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# The large-scale mixing and the estuarine circulation in the Skagerrak; calculations from observations of the salinity and velocity fields

By JOHAN RODHE, *Department of Oceanography, University of Gothenburg, Box 4038, S-40040 Gothenburg, Sweden*

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ABSTRACT

The flow in the estuarine circulation and the large-scale mixing in the Skagerrak are calculated from repeated observations of the salinity and velocity fields at two sections. The calculations are performed with salinity and time as independent variables and improved by the use of volume and salt conservation. The estimated magnitude of the upwelling of deep water associated with the estuarine circulation in the Skagerrak is $4 \cdot 10^5$ m$^3$ s$^{-1}$. This corresponds to a horizontally-averaged entrainment velocity of 2 m day$^{-1}$. The lateral mixing of salt from the open sea into the low-salinity water along the Swedish and Norwegian coasts is calculated to be about 1.3 kg s$^{-1}$ m$^{-1}$. The results of this paper can be used to calculate transports of other constituents in the sea water.

## 1. Introduction

The Skagerrak is the inner end of the Norwegian Trench (see Fig. 1). The trench cuts into the shelf sea (The North Sea) from the Norwegian Sea along the coast of Norway to the Baltic entrance between Sweden and Denmark. The trench has the topography of a large fjord. The sill depth is about 270 m and the maximum depth is a little more than 700 m. At the inner end, the trench has a narrow continuation into the Kattegat with a depth of about 100 m. Svansson (1975) has described the physical and chemical oceanography of the Skagerrak. In Rodhe (1987), there is a discussion of the velocity and salinity fields at two sections (see Fig. 1) in the Skagerrak using the same observations as those used here.

The annual mean fresh water supply to the Baltic, including the Kattegat, is $\sim 1.5 \cdot 10^4$ m$^3$ s$^{-1}$. To the Skagerrak there is an addition of $\sim 0.2 \cdot 10^4$ m$^3$ s$^{-1}$.

The salinity in the Skagerrak ranges from 20–25‰ in a thin surface layer in the south-east corner to about 35‰ in the deep water. The main part of the low salinity water follows the Swedish coast in the Baltic Current and the Norwegian coast in the Norwegian Coastal Current. Outside these currents there is a cyclonic circulation (on the average) with North Sea and Atlantic water entering along the Danish coast and leaving along the Norwegian coast. Aure and Sætre (1981) discuss how the different wind situations over the area can give rise to blockings of the outflow of Baltic water followed by outbreaks.

Fig. 2, from Rodhe (1987) show the mean salinity at section 1, its standard deviation and the mean value of the velocity component perpendicular to that section. From these figures, we note that the salinity distribution is such that waters of all salinities up to 35‰ are at times present over large areas in the surface layer and are thus available to wind mixing. The velocities in the horizontal circulation are such that the residence time for the water in the Skagerrak is of order 100 days. This means that wind mixing in the area could be treated as a statistically constant process, since the time scale of the wind

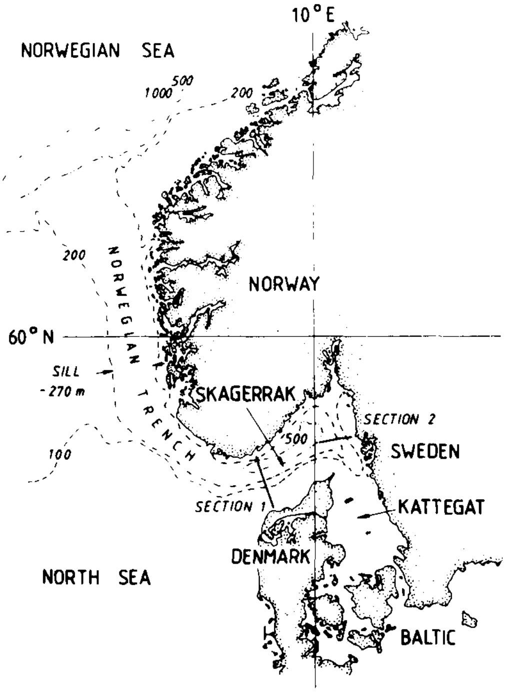


*Fig. 1.* Map over the eastern part of the North Sea including the Skagerrak with the observed sections 1 and 2.

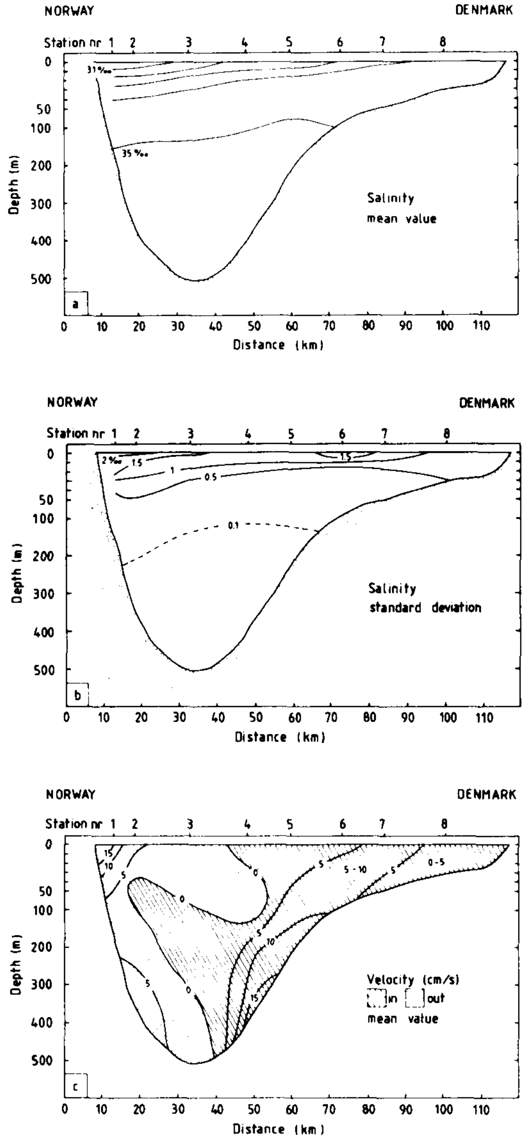


*Fig. 2.* Calculations from 22 observations of velocity and salinity at section 1: (a) salinity, mean value, (b) salinity, standard deviation and (c) velocity component perpendicular to the section, mean value, from Rodhe (1987).

variations are typically of order a week, i.e., much shorter than the residence time of the water in the Skagerrak.

Knowledge of large scale mixing in the sea is generally achieved by indirect methods. One way is to apply conservation criteria to tracer distributions. A simple and widely used example is the use of the Knudsen relations at a closed section in an estuary. In this case, observations of the salinity distribution, together with knowledge of the fresh water supply, make it possible to calculate the mixing of salt and the estuarine circulation inside the section. (The magnitude of the estuarine circulation is here defined as the flow of high salinity water, which is mixed up into the fresher surface water before it leaves the estuary.) It is possible to use the Knudsen relations only if there is a two layer flow through the section and the salinity and flow variations are uncorrelated. However, in general both velocity and salinity are functions of time and, in addition, salinity and velocity fluctuations can be expected to be correlated. To calculate the estuarine circulation and the mixing under these circumstances we have to make observations of both the velocity and the salinity with high resolution in time and space. At a wide section, it is hardly possible to make such observations with sufficiently high accuracy using the observation methods available at present. In this paper, we shall use conservation of volume and salt together with observations of the velocity field in a way that could be thought of as using "generalized Knudsen relations".

Walin (1977) relates the observations of the velocity at a section to the salinity instead of the horizontal and the vertical coordinates when describing the flow of water to estuaries. One advantage in doing this is that it becomes simple to include information such as conservation of water and salt, knowledge of the type of the mixing etc., into the calculations to improve the estimates based on the observations.

The calculations presented in this paper are based on the detailed observations of currents and hydrography in the Skagerrak reported in Larsson and Rodhe (1979) and discussed in Rodhe (1987).

In Section 2, there is a review of a framework for describing the water exchange in an estuary using salinity and time as independent variables. This is followed by a discussion of how the outflow through a control section depends on the salinity at different sections in the Baltic entrance region. In Section 3, the basic function $M(s)$ is determined at two sections in the Skagerrak. In Section 4, the magnitude of the estuarine circulation is calculated. In Section 5 the diffusive flux of salt between the coastal water and the open sea is calculated. In Section 6 the result is discussed in relation to the salinity distribution and in Section 7 there is a summary and some concluding remarks.

## 2. Salinity and time as independent variables

Walin (1977) introduced a method to describe exchange and mixing in a part of an estuary using salinity and time as independent variables. He applied the scheme to the inflow of deep water to the Baltic inside the sill. The salinity of the deep water was varying whereas the surface layer salinity was practically constant. In the present paper we shall look at the outflow of low salinity surface water outside the sill. In our case the salinity of the outflow is varying whereas the deep water has a practically constant salinity. We shall use the Walin (1977) method with a slight change of definitions which makes it more suitable to our situation.

In Fig. 3, the basic functions $V(s,t)$, $M(s,t)$, $G(s,t)$, $Q_0(t)$ and $F(s,t)$, are defined. We are looking at integrated functions defined for a part of an estuary limited by the coast, a vertical control section, the sea surface and the isohaline surface $s$.

For convenience, we also define two functions $m(s,t)$ and $v(s,t)$ related to the basic functions $M(s,t)$ and $V(s,t)$ through the relations:

$$m(s,t) = \frac{\partial M(s,t)}{\partial s} \tag{2.1}$$

$$v(s,t) = \frac{\partial V(s,t)}{\partial s}. \tag{2.2}$$

$m(s,t)\cdot ds$ is thus the outflow of water in the salinity interval $s$ to $s+ds$ and $v(s,t)\cdot ds$ is the volume of water in the same interval.

The reader is referred to Walin (1977) for the derivation of the conservation equations for volume and salt. With our definitions these equations become:

$$\frac{\partial V(s,t)}{\partial t} = Q_0(t) - M(s,t) - G(s,t), \tag{2.3}$$

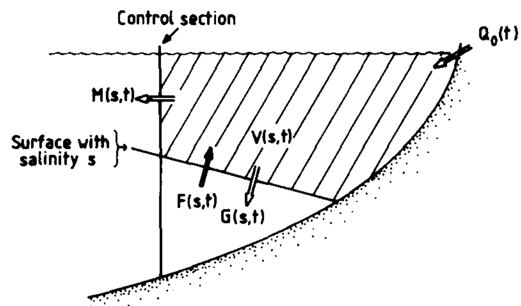


*Fig. 3.* Definition of the dependent integral variables in a part of an estuary, using salinity and time as independent variables. $V(s,t)$ is the volume of water with salinity lower than $s$. $M(s,t)$ is the volume transport, with salinity lower than $s$, out through the control section. $G(s,t)$ is the volume flow out through the isohaline surface $s$, i.e., the amount of water with salinity lower than $s$ which is, due to mixing, transformed to water of higher salinity. $F(s,t)$ is the diffusive flux of salt, per unit mass, in through the isohaline surface $s$. $Q_0(t)$ is the fresh water supply, expressed as volume flow.

$$\frac{\partial}{\partial t}\int_0^s s'\cdot v(s',t)\,\mathrm{d}s' = -\int_0^s s'\cdot m(s',t)\,\mathrm{d}s' - s\cdot G(s,t) + F(s,t). \quad (2.4)$$

The eqs. (2.3) and (2.4) relate the essentially non observable functions $G(s,t)$ and $F(s,t)$ to the functions $V(s,t)$, $M(s,t)$ and $Q_0(t)$ which, at least in principle, are possible to observe. The main advantage in using salinity and time as independent variables is that both the continuity eq. (2.3) and the salt diffusion eq. (2.4) are linear. This means that time averaging does not introduce any complications.

In the following, we shall use the time averages of these equations. We denote the time averaged functions as functions of salinity only. For a steady state mean the time averages of (2.3) and (2.4) become:

$$M(s) + G(s) = Q_0, \quad (2.5)$$

$$\int_0^s s'\cdot m(s')\,\mathrm{d}s' + s\cdot G(s) - F(s) = 0. \quad (2.6)$$

We can now formulate the conservation of volume and salt in the entire estuary. If $s = s_{\max}$ is the highest salinity occurring at the control section, then conservation of volume becomes:

$$M(s) = Q_0, \quad \text{for } s \geqslant s_{\max}. \quad (2.7)$$

The conservation of salt requires the average net flow of salt through the control section to vanish:

$$\int_0^s s'\cdot m(s')\,\mathrm{d}s' = 0 \quad \text{for s} \geqslant s_{\max}. \quad (2.8)$$

Integrating by parts and using (2.7), the expression for the conservation of salt becomes:

$$\int_0^s M(s')\,\mathrm{d}s' = Q_0\cdot s \quad \text{for } s \geqslant s_{\max}. \quad (2.9)$$

Now we can use the introduced variables to make a quantitative definition of the estuarine circulation $Q_e$. It is defined as the amount of inflowing high salinity deep water that is mixed into the outflowing low salinity surface water, i.e., the maximum value of the cross isohaline flow, from high to low salinity, inside the control section under consideration. $Q_e$ is thus given by the maximum negative value of $G(s)$. According to (2.5) it can be formulated:

$$Q_e = \max(M(s)) - Q_0. \quad (2.10)$$

Before turning to the calculations of the mixing in the Skagerrak we illustrate the shape of the function $M(s)$ that could be expected at different sections crossing the Baltic entrance between the Baltic and the Skagerrak. In Fig. 4 there is a sketch of the salinity structure and the flow of water in the mixing process.

In the Skagerrak (section A in Fig. 4), outside the shallow and narrow areas, we have a thin surface layer of brackish water overlaying the more or less homohaline deep water. The wind mixing is important over the large surface area and the mixing results in entrainment of water into the outflowing surface layer. We can expect outward mean flow for all salinities lower than the maximum salinity as a result of the dominating baroclinic forcing. $M(s)$ will thus increase monotonously up to the highest salinity and drop to $Q_0$ at the deep water salinity, see Fig. 5, curve A.

In the Danish Sounds (section B in Fig. 4), mixing is generated in both layers resulting in a two-way "entrainment flow". We can expect time variations of the flow and the salinity in both layers. A transient barotropic flow dominates here and water of all salinities can be transported in both directions through this region. The result is an increasing $M(s)$ at low salinities and a decreasing $M(s)$ at high salinities, see Fig. 5, curve B.

Inside the shallow Danish Sounds (section C in Fig. 4), there is a thick homogeneous surface layer with constant salinity. Incoming water with higher salinity than the surface salinity flows down the sloping bottom in an energetic gravity

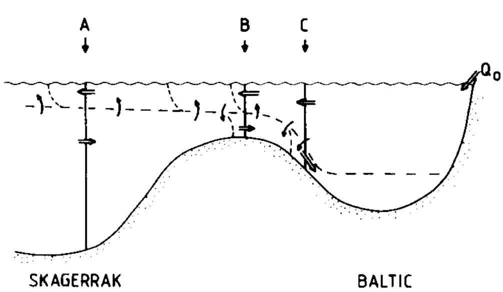


*Fig. 4.* Sketch of the salinity structure and the mean flow directions in the estuarine mixing process between the Skagerrak and the Baltic. A, B and C are the positions of three control sections discussed in the text.

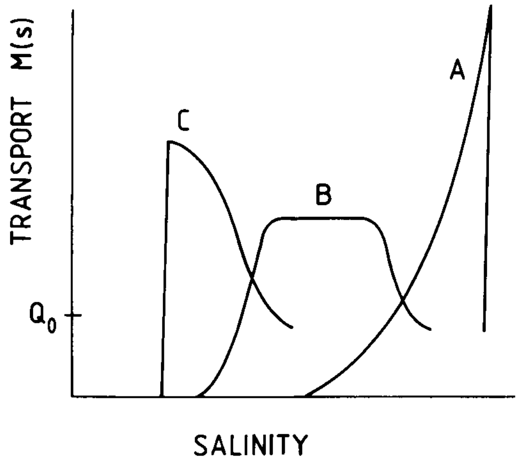


*Fig. 5.* Expected functions $M(s)$ at different positions, see Fig. 4, in the Baltic entrance area.

current, which entrains water from the surface layer. Once the high saline water is inside the sill it is trapped. $M(s)$ at a cross section in this area has a jump from zero to its maximum value at the surface salinity. For higher salinities there is a monotonous decrease of $M(s)$, see Fig. 5, curve C. $M(s)$ with this shape is observed in this area, see Walin (1981) (note his different definition of $M(s)$).

## 3. Determination of the transport function $M(s)$

Simultaneous observations of salinity, from water bottle samples, and velocity, using gelatine pendulums, have been performed at two sections in the Skagerrak, Fig. 1 (see Larsson and Rodhe (1979) for details). The observations were repeated 22 times (section 1) and 9 times (section 2) in the years 1975 to 1977. The number of velocity observations during each transect was about 80 (section 1) and 60 (section 2). From each set of observations a standard matrix representing the section was constructed by linear vertical interpolations and filling up to the surface with the uppermost and down to the bottom with the lowermost value. For the transport calculations every point in the matrix was assigned an area. The transport and salinity matrices were then used to construct the functions $M(s, t_n)$ ($n$ stands for the different transects). The mean transport functions, $M(s)$, were calculated as the average of $M(s, t_n)$. Two different mean functions were calculated for section 1. One using all observations and one using only the observations that were carried out in connection with those at section 2.

The mean functions $M(s)$ are presented in Fig. 6. The calculations are carried up to $s = 35‰$. We note that the deep water salinity is $35.0 \pm 0.2‰$ and that the circulation of water in this salinity range has little to do with the estuarine circulation. To show the variability of $M(s, t_n)$, the standard deviation of the mean for the calculations from section 2 is inserted. In addition, Table 1 shows $M(s, t_n)$ for a selection of salinities at section 2. We note here that, at this section, there are no observations from the last third of the year. However, since the seasonal variations

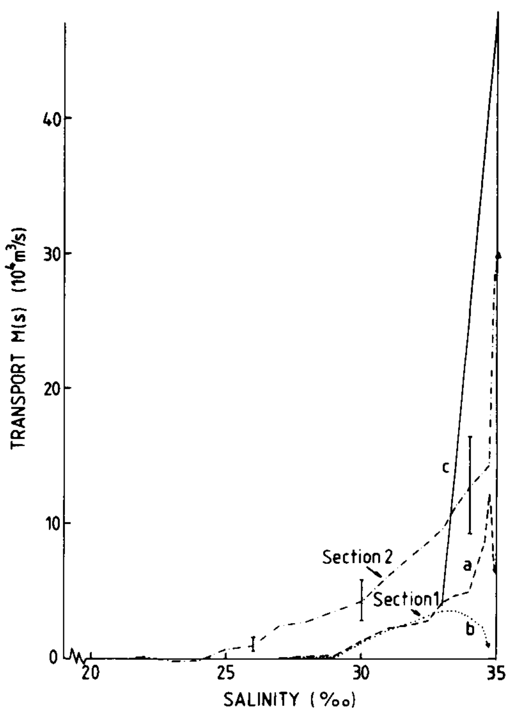


*Fig. 6.* Calculated functions $M(s)$ at section 1 and 2. Curve 1a and b are mean values of two different samples from the observations at section 1, 1c is an estimate of $M(s)$ discussed in the text. The standard deviation of the mean is inserted for some salinities in the curve representing section 2.

Table 1. *Transport $M(s,t)$ through Section 2 and wind velocities observed on the ship at the beginning and the end of the transects. Positive value on $M(s,t)$ means out from the Baltic (northward). Units: $10^3$ $m^3$ $s^{-1}$ for $M(s,t)$ and m $s^{-1}$ for wind velocities*

| | $M(s,t)$ | | | | | | Wind | |
|---|---|---|---|---|---|---|---|---|
| Date / Salinity | 25 | 27 | 29 | 31 | 33 | 35 | beginning | end |
| 25 Mar 1976 | −8 | −14 | −20 | −7 | −30 | −266 | SE 16 | SE 10 |
| 29 Apr 1976 | 0 | 0 | 27 | 85 | 203 | 314 | W 12 | NW 12 |
| 7 May 1976 | 22 | 46 | 6 | 0 | 5 | 281 | NW 7 | calm |
| 11 Jun 1976 | −6 | 52 | 110 | 126 | 140 | 110 | W 12 | W 8 |
| 5 Jul 1976 | 0 | 0 | 0 | 50 | 113 | 197 | W 7 | SW 8 |
| 14 Jul 1976 | 15 | 38 | 57 | 86 | 214 | 448 | S 9 | S 7 |
| 28 Aug 1976 | 40 | 50 | 46 | 62 | 15 | 15 | E 12 | NE 12 |
| 21 Jan 1977 | 10 | 29 | 55 | 72 | 87 | 326 | E 6 | E 7 |
| 24 Apr 1977 | 0 | 23 | 34 | 85 | 123 | 1247 | W 5 | W 7 |

in the large scale circulation in the Skagerrak seem to be of much less importance than the variations on the time scale of the local weather, a week or so (see Rodhe, 1987), we do not think this is serious.

Let us now analyse the calculated transport functions $M(s)$. We start with the inner section, no. 2. This section is not closed over the estuary, but due to the basic cyclonic circulation in the area, we can expect that the main part of the low salinity outflow from the Baltic passes through this section. The general character of the function $M(s)$, see Fig. 6, is what we could expect on this side of the sill to the Baltic. $M(s)$ is increasing up to the maximum salinity, showing net outflow of water of all salinities below the deep water salinity. Apart from the random wiggling of the curve, two distinct breaking points can be identified on the $M(s)$ curve. The transport starts abruptly at $s = 24‰$ and just below $s = 35‰$ there is another breaking point. The high values of $M(s)$ close to the maximum salinity is a measure of the transport of deep water in the basic cyclonic circulation in the area (cf. eq. (2.10)). At $s = 30‰$ there is a pronounced change in the slope of $M(s)$. Our interpretation is that this is an indication of the limit of the baroclinic coastal current, i.e., the Baltic Current at section 2 having salinities in the range 24 to 30‰. This does not imply that waters of these salinities are entirely confined to this current.

The fresh water outflow $Q_{0i}$ through the $i$th section can be calculated from the salt balance eq. (2.9). For section 2, we obtain:

$$Q_{02} = 1.7 \cdot 10^4 \text{ m}^3 \text{ s}^{-1}.$$

Here we have taken $s_{max} = 35‰$ but ignored the high values of $M(s)$ just below that salinity. The estimated value is a little higher than the mean fresh water supply to the Baltic and the Kattegat ($\sim 1.5 \cdot 10^4$ m$^3$ s$^{-1}$). The difference between the estimate of the fresh water flow through section 2 and the supply to the Baltic is about half of the fresh water supply to the southern North Sea, see Otto (1983). The amount of fresh water from that area entering the cyclonic circulation in the Skagerrak is difficult to estimate in any other way.

Now we turn to the results from section 1. This section closes the estuary. The conditions (2.5) and (2.9) should be satisfied by $M(s)$ at this section. Here we have two sets of calculations (see above). Both estimates of $M(s)$ start abruptly at $s = 29‰$. After this point they have approximately the same constant slope to $s = 33‰$. The behaviour up to this salinity is the same as that of the $M(s)$ function at section 2. Above this salinity the two curves differ from each other and from the structure of $M(s)$ at section 2, where $M(s)$ and its derivative increase with salinity. Furthermore, the conditions (2.5) and (2.9) are far from being satisfied. There is obviously a large missing outflow of water with salinity in the interval 33 to 35‰. This is probably due to poor horizontal resolution and technical problems with the current measurements at the outer edge of the

Norwegian coastal current. The current is very narrow here and the bottom slope is extremely steep.

Now we shall proceed with the calculations in a way that could be thought of as a generalization of the Knudsen relations. In these, we make a very strong assumption about the salinity distribution, namely the two-layer assumption that the inflowing water has one constant salinity and the outflowing water has another constant salinity. In this generalization, we shall make other assumptions about the correlation between the salinity and the transport, i.e., the shape of $M(s)$, at section 1. These assumptions are based on (i) the results from section 2, (ii) the discussion of the shape of $M(s)$ at different sections crossing the Baltic entrance and (iii) some consistent results achieved at section 1. The assumptions are as follows.

(a) Net outflow through the section of water of all salinities up to the maximum salinity, 35‰, i.e., a monotonously increasing $M(s)$. This is in accordance with the discussion of the shape of $M(s)$ and with the observations from section 2.

(b) The estimated $M(s)$ follows the observed one, 1a in Fig. 6, up to $s = 33‰$. The reasons to make this assumption are that the shape of $M(s)$ here is similar to the shape of $M(s)$ at the other section and that the two different calculations, 1a and 1b in Fig. 6, are practically identical below $s = 33‰$.

(c) From $s = 33‰$ we choose a straight line in such a way that the conservation of salt (2.9) is satisfied. The resulting end points of the line are:

$$M(s) = 4 \cdot 10^4 \text{ m}^3 \text{ s}^{-1} \quad \text{for } s = 33‰$$

$$M(s) = 48 \cdot 10^4 \text{ m}^3 \text{ s}^{-1} \quad \text{for } s = 35‰.$$

This choice is not the only possible since conservation of salt only determines the integral of the function.

Is this estimate of $M(s)$ at the highest salinities realistic? There is no reason to believe that the general shape of the $M(s)$ at this section should differ from that of $M(s)$ at section 2, i.e., we can expect the slope to increase in some way towards increasing salinity. Further, any other realistic function, with the same integral as eq. (2.9) requires, would only lead to an appreciable difference from the one chosen close to $s = 35‰$. The function $M(s)$ constructed this way is drawn in Fig. 6, curve 1c. This will be used in the discussions below.

We shall now use the functions $M(s)$ from the two sections to calculate the cross isohaline flow, $G(s)$, and the mixing of salt, $F(s)$, inside section 1 and section 2. We shall then use these results to calculate the upwelling of deep water to the surface layer in the Skagerrak, i.e., the estuarine circulation, and the lateral mixing between the coastal area and the open sea between the two sections.

## 4. Calculation of the estuarine circulation

The magnitude of the estuarine circulation inside a section is defined in (2.10). Using this we obtain for the estuarine circulation inside section 1, $Q_{e1}$:

$$Q_{e1} = 4.6 \cdot 10^5 \text{ m}^3 \text{ s}^{-1}.$$

Here we have taken the fresh water supply inside the section to be $0.2 \cdot 10^5$ m$^3$ s$^{-1}$ and the maximum salinity to be 35‰.

To calculate the estuarine circulation inside section 2 we exclude the peak on $M(s)$ just below 35‰. The reason for this is that $M(s)$ just below $s = 35‰$ measures the basic cyclonic circulation of deep water in the Skagerrak and not the flow associated with the estuarine circulation. We arrive at the following estimate of the magnitude of the estuarine circulation inside section 2, $Q_{e2}$:

$$Q_{e2} = 1.3 \cdot 10^5 \text{ m}^3 \text{ s}^{-1}.$$

We have taken the fresh water supply inside the section to be $0.2 \cdot 10^5$ m$^3$ s$^{-1}$ and the maximum salinity to be 35‰.

The increase in the estuarine circulation between the sections 2 and 1 is thus $3.3 \cdot 10^5$ m$^3$ s$^{-1}$. This amount of deep water is mixed into the low salinity surface water between the two sections. If we want to include all Skagerrak we have to estimate the increase in the estuarine circulation between the Kattegat and section 2. To do so we assume that the mixing per unit length in the direction of the coast is the same all the way. We take the distance between the border to the Kattegat and section 2 to be between a third and a fourth (dependent on how far from the coast line we measure the distance) of the distance between the two sections and arrive at an estimate of between 4.1 and $4.4 \cdot 10^5$ $m^3\ s^{-1}$ for that part of the estuarine circulation inside section 1 which occurs in the Skagerrak. The rest of the estuarine circulation inside section 1, i.e., between 0.2 and $0.5 \cdot 10^5\ m^3\ s^{-1}$, should be a very crude estimate of the magnitude of the inflow of high salinity water to the Kattegat. Other estimates of the rate of renewal of the deep water in the Kattegat give results in the same interval, see, e.g., Stigebrandt (1983). This gives evidence that the constructed part of the function $M(s)$ at section 1 may be a good estimate.

The upwelling of deep water associated with the part of the estuarine circulation that occurs in the Skagerrak, $Q_{eS}$, is according to the calculations above estimated to:

$$Q_{eS} = 4 \cdot 10^5\ m^3\ s^{-1}.$$

If we assume that this is distributed over the main part of the surface, as the salinity distribution indicates, then the entrainment velocity, $u_e$, to the surface layer can be estimated to be:

$$u_e = 2 \cdot 10^{-5}\ m\ s^{-1}.$$

This corresponds to $\sim 2$ m day$^{-1}$.

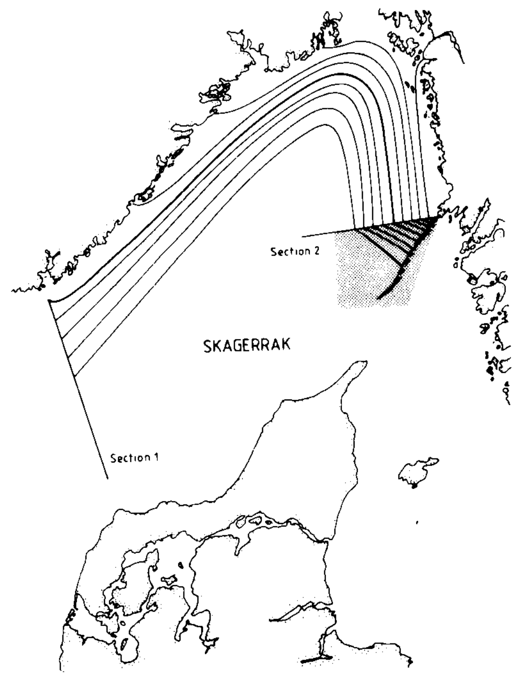


*Fig. 7.* A sketch of the salinity structure between the two sections used in the discussion of the mixing between the coastal water and the open sea. The shaded area shows a vertical section.

## 5. Calculation of the mixing between the coastal water and the open sea

In this section, we shall use the determined basic functions $M(s)$ to calculate the diffusion of salt through the isohaline surfaces between the two sections in the Skagerrak, i.e., a local $F(s)$. Fig. 7 shows a sketch of the salinity distribution between the two sections. The isohaline surfaces are practically aligned with the outflow of Baltic water in the coastal current. This means that the diffusion through the isohaline surfaces could be thought of as the effect of a lateral mixing between the coastal water and the open sea, whether the mixing takes place mainly through the part of the salinity surface that is close to the sea surface or it is distributed over the entire surface below the coastal water.

To make the calculation of the diffusion of salt between the coastal water and the open sea we reformulate the conservation equations (2.5) and (2.6) for a volume in the coastal current. We look at a sub volume in the estuary defined by the vertical sections 1 and 2 together with the isohaline surface $s$, the sea surface and the bottom between these two sections. The conservation equations of water and salt becomes:

$$G(s) = Q_0 - M_1(s) + M_2(s) \tag{5.1}$$

$$F(s) - s \cdot G(s) = \int_0^s s' \cdot m_1(s')\,ds' - \int_0^s s' \cdot m_2(s')\,ds', \tag{5.2}$$

where $G(s)$ and $F(s)$ are evaluated on the isohaline surface $s$ between the sections.

$Q_0$ is here the supply of fresh water between section 1 and 2. The indices 1 and 2 on the flow functions refer to the sections 1 and 2 respectively. We recall that the definitions of $M(s)$ and $m(s)$ are such that $M_1(s)$ and $m_1(s)$ are positive for flow out from the volume under consideration here, whereas $M_2(s)$ and $m_2(s)$ are positive for flow into the volume.

If we use (5.1) in (5.2), then the salt diffusion equation can be simplified to:

$$F(s) = s \cdot Q_0 + \int_0^s (M_2(s') - M_1(s'))\,ds'. \tag{5.3}$$

Table 2. *The volume flow G(s) and the diffusive flux of salt F(s) trough different salinity surfaces between the two sections in the Skagerrak*

| Salinity ‰ | Volume flow $G(s)$ $10^4$ m$^3$ s$^{-1}$ | Diffusive salt flux $F(s)$ $10^5$ kg s$^{-1}$ |
|---|---|---|
| 25 | 1.0 | 0.5 |
| 26 | 1.2 | 0.6 |
| 27 | 2.7 | 0.8 |
| 28 | 2.8 | 1.1 |
| 29 | 3.5 | 1.4 |
| 30 | 3.2 | 1.8 |
| 31 | 4.1 | 2.2 |
| 32 | 5.5 | 2.6 |
| 33 | 5.8 | 3.2 |

Results of calculations of the volume flow $G(s)$ and the diffusion of salt $F(s)$ through different salinity surfaces between section 1 and section 2 are given in Table 2. In the calculations $Q_0$ is taken to be $2 \cdot 10^3$ m$^3$ s$^{-1}$. (We note that $F(s)$ gives the salt flux numerically correct when SI units are used and the salinity is given in ‰.) We have performed the calculations in a salinity interval where we can use $M_1(s)$ based on the observations. Looking at Fig. 6 in relation to eq. (5.3) it becomes evident that $F(s)$ reaches its maximum value for a salinity just above $s = 33‰$, i.e., the salinity where we stopped the calculation. (Here we have to use curve c in Fig. 6.)

## 6. Discussion of the results of the mixing calculation

The observed changes in the salinity distribution and the function $M(s)$ downstream, i.e., from section 2 to section 1, is an effect of the mixing between the two sections. We shall now use some observations of these changes in a discussion of the results of the mixing, presented in Table 2, and estimate the diffusion of salt in an Eulerian sense by transforming the result to diffusion per unit length along the coast.

The diffusion is a surface process, therefore, we start by discussing how the area of the isohaline surfaces depends on the salinity in the region under consideration. From the sketch of the salinity distribution in Fig. 7, we see that variations with salinity of the area of the salinity surfaces depends on the salinity gradient along the coast and on the gradient out from the coast. But, since the coastal current is narrow and the salinity variations in the lateral direction has a more or less front like structure, we realise that the areas of the salinity surfaces are essentially proportional to how far along the coast they reach. If we, on the other hand, only consider that part of the salinity surfaces which is close to the sea surface to be important for the mixing, then this proportionality becomes even more strict.

The salinity distribution between the sections, sketched in Fig. 7, is not stationary. Instead, the salinity pattern moves up and down along the coast so that the area between the sections of a salinity surface changes with time. Which of the salinity surfaces that has its downstream end point at a section is given by the minimum salinity at the section. Fig. 8 shows the frequency distribution of the minimum salinity at the two sections. From this we can make the following conclusions.

(i) The surfaces with salinities above 33‰ reaches always all the way between the sections.

(ii) For salinities below 33‰, the mean exposed length along the coast of the salinity surfaces increases with salinity.

(iii) The mean increase in salinity between the sections is some 4 to 5‰.

The last conclusion is consistent with the observed change in the function $M(s)$ from section 2 to section 1. From Fig. 6, we see that in the

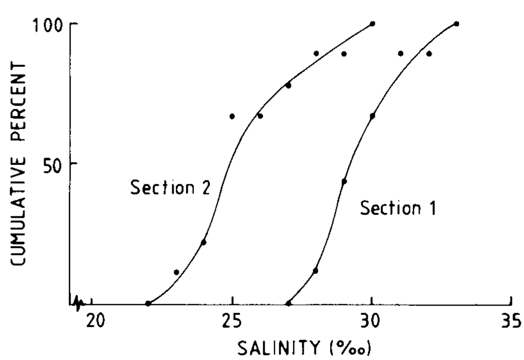


*Fig. 8.* Cumulative frequency of the lowest observed salinity at section 1 and 2.

low salinity range, there is an approximate relationship:

$$M_2(s) \approx M_1(s + \Delta s)$$

for a salinity difference, $\Delta s$, of 4–5‰.

According to (i) we can estimate the lateral mixing of salt per unit length along the coast, $F_L$, by dividing the diffusion of salt, $F(s)$, for $s = 33‰$ by the distance between the sections (about $2.5 \cdot 10^5$ m). According to Table 2 we then arrive at:

$$F_L = 1.3 \text{ kg s}^{-1} \text{ m}^{-1}.$$

The order of magnitude of the increase with salinity of $F(s)$ in Table 2 is in accordance with the estimate above. For the salinities just below 33‰ the increase per ‰ is about a sixth of the maximum $F(s)$. If the increase should be completely explained by an increase with salinity of the mean exposed length along the coast of the salinity surfaces, then, according to (ii) and (iii) we should expect the increase per ‰ to be between a fourth and a fifth of the maximum $F(s)$.

Our conclusion is that we have achieved a reliable estimate of the lateral mixing of salt into the outflowing low saline coastal current.

## 7. Summary and discussion

We have used repeated synoptic observations of salinity and velocity at two sections in the Skagerrak to calculate the magnitude of the flow in the estuarine circulation and the large scale mixing of salt in the area. Using salinity and time as independent variables, volume and salt conservation could be included to improve the estimates. The estimated magnitude of the flow in the upwelling of deep water to the surface layer, associated with estuarine circulation in the Skagerrak, is $4 \cdot 10^5$ m$^3$ s$^{-1}$. This corresponds to a horizontally averaged entrainment velocity of 2 m day$^{-1}$. Calculations of the cross isohaline flow and mixing of salt in the coastal water between the two sections are performed. From these the lateral (perpendicular to the outflowing low salinity water along the Swedish and Norwegian coasts) diffusion of salt is estimated to 1.3 kg s$^{-1}$ m$^{-1}$.

The results presented in this paper could be used to calculate the large scale upwelling of other constituents in the sea water than salt and the mixing of them between the coastal area and the open sea according to the theory by Walin (1977). To do so the concentration of the constituent should be related to the salinity of the water. This relation can of course vary with time and location. The restrictions on the time and lengths scales are given by the mixing time scale and the advection length in the basic cyclonic circulation in the Skagerrak during this time. Since most of the mixing is likely to depend on the local winds of the day to day weather, a lower bound on the time scale should be a few weeks corresponding to a length scale of say 300 km (taking the velocity in the basic circulation to be 0.2 m s$^{-1}$ and the time to be 2 weeks).

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