The Geostrophic Poleward Flux of Angular Momentum in the Month of January 1949[1]

By YALE MINTZ, University of California

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Abstract

From sea level, 700, 500, 300 and 200 mb daily synoptic charts over the whole of the northern hemisphere and 100 mb charts over three fourths of the hemisphere, the mean geostrophic poleward flux of angular momentum is computed for January 1949. A northward total flux is found in all latitudes, with the maximum at latitude 32 1/2° N of 5.4×10^{16} t m^2 s^{-2}. Four fifths of this flux occurs between the 500 and 100 mb levels. From the meridional convergence of the flux, the mean zonal surface stress is computed. The sign of this derived stress is found to be in good agreement with the distribution of the surface zonal wind: its average magnitude, in the region of the surface westerlies, is 1.2 dynes cm^{-2}.

1. Introduction

The total poleward flux of angular momentum across the vertical surface σ_φ, at the latitude φ, can be expressed as the sum of terms, one of which is the mean geostrophic poleward flux of geostrophic angular momentum,

$$\overline{J} = \int_{\sigma_\varphi} r \overline{\varrho u_g v_g} \, \delta \sigma_\varphi \tag{1}$$

where r is the distance from the earth's axis of rotation, ϱ density, u_g and v_g the zonal and meridional components of the geostrophic wind, $\delta\sigma_\varphi$ element of vertical surface in latitude φ, and the superior bar denotes the mean of $(\varrho u_g v_g)$ taken with respect to time. That this term is one of the most important, if not the most important, in the total poleward flux of angular momentum will be shown in section 4.

Utilizing the hydrostatic relation, to a close approximation equation (1) can be written

$$\overline{J} = - \int_{\tilde{\overline{p}}_0}^{0} \delta p \frac{r^2}{g} \int_0^{2\pi} \overline{u_g v_g} \, \delta \lambda \tag{2}$$

where g is the acceleration of gravity, $\delta\lambda$ element of longitude, δp element of pressure, and $\tilde{\overline{p}}_0$ is sea level pressure averaged with respect to time and longitude. In equation (2)

$$\frac{r^2}{g} \int_0^{2\pi} \overline{u_g v_g} \, \delta \lambda = \overline{j} \tag{3}$$

is the mean geostrophic poleward flux of angular momentum per unit vertical pressure increment. $\overline{j}$ is easily evaluated, for any pressure p, from daily circumpolar synoptic charts.

2. Computation of the momentum flux and its convergence, per cb layer

For the month of January 1949, $\overline{j}$ was evaluated from daily charts of the 700, 500, 300

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200 and 100 mb isobaric surfaces over the northern hemisphere. The sea level and 500 mb charts used were the Northern Hemisphere Historical Weather Maps, Sea Level and 500 Millibar, published by the Headquarters, Air Weather Service. The 700 mb charts used were the Northern Hemisphere 700 Millibar Charts, analyzed by the Extended Forecast Section of the U. S. Weather Bureau, and extended over India. For the 300, 200, and 100 mb levels, daily northern hemisphere charts were analyzed specifically for this investigation. The 300 and 200 mb level maps extended around the whole of the northern hemisphere, and the 100 mb maps covered all except the Asiatic quadrant from 30° E to 120° E. These high level circumpolar synoptic charts were made under the direction of Dr Jakob Bjerknes, with Dr Adam Kochanski as chief analyst. A detailed description of the construction of the maps has been given by KOCHANSKI (1950). In the construction of the maps emphasis was placed on systematically incorporating into the contour height analysis all available upper-air winds (pibals, rabals, and rawins) in accordance with the geostrophic wind relation.

From the upper level maps the contour heights to the nearest 25 feet (and from the sea level maps the pressures to the nearest millibar) were tabulated for every 5° latitude longitude intersection, from latitude 15° to 80° N, for each day of January 1949, and the data entered on IBM punch cards: 180,000 in number. Using the differences between the heights (or pressures) at the adjacent latitude longitude intersections to find u_g and v_g, $\overline{J}$ was computed from the cards by IBM machines. The IBM computations were done under the direction of Dr E. C. Yowell by the Institute for Numerical Analysis (National Bureau of Standards) on the U.C.L.A. campus. To find the geostrophic wind at sea level, mean densities for the latitude circles were used. To obtain the total flux at the 100 mb level, the flux in the missing Asiatic quadrant was estimated by graphical extrapolation of the flux at the lower levels of the same sector. For the pressure surfaces which are intersected by the ground, the integration of $(\overline{u_g v_g})$ in (3) should properly be taken only over those longitudes which lie between the continents and between the mountains. But examination of zonal cross-sections of $(\overline{u_g v_g})$ showed that the error in $\overline{J}$ introduced by computing $\overline{J}$ over all longitudes (through solid earth as well as through the air) is a small one, and the latter procedure, because it could be handled easily by the IBM machines, was therefore used.

The results of the computations of the mean geostrophic poleward flux of angular momentum, per centibar layer, are shown in Table 1 and in fig. 1. The greatest poleward flux of angular momentum takes place at 200 mb and 30° N. The level of maximum flux descends somewhat with increasing latitude, from above 200 mb at 20° N to about 300 mb at 70° N. The latitude of maximum flux decreases with increasing elevation, from 42° N at sea level to 28° N at 100 mb. Everywhere

Table 1

The mean geostrophic poleward flux of angular momentum, per centibar layer, in January 1949.

$$\frac{r^2}{g}\int_0^{2\pi} \overline{u_g v_g}\, \delta\lambda, \text{ in } 10^{14} \text{ t m}^2 \text{ s}^{-2} \text{ cb}^{-1}$$

	75° N	70°	65°	60°	55°	50°	45°	40°	35°	30°	25°	20° N
100 mb	−0.1	0.0	0.4	1.1	1.6	2.5	3.5	4.5	5.5	7.0	7.0	6.0
200 mb	0.0	0.0	0.6	1.5	2.7	5.0	7.5	10.4	13.6	15.2	13.5	9.0
300 mb	0.0	0.1	0.7	1.9	3.6	6.2	8.8	10.8	13.1	13.7	10.3	4.8
500 mb	0.0	0.0	0.2	0.7	1.9	3.3	4.5	5.5	5.6	4.7	3.3	1.8
700 mb	0.1	0.1	0.0	0.2	0.6	1.0	1.4	1.9	2.4	2.0	1.5	1.1
Sea level	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.7	0.5	0.3	0.5	1.0

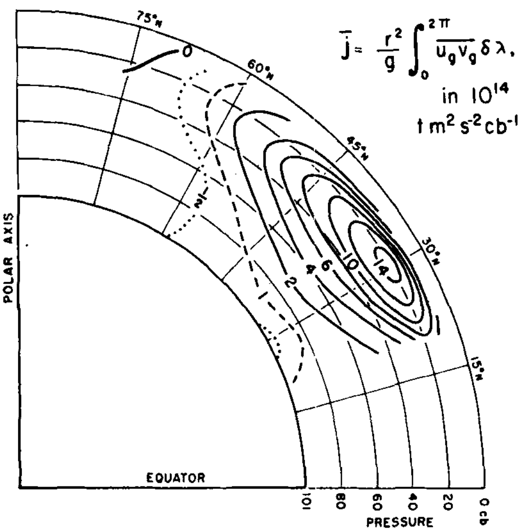

Fig. 1. The mean geostrophic poleward flux of angular momentum, per centibar layer, in January, 1949.

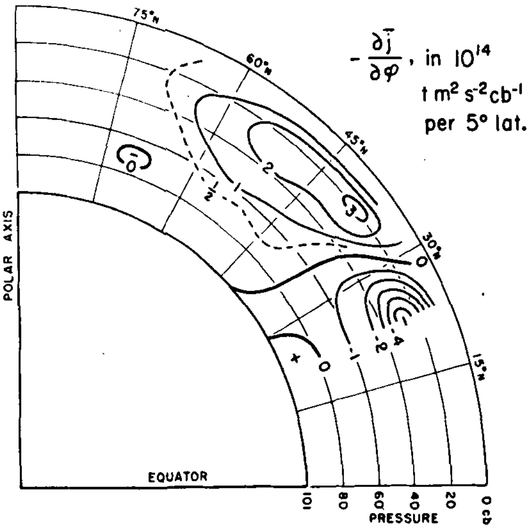

Fig. 2. Meridional convergence of the geostrophic poleward flux of angular momentum, per cb layer, in January, 1949.

within the region of the analysis the flux is toward the pole, except for very small negative transports in the highest levels at 75° N.

The meridional convergence of the geostrophic flux of angular momentum, per centibar layer, is shown in fig. 2. In the tropical and subtropical part of the atmosphere there is divergence of the flux, in the middle and higher latitudes convergence. The line of zero convergence, which is the axis of maximum flux, slopes equatorward with increasing elevation.

Fig. 3 shows the mean geostrophic zonal wind averaged over all longitudes, $\tilde{u}$, in January 1949. From a comparison of figs. 2 and 3, it is seen that in the middle and higher latitudes there is convergence of $\bar{j}$ where the winds are westerly; but that in the sub-tropical latitudes there is strong divergence of $\bar{j}$ in a region of strong westerlies. Obviously, in this sub-tropical region the geostrophic poleward angular momentum flux is acting to diminish the westerly wind, $\tilde{u}$. Here, $\tilde{u}$ must be maintained by some other mechanism; either by convergence of the non-geostrophic terms in the total poleward angular momentum flux, or by convergence of the vertical momentum flux within the atmosphere, or by both. Which mechanism is the important one in this region cannot now be determined. In order to have convergence of the vertical momentum flux in this region, continuity considerations require that the flux itself be directed upward against the vertical gradient of $\tilde{u}$. Synoptic studies in

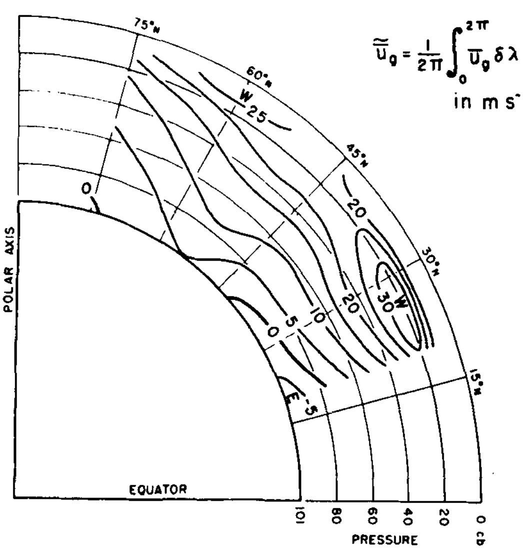

Fig. 3. The geostrophic zonal wind averaged for the month and for all longitudes, in January, 1949.

fact show[2] that the individual anticyclonic cells in this region have general ascending motion in their western parts, where u is large, and descending motion in their eastern parts, where u is small or negative, thereby producing an upward momentum flux opposite in direction to the gradient of $\tilde{u}$. But the magnitude of this flux is still unknown.

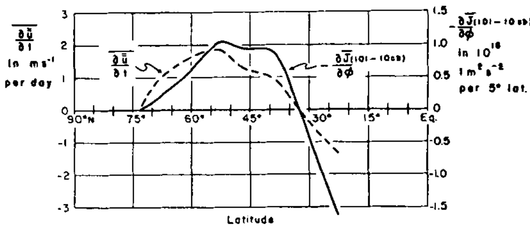

Fig. 5. Convergence of the total geostrophic poleward flux of angular momentum, in t m² s⁻² per 5° latitude, and in m s⁻¹ per day change of zonal wind speed between 101 and 10 cb, in January, 1949.

3. The total momentum flux, and its convergence, from sea level to 10 cb

To find the total geostrophic angular momentum flux, $\bar{J}$, across the entire vertical surface at the latitude φ, the flux per unit isobaric layer must be integrated to the top of the atmosphere, where p approaches zero. The practical upper limit, however, was 10 cb, so that only the flux in the lowest 90 % of the mass of the atmosphere was computed. The total poleward flux in this layer, $\bar{J}_{(101-10\text{ cb})}$, was found from $\bar{j}$ by graphical integration with the results shown in fig. 4.

The total flux, $\bar{J}_{(101-10\text{ cb})}$, is positive at all latitudes. The maximum flux is at latitude 32 1/2° N and equal to 5.4×10^{16} t m² s⁻². As may be seen in fig. 1, 80 % of this maximum flux occurs between the 50 and 10 cb levels.

From $\tilde{\bar{u}}$ we find that, in the mean for January 1949, the total relative angular momentum in the atmosphere north of latitude 32 1/2°, from 101 to 10 cb, was 5.1×10^{22} t m² s⁻¹. A constant poleward angular momentum flux across latitude 32 1/2° of 5.4×10^{16} t m² s⁻² would therefore suffice to bring the atmosphere from a state of rest to the state of zonal flow shown in fig. 3 in 9.4×10^5 seconds, or 11 days.

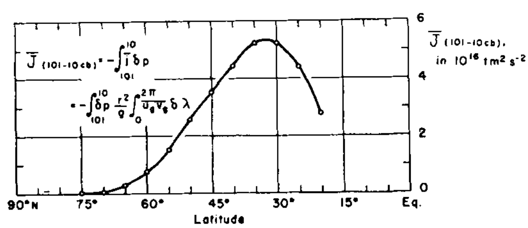

Fig. 4. The total geostrophic poleward flux of angular momentum, from 101 to 10 cb, in January, 1949.

[2] From work in progress on the three-dimensional structure of the North Pacific Anticyclone, Sub-tropical Meteorology Project, Department of Meteorology, University of California, Los Angeles.

Integration of $-\dfrac{\partial \bar{j}}{\partial\varphi}$ from 101 to 10 cb, or differentiation of $\bar{J}_{(101-10\text{ cb})}$ with respect to latitude, yields $-\dfrac{\partial \bar{J}_{(101-10\text{ cb})}}{\partial\varphi}$, the meridional convergence of the total flux, as shown by the solid curve in fig. 5. The maximum total flux convergence is at latitude 52° N and is equal to 1.0×10^{16} t m² s⁻² per 5° latitude. The total flux convergence is zero at 32 1/2° N and approaches zero near 75° N. South of 32 1/2° N there is flux divergence.

Dividing $-\dfrac{\partial \bar{J}_{(101-10\text{ cb})}}{\partial\varphi}$ by the mass of air in each latitude ring from 101 to 10 cb and by the moment, r, enables one to express the flux convergence in terms of its influence on the zonal wind speed. This is shown by the broken curve in fig. 5, where $\tilde{\bar{u}}$ is the instantaneous zonal wind averaged over all longitudes and over pressure from 101 to 10 cb (the 101—10 cb weighted zonal index). The maximum convergence of the geostrophic angular momentum flux, thus expressed in terms of its effect on $\tilde{\bar{u}}$, is 1.9 m s⁻¹ per day, at 53° N. It should be emphasized that fig. 5 shows the flux convergence averaged for an entire month. At any given time there may be large deviations from the monthly average. The day to day variation of the flux convergence, and its correlation with the observed day to day change of weighted zonal index, is being investigated and will be reported upon at a later time.

4. Computation of the mean surface stress from the geostrophic angular momentum flux

Neglecting the non-geostrophic terms in the total poleward flux of angular momentum, the zonal stress of the atmosphere on the earth, averaged with time and over all longitudes, is

$$\bar{\bar{\tau}}_{x,s} = -\frac{1}{2\pi r^2 \delta}\left(\frac{\partial \bar{J}}{\partial \varphi} + \frac{\partial \overline{M}}{\partial t}\right) \qquad (4)$$

where M is the angular momentum per unit latitude ring, and δ is length of unit latitude. Taking $\frac{\partial \bar{J}_{(101-10\text{ cb})}}{\partial \varphi}$ from section 3 above, and $\frac{\partial \overline{M}_{(101-10\text{ cb})}}{\partial t}$ from the change of $\tilde{\bar{u}}_g$ between the first and last day of the month, $\bar{\bar{\tau}}_{x,s}$ was computed for January 1949. (The change of both relative and absolute angular momentum due to net change of mass was very small and was neglected). The results of the computation are shown by the broken curve in fig. 6.

The fact that in this particular month, January 1949, $\frac{\partial \overline{M}_{(101-10\text{ cb})}}{\partial t}$ was negative between latitudes 35° and 52° N and positive south of 35° N was fortuitous. At 40° N, for example, $\tilde{\bar{u}}$ was 5 1/2 m s^{-1} smaller on January 31 than on January 1, 1949. But if this calendar month had begun a few days earlier or later, $\frac{\partial \overline{M}_{(101-10\text{ cb})}}{\partial t}$ might have been reduced to zero or changed in sign. In any event, $\frac{\partial \overline{M}}{\partial t}$ will approach zero as the time over which the mean is taken increases continuously without limit.

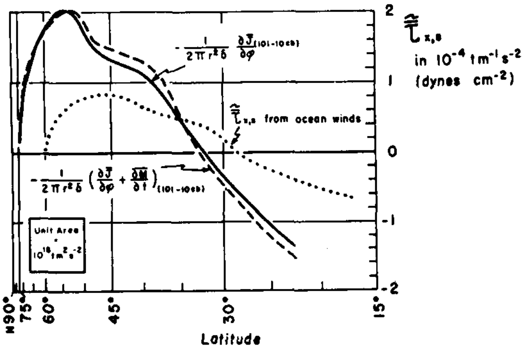

Fig. 6. The mean zonal stress of the atmosphere on the earth.

The solid and broken curves are computed from the convergence of the geostrophic poleward angular momentum flux and the momentum change of the atmosphere in January 1949.

The dotted curve is the mean stress estimated directly from the surface winds over the Atlantic and Pacific Oceans, January Normal (Priestley, 1951).

Arna in the figure is proportional to total torque on earth. Unit area equals 10^{16} t m^2 s^{-2} when $\bar{\bar{\tau}}_{x,s}$ is the mean stress over all longitudes.

The mean surface stress in January 1949, from (4), if we assume $\frac{\partial \overline{M}}{\partial t} = 0$ and $\bar{J} = \bar{J}_{(101-10\text{ cb})}$, is shown in Table 2 and by the solid curve in fig. 6. (In fig. 6, $\bar{\bar{\tau}}_{x,s}$ is plotted against $\int_0^{\varphi} \cos^2\varphi\, \delta\varphi$, so that between any two latitudes the area under the curve is proportional to the torque of the atmosphere on the earth. By construction, then, the area under the solid curve from 32 1/2° to 90° N equals the value of $\bar{J}_{(101-10\text{ cb})}$ at 32 1/2° N, or a torque of 5.4×10^{16} t m^2 s^{-2}).

Between 32 1/2° N and 75° N $\bar{\bar{\tau}}_{x,s}$ was positive, with the average value of the westerly

Table 2

The mean zonal stress of the atmosphere on the earth, computed from the geostrophic poleward flux of angular momentum, in January 1949

$$-\frac{1}{2\pi r^2 \delta}\frac{\partial \bar{J}_{(101-10\text{ cb})}}{\partial \varphi}, \text{ in } 10^{-4} \text{ t m}^{-1}\text{ s}^{-2} \text{ (or dynes cm}^{-2}\text{)}$$

Latitude	72 ½°	67 ½°	62 ½°	57 ½°	52 ½°	47 ½°	42 ½°	37 ½°	32 ½°	27 ½°	22 ½° N
Stress	0.1	1.1	1.6	1.9	2.0	1.5	1.2	0.9	0.0	— 0.7	— 1.4

stress of atmosphere on earth equal to 1.2×10^{-4} t m^{-1} s^{-2} (1.2 dynes cm^{-2}). The maximum $\tilde{\tau}_{x,s}$ 2.0×10^{-1} t m^{-1} s^{-2}, was at 53° N; which was unexpectedly far to the north as the mean sea level and lower level tropospheric winds for all Januaries have their westerly maxima between latitudes 35° and 40° N (MINTZ and DEAN, 1951, fig. 47). But subsequent examination of the zonal winds of January 1949 showed that this was in fact an unusual month. Fig. 3 shows that the sea level and lower tropospheric zonal winds, in this particular month, did indeed have their maxima near latitude 53°. The fact that in January 1949 $\tilde{\tilde{\tau}}_{x,s}$ was zero near 32 1/2° N, whereas the sea level zonal wind was zero near 43° N, is also not inconsistant; as between 32 1/2° and 43° N, where the sea level winds are easterly, many mountains and plateaus extend up as high as 700 and 500 mb into strong westerlies.

The above computed values of $\tilde{\tilde{\tau}}_{x,s}$ are about twice as great as the January normal zonal wind stress over the Atlantic and Pacific Oceans as estimated directly from the surface winds by PRIESTLEY (1951), and about one and a half times as great as the normal wind stress on the oceans as estimated by PRITCHARD and SAUR (1948) and COCHRANE and OSBORN (1950). Priestley's values of the normal January zonal wind stress, averaged for the Atlantic and Pacific Oceans, are shown in figure 6 by the dotted curve. The fact that the latitudes of zero and of maximum zonal stress, computed from the momentum flux convergence of January 1949, are displaced some seven to eight degrees to the north of the corresponding features of the normal wind stress on the oceans probably has its explanation, in part, in the unusual character of this particular month, as shown by fig. 3.

Fig. 6 indicates that the geostrophic angular momentum flux convergence is of the same order of magnitude as the torque due to the surface wind stress. For a number of reasons it is not profitable, at this point, to press for a closer comparison than order of magnitude. First, the geostrophic flux in January 1949 may differ from the January normal geostrophic flux. Secondly, in relative magnitude the mean wind stress on the oceans is almost certainly smaller than the mean stress over the whole earth: although over the oceans the zonal surface winds are much stronger than over the continents, over the continents the influence of the large mountain barriers must be taken into account.[1] Thirdly, all direct estimates of the absolute surface stress over land and sea are subject to considerable error. Indeed, one might with equal if not greater reliability assume the geostrophic wind from the outset and from the geostrophic poleward flux of angular momentum deduce what the mean surface stress, and the mean coefficient of surface resistance, must be. But the computed order of magnitude of $\frac{\partial \bar{J}}{\partial \varphi}$, and its relative distribution with latitude, leave no doubt that of the total poleward flux of angular momentum — which maintains the zonal motion of the atmosphere against the torques of surface friction — the geostrophic flux is an important, perhaps the most important, part.

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[1] The cumulative zonal pressure drop across the very large scale topographic features of North America is being computed from surface (station level) pressure observations and will be described in a subsequent report.

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