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A note on the locus of a shelf front

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ABSTRACT

It is suggested that the locus of a shelf front is where the water depth is equal to the thickness of the tidal frictional bottom boundary layer. From this, it follows that the locus of the front should be given by a critical value of h/U_t, where h is the water depth and U_t is the tidal stream amplitude. This result differs from that suggested by Simpson and Hunter who argued that a critical value of h/U_t^3 determines the locus of the front. The parameterization suggested here implies that the locus of a shelf front (1) does not adjust in response to the seasonal heating cycle and (2) responds only weakly to the spring-neap cycle. Utilizing established empirical constants, we predict that the locus of a shelf front should be where $h/U_t \approx 80$ s. All these predicted features conform well with observed properties of shelf fronts. The bottom boundary layer thickness conformant with these results is equal to $\lambda(C_d \cdot U_t^2)^{1/2}/f$, where $\lambda = 0.20$ and $C_d = 0.003$.

1. Introduction

Thermal shelf fronts were first analysed by Simpson and Hunter (1974). On the offshore, deeper side of the front the water is thermally stratified while on the landward, shallower side, turbulence generated by the tides keeps the water vertically homogeneous. In essence, Simpson and Hunter suggested that a shelf front is located where a length formed from the tidal stream amplitude U_t and the buoyancy flux B through the sea surface attains a certain value. They argued that B may be considered as constant during the summer season. Under this premise, the locus of the front should be defined by a critical value of h/U_t^3, where h is the water depth.

Below it is suggested that the locus of a shelf front is where the tidal turbulent boundary layer becomes thinner than the water column and 'dives' below the sea surface. This is in accordance with the mixed layer dynamics in the seasonal pycnocline model developed by the present author (Stigebrandt, 1985). Thus, heating (positive buoyancy flux) should play no major dynamical role for the shelf front but acts mainly to create higher temperatures seaward of the front which make the front visible in, e.g., satellite infrared imagery.

It is well-known (e.g., Landau and Lifshitz, 1959) that fluctuating currents like the tides have frictional boundary layers at the sea floor with thicknesses $L_E = \lambda \cdot u_*/\omega$ where u_* is the friction velocity, $\rho \cdot u_*^2$ equals the bottom stress and ω is the angular frequency of the fluctuating current. This kind of frictional boundary layer is similar to the Ekman layer (Ekman, 1905) the thickness of which is $L_E = \lambda \cdot u_*/f$, where f is the Coriolis parameter. The empirical constant λ has recently been estimated to be equal to about 0.2, both for tidal currents (Mofjeld and Lavelle, 1984) and for wind-driven surface currents (Stigebrandt, 1985). It is apparent that the turbulent bottom layer caused by a fluctuating tidal current may in sufficiently deep water not reach the sea surface, while in sufficiently shallow water, the turbulent layer will fill the whole water column.

Under conditions with unstratified water and no buoyancy flux through the sea surface, L_E is the thickness of the frictional boundary layer at the air-sea interface caused by the wind stress. In periods with positive buoyancy flux through the sea surface, the vertical penetration of the wind-generated turbulence may be reduced by the buoyancy flux and the turbulent well-mixed surface layer may be thinner than L_E. In this case, the equilibrium thickness of the well-mixed layer is equal to the Monin-Obukhov length L_{MO}. Below we look more closely into the dynamics characterized by the two length-scales L_E and L_{MO} on continental shelves in the heating period.

The idea that the locus of a shelf front is where the water depth is equal to the thickness of the tidal frictional bottom boundary layer was briefly discussed by Garrett et al. (1978). The λ they used to estimate the thickness of the boundary layer ($\lambda = 0.4$, adopted from literature) led them to reject this possibility since it gave a thickness of the boundary layer about twice that observed at the locus of the front. However, the idea has later been put forward by several other authors (see Loder and Greenberg (1986)).

We should also mention the work of Weatherly and Martin (1978) who discussed and modelled the thickness of the benthic boundary layer in the presence of stratification. They proposed a boundary layer thickness for steady flow which involved the buoyancy frequency N. The mixed layer they observed often had its maximal thickness which was about half that of an Ekman layer with $\lambda = 0.4$ (a value they adopted from literature).

It is a strange coincidence that the boundary layer thicknesses observed both by Garrett et al. (1978) and Weatherly and Greenberg (1978)[1] under very different conditions, both happen to be half that of the expected Ekman layer thickness with $\lambda = 0.4$. This coincidence indeed points to the possibility that the λ-value used is overestimated by a factor of two.

2. Does Monin-Obukhov or Ekman dynamics determine the locus of shelf fronts?

The Monin-Obukhov length is $L_{MO} = 2m_0 u_*^3/B$, where m_0 is an empirical constant (~ 0.6, see, e.g., Stigebrandt, 1985) and u_* is the frictional velocity. If the turbulence is caused by tides, we expect $u_*^2 = C_d U_t^2$, where U_t is the depth mean tidal speed and C_d is the drag coefficient for tidal flow over the bottom ($C_d \sim 0.003$). In essence, Simpson and Hunter (1974) reasoned that the shelf front should be located where a Monin-Obukhov type of length, based upon the tidal stream amplitide, is equal to the water depth. Assuming $B =$ constant, it then follows that the locus of a front should be given by a critical value of h/U_t^3, as suggested by Simpson and Hunter.

If only heating contributes to the buoyancy flux, this may be computed from the formula $B = g\alpha H/(\rho C)$, where ρ is the density of the water, α is the coefficient of heat expansion, C is the specific heat, g is the acceleration of gravity and H is the heat flow through the sea surface. As typical values, one may use $B = 4 \cdot 10^{-8}$ [m^2 s^{-3}], which is obtained if $H = 100$ W m^{-2} at water temperatures typical for the Irish Sea in the heating period ($\alpha = 17 \cdot 10^{-5}$), and $\omega = 0.00014$ which applies to semi-diurnal tides.

L_{MO} varies tremendously with U_t, since it is proportional to the (friction velocity)^3, while L_E varies much less since it is directly proportional to the friction velocity. If we use the figures introduced above, we find that L_E is one or more orders of magnitude smaller than L_{MO} for the tidal velocities of interest (~ 1 m s^{-1}). It then follows that primarily the penetration depth of the turbulent boundary layer, given by L_E, should determine the locus of a shelf front. The front will be found where the turbulent bottom layer 'dives' below the sea surface. This result is in conflict with Simpson and Hunter (1974). After all, they did explain the locus. This implies that the m_0 value corresponding to their critical value of h/U_t^3 must be at least one order of magnitude smaller than the value applicable to seasonal thermoclines.

Utilizing the recently determined $\lambda = 0.20$ and the generally accepted $C_d = 0.003$, we may predict that value of h/U_t which gives the water depth h ($= L_E$) where one should expect to find the tidal front. We obtain $h/U_t = \lambda C_d^{1/2}/\omega = 0.2 \cdot 0.003^{1/2}/0.00014 \sim 80$ s. Actually this value of h/U_t locates the fronts in the Irish Sea and the Gulf of Maine region at about the correct water depth, *cf.* Simpson and Hunter (1974) and Loder and Greenberg (1986), respectively. Note that here we have used well-founded values of the empirical constants λ and C_d to determine the critical value of h/U_t: no tuning using shelf front data is involved. Loder and Greenberg (1986) found that the h/U_t criterion located the fronts in the Gulf of Maine region better than the Simpson and Hunter criterion. In fact, Loder and Greenberg (1986) found that h/U_t should be equal to about 80 s which is what we actually predict.

3. The response of shelf fronts to time-varying forcing

The excursion of a front in the tidal spring-neap cycle can be estimated as follows. Like Simpson and Bowers (1981), we assume that the depth mean tidal stream amplitude varies over the spring-neap cycle (of frequency σ) according to

$$U_t = u_2(1 + e \cos(\sigma t)).$$

Below we use the subscripts s and n to denote mean spring and mean neap conditions, respectively. Since, according to Simpson and Hunter (1974), $h/U_t^3 = \text{const.}$, we obtain

$$h_s = h_n U_{ts}^3/U_{tn}^3 = h_n(1 + e)^3/(1 - e)^3.$$

For a representative $e = 0.3$ (Simpson and Bowers, 1981), one obtains $h_s/h_n = 6.4$. This 'equilibrium' value is about 5 times larger than the observed h_s/h_n. Simpson and Bowers argued that when the front advances into deeper water during increasing tidal stream amplitudes, the turbulence on the well-mixed side of the front not only has to mix the buoyancy supplied through the sea surface into the whole water column, but also has to homogenize the stratified water encountered. This would slow down the advance of the front into deeper water. Simpson and Bowers (1981) estimated that this effect would decrease the spring-neap oscillation of the front locus to about half of the equilibrium oscillation. However, this is still about 2.5 times the amplitude of the observed oscillation. Simpson and Bowers (1981) then introduced variable efficiency of vertical mixing such that the efficiency is reduced as stratification becomes established. This allowed the authors to obtain a good fit between data and predictions.

If the locus of a shelf front is determined by the thickness of the frictional bottom boundary layer, the spring-neap front oscillation should be given by

$$h_s/h_n = U_{ts}/U_{tn} = (1 + e)/(1 - e).$$

Again using $e = 0.3$, one obtains the equilibrium $h_s/h_n = 1.86$ which is about 1.4 times the amplitude of the observed oscillation. In our case, stratification would also again tend to decrease h_s/h_n compared to the equilibrium value. Thus we may conclude that a shelf front controlled by the thickness of the frictional bottom boundary layer ($h/U_t = \text{const.}$) would display a spring-neap oscillation close to that observed.

The annual cycle of the locus of a shelf front should, if governed by Monin-Obukhov dynamics, develop at great water depths in early spring when the buoyancy flux is weak. The front should later move towards shallower water and reach its shallowest position in summer when the buoyancy flux is a maximum, and in autumn retreat towards greater water depths. However, if the thickness of the frictional bottom boundary layer governs the locus of a shelf front, the front should be at the same location all the year and no seasonal adjustment in response to a changing buoyancy flux is expected. To the best of the present authors knowledge, there are no published observations showing systematic, seasonal movements of fronts.

Through the introduction of a variable mixing efficiency (in principle a remarkably small and varying m_0) Simpson and Bowers (1981) managed to account for the observed weak spring-neap adjustment of fronts. We would, however, favour the much simpler explanation presented here.

4. Conclusions

It has been demonstrated that the locus of a shelf front should be determined by the thickness of the tidal frictional boundary layer and not by the Monin-Obukhov length. From this, it follows that there should be no change of the locus of a front in response to a varying buoyancy flux. Furthermore, there will only be a weak adjustment of the front position to the spring-neap tidal cycle. These predicted properties are in accord with observations. In general, positive buoyancy fluxes should have very little to do with the actual locus of a front. Buoyancy fluxes are, however, essential in making shelf fronts visible.

The fact that $h/U_t \approx 80$ s actually locates shelf fronts corrctly in the Irish Sea and in the Gulf of Maine region may be taken as further evidence for the correctness of the value of λ (≈ 0.20) provided one accepts $C_d \approx 0.003$. With $\lambda = 0.20$, it also appears that the thickness of the benthic boundary layers described in Weatherly and Martin (1978) at least at times may have been set by Ekman dynamics.

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REFERENCES

Ekman, V. W. 1905. On the influence of the earth's rotation on ocean currents. *Arch. Math. Astron Phys., Vol. 2, No. 11.*

Garrett, C. J. R., Keeley, J. R. and Greenberg, D. A. 1978. Tidal mixing versus thermal stratification in the Bay of Fundy and Gulf of Maine. *Atmosphere-Ocean 16*, 403–43.

Landau, L. D. and Lifshitz, E. M. 1959. *Fluid mechanics. Course in theoretical physics, vol. 6.* Oxford: Pergamon Press, 536 pp.

Loder, J. W. and Greenberg, P. D. 1986. Predicted positions of tidal fronts in the Gulf of Maine region. *Continental Shelf Res. 6*, 397–414.

Mofjeld, H. O. and Lavelle, J. W. 1984. Setting the length scale in a second-order closure model of the unstratified bottom boundary layer. *J. Phys. Oceanogr. 14*, 833–839.

Simpson, J. H. and Hunter, J. R. 1974. Fronts in the Irish Sea. *Nature 250*, 404–406.

Simpson, J. H. and Bowers, D. 1981. Models of stratification and frontal movement in shelf seas. *Deep-Sea Res. 28*, 727–738.

Stigebrandt, A. 1985. A model for the seasonal pycnocline in rotating systems with application to the Baltic Proper. *J. Phys. Oceanogr. 15*, 1392–1404.

Weatherly, G. L. and Martin, P. J. 1978. On the structure and dynamics of the oceanic bottom boundary layer. *J. Phys. Oceanogr. 8*, 557–570.