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Interpolation of wind, temperature and humidity values from model levels to the height of measurement

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ABSTRACT

A new analytical technique is proposed to interpolate data between the lowest model level and the earth's surface in numerical weather prediction systems. The data can be interpolated to any height and especially to the one of routine measurements. The method can therefore be applied either to the verification of forecasts or to the determination of "observation minus guess" increments in data assimilation. The procedure assumes that a Monin-Obukhov-type flux calculation has been previously performed when integrating the model.

1. Introduction

Most of routinely observed data in the planetary boundary layer (PBL) are obtained at heights of 2 m (temperature and humidity) and 10 m (wind). We have to use a precise but rapid diagnostic calculation of the vertical profiles of wind temperature and humidity in the PBL in order to verify numerical weather predictions (NWP) against such observed values and also to be able to compute the increments needed by data assimilation procedures (i.e., observation minus the guess field value provided by a model forecast). This is necessary since, for computational efficiency reasons, the vertical resolution of today's models is still insufficient to allow for prognostic levels at these low altitudes above the ground. For instance, the operational hemispheric large-scale and limited area meso-scale forecasting models of the French Weather Service (Direction de la Météorologie Nationale (DMN)) have 43 m and 19 m heights, respectively, for their lowest level. Furthermore, the strongly varying vertical gradients of the variables inside the PBL do not allow any use of polynomial ad hoc interpolation techniques (for example splines) like those used in the free atmosphere when going from pressure surfaces to sigma surfaces (and conversely). One must thus make use of the additional information provided by the surface fluxes of momentum, heat and moisture computed during the model run and also by the roughness length (z_0) chosen in the model as representative for the grid box area.

It is theoretically possible to search which values of the atmospheric parameters at the measurement height would give surface energy and momentum fluxes equal to the model-diagnosed ones when performing a direct computation using them as input. However, the analytical solutions to these direct formulations are very complex as soon as they differ from the logarithmic profile of the neutral case. This implies an iterative and therefore not cost-effective solution to this problem. Thus, in NWP operational applications, it is preferred to use approximate non-iterative algorithms. From January 1985 to January 1986, such a solution, described by Louis (1983) and used at ECMWF since 1982, was applied in the operational models at DMN. This method first searches which is the modified roughness length z_0' that, for a neutral case, would have given the same surface fluxes as those already computed. The vertical interpolation is then performed with the logarithmic profile characterized by this z_0'. However, we discovered that this technique has a systematic bias towards too warm temperatures and that this bias can even become of the order of several degrees in the case of strong surface inversions during the winter. This weakness was traced back to the fact that the previously described technique ignores the information contained in the value of z_0. We then designed the new technique that will now be described and that has worked satisfactorily in operations since.

2. Notation

We rotate the coordinate so that the wind u becomes parallel to the x-axis. No vertical rotation of the wind direction is considered, since we suppose that the lowest model layer is in the constant flux layer. The thermodynamic variable we will use is the dry static energy $s = C_p T + \phi$, but the potential temperature θ could be used as well. The interpolation weights that will be obtained for the dry static energy can be immediately applied to the specific humidity q. We will use the following notation:

z_0: roughness length
Z: measurement height
Z_1: lowest model level's height
z: current height
u, s: variables
u_*, s_*: friction values
$\tilde{s}(\tilde{u} = 0)$: surface value
$\kappa = 0.4$: Von Karman's constant
C_N: neutral surface exchange coefficient
C_D: drag coefficient
C_H: heat surface exchange coefficient.

3. Mathematical formulation

The Monin-Obukhov (1954) equations read:

$$\frac{\partial u}{\partial z} = \frac{u_*}{\kappa(z+z_0)} \varphi_D\left(\frac{z+z_0}{L}\right), \tag{1}$$

$$\frac{\partial s}{\partial z} = \frac{s_*}{\kappa(z+z_0)} \varphi_H\left(\frac{z+z_0}{L}\right), \tag{2}$$

$$L = \frac{C_p T}{g\kappa} \frac{u_*^2}{s_*}. \tag{3}$$

Except with very simplified expressions for φ_D and φ_H, there is no direct explicit solution to this problem of three equations with three unknowns (Louis, 1979) and thus we cannot use exact φ_D and φ_H values for our purpose. We will here bypass the details of how to solve the system of equations (1), (2) and (3), since it is irrelevant to the application of our interpolation technique. The operational models at the French Weather Service utilize the 1983 version of the ECMWF PBL scheme described by Louis et al. (1982), and the numerical example of Section 6 will be based on it.

All we need to know are the three values of the surface exchange coefficients relative to the height Z_1:

$$C_N = \left[\frac{\kappa}{\ln\left(\frac{Z_1 + z_0}{z_0}\right)}\right]^2, \tag{4}$$

$$C_D = \frac{u_*^2}{[u(Z_1)]^2}, \tag{5}$$

$$C_H = \frac{u_* s_*}{u(Z_1)[s(Z_1) - \tilde{s}]}. \tag{6}$$

u_* and s_* are obtained from the surface fluxes for heat and momentum by the expressions:

$$-\rho \overline{u'w'} = \rho u_*^2, \tag{7}$$

$$-\rho \overline{s'w'} = \rho u_* s_*. \tag{8}$$

The problem can now be formulated in the following way: knowing C_D, C_N and C_H on the one hand, and $u(Z_1)$, $s(Z_1)$ and $\tilde{s}$ on the other, we will compute $u(Z)$ and $s(Z)$ without a-priori knowledge of the exact form of the φ_D and φ_H functions.

4. The interpolation technique

To solve our problem, we prescribe a simple analytical form for the $\varphi_{D/H}$ functions with one single arbitrary parameter $\alpha_{D/H}$. This parameter will be determined in order to obtain the right values of C_D (or C_H) when (1) and (2) are integrated from 0 to Z_1. The function which is now completely defined, $\varphi_{D/H}(\alpha_{D/H}, z/L)$ is finally used for the solution of the problem by integrating (1) and (2) from 0 to Z.

The choice of the approximated formulation for φ has to achieve two goals:

the solution must be analytical without approximation;

the function should fit as closely as possible the unknown function implicitly used in the direct calculations.

We chose:

$\varphi_{D,H} = 1 + \alpha_{D/H} z/L$ for the stable case and

$\varphi_{D,H} = 1/(1 - \alpha_{D/H} z/L)$ for the unstable case.

If the choice for the stable case conforms to the two requirements, the one for the unstable case should rather have been $1/(1 - 3\alpha_{D,H} z/L)^{1/3}$ for the second goal. Unfortunately, the first condition cannot be met in that case (there exists an analytical integral of $\int \varphi(x)\,d\ln x$, but x occurs more than once in the result and cannot be analytically eliminated between the two resulting equations). Nevertheless, a numerical check in asymptotic cases (not shown here) indicates that this disadvantage is not too serious.

5. The interpolation formulae

We set:

$$b_N = \frac{\kappa}{\sqrt{C_N}}, \quad b_D = \frac{\kappa}{\sqrt{C_D}}, \quad \text{and } b_H = \frac{\kappa\sqrt{C_D}}{C_H}.$$

5.1. *Stable case*

$$\varphi_{D/H}(z/L) = 1 + \alpha_{D/H} z/L. \tag{9}$$

Integrating equation (1) from 0 to Z_1 gives for u at the level Z_1:

$$u(Z_1) = \frac{u_*}{\kappa}\left[\ln\left(\frac{Z_1 + z_0}{z_0}\right) + \frac{\alpha}{L} Z_1\right], \tag{10}$$

and, using (4) and (5), (10) becomes

$$\frac{\alpha}{L} = \frac{1}{Z_1}(b_D - b_N). \tag{11}$$

Integrating again eq. (1), but from 0 to Z, gives

$$u(Z) = \frac{u_*}{\kappa}\left[\ln\left(\frac{Z + z_0}{z_0}\right) + \frac{\alpha}{L} Z\right]. \tag{12}$$

Substitution of (11) in (12) leads to the result

$$u(z) = \frac{u(Z_1)}{b_D}\left[\ln\left(1 + \frac{Z}{Z_1}(e^{b_N} - 1)\right) - \frac{Z}{Z_1}(b_N - b_D)\right], \tag{13}$$

and replacing (1) by (2) and (5) by (6), equation (13) for the wind becomes for the dry static energy:

$$s(z) - \tilde{s} = \frac{s(Z_1) - \tilde{s}}{b_H}\left[\ln\left(1 + \frac{Z}{Z_1}(e^{b_N} - 1)\right) - \frac{Z}{Z_1}(b_N - b_H)\right]. \tag{14}$$

5.2. *Unstable case*

$$\varphi_{D,H}(z/L) = (1 - \alpha_{D,H} z/L)^{-1}. \tag{15}$$

Following the same principle we get instead of equations (10), (11) and (12):

$$u(Z_1) = \frac{u_*}{\kappa}\ln\left[\frac{Z_1 + z_0}{z_0}\,\frac{1 - (\alpha/L) z_0}{1 - (\alpha/L)(Z_1 + z_0)}\right], \tag{16}$$

$$\frac{\alpha}{L} = \frac{1}{Z_1}\,\frac{e^{b_N} - 1}{e^{b_D} - 1}(e^{b_D - b_N} - 1), \tag{17}$$

$$u(Z) = \frac{u_*}{\kappa}\ln\left[\frac{Z + z_0}{z_0}\,\frac{1 - (\alpha/L) z_0}{1 - (\alpha/L)(Z + z_0)}\right]. \tag{18}$$

Thus, the equivalent of eq. (13) is more complicated but still analytical:

$$\begin{aligned}
u(Z) &= \frac{u_*}{\kappa}\ln\left[\frac{Z + z_0}{z_0} \times \frac{Z_1(e^{b_D} - 1) - z_0(e^{b_N} - 1)(e^{b_D - b_N} - 1)}{Z_1(e^{b_D} - 1) - (Z + z_0)(e^{b_N} - 1)(e^{b_D - b_N} - 1)}\right] \\
&= \frac{u(Z_1)}{b_D}\ln\left[\left(1 + \frac{Z}{Z_1}(e^{b_N} - 1)\right) \times \frac{(e^{b_D} - 1) - (e^{b_D - b_N} - 1)}{(e^{b_D} - 1) - (e^{b_D - b_N} - 1)\left(1 + \frac{Z}{Z_1}(e^{b_N} - 1)\right)}\right] \\
&= \frac{u(Z_1)}{b_D}\ln\left[\frac{1 + \frac{Z}{Z_1}(e^{b_N} - 1)}{1 + \frac{Z}{Z_1}\frac{(e^{b_N} - 1)(e^{b_D - b_N} - 1)}{(e^{b_D - b_N} - e^{b_D})}}\right] \\
&= \frac{u(Z_1)}{b_D}\left[\ln\left(1 + \frac{Z}{Z_1}(e^{b_N} - 1)\right) - \ln\left(1 + \frac{Z}{Z_1}(e^{b_N - b_D} - 1)\right)\right],
\end{aligned} \tag{19}$$

and the application to dry static energy can readily be derived

$$s(Z) - \tilde{s} = \frac{s(Z_1) - \tilde{s}}{b_H}\left[\ln\left(1 + \frac{Z}{Z_1}(e^{b_N} - 1)\right) - \ln\left(1 + \frac{Z}{Z_1}(e^{b_N - b_H} - 1)\right)\right]. \tag{20}$$

The formulations (13), (14), (19) and (20) fulfil the requirements put down in Section 3. We notice the similarity of eqs. (13) and (19) (or (14) and (20)): a first term corresponding to the logarithmic profile and a stability dependent correction term. We can easily verify that the asymptotic behaviour of these corrections is consistent around neutrality:

$$\ln\left(1+\frac{Z}{Z_1}\left(e^{b_N-b_D}-1\right)\right) \simeq \frac{Z}{Z_1}(b_N-b_D)$$

if $b_D \to b_N$.

6. A numerical application

In this section, we give two idealized examples of a numerical application, one for a stable situation, one for an unstable one (the neutral case is not interesting, since all methods give the correct logarithmic profile). The chosen values are the following: $Z_1 = 30$ m, $z_0 = 0.005$ m, $u(Z_1) = 6$ m/s, $\tilde{s} = 300{,}000$ J/kg, $s(Z_1) = \tilde{s} \pm 5000$ J/kg. Fig. 1 shows the normalized vertical profiles of $(s(Z)-\tilde{s})/(s(Z_1)-\tilde{s})$ as a function of $\ln(1+Z/z_0)/\ln(1+Z_1/z_0)$ in both stable and unstable situations for the three cases: truth, old method, new method. We notice the bias

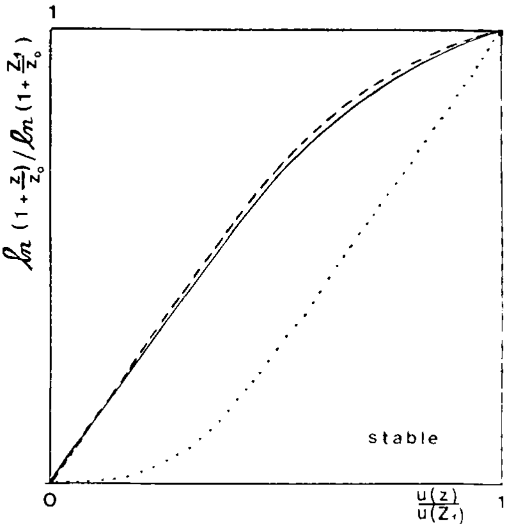

Fig. 2. Same as Fig. 1 but in the stable case only. The adimensionalized dry static energy difference is replaced here by the adimensionalized wind speed.

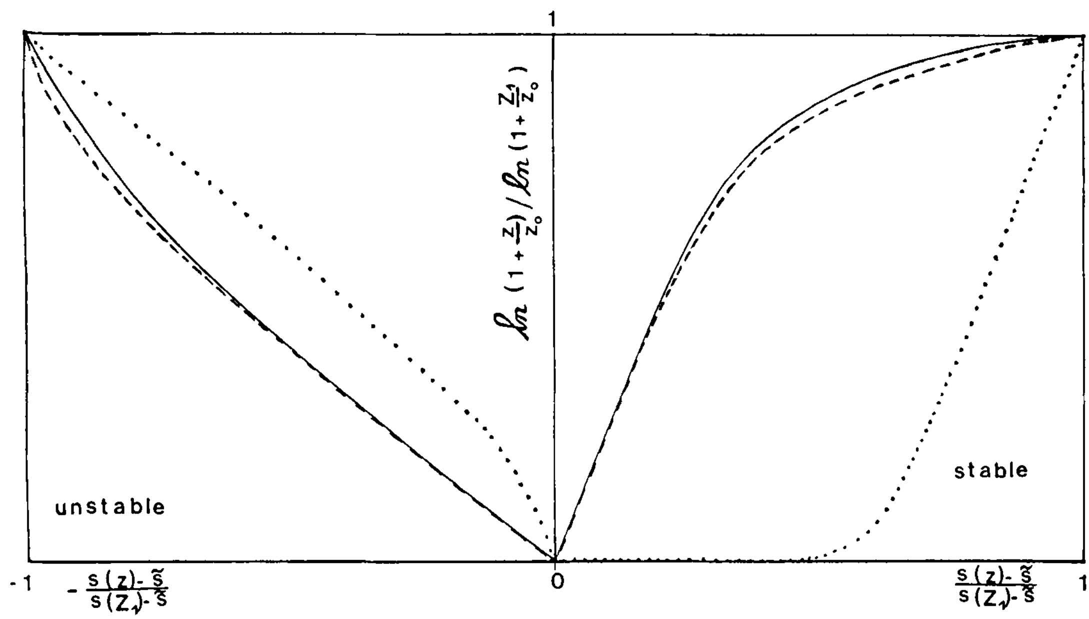

Fig. 1. Adimensionalized vertical profiles of dry static energy differences between atmosphere and surface plotted against adimensionalized logarithmic heights (scaled by the roughness length z_0). For numerical values see text. Continuous line = "truth"; dashed line = new method; dotted line = old method. Stable case on the right and symmetrical unstable case on the left.

associated with the previous method and the good quality of the new technique when compared with the "truth" (computed from the Louis et al. (1982) scheme through an iterative procedure). Figs. 2, 3 refer to the wind profiles for the stable and unstable cases, respectively. Here also the benefit of the new technique is evident. In all figures, a logarithmic profile would be represented by a straight diagonal line. Obviously a 5 K difference between the surface and 30 m is important but not uncommon and helps to clearly show the discrepancies between the different profiles that would be less apparent with more realistic Δs values (indeed the old technique is quite good as long as $|\Delta s|$ remains small).

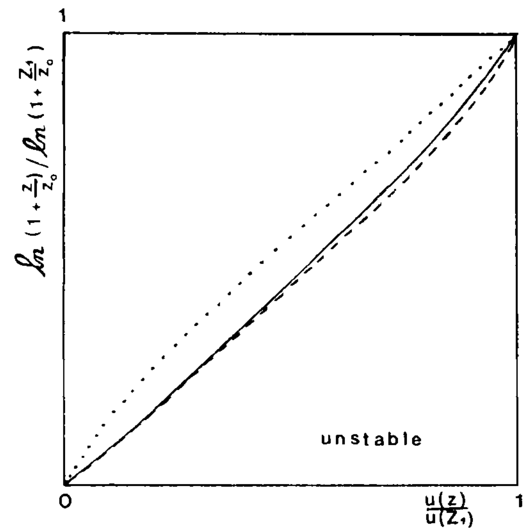

Fig. 3. Same as Fig. 2 but in the unstable case.

7. Conclusion

We have presented in this short note a new analytical technique of vertical interpolation within the surface layer that is simple, physically coherent with the Monin-Obukhov theory and that can be applied as soon as C_N, C_D and C_H are available from any direct model computation (independently of how these are obtained). The method is more precise than the one previously used at the French Weather Service and that was using only C_D and C_H. It has proved robust and reliable in its diagnostic and data-assimilation applications since January 1986. It has also replaced the old technique at ECMWF since April 1987 (Blondin, personal communication).

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