2). This matrix is, in effect, equivalent to, and provides an explanation for the success of the empirical linear prediction matrix described by Roads (1990).

The modified persistence forecasts with the parameterized linear operator are denoted as "parameterized modified persistence" and the skills are shown in Fig. 5. The barotropic and baroclinic "parameterized modified persistence" skills are given by the curves labeled with "A" in Fig. 5. The curves marked with "C" in Figs. 5b and 5c denote the skill by the "parameterized modified persistence" forecasts in the coupled model. Here we chose "M" as a baseline to compare the skill of the forcing parameterization, since eq. (22) should include the effects of initial forcing as well.

We see from Fig. 5 that parameterized forecasts show substantial improvements in these models. The skill increases from 0.28 (0.42) of modified persistence to 0.52 (0.6) for the monthly

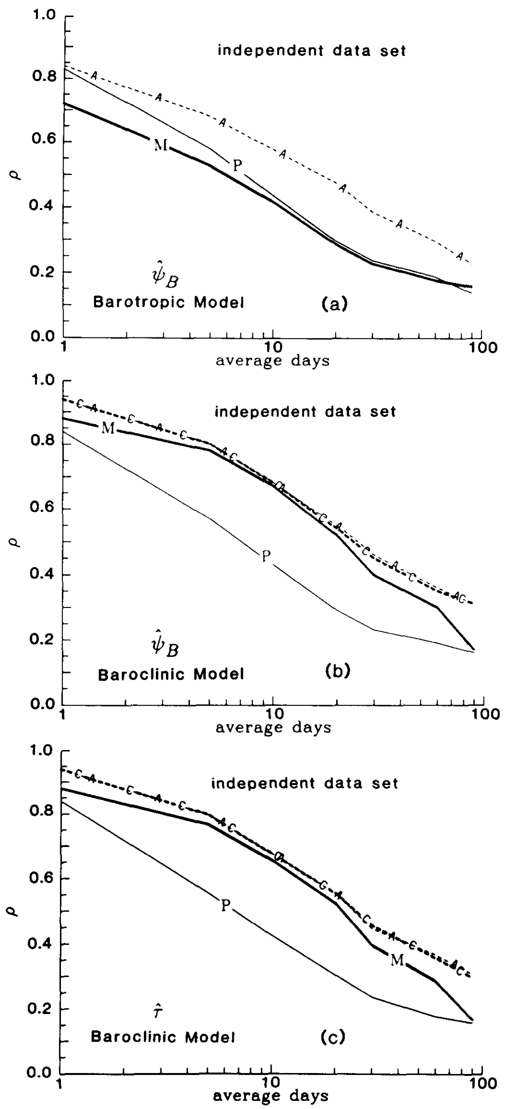


*Fig. 6.* As in Fig. 5, except the independent data set is used. "P" denotes persistence forecasts.

"parameterized modified persistence" barotropic (baroclinic) models. Although the parameterizations are better in the baroclinic case than in the barotropic case (higher multiple correlation coefficient of the parameterization in the former), the relative improvement seems to be better in the barotropic model.

A concern about this empirical approach is that the parameterizations were developed from a dependent data set of limited data. Thus, to further test this parameterization we evaluated another independent 1825 daily data set. This data set was simply a continuation of the previously analyzed 5 year data set as described by Roads (1990). The second data set had some slight differences in climatology, but for these tests we used the climatology (basic states) and parameterizations developed from the first set. The results are given in Fig. 6, which should be compared to Fig. 5. As shown, the modified persistence forecasts are better than persistence in the baroclinic model (Figs. 6b–c), but are slightly worse than persistence in the barotropic model (Fig. 6a). "Parameterized modified persistence" forecasts show some improvements, although not as spectacular as in the dependent data set. Still, the skills of monthly parameterized forecasts rise from about 0.24 for persistence forecasts to more than 0.4 for both the barotropic and baroclinic models. The results predicted by coupled or non-coupled models do not show any further improvement for these independent predictions.

In summary, time mean forecasts with linear models can be improved further in various ways with the parameterization of the transient eddy fluxes being one of the important improvements.

## 6. Conclusions

A linear prediction model based upon linear operators acting on anomalous initial states has been developed for forecasts of time mean anomalies. The model skills are first evaluated from a dependent data set, which is derived from a two-level GCM and compared to persistence forecasts. Although the major skill for short-term forecasts is due to persistence, the addition of linear dynamics ("modified persistence") notably improves the skill over persistence forecasts. In particular, forecasts in the baroclinic model are much better than they are in the barotropic model. This also indicates that inclusion of linear baroclinic terms is helpful to the making of forecasts. Further improvements can be obtained by taking into account the initial forcings. For monthly and seasonal forecasts, however, the skills for all modified models are only increased slightly over the skills achieved for persistence models.

The a posteriori forcings play a vital role for time mean predictions, especially for long term forecasts. Although the specified transient momentum flux convergences contribute skill to the forecasts in the barotropic model, the "unknown forcing" terms describing the baroclinic processes are far more important. In the baroclinic model, both the baroclinic transient eddies and the surface temperature anomalies have a very significant impact on the forecasts.

Some approaches for improving the time mean forecasts were proposed. A coupled model which included the surface temperature equation gave forecast skills only slightly better than those obtained by using an uncoupled model. Similar to the forecasts of atmospheric anomalies, the skill of surface temperature forecasts is greatest for short term averages but decreases rapidly and approaches the skill of persistence forecasts for longer time scale forecasts. Therefore, although the specified surface temperature does have a significant impact on the longer time scale forecasts, the prediction of the surface temperature, and hence the atmospheric streamfunctions by a linear operator, are still limited, even with a coupled model.

The analyses here also found that the effect of transient fluxes cannot be simply represented by a simple damping relationship. An empirical method for improving long term forecasts was suggested. This empirical method parameterizes all the "unknown forcings" in terms of all available waves via a least-square fit. The parameterization is a time invariant, but spatially dependent, coefficient matrix. The matrix is developed empirically from calculated forcings and "observed" flows. The formulation can be applied in practice for parameterizing all excluded forcings (from the residuals), even if we don't have any knowledge of how these forcings are formulated. The parameterized forecasts show significant improvement in the dependent data set, but the skill is not as high in the independent set. We speculate that the reason that the skill is limited for the independent set is because we chose the parameterization from all data indiscriminately rather than applying eq. (23) to various weather regimes. A classification method, perhaps similar to Vautard and Legras (1988), could be adopted to yield more significant relationships between time-mean and transient flows. The results of this study strictly apply to the simple two-level GCM study; however our conclusions provide guidance, we hope, to the development of more complex anomaly models.

## 7. Acknowledgments

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## 8. Appendix

### 8.1. *Baroclinic and coupled model equations*

The baroclinic anomaly equations (3) and (4) can be written in the following matrix form:

$$\frac{\partial A\psi'}{\partial t}+B\psi'=F, \tag{A1}$$

Here

$$\psi'=\begin{pmatrix}\psi_1'\\ \psi_3'\end{pmatrix},$$

$$F=\begin{pmatrix}F_1\\ F_3\end{pmatrix},$$

$$A=\begin{pmatrix}\nabla^2-\lambda^2, & \lambda^2\\ \lambda^2, & \nabla^2-\lambda^2\end{pmatrix}, \tag{A2}$$

$$B=\begin{pmatrix}L_1, & \mathscr{L}_1\\ \mathscr{L}_3, & L_3\end{pmatrix}. \tag{A3}$$

and

$$F_1=-2\lambda^2K_T\gamma T'-J_{t1}', \tag{A4}$$

$$F_3=2\lambda^2K_T\gamma T'-J_{t3}', \tag{A5}$$

$$J_{t1}'=-J'(\psi_1,q_1'), \tag{A6}$$

$$J_{t3}'=-J'(\psi_3',q_3'), \tag{A7}$$

$$L_1\psi_1'=J(\bar\psi_1,\nabla^2\psi_1'-\lambda^2\psi_1')+J(\psi_1',\bar q_1)+K\nabla^6\psi_1'-\lambda^2(K_T+K_R)\,\psi_1', \tag{A8}$$

$$L_3\psi_3'=J(\bar\psi_3,\nabla^2\psi_3'-\lambda^2\psi_3')+J(\psi_3',\bar q_3)+K\nabla^6\psi_3'+K_s\nabla^2\psi_3'-\lambda^2(K_T+K_R)\,\psi_3', \tag{A9}$$

$$\mathscr{L}_1\psi_3'=J(\bar\psi_1,\lambda^2\psi_3'+\lambda^2(K_T+K_R)\,\psi_3', \tag{A10}$$

$$\mathscr{L}_3\psi_1'=J(\bar\psi_3,\lambda^2\psi_1'+\lambda^2(K_T+K_R)\,\psi_1'. \tag{A11}$$

By using the same procedures described previously for the barotropic model, the baroclinic linear prediction model becomes:

$$\hat\psi=[A+\tfrac{1}{2}\Delta t(Q-1)B]^{-1}\times\left\{(A-\Delta tB)\,\psi'(0)+\Delta tF(0)+\frac{\Delta t}{Q}\sum_{n=1}^{Q-1}(Q-n)[F(n\,\Delta t)-B\psi''(n\,\Delta t)]\right\}. \tag{A12}$$

Similarly, if the surface temperature equation is added to the above model in order to make the coupled atmosphere-ocean model then the coupled linear prediction model can be written as

$$\hat\psi_c=\left[A_c+\frac{\Delta t}{2}(Q-1)\,B_c\right]^{-1}\times\left\{(A_c-\Delta tB_c)\,\psi_c'(0)+\Delta tF_g(0)+\frac{\Delta t}{Q}\sum_{n=1}^{Q-1}(Q-n)[F_g(n\,\Delta t)-B_c\psi_c''(n\,\Delta t)]\right\}, \tag{A13}$$

where

$$A_c = \begin{pmatrix} \nabla^2 - \lambda^2, & \lambda^2, & 0 \\ \lambda^2, & \nabla^2 - \lambda^2, & 0 \\ 0, & 0, & \gamma \end{pmatrix},$$

$$B_c = \begin{pmatrix} L_1, & \mathscr{L}_1, & 2\lambda^2 K_T \gamma \\ \mathscr{L}_3, & L_3, & -2\lambda^2 K_T \gamma \\ -\frac{1}{2}K_g, & \frac{1}{2}K_g, & \gamma(K\nabla^4 + K_g) \end{pmatrix},$$

$$\psi_c = \begin{pmatrix} \psi'_1 \\ \psi'_3 \\ T' \end{pmatrix},$$

$$F_g = \begin{pmatrix} -J'_{t1} \\ -J'_{t3} \\ 0 \end{pmatrix}.$$

### *8.2. Parameterized linear operators*

The baroclinic parameterized model in the baroclinic case is written as (A1), except that the vector $F$ is replaced by $\alpha_B \psi'$, where

$$\alpha_B = \langle F\psi'^{T} \rangle \langle \psi'\psi'^{T} \rangle^{-1}.$$

## REFERENCES


Chao, J. P., Guo, Y. F. and Xin, R. N. 1982. A theory and method of long-range numerical weather forecasts. *J. Met. Soc. Japan 60*, 282–291.

Chao, J. P., Wang, X. X., Chen, Y. Y. and Wang, L. Z. 1986. Monthly and seasonal numerical forecasts by using the anomaly ocean-atmosphere coupled filtered model. *Advances in Atmospheric Sciences 3*, 139–149.

Chen, S.-C. and Trenberth, K. E. 1988. Forced planetary waves in the Northern Hemisphere winter: Wave-coupled orographic and thermal forcings, *J. Atmos. Sci. 45*, 682–704.

Group of Long-Range Numerical Weather Forecasting, 1977: On the physical basis of a model of long-range numerical weather forecasting. *Scientia Sinica 20*, 377–390. —, 1979: A filtering method for long-range numerical forecasting. *Scientia Sinica 22*, 661–674.

Green, J. S. A. 1970. Transfer properties of the large-scale eddies, and the general circulation of the atmosphere. *Q. J. Roy. Met. Soc. 96*, 157–184.

Held, I. M. 1983. Stationary and quasi-stationary eddies in the extratropical troposphere: Theory. In: *Large-scale dynamical processes in the atmosphere*, (Eds. B. J. Hoskins and R. P. Pearce), Academic Press, 127–168.

Hendon, H. H. 1986. Time-mean flow and variability in a nonlinear model of the atmosphere with orographic forcing. *J. Atmos. Sci. 43*, 433–448.

Hoskins, B. J. 1983. Modeling of the transient eddies and their feedback on the mean flow. In: *Large-scale dynamical processes in the atmosphere*, (Eds. B. J. Hoskins and R. P. Pearce), Academic Press, 169–199.

Lau, N.-C. 1979. The observed structure of tropospheric stationary waves and the local balances of vorticity and heat. *J. Atmos. Sci. 36*, 996–1016.

Lindzen, R. S. and Farrell, B. 1980. The role of polar regions in global climate, and a new parameterization of global heat transport. *Mon. Wea. Rev. 108*, 2064–2079.

Miller, A. J. and Roads, J. O. 1990. A simplified coupled model of extended-range predictability. *J. Clim. 3*, in press.

Miyakoda, K. and Chao, J. P. 1982. Essay on dynamical long-range forecasts of atmospheric circulation. *J. Met. Soc. Japan 60*, 290–307.

Nigam, S., Held, I. M. and Lyons, S. W. 1986. Linear simulation of the stationary eddies in a GCM. Part I. The "no-mountain" model. *J. Atmos. Sci. 43*, 2944–2961.

Nigram, S., Held, I. M. and Lyons, S. W. 1988. Linear simulation of the stationary eddies in a GCM. Part II. The "mountain" model. *J. Atmos. Sci. 45*, 1433–1452.

Opsteegh, J. D. and Vernekar, A. D. 1982. A simulation of the January standing wave pattern including the effects of transient eddies: *J. Atmos. Sci. 39*, 734–744.

Reinhold, B. B. and Pierrehumbert, R. P. 1982. Dynamics of weather regimes: Quasi-stationary waves and blocking. *Mon. Wea. Rev. 110*, 1105–1145.

Roads, J. O. 1987. Predictability in the extended range. *J. Atmos. Sci. 44*, 3495–3527.

Roads, J. O. 1989. Linear and nonlinear atmospheric responses to surface temperature anomalies. *J. Clim. 2*, 1014–1046.

Roads, J. O. 1990. Linear model predictions of time averages. *J. Clim. 3*, 317–336.

Simmons, A. J., Wallace, J. M. and Branstator, G. W. 1983. Barotropic wave propagation and instability, and atmospheric teleconnection patterns. *J. Atmos. Sci. 40*, 1363–1392.

Stone, P. H. 1972. A simplified radiative-dynamical model for the static stability of rotating atmospheres. *J. Atmos. Sci. 29*, 405–418.

Valdes, P. J. and Hoskins, B. J. 1989. Linear stationary wave simulations of the time-mean climatological flow. *J. Atmos. Sci. 46*, 2509–2527.

Vautard, R., Legras, B. and Déqué, M. 1988. On the source of midlatitude low-frequency variability. Part I: Statistical approach to persistence. *J. Atmos. Sci. 45*, 2811–2843.

Vautard, R. and Legras, B. 1988. On the source of midlatitude low-frequency variability. Part II: Nonlinear equilibration of weather regimes. *J. Atmos. Sci. 45*, 2845–2867.

Youngblut, C. and Sasamori, T. 1980. The nonlinear effects of transient and stationary eddies on the winter mean circulation. *J. Atmos. Sci. 37*, 1944–1957.