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The Effect of Winds and Tides on the Length of the Day

By YALE MINTZ[1] and WALTER MUNK[2], University of California

Abstract

The seasonal variation in the zonal winds of the atmosphere are reexamined and estimated to account for a January to July change of 0,5 milliseconds in the length of the day. Thus only one third of the change in length of the day reported by the astronomers is explained by angular momentum transfers between the earth and atmosphere. The corresponding effect of the varying equilibrium body tide of the earth caused by changes in the sun's distance and longitude is negligible.

Introduction

Slight seasonal deviations of astronomic time from accurate clock time have been reported by various astronomers, notably Stoyko, Finch and Uhink. If these variations are to be ascribed to seasonal variations in the earth's angular velocity, as has been suggested, then the length of the day varies by two milliseconds between early spring and late summer and by 1.5 ms between mid-January and mid-July. It is not yet quite certain whether this interpretation should account for all of the observed effect.

In a previous paper (MUNK and MILLER, 1950) it was shown that ocean currents and shifts in air and water masses could play only a relatively minor role, whereas the seasonal variations in zonal winds were estimated to account for a variation of 1.6 milliseconds in the length of the day. From new information on zonal winds (HUTCHINGS, 1950; MINTZ and DEAN, 1951) we now estimate this effect to be only one third as large. We have also computed the effect of seasonal variations in the earth's moment of inertia due to earth tides. This effect may account for some of the reported semi-annual component, but only a very small part of the annual component.

Astronomic observations

Measurements by Stoyko, Finch and Uhink give, on the average (STOYKO, 1951),

$$\Delta t = 58 \sin(\odot + 28°) + 8.3 \sin 2(\odot + 146°),$$

for the difference in milliseconds between mean annual time and astronomic time, as function of the sun's longitude $\odot$. We shall assume this to be due entirely to variations in the earth's angular velocity. Differentiation yields

$$\triangle T_{\text{astr}} = \frac{2\pi}{365}\frac{d(\triangle t)}{d\odot} = 1.00 \cos(\odot + 28°) + 0.29 \cos 2(\odot + 146°)$$

for the anomaly in milliseconds in the length

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of the day.[1] The three observers differ on the average by 5 per cent with regard to the amplitude of the annual term, and by 24 per cent with regard to the semi-annual term. For the longest day, $\triangle T = 0.99$ ms, $\odot = 3°$ (March 24); for the shortest day, $\triangle T = -1.23$ ms, $\odot = 137°$ (Aug. 7).

Earth tides

The fluctuation in the length of the day is due partly to seasonal changes in the semi-diurnal tides. There is a slight additional protuberance, on the average, in the equatorial region at the expense of the polar regions, and a resulting increment in the earth's moment of inertia by

$$\delta I = -k\, I \frac{M'}{M} \frac{r^3}{r'^3} (2-3 \cos^2 \Theta)$$

over what it would be if the sun were absent. Here $I = (1/3)\, Mr^2$ is the earth's moment of inertia, M its mass and r its radius; M', r' and Θ refer to the mass, the distance and the declination of the sun; $k = 0.27$ is Love's number. The essential effects here are variations $\triangle(\delta I)$ which occur because of seasonal changes in the heliocentric distance, by

$$\triangle r' = -0{,}0168\, r'_0 \sin(\odot + 168°),$$

and because of changes in the sun's longitude according to

$$\Theta = 23.5° \sin \odot$$

due to the obliquity of the ecliptic. The resulting variations in T, the length of the day, in ms, is

$$\triangle T_{\text{tides}} = \frac{\triangle(\delta I)}{I}\, T = 0.02 \cos(\odot + 73°) + 0.15 \cos 2\odot.$$

Certain modifications may be required to allow for the difference in the yielding of land and sea to tidal forces.

The obliquity of the ecliptic, which leads to the semi-annual term in the above expression, can account for half the amplitude of the reported semi-annual effect, but not all, even if allowance is made for the large observational error. There is no agreement with regard to phase. Prof. JEFFREYS (1928) drew attention to this possible role of the obliquity on accurate time-keeping before this effect was discovered. A somewhat larger effect on $\triangle T$ must be expected from the variation in lunar tides, but because of the short period of this variation the effect on the cumulative time anomaly Δt is relatively small and has not yet been observed.

Subtracting the tidal effect from the observed effect leaves

$$\triangle T_{\text{astr}} - \triangle T_{\text{tides}} = 0.98 \cos(\odot + 27°) + 0.27 \cos 2(\odot + 131°)$$

to be accounted for by other causes. The difference mid-January minus mid-July equals 1.5 ms.

The effect of zonal winds

Because the total angular momentum of the earth and atmosphere are conserved, it follows that anomalies in the length of the day due to variations in zonal winds (MUNK and MILLER, 1950) are given by

$$\triangle T_{\text{wind}} = \frac{\triangle(I_a \omega_a)}{I \Omega_e}\, T,$$

where

$$I_a \omega_a = \frac{2\pi r^3}{g}\, p_0 \int_{-\pi/2}^{\pi/2} \tilde{u} \cos^2 \phi\, \delta\phi$$

is the angular momentum of the atmosphere, $I\Omega_e$ the angular momentum of the planet earth, g gravity, ϕ latitude, and where

$$\tilde{u} = \frac{1}{2\pi} \int_0^{2\pi} \delta\lambda\, \frac{1}{p_0} \int_{p_0}^{0} u\, \delta p$$

is the zonal wind averaged over all longitudes λ and pressures p.

In fig. 1, $(\tilde{u}_{\text{Jan}} - \tilde{u}_{\text{July}})$ is plotted against $\int_0^{\phi} \cos^2 \phi\, \delta\phi$. The area under the curve is therefore proportional to $(I_a\omega_a)_{\text{Jan}} - (I_a\omega_a)_{\text{July}}$.

[1] Because of a numerical error STOYKO (1951) obtains only 0.17 ms for the amplitude of the semi-annual term.

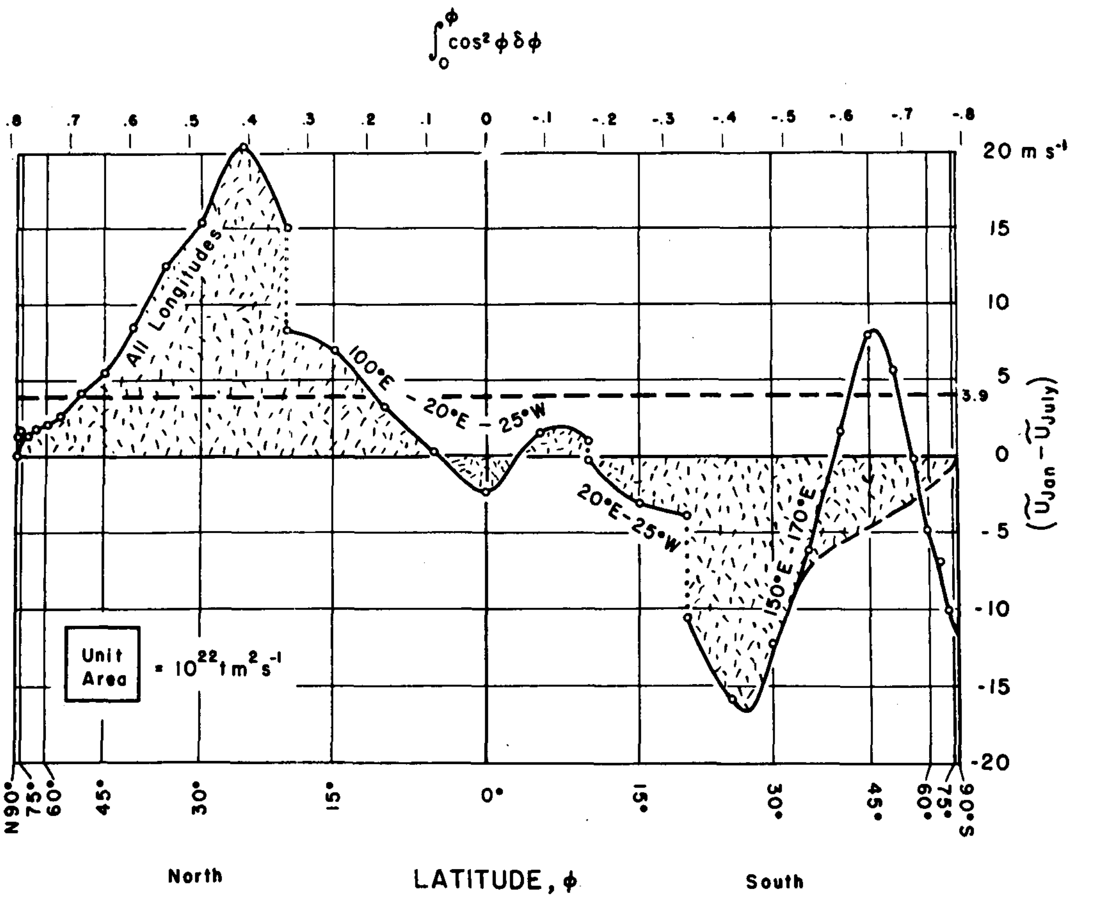

Fig. 1. The seasonal change of zonal wind, as a function of latitude ϕ. The mean zonal wind $\tilde{u}$ is averaged from 101 to 0 cb and over all longitudes. The abscissa has been chosen as a particular function of ϕ for which the area under the curve is proportional to change in angular momentum: unit area equals 10^{22} t m^2 s^{-1}, corresponding to 0.15 milliseconds in the length of the day. In order to account for a reported change of 1.5 ms, the mean value of the curve should equal 3.9 m s^{-1}, as indicated by the dashed line.

In order to account for a difference of 1.5 ms, the mean value of $(\tilde{u}_{Jan} - \tilde{u}_{July})$ should equal 3.9 m sec^{-1}, as shown.

For the latitude belt 20°—90° North, $(\tilde{u}_{Jan} - \tilde{u}_{July})$ was computed from cross sections of the mean January and July geostrophic zonal winds averaged over *all* longitudes (MINTZ and DEAN, 1951), between pressures 101 and 5 cb (instead of sea level pressure p_0 and 0 cb), with the results shown in fig. 1.

For the latitude belt 20°—75° South, $(\tilde{u}_{Jan} - \tilde{u}_{July})$ was computed from summer and winter cross sections of geostrophic zonal winds in the longitudes of New Zealand (HUTCHINGS, 1950), from 101 to 5 cb; and east coast Australia (LOEWE and RADOK, 1950), from 101 to 15 cb, with the average for the two longitudes shown in the figure by the solid curve. The subtropical part of this curve was assumed to approximate conditions averaged over all longitudes of the southern hemisphere, [although this assumption is not without its uncertainties, as in the northern hemisphere the variation of $(\tilde{u}_{Jan} - \tilde{u}_{July})$ with longitude is as great as $(\tilde{u}_{Jan} - \tilde{u}_{July})$ itself; and in the southern hemisphere, at 30° S, $(\tilde{u}_{Jan} - \tilde{u}_{July})$ is 6 m sec^{-1} greater at 170° E than at 150° E.] The temperate and higher latitude part of the solid curve shows the seasonal change of zonal wind in the region of the Ross Sea low (and upper air trough), and was not assumed to approximate conditions averaged over all longitudes. A plausible estimate of $(\tilde{u}_{Jan} - \tilde{u}_{July})$ averaged over all longitudes is given in the figure by the broken curve.

In the tropical latitudes $(\tilde{u}_{Jan} - \tilde{u}_{July})$ was computed from cross sections of summer and winter mean pilot balloon winds at longitudes 20° E and 25° W (EKHART, 1941), from 101 to 30 cb; in addition, at latitudes 20° N to 10° S the January and July mean pilot balloon zonal winds at longitude 100° E, (MINTZ and DEAN, 1951), from 101 to 5 cb were used. The curves in the figure show the average results, with the longitudes indicated. It is uncertain how well these curves represent all longitudes. The tropospheric circulation in the tropics is divided into pronounced horizontal cells and there are large variations of zonal wind, and of seasonal change of zonal wind, with longitude. Moreover, pilot balloon observations are selective with respect to cloudiness and hence with respect to the zonal component of the wind. We note that the curves between 20° N and 20° S reveal only a small net effect. This result might be expected from the fact that at these latitudes the percentage of the earth's surface that is covered by land, the heights of the mountain barriers, and the seasonal change of the zonal wind at the ocean surface are about the same in the two hemispheres.

Table 1. Seasonal change of atmospheric angular momentum and the resulting changes in the length of the day, for stated latitude belts.

	90°N—20° N	20°N—20° S	20° S—90° S	90°N—90° S
$(I_a \omega_a)_{Jan} - (I_a \omega_a)_{July}$, in $10^{22} t\ m^2\ s^{-1}$	8.0	1.3	— 5.9	3.4
$(\Delta T_{wind})_{Jan}$ — — $(\Delta T_{wind})_{July}$, in millisec	1.2	0.2	— 0.9	0.5

By integration of the shaded area in the figure, the values in Table 1 were computed. According to these figures the winds account for only one third of the reported seasonal variation in the length of the day.[1] If the section along 150° E and 170° E in the southern hemisphere is used without adjustment (Fig. 1, solid curve), the winds would account for 0.8 ms, half the reported effect. Because of the large uncertainties in the calculations for tropical and southern latitudes, our estimates may be greatly in error. It is not impossible that instead of accounting for one third the reported effect the winds may account for only a negligible part, or for more than half of it. From an inspection of Figure 1 it does not seem likely, however, that the winds can account for all of the reported variation.

Thus in spite of the fact that our knowledge of the wind field is based on larger numbers of observations than are available for any other kind of geophysical quantity, the result is uncertain, and will remain so for some time to come.

No improvement is to be found by resorting to estimates of the surface zonal wind stress (VAN DEN DUNGEN, et al., 1950). A six month change of 1.5 ms in the length of the day corresponds to an average rate of change of angular momentum of $0.7 \times 10^{16}\ t\ m^2\ s^{-2}$. But the total westerly torque and the total easterly torque of the atmosphere on the earth are each of the order of $10 \times 10^{16}\ t\ m^2\ s^{-2}$, (MINTZ, 1951). To come within 25 per cent of the seasonal change in length of the day, the surface torques would have to be computed to an accuracy of one part in one hundred!

Conclusions

Astronomic observations indicate that the mean length of the day in January exceeds that in July by 1.5 milliseconds. It is however possible that this difference might be smaller, due to periodic errors in the right ascension system of the fundamental catalogue of star places. More information on this point will be available within the next few years, when the new Greenwich Photographic Zenith Tube comes into action.[1] We may also expect increased accuracy from the improvement of clocks, possibly the use of atomic clocks.

[1] Previous computations by MUNK and MILLER (1950) gave too high a value principally because in the northern hemisphere they used the published sections along 100 E° and 80° W, where, as it now turns out, the variations in the zonal winds are larger than at all other longitudes.

[1] We are indebted to the Astronomer Royal for information concerning these developments.

Our present estimate is that the winds account for 0.5 ms, but this estimate may be considerably in error due to uncertainties concerning the tropical southern hemisphere winds. The circulation of the ocean, changes in sea level and shifts in air masses might possibly contribute another 0.5 ms, but this is by no means established (MUNK and MILLER, 1950). If in each case we put down the largest possible number, the combined effect might add up to 1.5 ms, but we do not suggest that this is the case.

One might hope to return to this problem when more complete meteorologic data are available, and to compute the effect for each month of the year. The principal needs are for station level pressure variations (not reduced to sea level), and for southern hemisphere upper air winds over the open sea.

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