A Synoptic-Aerological Study of Interaction between Vortices

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Abstract

The displacement and intensity changes of a cyclone and anticyclone over the Northeastern Atlantic in October 1945 are investigated. An attempt is made to explain the motion of the cells as an analogue to the vortex pair in hydrodynamics. The observed paths and translational velocities are compared with those computed from theoretical relationships derived on simplifying assumptions involving a.o. conservation of absolute vorticity and an hypothesis of equivalent vortex strength. With reference to the third day after the genesis of the system, the result of this comparison is satisfactory and is considered significant in view of a concurrent transient reduction of horizontal divergence inferred from observed variations in the thermal structure of the cyclone.

The genesis and behaviour of the system of vortices is also discussed in relation to a recent theory of composite baroclinic vortices. In part this theory is confirmed by observational evidence as e.g. an accelerated equatorward drift of the cyclone in its early stage and an intensification of its upper part in a later stage when the centre of the system of vortices is stationary.

1. Introduction

During the last three decades theoretical discussion of dynamical problems in extratropical synoptic meteorology has been largely concerned with the genesis, development and and movement of disturbances in the pressure field associated with fronts and wave-like flow patterns in the middle and upper troposphere. In recent years the interest in atmospheric vortices, especially their motion across the surface of the earth, has been strongly revived, chiefly because of the important rôle they play as links in the general circulation. Synoptic-aerological descriptions of long-lived cyclonically rotating high-tropospheric pools of cold air breaking away from the circumpolar whirl of westerlies (Scherhag, 1948), were followed by studies of these vortices — and their counterparts, warm anticyclones to the north of the maximum upper westerlies — with a view to throwing light on their part in the mechanism of the meridional transport of heat and zonal angular momentum (UNIVERSITY OF CHICAGO 1947; PALMÉN 1949).

The dynamics of vortices of this kind has been also explored from a theoretical viewpoint by other writers employing simplified kinematic models (see references in next section). The present investigation is a synoptic aerological study of an observed remarkable motion of vortex-like systems: of a Low and a High moving in quasi-circular paths of unequal radii about a fixed point. For a preliminary view of the displacement of the pressure cells, the reader may at this point turn to the 500 mb contour charts (Fig. 3), cells A_3 and C_3, and to the track diagrams (Figs. 5 and 6). The representation of the observational data and the proposed explanation of the phenomenon as an analogue to the two-vortex model in hydrodynamics will be facilitated by a prior review of recent theories of large-scale vortex motion in the atmosphere.

2. Review of theories

In an attempt to clarify the problem of the dynamical control of pressure in the atmosphere from a principal standpoint, PRIESTLEY (1948) has a.o. dealt with adjacent large-scale, quasi-circular baroclinic cyclones and anticyclones centred in a latitude circle. By using analytical methods similar to those employed in a theory of the wave-cyclone (BJERKNES and HOLMBOE 1944) and based on the semi-empirical concept of local gradient wind motion, the displacement in the line of centres or the zonal drift of such systems is derived from an appropriate approximation of the field of horizontal divergence.

The theory of motion and mutual interaction of circular vortices of various standard types, i.e. of different radial pressure profiles, has been developed by JAMES (1950). The velocity is taken as the sum of geostrophic, cyclostrophic and isallobaric components, which is however not a general truth.[1] A theory of quasi-geostrophic vortex interaction is forwarded, based on the hypothesis that in mutual interaction the pressure profile of individual vortices is conserved, and permits vortex interaction to be considered as the motion of attracting and repelling "centres of action". Pertinent to the problem in the present study is, in particular, the author's conclusion that "*if two vortices are rotating about some point with constant separation, such motion could persist without violating the energy equation, as the mutual energy is a function of the separation only*". Although the equation of energy balance for the two-vortex case, as given by James, may well serve as a basis for a quantitative verification of the vortex paths in the present case, it is perhaps more desirable to take recourse to a hydrodynamic theorem due to VON HELMHOLTZ (1858) and LORD KELVIN (1866), revealing in the same manner an exact analogy to the similar equations in the dynamics of attracting bodies and electric charges.

However, as the movement of atmospheric vortices is observed in a system rotating with the earth, certain effects must be entirely due to its rotation and its spherical shape. It has been shown (DAVIES 1948, ROSSBY 1948, 1949) that *solitary* vortices with vertical axes in an environment at rest relatively to the earth are subjected to a force directed polewards in the case af a cyclone, equatorwards in the case of an anticyclone. This force is readily determined when it is assumed that the lateral extent of the vortex be small (viz. its radius of cross section R small in comparison with the earth's radius a) and that the motion inside the vortex as well as the escape motion in the environment be non-divergent. The force is then proportional to the gradient of absolute vorticity Z_0 in the environment, which in this case is equal to the (mean) variation of the Coriolis parameter with latitude,

$$\partial Z_0 / a \partial \varphi = \partial f / a \partial \varphi = 2\,\Omega \cos \varphi / a \equiv \beta$$

ROSSBY found for this force, per unit square of a horizontal slice of unit thickness, acting on a vortex in *solid rotation* having vorticity $\zeta = 2\,\omega$, where ω is the angular velocity (counted positive for cyclonic rotation):

$$Fy = \frac{1}{4} R^2 \omega \beta = \frac{1}{8} R^2 \zeta \beta \qquad (1)$$

In the derivation of this expression it is implied that the velocity has a jump at the outer boundary of the vortex from the value ωR to zero. It is noteworthy that when an appropriate annular zone of lateral shear at the outer boundary is taken into consideration the result is almost the same, since e.g. in the case of a cyclone the factor $1/4$ is replaced by $5/16$. This has been shown by KUO (1950) who points out that these, however, are only approximate results, because rigorous solutions satisfying the vorticity equation for non-divergent motion in a barotropic fluid are only possible if the absolute vorticity ($Z_0 = f + \zeta_0$) in the environment of the vortex is uniformly distributed — whereas in the seemingly simple case of a solitary vortex in an environment at rest this is clearly *not* satisfied. When $\partial Z_0 / a \partial \varphi = 0$, but f and ζ

[1] From an analysis of the relation between the ageostrophic wind component and the isallobaric field it is possible to show (BERSON 1939) that the isallobaric and cyclostrophic components are mutually interdependent.

are linear functions of latitude so that $\beta =$ const. and $\zeta_0 = \zeta_{00} - \beta a \delta\varphi$ where ζ_{00} is the vorticity at the centre of the vortex, the force driving a vortex with solid boundary across the latitude has two principal components: one, a rotor effect, stems from the motion of the vortex relative to the environment and from the circulation around the vortex; the other is due to the distribution of relative vorticity in the surrounding fluid. (In the case of a superimposed basic zonal current, this distribution would then correspond to a poleward decreasing cyclonic, or increasing anticyclonic lateral shear $\partial U_0/a\partial\varphi$. The corresponding force F_x is in this case due to the rotor effect only.)

It is to be noted that although in the above theory it is assumed that the vortex possesses a solid boundary, the motion *inside* the vortex may be of any kind, provided it has the same (relative) vorticity as the surrounding fluid. Yet even if its vorticity differs from that of the latter — as is usually the case in the atmosphere — *the effect of a superposed circulation is only to give an irrotational cyclic motion to the surrounding fluid.* This irrotational motion is represented by an additional logarithmic stream function, $\Psi_1 = 1/2\ R^2(\zeta - \zeta_0)\ \ln r$, where r is the distance from the centre of the vortex. This stream function satisfies the vorticity equation and the boundary condition that $\Psi_1 =$ const when $r = R$.

In the next section it will be seen that the earlier mentioned vortex theory due to VON HELMHOLTZ and KELVIN, which permits one to determine the velocity at every point in an incompressible fluid containing a discrete number of vortex filaments and at rest at infinity, may be taken as both a simplification and generalization of the foregoing treatment; for if it be assumed that no basic current exists and the effect of the earth's rotation be disregarded (U_0 and β vanishing), the rotor effect as well as the effect from the distribution of vorticity in the environment vanish. We are left with the reaction to the surrounding irrotational circulation given by the stream-function $\Psi = 1/2\ R^2\zeta\ \ln r$ and so, between two or more adjacent vortices, only with their interaction. Presumably in the case of two or more vortices, for these conditions there also ought to exist an effect on each from the variation of the Coriolis parameter with latitude; but if we arbitrarily disregard the relevant forces Fy, our problem is reduced to a simple kinematic vortex interaction in a rotating system with constant Coriolis parameter, i.e. on a rotating disc.

There is another important aspect of the displacement of solitary vortices across the latitude circles, due to a non-uniform distribution of absolute vorticity in the environment. Because of the latter the vorticity and thus also the flow in the vortex must readjust themselves whereby a certain amount of energy will be released. This *kineto-potential* energy supply is, as ROSSBY has shown, not necessarily converted into translational energy only, but provides (in favourable conditions) for the creation of *rotational* energy as well. A baroclinic system, as e.g. an anticyclonically rotating cold dome, will sink and spread out along the surface of the earth during its displacement equatorward and a cyclonic circulation above the sinking dome will be created. A verification of this process from synoptic and statistical evidence has been given (ROSSBY, 1949 and KOO, 1950). In the present study of vortex motion there is certain evidence that the rotary motion of the pressure cells has been preceded by a process of this kind. Data supporting this conclusion will be given in § 7.

3. Application of "vortex pair" theory

If in particular we restrict our attention to two rectilinear vortex filaments in an incompressible non-viscous fluid at rest at infinity, it is possible to derive relationships for the displacement of the filaments relative to a fixed point. When the strengths of the vortices, whose separation D is large in comparison with their radius of cross section ($D \gg R$), are equal and of opposite sign, viz. $I_1 = 2\pi R^2\omega = -I_2$, the system is called a "*vortex pair*" par excellence. The filaments in this case propel each other in a direction perpendicular to the line of their centres. In the following we shall use the term "vortex pair" even for the case when $|I_1| \neq |I_2|$.

We shall next give a brief derivation of the relationships for vortex filaments which will be subsequently applied to large-scale pressure systems. The deductions will be based mainly on a summary of the HELMHOLTZ-KELVIN theory given by LAMB (1932), but the notations are altered so as to bring them into line with accepted meteorological conventions.

When the motion is in two dimensions (rectilinear vortices) the Cartesian components of the velocity, $\boldsymbol{V}_2$, are u and v and are functions of x and y only. The vortex lines are straight lines parallel to the z axis and the vorticity $\zeta = \partial v \mid \partial x - \partial u \mid \partial y$. The velocity outside the vortex filaments is determined by a streamfunction Ψ, as the motion is assumed to be nondivergent, whence

$$\boldsymbol{V}_2 = \boldsymbol{k} \times \boldsymbol{\nabla}_2 \Psi \tag{2}$$

where $\boldsymbol{k}$ is the unit vertical vector directed along the positive z-axis (pointing upward) and $\boldsymbol{\nabla}_2$ is the horizontal ascendent. The stream function is found from a solution of the differential equation

$$\Delta_2 \Psi = \zeta \tag{3}$$

where Δ_2 denotes the two-dimensional Laplacian operator. This solution is in virtue of the assumptions mentioned in the preceding section

$$\Psi = \frac{1}{2\pi} \iint \zeta' \ln r\, dx' dy' + \Psi_0 \tag{4}$$

where ζ' is the vorticity at the point (x', y') at which the volume integral is taken (extended over that part of the two-dimensional space in which $\zeta' \neq 0$) and r is the distance between that point and the point (x, y) at which the velocity $\boldsymbol{V}_2$ is required; Ψ_0 is a complementary function which is any arbitrary solution of $\Delta_2 \Psi_0 = 0$ which satisfies the boundary conditions. In virtue of the restrictions imposed upon the fluid (u and v assumed vanishing at infinity), Ψ_0 reduces to a constant and one obtains then for the velocity

$$\boldsymbol{V}_2 = \frac{1}{2\pi} \iint (\boldsymbol{k} \times \boldsymbol{\nabla}_2 \ln r)\, dx' dy' \tag{5}$$

or

$$u = -\frac{1}{2\pi} \iint \zeta' \left[\frac{y - y'}{r^2}\right] dx' dy'$$

$$v = \frac{1}{2\pi} \iint \zeta' \left[\frac{x - x'}{r^2}\right] dx' dy'$$

These relations show that any vortex filament situated at (x', y') and possessing strength $\Gamma' = \zeta\sigma'$, where σ' is the small cross section $\delta x' \delta y'$, contributes to the motion at (x, y) a velocity

$$\boldsymbol{V}_2 = \frac{\Gamma'}{2\pi} (\boldsymbol{k} \times \boldsymbol{\nabla}_2 \ln r) \tag{6}$$

being thus directed at right angle to the line joining the points (x', y') and (x, y) and having the value $\Gamma'/2\pi r$ (See fig. 1 — The

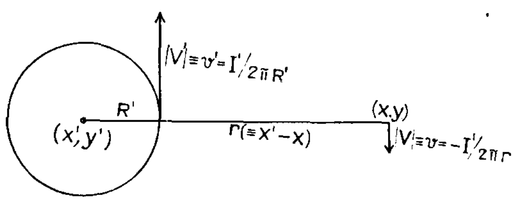

Fig. 1. Illustrating the motion at a point (x, y) due to a neighbouring vortex filament at (x', y'). For simplicity the line joining these points is made to coincide with the x-axis.

vector $\boldsymbol{k}$ turns the vector $\boldsymbol{\nabla}_2 r$ counterclockwise by 90° in our right-handed system of coordinates).

Taking the integral of the product of vorticity and velocity, extended over the sections of all the vortices present in the fluid, viz. $\iint \zeta \boldsymbol{V}_2 dx dy$ and substituting from (5), it is easy to show that this integral vanishes identically, whence

$$\Sigma\, \Gamma' u = \Sigma\, \Gamma' v = 0 \tag{7}$$

These equations permit one to fix the "*centre of inertia*" of the system of vortices which is analogous to the centre of a film of substance distributed over the xy-plane with the surface density ϱ (corresponding to ζ). This analogy is already embodied in the equation for the streamfunction, eq. (4), which is in form identical with the corresponding equation in the theory of attraction (between gravitating rods).

The coordinates of the centre of inertia are readily obtained from equations (7). In view of the assumptions of nondivergence and barotropy the vorticity is individually conserved or $d\zeta/dt = 0$; and the cross sections are unchanged during the displacements of the filaments. Therefore the strengths I' are also invariants so that, as $u = \frac{dx}{dt}$, $v = \frac{dy}{dt}$ we have

$$\Sigma I'x = \text{constant}$$

or

$$x_i \Sigma I' = \Sigma I'x$$

whence

$$x_i = \frac{\Sigma I'x}{\Sigma I'} \text{ and similarily } y_i = \frac{\Sigma I'y}{\Sigma I'} \tag{8}$$

the point (x_i, y_i) representing the centre of inertia.

Referring back to the case of *two* vortex filaments of strengths I_1 and I_2 resp., it is evident from the foregoing that the motion of each filament must be entirely due to the other and is therefore perpendicular to the line joining their centre (compare also Fig. 1). The two filaments remain at a constant distance apart and *rotate about the centre of inertia which is fixed*. It is easy to prove that the angular velocity of this rotation is determined by the identity

$$\omega^\star \equiv \frac{I_1 + I_2}{2\pi D^2} = \frac{R_1^2\omega_1 + R_2^2\omega_2}{D^2} \tag{9}$$

where D is the separation of the centres. It follows from eq. (8), after rewriting it for the two-vortex case, that if the sense of rotation in the two vortices is the same the centre of inertia lies *between* the filaments whereas in the case of opposing rotation it lies *beyond* one of these, in the line joining them (extended). It is readily seen that the filaments of a "vortex-pair" par excellence will propel each other in the direction perpendicular to the line joining their centres, since in this case $\omega^\star = 0$.

Application of vortex pair theory and in particular equation (9) to barotropic vortices with vertical axes in the atmosphere seems a priori feasible when, as required by one of the premises underlying the theory, $D \gg R$ and when $\omega \gg f/2$. Attempts have in fact been made in this direction with a view to explaining certain characteristics of the motion of small atmospheric vortices such as dust-whirls a.o. (Williams, 1948).

Serious objections may be raised against any attempt to apply the above relations to phenomena characterized through that simultenaously ω and $f/2$, as well as D and R are respectively of the same order of magnitude. This is true for most large vortex-like pressure systems of comparable size and so also for the cells A_3 and C_3 in the present investigation. When ω and $\Omega \sin \varphi$ $(=f/2)$ are of the same o.o.m., equations (8) and (9) are still applicable, provided R is much smaller than the earth's radius so that the variation of the Coriolis parameter with latitude may be neglected. We have then to insert in the place of the strength I the absolute strength $I_a = 2\pi R^2 (\omega + f/2)$, and for the angular velocity of rotation about the inertia centre to take the absolute angular velocity, viz. $\omega_a^\star = \omega^\star + f/2$, whence from eq. (9)

$$\omega^\star = \frac{(I_a)_1 + (I_a)_2}{2\pi D^2} - \Omega \sin \varphi \tag{9a}$$

However, when D and R are actually of the same o.o.m., in view of the proximity of the cells the velocity field, resulting from a superposition of the rotational field of one cell and a circulation pertaining to the environment of the other, will be rather complex. Strictly the application of the above relation would be permissible, if it could be shown from observational data that the velocity field is in fact composed of these components. An investigation of this kind would, however, ultimatively lead to the well known concept of a vortex-like disturbance being *steered* by the velocity field of an adjacent circulation system. When the latter itself is being displaced at a comparable rate — as is in fact the case in our "vortex pair" — not much could be profited from following up this line of approach.

The above elucidated difficulties, as well as others arising from the non-uniform distribution of vorticity within the pressure cells, may be overcome by making a hypothesis based on the exact correspondence between the centre of a film of mass and the centre of inertia of a system of (rectilinear) vortices, to which we have referred earlier. We shall assume that with regard to the specific interaction of two vortices, viz. their rotation about a fixed point, there is an equivalence between filaments of cross section πR^2, $(R \ll D)$ possessing vorticity ζ, and large adjacent

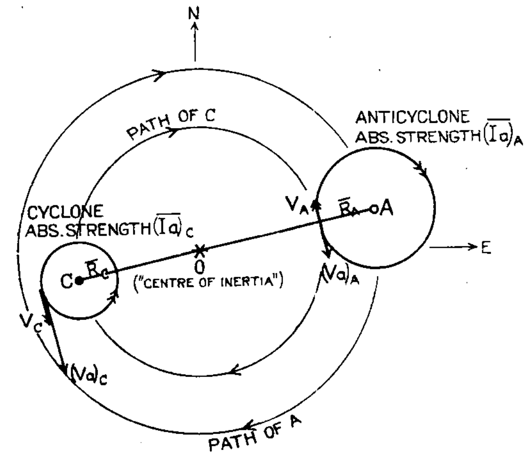

Fig. 2. Schematic representation of the path of circular vortices with vertical axes (cyclone centered at C, anticyclone at A) about their "centre of inertia" (at o). The ratio of the absolute vortex strengths is assumed as $I_{aC}/I_{aA} = 1.5$, the ratio of the radii of cross sections as 0.67. From equations (8) one obtains for the ratio AO/AC the value 0.6 and for the ratio of the absolute peripheral velocities V_{aC}/V_{aA} the value 2.25. V_C and V_A are the peripherical velocities relative to the earth's surface (not drawn to scale).

circulation systems with cross section $\pi \bar{R}^2$, ($\bar{R} \approx D$), possessing an average vorticity $\bar{\zeta} = \zeta \dfrac{\pi R^2}{\pi \bar{R}^2} = 2\,\omega\,(R/\bar{R})^2$. This hypothesis may be also formulated as follows: "*Arbitrary vortices (solid bodies) may be contracted to filaments (gravitating rods) with strength (mass) equalling that of the former, without affecting thereby the interaction with (attraction exerted on) a second vortex (body) situate at finite distance from the centre of the first.*"

Accepting this hypothesis we may apply equations (8) and (9 a) to adjacent irregular systems such as the cyclonic and anticyclonic cells C_3 and A_3 of the present study. Giving the corresponding equivalent quantities $\bar{I}$, $\bar{\omega}$ and $\bar{R}$ the subscripts A and C (anticyclone and cyclone), equation (9 a) may be written as

$$\omega^\star = \frac{(\bar{I}_a)_A + (\bar{I}_a)_C}{2\,\pi D^2} - \Omega \sin \bar{\varphi}$$

$$= \frac{(\bar{\omega}_A + \bar{f}/2)\,R_A{}^2 + (\bar{\omega}_C + \bar{f}/2)\,\bar{R}_C{}^2}{D^2} - \Omega \sin \bar{\varphi} \qquad (10)$$

where $\bar{\varphi}$ is the latitude at which the inertia centre (point x_i, y_i) is situate. The geometry of the vortex pair and of the paths of its cells are schematically illustrated in fig. 2.

In order to illustrate the usefulness of the theory by means of a concrete example, let us for simplicity assume that there are two vortices with equal cross sections and that their separation is 3 $\bar{R}$, the cyclone having an equivalent vorticity equalling twice the Coriolis parameter, or $\bar{\omega}_C = \bar{f}$, the anticyclone having an equivalent vorticity equalling one half of the critical (minimum) value — which is obtained from the criterion for inertial instability of horizontal flow, viz. $\omega_a = 0$, thus $\bar{\omega}_A = -\bar{f}/4$. With these assumptions it follows from equation (10) that the cells would rotate about their inertia centre with an angular velocity $\omega^\star = -0.3\,\bar{f}$ or, taking the latitude as 55°, $= -0.35 \times 10^{-4}$ sec^{-1}. The vortices move therefore in *anticyclonic paths* of unequal radii. (The radii are the respective distances from the vortex centres to the inertia centre, and are readily calculated from the relations (8)). The corresponding period of revolution is for this case appr. 50 hours, a rather low value but of a reasonable order of magnitude.

The choice of the values for the dimension ratios $\bar{R}_A/\bar{R}_C$ and $\bar{R}/D$ and of the rotation ratios $\bar{\omega}_A/\Omega \sin\varphi$ and $\bar{\omega}_C/\Omega \sin\varphi$ does not affect the sense of curvature of the path, because for a wide range of values for these ratios one obtains negative $\omega^\star$ and thus *anticyclonic paths.* E.g. with prescribed values for the first three of the above ratios, taken as in the above example, *cyclonic paths* would result for an equivalent angular velocity of the cyclonic vortex exceeding the value 15/4 $\bar{f}$. The corresponding equivalent absolute vorticity $\bar{Z}$ would be 8.5 $\bar{f}$, but such high values are not observed in extratropical cyclones. Clearly, a certain combination of a reasonably large ratio ω_C/Ω sin $\bar{\varphi}$ with a large ratio $\bar{R}_C/D$ may also reverse the sign of $\omega^\star$, so that a cyclonic path would result. We shall not here go into further details regarding this question, but it may be useful to compute roughly the angular velocity in the case of a cyclone being "captured" by a larger anticyclone (both cells, however, being of comparable seize). This is not unlike the kind of system observed in the present case. Assuming $R_A = 2\,\overline{R}_C$, $D \approx \overline{R}_A + \overline{R}_C$, $\overline{\omega}_A = -\overline{f}/4$ and $\overline{\omega}_C = \overline{f}/4$, one obtains for $\omega^\star$ approx. $-f/3$ or very nearly the same value as in the first exemple above and, incidentally, corresponding roughly to the mean values obtained in the calculations from the observational data, given below.

4. The flow pattern—"vortex pair" of October 4—7, 1945

The purpose of presenting the observational data in this, and the results of the calculations in the following, section is to show that the displacement of pressure systems may be determined to a large extent by the kind of interaction discussed in the foregoing. In this connection it is necessary to stress that the mutual dependency of the velocity field in the environment of adjacent vortices and of their displacement is usually overshadowed, not only by the effects which have been discussed in par. 2, but also by processes which cannot be accounted for by any theory of rectilinear (barotropic) vortices. In view of baroclinity in the pressure field, almost always present, such processes can only be understood from a study of the distribution of motion and mass (temperature) in three dimensions; and over an area exceding by far the area covered by the system as a whole. Nevertheless, the present case appears to bring out the mutual dependency rather clearly; it lends itself also for a discussion of the discrepancy between the proposed theory and the observed displacements (see §§ 6 and 7).

In order to provide a basic orientation with regard to the large-scale flow patterns constituting the environment of the "vortex pair", the analysis is chiefly restricted to the 500 mb level. However, the composite cross section through the cyclonic cell, given in § 7, will throw some light on the influence of the baroclinity and vertical motion concurring with its displacement.

From an inspection of the 500 mb charts (fig. 3[1]) and the track diagrams (figs. 5 and 6) the principal changes over the area covered by the charts may be summarised as follows:

1. The flow pattern in *middle latitudes*, between 40° and 60° N approx., is featured by *almost complete absence of zonal motion*. There are two quasi-stationary depressions with cold cores, C_a and C_b, centred over the Northern Adriatic Sea and in the vicinity of the Azores resp. The longitudinal distance of these centres is about 50°, corresponding to the wave-length of a highly unstable planetary wave-train of whose growth the cells C_a and C_b are presumably a final product[2]. They are separated by a ridge R_a to the west of Spain, possessing a feeble anticyclonic cell A_a.

2. In *subpolar latitudes*, north of 60° N, there is on 3 Oct. (marking the beginning of the four-day period investigated) an intense quasi-zonal, strongly baroclinic current on which a series of short waves is superimposed (troughs t_1, t_2, t_3; ridge r_3). The first two of these disturbances propagate rapidly downstream without growth of amplitude. The third, situated southeast of Greenland and possessing an extremely short wave-length, undergoes rapid amplitude growth between 3 and 4 Oct., and small circulation cells are simultaneously formed in both the ridge r_3 and the trough t_3. These cells are indicated on the chart of 4 Oct., and the following, as A_3 and C_3. Subsequently C_3 breaks away from the main current at a point near Iceland and moves rapidly almost due south across Ireland, subsequently southwest, swinging in a semicircular, *anticyclonic path* to a point not far from its initial position (Fig. 5). The anticyclonic cell A_3 remains during its movement at an almost constant distance from the cyclonic cell C_3 so that these two cells constitute an anticyclonically rotating "vortex pair", if the translation between 4 and 5 Oct. is eliminated (see Fig. 6).

It is noted that the anticyclone grows in size rather rapidly and that its movement cannot be entirely divorced from that of the ridge belonging to the initial wave

[1] These charts have been drawn with the aid of all available data (D. W. R., Meteorological Office, London, 1945; Historical Weather Maps, H. Q., U. S. Army Air Forces, 1945; Swedish Meteorological and Hydrological Institute, Yearbook 1945). — Some of the data had not been taken into account in the U. S. charts for reasons unknown to the writer who, incidentally, came upon the situation whilst on duty at the Upper Air Section of the Meteorological Office, Dunstable.

[2] Compare the theory of long-wave instability a. o. CHARNEY (1947), BERSON (1950).

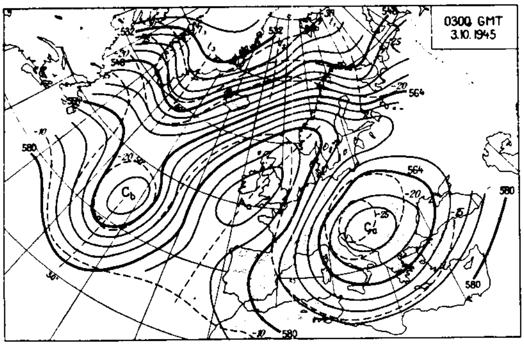

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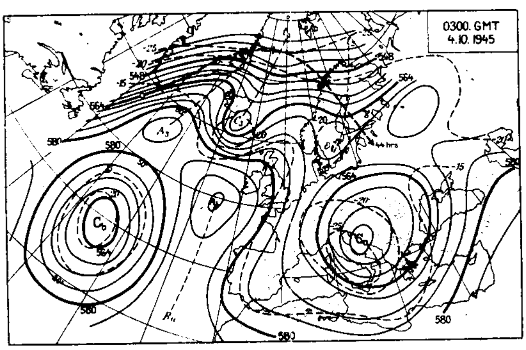
Fig. 3. Contours and isotherms of the 500 mb surface. The thick hatched line on the charts for 4 and 6 October is a constant absolute vorticity trajectory; the number at its endpoint denotes the time lapse (in hours) from the starting point.



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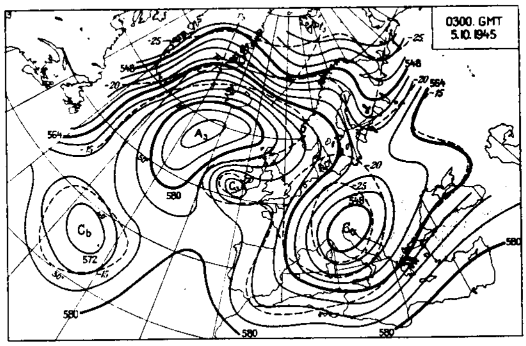

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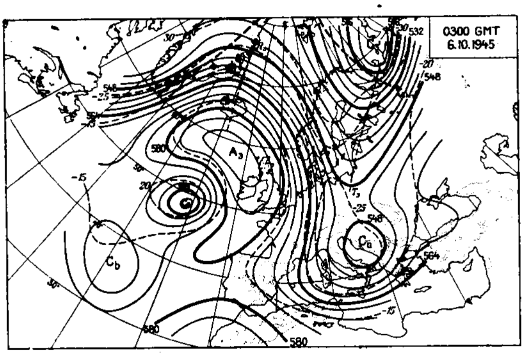

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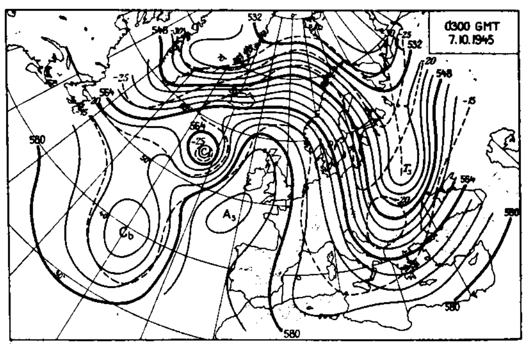

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disturbance r_3, t_3. The latter attains on 6 Oct. the dimensions of a *long wave* (R_3, T_3) which, as it grows considerably in amplitude, leads to the formation of a deep cyclone farther downstream in the White Sea area (C_3') and also to a strong connection of the northerly current in the rear of C_3' with the current in the rear of the quasi-stationary middle-latitude cyclone C_a. This in turn is finally, on 7 Oct., absorbed in the slowly progressive wave disturbance R_3, T_3 (The cell C_3' is no longer visible on the chart of 7 Oct. and the position of the northern portion of the trough T_3 is uncertain because of sparse observations.)

On the charts for 4 and 6 Oct. the dotted lines represent Constant Absolute Vorticity Trajectories originating on, or very close to, the south tip of Greenland at which the inflexion point in a periodically intensifying southwesterly current of jet-stream strength is situate. The trajectory of 4 Oct. gives a rather strikingly good verification for the movement and the marked amplification of the initial wave-disturbance r_3, t_3. Note that the southward bending portion of this trajectory predicts the formation of a northwesterly jet-stream after just under two days across Fennoscandia. The charts of 6 Oct. exhibits this current clearly, the central part, with maximum velocity, being only slightly shifted to the southwest, i.e. to Southern Norway and Denmark. This will be of importance in the detailed investigation into the behaviour of the system A_3, C_3; the verification of the *C. A. V* trajectory — originating at the upstream part of the circumpolar zonal current covered by the chart — suggests that in the middle troposphere the absolute vorticity vested in this current is largely conserved. The mass divergence which *is* however associated with the amplification of the wave disturbance, contributes to the growth of the anticyclonic cell A_3. To this extent one must *a priori* expect somewhat inexact results from an application of the relationships derived in the previous section; for these were deduced on the fundamental assumption that the absolute vorticity of each vortex filament be conserved. We shall now proceed to test the validity of these relationships.

5. Barrier, centre of inertia and equivalent vortex strength

It is evident from a closer study of the 500 mb charts that the system A_3, C_3 cannot be isolated from the environment in such a manner that the kinematic requirements of the theory are strictly satisfied. It is implied in these a.o. that the velocity field is a simply connected region which is true to the extent to which the atmosphere, as an infinitely thin envelope overlaying the earth, rotates with the angular velocity of the latter, whereby this imparts on all fluid filaments a spin, about vertical axes, equal to the Coriolis parameter. Because of the superimposed complex field of relative (horizontal) motion one cannot then any longer define a boundary enclosing the system A_3, C_3 such that this boundary would represent a "*diaphragm*" or a "*barrier*"[1] to the air masses external to the boundary; furthermore the motion at some distance from the cells is not everywhere irrotational.

Nevertheless, it is thought possible to overcome these difficulties arising from departures between the observed field and the model, by adopting the following subsidiary hypothesis. If the system A_3, C_3 were to lie wholly *within* a region of quasi-circular cross section at whose outer boundary the relative vorticity is small in comparison with the vorticity observed in the cells; and if this region were surrounded by an annular zone of some width in which the absolute vorticity is constant, then the (inner)boundary may be regarded as a diaphragm so that interaction between the fields internal and external to that boundary will be negligible.

The vorticity charts, Fig. 4,[2] show a thin dotted curve drawn in such a manner that it coincides as far as possible with an isoline of zero relative vorticity and at the same time encloses the vorticity fields pertaining to the cells A_3 and C_3. It is seen that this curve is semicircular and that the inequalities of vorticity in an annular zone surround-

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[1] A diaphragm is a surface (or curve in the two dimensional case) drawn across the region and limited by the line (or points) in which it meets the boundary.

[2] The vertical component of relative vorticity has been approximated by a value obtained with grid methods from the geostrophic windfield, i.e. as a function of the two-dimensional Laplacian of contour height. It is noteworthy that the dependency on the geographical latitude is practically eliminated if a conical orthomorph chart projection (at 50° lat.) is used, since then the product of the square of the projection factor into the sine of the latitude is nearly constant, varying between 0.74 at 35° and 0.78 at 80°. — A closer approximation can be obtained from the gradient wind field. The effect of the curvature of the isobars (streamlines) and of the movement of the system on the value of the vorticity has been assessed applying the BLATON-PETTERSSEN (1938, 1940) formula for the radius of a parcel trajectory, $1/\varrho = 1/\varrho_i\ (1 - c/G \cos\gamma)$, where c is the velocity of translation of the system, ϱ_i the curvature radius of the isobar and γ the angle between the translation direction and the wind direction. The percentage correction for isobar curvature, which is added in order to obtain the path curvature, can be directly read off a diagram (Fig. 7, p. 183). The method is one of successive approximation since in the above formula the "true wind" is approximated by the gradient wind (G). The values converge rapidly in most instances, after several successive approximations.

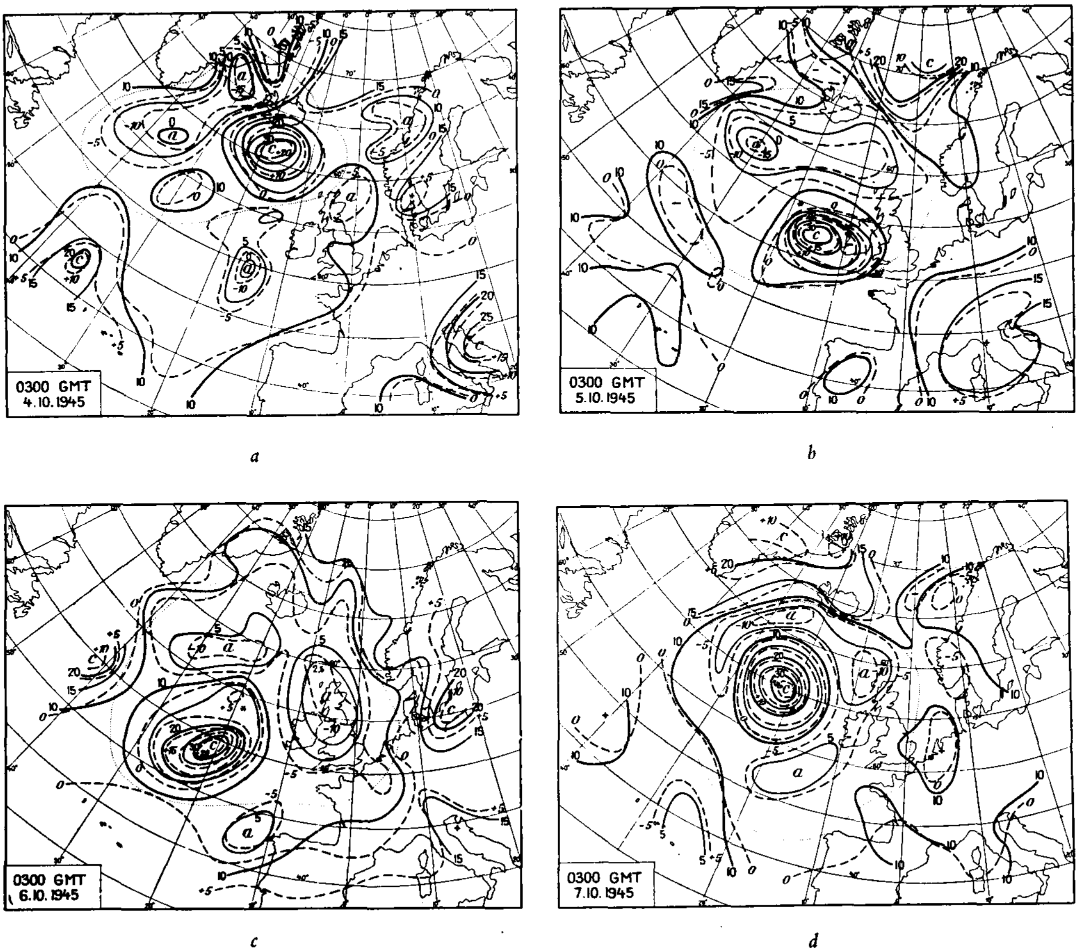

Fig. 4. Distribution of the vertical component of relative geostrophic vorticity (dashed curve) and of absolute vorticity (solid curve) in the 500 mb surface. The values of the isolines are in units of $10^{-5}\,\mathrm{sec}^{-1}$. The dotted semicircular curve is the "barrier" bounding the system A_3, C_3.

ing it are in some parts negligible and in other parts considerably less than the inequalities inside the curve. On the charts for 5 and 6 Oct. this condition is more closely fulfilled than on the charts for 4 and 7 Oct.

The flow pattern within the dotted curve may be described as follows: The cell C_3 resembles a *moon-shaped low pressure area*[1] denting, as it were, the cell A_3 at its convex part in which the strongest cyclonic vorticity is situate and at its concave part being separated from the environment by a col. Only on 6 Oct. the cell C_3 resembles a more perfect circular vortex, a larger number of closed semicircular contour lines being observed. The anticyclonic system has throughout an ellipsoidal cross section with at least two maxima of anticyclonic (relative) vorticity.

With a view to applying eq. (10) to these pattern, the vortical strengths $(I_a)_{A3}$ and $(I_a)_{C3}$ (henceforth I_{aA} and I_{aC}) are computed by taking for $\overline{R}_A$ and $\overline{R}_C$ the radii of circles whose surfaces are equal to the surfaces bounded by the isolines $\zeta = 0$. The centres of the *equivalent vortices* will, however, be identified with the highest and lowest pressure contour heights resp. In the case of the cyclonic cell this point is very

[1] See also p. 186.

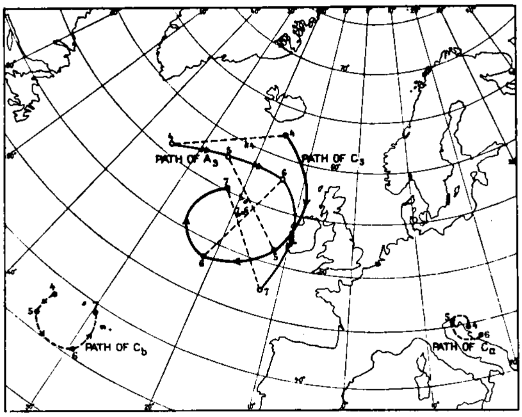

Fig. 5. The paths of cells A_3 and C_3 over the Northeastern Atlantic and the path of the (quasi-stationary) large-scale cold-core cyclones C_a and C_b between 4 and 7 October 1945. The asterisks give the position of the "centre of inertia" of the system A_3, C_3.

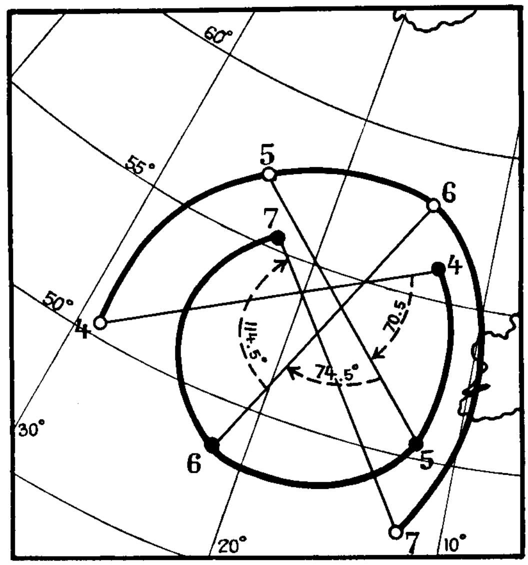

Fig. 6. The paths of cells A_3 and C_3 between 4 and 7 October 1945 that would result through the elimination of the translatory motion of the "centre of inertia". (Actually this motion is southward, whereas in this diagram the northward displacements in the negative time axis have been subtracted.)

close to the point at which the vorticity has a maximum, whereas in the case of the anti-cyclonic system this point ought to be found half way between the two principal minima of absoluted vorticity. It is seen, from the contour charts and the vorticity charts, that the latter condition is satisfied again on 5 and 6 Oct. only.

The coordinates of the inertia centre — this point being referred to the area within the "barrier" (dotted curve) — can be directly computed from equations (8). In Figs. 4 & 5 this point is marked by an asterisk. It is remarkable and gratifying to note that it lies very close to the line joining the centres of the low and high pressure areas.

6. Computed and observed displacements

The movement of the centre of inertia across the chart indicates the *drift* of the system A_3, C_3 as a whole. This drift is considerable between 4 and 5 Oct. only. During the following three days the centre of inertia is very nearly fixed relative to the earth's surface.

We may eliminate the *translation* of the system by shifting the inertia centre from the positions observed on 4, 5 and 6 Oct. onto the position observed on 7 Oct. and by shifting the centres of the cells A_3 and C_3 in parallel directions the corresponding distances. In this manner we obtain the paths of the cells about a fixed centre of inertia. These paths are shown in Fig. 6. It is seen that they are nearly concentric circles of unequal radii, as predicted by the theory for vortex filaments.

The quantities which enter the equations (8) and (10) are assembled in the first group of table I. The second group contains the absolute and relative angular velocities computed from eq. (10) and the corresponding periods of revolution $\tau^\star$ in pendulum days. The linear path velocities of the cells have been also computed, using the relations $V^\star = \omega^\star d$ where d is the distance from the center of inertia. The values of $\omega^\star$ and $\tau^\star$ have been computed either with the aid of geostrophically determined vortical strengths $\bar{I}$ (first row) or with vortical strengths obtained by applying an over-all correction for curvature of trajectory (second row; values in brackets). The method of correction is very briefly outlined in the footnote on p. 180.

The third group of table I contains the

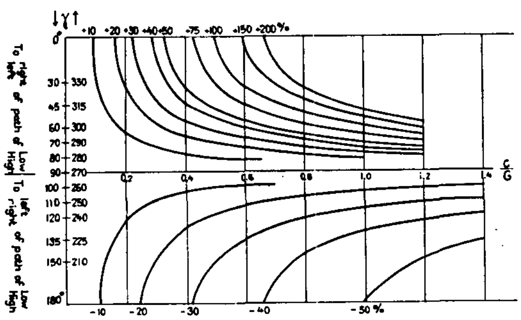

Fig. 7. Diagram for computing the trajectory curvature. The percentage correction of the isobar curvature (expressed through the radius of curvature ϱ_i) is given as function of the ratio between translation velocity, c, of the pressure system and the gradient wind velocity, G, and of the angle γ between the direction of translation and the direction of the (gradient) wind.

quantities signifying the observed rotary motion of the cells. The values in brackets denote the true linear path velocities of the cells resulting from both the rotary motion and the translation of the system as a whole.

The data in the table bring out the following noteworthy points:

i) the equivalent absolute vorticity $\overline{Z}$ pertaining to the cells is very nearly constant during the life of the system;

ii) the radii $\overline{R}$ increase from the first to the third day, i.e. during the southward displacement of the system, but undergo little change between the third and fourth day when the system shows no drift. The increase of the radii is responsible for the strong increase, proportional to their square, of the strengths $\overline{I}_a$;

iii) the relative vortex strength $\overline{I}_c$ increases during the southward displacement of the cyclone and decreases during its northward motion. This is partly a consequence of the corresponding variations mentioned in i) and ii). (The variation in vortex strength of a cyclone drifting across latitude circles may be analogous to the change of curvature, involving change if relative vorticity, along a *C. A. V.* trajectory drawn from a midpoint in a laterally uniform straight and broad current);

iv) the cell separation D is remarkably constant in the course of the last three days of the period during which the drift of the system as a whole is small or nil;

v) the order of magnitude of computed and observed average values of ω^* agrees. Numerical agreement is significant only between the value computed on 5 Oct. and that assessed from the displacement between 5 and 6 Oct.; and

vi) in all instances with the exception of 4 Oct. the use of corrected vorticity improves the agreement between observed and computed quantities.

In view of the large standard deviation of the computed average value of ω^* the numerical agreement recorded at a particular instance may be regarded as fortuitous. A closer study of the observational evidence given heretofore, as well as of additional data, is therefore indicated. It will be necessary to show that the fundamental and subsidiary assumptions leading to eq. (9 a) are more nearly fulfilled at that instance than at others. The practical value of such a test may be measured by setting it against the additional hypotheses we had made in the foregoing, i.e. the concept of equivalent vortex strength and the roughly sketched determination of a "barrier".

7. The deviation from non-divergent flow

We shall first deal with *subsidiary* assumptions. As to the centre of the vortices, it is seen from the 500 mb charts that on 4, 6 and 7 Oct. the identification of the anticyclonic centre with the centre of the cell A_3 is rather ambiguous. On the other hand, on the 5th the flow pattern over the whole area resembles more closely a "vortex pair" than on the other days (although on the 6th the cyclonic cell itself resembles more closely a circular vortex), since the vorticity chart of the 5th exhibits a more regular arrangement of the isolines of negative relative vorticity than on the other days. The centre of the high pressure area lies near the midpoint between the principle minima in the vorticity field on both the 5th and 6th, but on the latter day the existence of a third minimum detracts from the validity of the assumptions. On the 7th the cyclone is surrounded by an annular zone of anticyclonic vorticity, so that the distribution of vorticity is little suggestive of a "vortex pair". Because of some of these discrepancies the requirement that the separation of the centres

Table I

Group	Quantity	Symbol		4 Oct.	5 Oct.	6 Oct.	7 Oct.	Average 4—7 Oct.	Unit and Dimension
Computed equivalent vorticity, equivalent vortex strength and parameters of vortices	Absolute vorticity	$\bar{Z}$	*A*	6.7	6.4	6.4	7.0	6.7	10^{-5} sec^{-1}
			C	20.1	18.9	18.7	18.4	19.0	10^{-5} sec^{-1}
	Radius of cross section	$\bar{R}$	*A*	890	1,000	1,150	1,140	1,050	km
			C	450	490	600	510	510	km
	Separation of vortices	D		1,220	1,100	1,120	1,080	1,130	km
	Latitude of inertia centre	$\bar{\varphi}$		59.7	54.8	53.6	53.4	55.8	°N
	Absolute vortex strength	$\bar{I}_a$	*A*	16.8	21.6	27.9	28.5	23.7	10^7 m^{2}sec^{-1}
			C	12.6	14.2	21.1	15.1	15.7	10^7m^2 sec^{-1}
	Relative vorticity	$\bar{\zeta}$	C	7.9	7.5	7.7	6.4	7.4	10^{-5} sec^{-1}
	Relative vortex strength	$\bar{I}$	C	5.0	5.6	8.7	5.2	6.1	10^7m^2 sec^{-1}
Quantities determining motion of vortices, computed from theoretical relationships	Abs. ang. vel. of revolution	$\omega_a^\star$		3.2	4.7	6.2	5.8	5.0	10^{-5} sec^{-1}
	Rel. ang. vel. of revolution	$\omega^\star$		— 3.1 (— 3.4)	— 1.3 (— 1.6)	+ 0.4 (— 0.1)	0.0 (— 0.4)	— 1.0 — 1.4	10^{-5} sec^{-1} 10^{-5} sec^{-1}
	Period of revolution	$\tau^\star$		2.0 (1.9)	4.6 (3.7)	14.7 (59)	— (14.6)		pendulum days
	Linear vel. of cells in reduced paths (see Fig. 6)	$V^\star$	*A*	24.8 (27.2)	6.6 (8.2)	(— 2.2) (0.5)	0.0 (3.2)		m sec^{-1} m sec^{-1}
			C	13.6 (15.0)	7.3 (9.0)	(— 2.5) (0.6)	0.0 (1.2)		m sec^{-1} m sec^{-1}
Quantities signifying observed motion of pressure cells *A* and *C*				4—5 Oct.	5—6 Oct.	6—7 Oct.			
	Rel. ang. vel. of revolution	$\omega^\star$		— 1.9	— 1.5	— 2.3		— 1.9	10^{-5} sec^{-1}
	Period of revolution	$\tau^\star$		3.2	3.9	3.0		3.4	pendulum days
	Linear vel. of cells in reduced (true)paths	$V^\star$	*A*	12.4 (7.5)	8.2 (7.9)	13.1 (13.3)		11.2 (9.6)	m sec^{-1} m sec^{-1}
			C	9.3 (15.6)	8.7 (9.4)	11.5 (11.7)		10.2 (12.2)	m sec^{-1} m sec^{-1}

of the vortices be constant is comparatively well established on the basis of the pressure centres between the 5th and the 6th only. Regarding the determination of the "barrier", it has been earlier mentioned that this is more realistic on the 5th and 6th than on the other two days, especially the 4th.

Concerning the *basic* assumption, viz. the conservation of absolute vorticity, it may be stated that the constancy of the equivalent vorticity $\overline{Z}$ points to the absence of any larger injections of momentum from without the area bounded by the "barrier". This conclusion is valid only to the extent to which that area remains fixed in size during the period in question. Actually it increases in size from the 4th to the 6th, whereafter it decreases markedly. Summarising these inferences regarding the preference given to the results obtained for 5 Oct., it is seen that on this date the subsidiary requirements of the theory are more nearly satisfied whereas no definite conclusions can be drawn with respect to the basic requirement.

It is however possible to obtain an idea of the presence of horizontal mass divergence from a consideration of the system's *baroclinity*. As far as the low-pressure area is concerned the available data suffice for a threedimensional treatment.

For each day two perpendicular cross sections have been drawn. These run very nearly through the centre of the cell C_3. They are simultaneously represented in the perspectively drawn cross section diagram, fig. 8 a, giving the position of the contours of the isentropes 293° and 313° K (potential temperature) in two perpendicular, vertical planes. Fig. 8 b gives the potential temperatures at standard isobaric surfaces used in the construction of the complex cross section. (Characteristic points provided by the radio soundings and weather flights have also been used.[1])

The slope of the isentropes with respect to the isobaric surfaces gives an indication of the vertical shear and thus of the change of the cyclonic field with height. The tangent of the slope of these two sets of surfaces, m_Θ and m_p are related to each other by the equation

$$m_\Theta = m_p + f\Lambda/S \qquad (11)$$

where, as before, f is the Coriolis parameter, S the static stability and Λ the vertical shear of the (geostrophic) wind. Since in the present case, as in most instances, m_Θ is of higher order of magnitude than m_p, the value of m_Θ is directly proportional to the shear. (The variations in f and S from one day to the next are in the present case sufficiently small so that we may simply write $\Lambda = m_\Theta \cdot \text{constant.}$)

The cross sections show that over the larger part and especially in the central portions of the depression, the isentropes are *descending* between the 4th and 5th, which indicates *subsiding* motion in the middle and upper troposphere whence, by virtue of the continuity equation, the horizontal divergence is there positive. The divergence concurs with a decrease in the (vertical) baroclinity as a result of which the contour gradient at the 500 mb level has decreased. A simultaneous weakening of the gradient in the cyclonic field at M. S. L. (not reproduced here) contributes also a little to that decrease. Between the 5th and 6th a *lifting* of the isentropic surfaces, associated with an increase of baroclinity, takes place. It is more marked in the middle troposphere as indicated by the changes of the 313° K isentrope. The strong local downward bulge of this isentrope in the section 44.7° N, 17.5° W to 54.9° N, 21.8° W is due to the high potential temperatures recorded at 58.3° N, 18.0° W — see Fig. 8 b. It could not be ascertained whether the data are faulty or whether the phenomenon is real. There is also a lowering of the 313° K isentropic surface in the right forward part of the cross section, but the ascent on 5 Oct. in this part (Penzance) falls actually outside the cyclonic system C_3. Over the central parts of the cross section both isentropes have anyhow ascended between the 5th and 6th. During the following 24 hours the height of the lower of these surfaces has hardly changed whilst the upper one continued its ascent, at an even increased rate. The baroclinity has also further increased, although this increase does not show up as an increase of the cyclonic circula-

[1] Fortunately, starting or terminal points of weather reconnaissance flights on 6 and 7 Oct. 1945 were situated very close to the cold core of the depression which could be ascertained from intermediate temperature data for 50 km distance apart, between start and terminal (ascent and descent of aircraft).

tion at the 500 mb level because by 7 Oct. the cyclonic circulation at M. S. L. has almost vanished.

It is seen that the interdiurnal height variations of the isentropic surfaces in the low and middle-troposheric layer of the depression change sign on the 5th, so that it may be assumed that during the hours preceeding and following 03:00 GMT on this day they undergo little height variation. Taking the corresponding variations of potential temperature (indicated by the observed height variation of the Θ-surfaces discussed above) to be chiefly produced by vertical motion, the following conclusions are permissible: *marked horizontal divergence* has ocurred in the middle troposphere during the first part of the 24 hours following 03:00 GMT, 4 Oct., coinciding with the most rapid *southward* displacement of the cell C_3 as well as of the "vortex pair" as a whole; *convergence* has set in some time after 03:00 GMT, 5 Oct. and has been rather strong between the 6th and 7th when the *northward* motion of the cyclone occurred; and the amounts of convergence or divergence have been very small at the 500 mb level at 03:00 GMT, 5 Oct. or near to this time.

The above results, obtained by a qualitative analysis of the temperature field of the cyclonic cell, have thus provided the additional evidence required for ascribing some measure of significance to the agreement between the observed and computed values of $\omega^{\star}$ for 5 October 1945.

In conclusion a few remarks on the genesis of the cyclonic cell C_3 are indicated. Certain structural and thermal properties of the "vortex pair", as observed on 4 Oct., suggest that the primary factor in the formation of the low-pressure area was the existence of an anticyclonically rotating cold core to the northwest of the trough t_3 on 3 Oct., since on the following day one can discern a shallow layer of cold air under the cyclone, indicated by the very low values of potential temperature at the 1,000 mb and 950 mb (750 m) levels in the ascent at Meeksfield on the 4 Oct. (see Fig. 8 b, left upper part of cross section). This sounding also gives extremely relative humidities (about 10%) between 950 and 750 mb suggesting that the air below an inversion at 750 mb, is part of a cold dome having undergone strong subsidence and spreading out.

Attention is drawn to two further points believed to be characteristic of the early stages in the life-history of the "vortex pair". The first point concerns the shape of the depression; as earlier mentioned, the low-pressure area has a moon-shaped configuration as the charts for 4 and 5 Oct. show clearly. The second point concerns the displacement of the system as a whole. The southward displacement is rapid between the 4th and 5th, but a closer study of the intermediate charts (M.S.L. and upper air) shows that between 3 and 4 Oct., 18:00 GMT this displacement is also accelerated.

The properties of the vortex pair in its early stages thus bear a distinct relation to the theoretical model of a *composite vortex* given by Rossby in the earlier cited paper (1949): the creation of a moonshaped low-pressure area to the east of the high pressure area is essentially the result of a deformation of the streamlines pertaining to an initially anticyclonic circular vortex with vertical axis, under the influence of an uniformly accelerated movement equatorward; this displacement in turn arising from the action of the force F_y which has been discussed briefly in § 2. The model of the composite vortex is reproduced in Fig. 9. It is to be noted that the theory for the creation of a complex vortex in this manner is based on the same fundamental assumptions as those used earlier in this paper and on the additional requirement that the total kinetic energy remains constant.

As to the aspects of the (two-dimensional) theory in application to a baroclinic anticyclone, Rossby has given an example which was analysed by C. C. Koo (see also Koo, 1950). This shows such a complex vortex over the Western United States, the depression at M.S.L. having the typical shape, a sinking of the cold core of the anticyclone and an increasing separation with time of the high-pressure center at the ground from the center of the cold core aloft. As to our system A_3, C_3 the data reveal that on 4 Oct. there is a considerable tilt westward of the axis of the cell C_3 between M.S.L. and the 500 mb level, although this tilt is not indicated by the distribution of temperature at the 500 mb level. In the later stages it is not possible to discern much resemblance with the composite vortex model, as the system $A_3\,C_3$ consists then

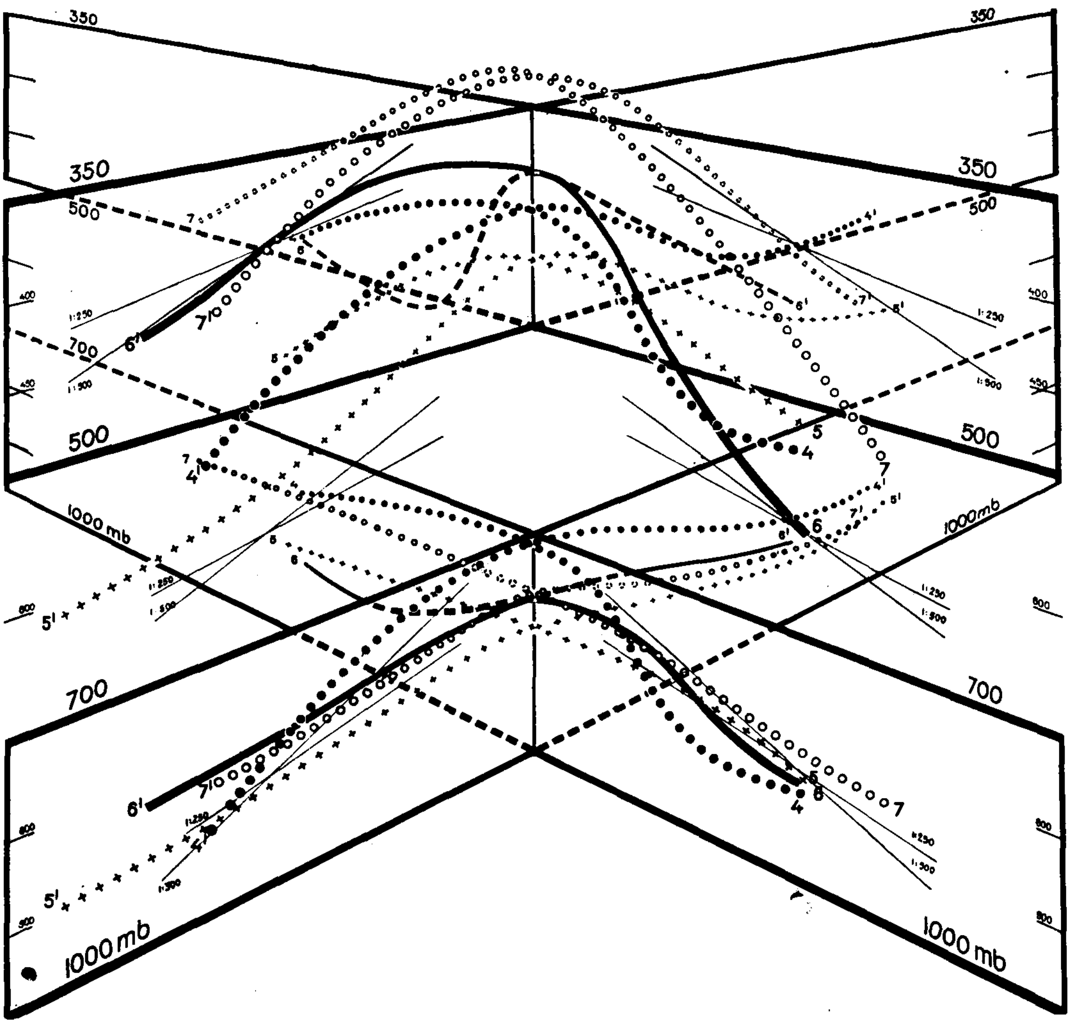

Fig. 8 a. Composite cross section showing the distribution of potential temperature through two selected isopleths, viz. the isentropes 293° K and 313° K, in the region of the cyclone C_3. Two perpendicular cross sections, intersecting in the vertical plane passing through the cold core of the depression, are shown in perspective. Numbers at the isentropes are calendar days. To facilitate in reading the diagram, the portions of the isentropes passing behind the vertical sheets 1,000—700 mbs and 500—350 mbs (which should be thought of as material walls) are indicated by primed numbers. The thin straight lines give the slopes of the isentropic with respect to the isobaric surfaces.

essentially of a cold cyclone and a warm anticyclone whose axes above some low level are quasi-vertical.

It is, however, indicated to draw attention to a further point bearing on the theory of composite vortices. Rossby points out that the continued deepening of the upper cyclone (verified in the present case by the high value of the vortex strength on the 6 Oct., see table I) would finally result in a vanishing of the Force F_y which drives the composite system southward. "*After surface friction had had a chance to reduce still further the anticyclonic circulation near the ground, the resultant force may ultimatively become positive, so that the composite vortex, or at least its upper cyclonic part, finally returns slowly to higher latitudes. It is difficult to analyse this*

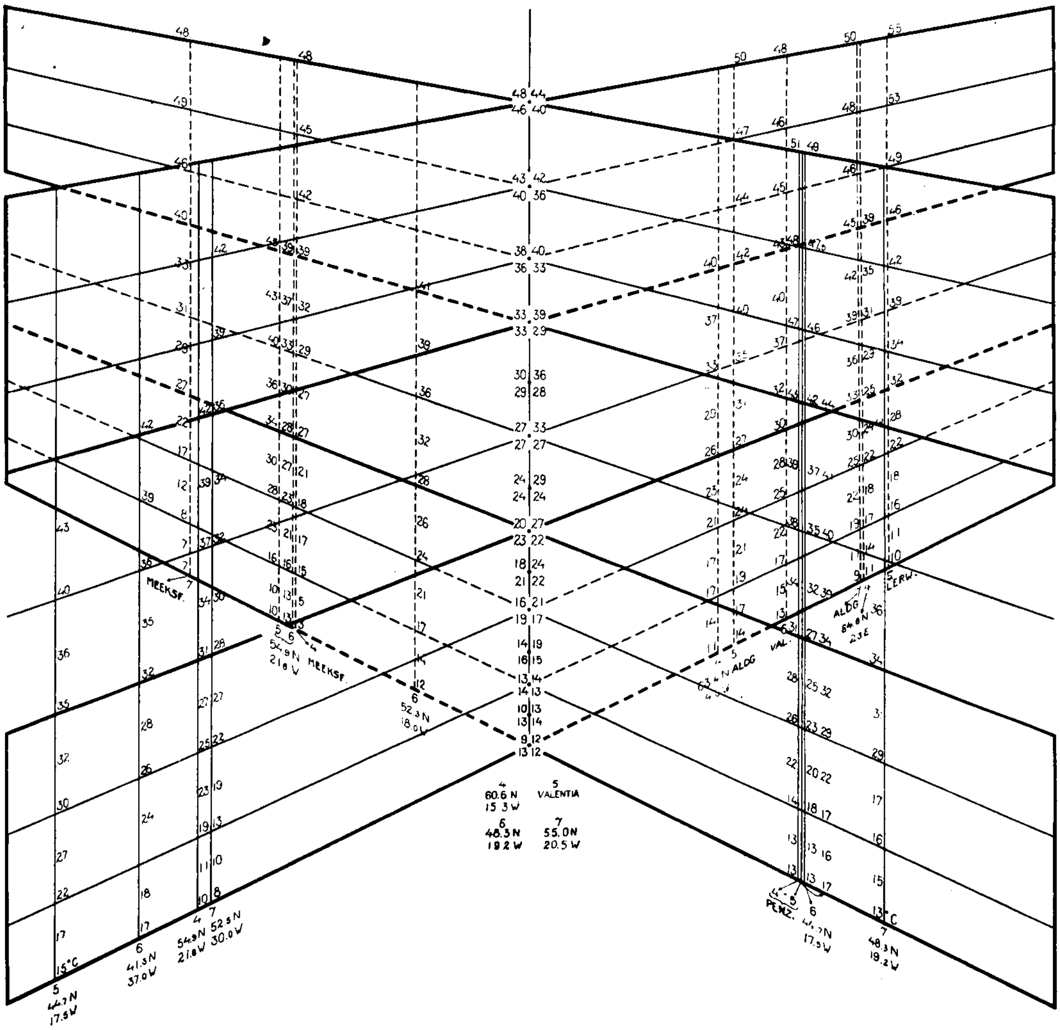

Fig. 8 b. Cross sections as in fig. 8 a. Vertical lines give the position of radiosonde or reconnaissance aircraft soundings, projected onto the corresponding cross section; numbers denote potential temperature at standard pressure.

behaviour of composite vortices quantitatively since it appears almost impossible to determine the extent to which the upper cyclonic vortex participates in the gradual displacement of the entire composite vortex, but it is evident that cold anticyclonic domes in their displacement southward must reach an equilibrium latitude at which the force F_y vanishes." Evidently, the model of a composite vortex makes no proviso for the genesis of a countercell to the upper cyclonic vortex, viz. an upper warm anticyclone. In the present case both these vortices had been generated almost simultaneously, as we have seen earlier, but it is the cyclonic circulation at the ground which was gradually reduced in the course of the rotary motion of the upper vortices; the anticyclonic base of the cold dome near Iceland collapsed almost at once during the first 24 hours. It appears from the study of this particular case that irrespective of the southward drift of the entire system (measured by the drift of the inertia centre, see Fig. 5), the motion of its cells and thus in particular the motion of the upper cyclone can only be

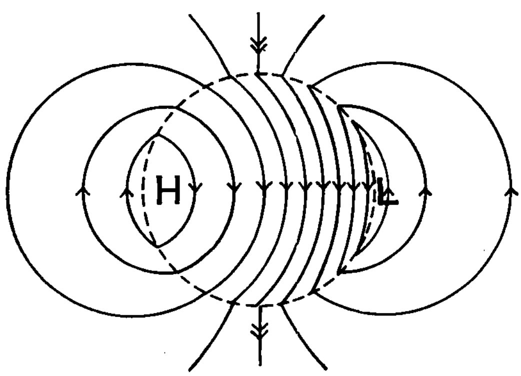

Fig. 9. Composite streamlines in the horizontal plane inside and outside an anticyclonically rotating, solid vertical vortex which is undergoing a uniformly accelerated motion southward. The ratio of the translational velocity to the peripheral tangential velocity of the vortex is assumed to be equal unity. (From C.-G. ROSSBY, 1949.)

understood as an *interaction between vortices possessing a circulation (cyclic irrotational motion) in their environment.* It must be assumed that this circulation has been communicated to the environment rather rapidly, but in a manner which cannot be easily explained. It is possible that lateral friction is largely responsible for this process.

In this study no mention has so far been made of the possible effects of heat transfer and of the transfer of momentum or vorticity in the vertical. In the particular case analyzed, they must be of a secondary nature because the circulation cells (vortices) are conserved in rather deep atmospheric layers for a number of days. If such transfers were of importance, the assumption of the quasi-constant absolute vorticity of individual vortex filaments underlying our theory would be entirely wrong, and the vorticity $\bar{Z}$ would show large variations. No such changes were observed. As to the vortex strength I_a, its variation may be associated partly with horizontal divergence, partly with a limited interaction between the masses inside and outside the "barrier" (dotted curve on Fig. 4). To support this conclusion it should be mentioned that between 7 and 8 Oct. the cyclonic cell has entirely disappeared from the 500 mb chart, suggesting that after the 7th heat and momentum transfer processes may have had a dominating effect. The cyclonic vortex still exists on the 8th in the uppermost troposphere, which can be ascertained from a detailed analysis of the 300 mb chart and other data; remnants of this vortex are found at this level to the north of Scotland. Simultaneously the anticyclone has moved considerably to the eastward; it seems that the entire system has undergone a translation eastward with the result that for the first time since the initial day (3 Oct.), a zonal flow is established over Northwestern Europe.

Postscript. It is frequently observed on the surface weather map that two cyclonic systems as e. g. a central low, usually in the stage of filling, and a secondary mostly in the stage of deepening, have a tendency to rotate around each other in cyclonic paths or just in an opposite manner to the motion observed in the October 1945 case. However, in such situations one of the vortices only, i. e. the central low extends at least to the upper part of the troposphere. The system as a whole is markedly baroclinic and interaction with cold and warm air currents in the environment can hardly be ruled out.

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